Upper critical field of superconducting anisotropic polycrystals

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We present a theory of the upper critical field for