## Superconductivity transition temperature enhancement due to Peierls instability

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The superconducting transition temperature  $(T_c)$  is theoretically shown to rise as a result of the presence of the Peierls instability within a simple incomplete-nesting model in two dimensions. It is demonstrated that  $T_c$  can exceed by an order of magnitude that in the absence of the Peierls transition under certain conditions. The implication of this is discussed for the high- $T_c$  superconductors  $La_{2-x}M_xCuO_4$ .

Much attention has been focused on layered perovskite materials since Bednorz and Müller<sup>1</sup> indicated the possibility of raising the superconducting transition temperature  $(T_c)$  in La-Ba-Cu-O systems, and subsequently Uchida *et al.*  $2$  identified the structure of these systems. There have already accumulated many experimental There have already accumulated many experimental<sup>3-10</sup><br>and theoretical<sup>11-18</sup> works, the number of which is rapidly increasing.

Key experimental findings are summarized as follows. (i) The reference system  $La_2CuO_4$  exhibits a structural transition from a tetragonal (high-temperature) to an orthorhombic (low-temperature) phase at 520 K, accompanied by a metal-insulator transition. (ii) Dilute Sr or Ba doping as low as  $x = 0.05$  in  $La_{2-x}(Ba, Sr)_xCuO_4$  induces superconductivity<sup>3</sup> and  $T_c$  reaches a maximum at around  $x = 0.15$ . (iii) Two-dimensionality (2D) of the crystalline and band structures is important to enhance  $T_c$ as evidenced by a systematic study<sup>4</sup> of the lattice-constant changes in the pseudoternary compound  $\text{La}_{2-x}(\text{Sr},\text{Ba},\text{Ca})_x\text{CuO}_4$ . (iv) The substantial orthorhombic component in x-ray measurements<sup>3</sup> persists up to  $x \approx 0.34$  in La<sub>2 - x</sub>Ca<sub>x</sub>CuO<sub>4</sub>, and a system with the orthorhombic structure even at room temperature exhibits superconductivity.<sup>5</sup> As quoted by Weber,<sup>14</sup> Fleming Batlogg, Cava, and Rietman<sup>10</sup> observe an orthorhombi distortion in many superconducting samples even below  $T_c$ . [However, Jorgensen et al.<sup>9</sup> observe no structural transition for  $x(Ba) = 0.15$ .

The calculated electronic band structure  $11-13$  for the undistorted tetragonal phase at  $x = 0$  is extremely simple and essentially 2D with half filling near the Fermi surface (FS) reflecting the layered crystal structure. The nearly perfect nesting feature is evident and responsible for the tetragonal-orthorhombic transformation associated with the FS-instability-derived metal-insulator transition.<sup>9</sup> Dilute doping of Ba or Sr lowers the Fermi level away from the half filling, deteriorating the nearly perfect nesting situation. Although the degree of incompleteness in nesting is dificult to estimate quantitatively at the present stage, we can expect a certain nesting situation to be preserved in the dilute doping region as in Refs. l6 and 17.

The purpose of the present paper is to show the  $T_c$ enhancement due to the Peierls transition or chargedensity wave (CDW) induced by the FS instability within a BCS framework. This is contrary to a general belief that the CDW simply kills superconductivity by opening up the Peierls gap at the FS. This is not always true. Needless to say if the nesting is complete, superconductivity never occurs via a second-order phase transition. However, if the nesting is incomplete as often happens in real materials, then under certain conditions  $T_c$  can be substantially enhanced as we will demonstrate below. Physically it is easy to see this: Near the boundary between the Peierls gapped and ungapped regions on the FS, the gap-edge singularities with diverging density of states (DOS) always just cross the Fermi level. This increases the effective DOS near the FS within the Debye-cutoffenergy shell region and leads to  $T_c$  enhancement in some cases. Such a boundary region on the FS can exist whenever the nesting for charge- and spin-density-wave formation is incomplete.

We start with the tight-binding Hamiltonian including the nearest-  $(t_0)$  and next-  $(t_1)$  nearest-neighbor transfer integrals on a 2D square lattice (lattice spacing of 1) (as we will see in the end, the  $T_c$  enhancement is not confined to strictly 2D and is also expected to occur in 3D.) We use a mean-field approximation to treat the CDW order parameter W and the superconductivity gap parameter  $\Delta$ on an equal footing; then the total mean-field Hamiltonian is given by

$$
H = H_0 + H_{CDW} + H_{BCS} \tag{1}
$$

where

$$
H_0 = \sum_{k,\sigma} \varepsilon(k) C_{k\sigma}^{\dagger} C_{k\sigma} ,
$$
  
\n
$$
H_{\text{CDW}} = W \sum_{k,\sigma} (C_{k+Q,\sigma}^{\dagger} C_{k\sigma} + \text{H.c.}) ,
$$
  
\n
$$
H_{\text{BCS}} = \Delta \sum_{k} (C_{k\uparrow}^{\dagger} C_{-k\downarrow}^{\dagger} + \text{H.c.}) ,
$$

with the self-consistent equations

$$
W = -u \sum_{k,\sigma} \langle C_{k}^{\dagger} + \varrho, \sigma C_{k\sigma} \rangle ,
$$
  

$$
\Delta = -g \sum_{k} \langle C_{k}^{\dagger} C_{k}^{\dagger} + \varrho, \sigma C_{k\sigma} \rangle ,
$$
 (2)

where  $\varepsilon(k) = \gamma_k + \delta_k$ ,  $\gamma_k = -t_0(\cos k_x + \cos k_y)$ , and  $\delta_k$  $=-t_1 \cos k_x \cos k_y$ . The nesting vector  $Q=(\pi, \pi)$ . The chemical potential is fixed to be zero, thus we are considering an electron filling of slightly less than half. (See Fig. <sup>1</sup> for the FS of the undistorted phase. ) We assume

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two independent attractive interactions  $u$  and  $g$  coming from appropriate electron-phonon couplings as in other works.<sup>19</sup> It is easy to diagonalize (1). We obtain the coupled self-consistent equations from (2),

$$
1 = gT \sum_{\omega_n} \sum_k (\omega_n^2 + \gamma_k^2 + \delta_k^2 + \Delta^2 + W^2) / D(\omega_n) , \quad 1 = uT \sum_{\omega_n} \sum_k (\omega_n^2 + \gamma_k^2 - \delta_k^2 + W^2 + \Delta^2) / D(\omega_n) ,
$$
 (3)

$$
D(\omega_n) = (\omega_n^2 + \gamma k^2 + \delta_k^2 + W^2)^2 - 4\delta_k^2(\gamma_k^2 + W^2) \tag{4}
$$

which completely determine  $W$  and  $\Delta$  once the system's parameters are chosen.

We obtain the equation for  $T_c$  by putting  $\Delta \rightarrow 0$  in (3) and (4) as

$$
\frac{1}{g} = T_c \sum_{\omega_n} \sum_k \frac{\omega_n^2 + \gamma_k^2 + \delta_k^2 + W^2}{[\omega_n^2 + (\delta_k + \sqrt{\gamma_k^2 + W^2})^2][\omega_n^2 + (\delta_k - \sqrt{\gamma_k^2 + W^2})^2]} \tag{5}
$$

This yields  $T_c$  as a function of a given W or implicitly the Peierls transition temperature  $(T_P)$  for  $t_0$ ,  $t_1$ , and the Debye cutoff parameter  $\omega_D$  chosen.

Before going into detailed calculations we briefly sketch the FS in the CDW phase: From the inspection of the quasiparticle spectrum,  $E_k^2 = (\delta_k \pm \sqrt{\gamma_k^2 + W^2})^2$ , formed by the CDW, it is easy to see the new FS as shown in Fig. 1. The CDW gap tends to make the original rounded FS square up; more precisely, the CDW induces the gaps at around the point B,  $(k_x, k_y) = (\pi/2, \pi/2)$ , and its equivalent points [henceforth we only consider the Brillouin zone sectioned by the triangle:  $(0,0)$ ,  $(\pi, 0)$  and  $(\pi/2, \pi/2)$ ]. On the straight line AB the CDW gap appears and simultaneously the remaining ungapped FS (denoted by the dotted curve  $AC$ ) moves toward the X point  $(\pi, 0)$ , where the position of C is given by  $\frac{1}{g} = T_c \sum_{\omega_n} \sum_{k=1}^{\infty} \frac{\omega_n^2 + \gamma_k^2 + \delta_k^2 + W^2}{[\omega_n^2 + (\delta_k + \sqrt{\gamma_k^2 + W^2})^2][\omega_n^2 + (\delta_k - \sqrt{\gamma})^2]}$ <br>This yields  $T_c$  as a function of a given W or implicitly the Peierls transition temperature  $(T_P)$  for  $t_0$ ,  $t_1$ , and t

$$
k_{x}(C) = \cos^{-1}\left(\frac{\sqrt{t_1^2(t_0^2 + W^2) - t_0^2 W^2} - t_0^2}{t_0^2 - t_1^2}\right), k_{y}(C) = 0.
$$

When  $W > W_{cr} = t_1$ , the point A coincides with X; thus the FS disappears and the system becomes insulator.

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FIG. 1. The Fermi surface (thick curve:  $t_0/2\pi T_{c0} = 5000$ ,  $t_1/2\pi T_{c0} = 500$ , and  $W/2\pi T_{c0} = 400$ ) and the nesting vector  $Q = (\pi, \pi)$  (arrow). The square denotes the Fermi surface in the half-filled case  $(t_1 = 0)$ . The dotted curves indicate the Fermi surface in the CDW state where the Periels gap opens on the line AB.

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There is no way to have superconductivity in this case. When  $W < W_{cr}$ , there always exists such a point A whose position is  $k_x(A) = \pi/2 + \sin^{-1} \sqrt{W/t_1}$  and  $k_y(A) = \pi/2$  $-\sin^{-1}\sqrt{W/t_1}$ . This is the very point on which the CDW gap-edge singularity is exactly situated at the Fermi level.

The results of the numerical computations of (5) are displayed in Figs. 2 and 3, where  $T_{c0}$  is the superconducting transition temperature in the absence of the CDW  $(W=0)$ . Although we have not exhausted all possible parameter space to seek the maximum enhancement as yet, it was easy to find large- $T_c$ -enhancement cases where the enhancement factor increases by an order of magnitude by optimizing the parameters within physically reasonable values. It should be noted from Fig. 2 that  $T_c$  is depressed in small- $W$  regions where the decrease on the FS area is effective and as the enhancement increases the initial depression of  $T_c/T_{c0}$  also increases. Figure 3 shows the trace of the  $T_c$ -enhancement maxima as a function of  $t_1$ 

 $(3)$  $\frac{1}{0.5}$  $1.0$ //t<sub>1</sub> FIG. 2. Changes of the superconducting transition temperature:  $T_c$  relative to that in the absence of the Peierls transition  $T_{c0}$  as a function of  $W/t_1$  for selected values of the parameters. (1)  $t_0/2\pi T_c 0 = 10000$ ,  $t_1/2\pi T_{c0} = 1000$ , and  $\omega_D/2\pi T_{c0} = 300$ .

(2) 1000, 100, and 300. (3) 100, 10, and 30.





FIG. 3. Trace of the enhancement maximum  $T_{\text{cmax}}/T_{c0}$  as a function of  $t_1/2\pi T_{c0}$ , where  $t_0/2\pi T_{c0} = 5000$  and  $\omega_D/2\pi T_{c0} = 300$ are fixed.

under a fixed  $t_0/2\pi T_{c0}$  (=5000). Again we see that the enhancement occurs over a rather large parameter space, not just at the accidental coincidence of some parameter values.

As is seen from Fig. <sup>1</sup> we have several conflicting factors which raise  $T_c$ ; namely, (a) as W increases, the point A moves toward  $X$  and, simultaneously, C moves toward X. Since X corresponds to the saddle point in the equalenergy surface characterized by the logarithmic van Hove singularity (this is only true for the 2D system), the system gains additionally a large DOS at around  $X$  by the confluence of the two singularities. (b) On the other hand, the increment of  $W$  leads to the shrinkage of the  $FS$ area, losing the DOS. Therefore  $T_c/T_{c0}$  tends to vanish ultimately at  $W \rightarrow W_{cr}$  ( $\equiv t_1$ ) as shown in Fig. 2.

According to the present theory, to attain maximum  $T_c$ enhancement in  $La_{2-x}M_xCuO_4$  and related materials it is advisable to tune the Fermi level, the Peierls temperature  $(T_P \text{ or } W)$ , and the band parameters  $(t_0 \text{ and } t_1)$  to optimize these parameters by doping and pressure. We do not attempt to evaluate a realistic maximum  $T_c$  value because there are several unknown parameters involved. We believe, however, that we could explain a high  $T_c$  within the range of  $\sim$  100 K, which is the maximum observed so far, if we take, for example,  $T_{c0} \approx 13$  K from the other perovskite material  $BaPb_{1-x}Bi_xO_3$ . Neither exotic mechinisms<sup>15,17</sup> nor an anomalously large electron-phonon coupling<sup>14</sup> are needed.

We have demonstrated that contrary to a general belief that the Peierls instability suppresses superconductivity, it can indeed raise the superconducting transition temperature  $T_c$  by producing a coincidence of the CDW gap-edge singularity with the Fermi level. This situation always occurs somewhere on the Fermi surface whenever the nesting for the density wave is incomplete even in the three-dimensional band. The two-dimensional case more effectively gains a high density of states at the Fermi level by the confluence of the two singularities as mentioned above. It should be stressed that the present  $T_c$  enhancement does not rely solely on a special feature of the logarithmic van Hove singularity, which is quickly diminished as we take into account the interlayer coupling, characteristic in the 2D band structure with nearly half filling.

Finally we point out that we can show a similar  $T_c$ enhancement in the case<sup>20</sup> where anisotropic superconductivity with odd or even parity and the spin-density wave compete.

Note added in proof. A similar argument of the  $T_c$ enhancement is done by Yu. V. Kopaev and A. I. Rusinov, Phys. Lett. 121A, 300 (1987).

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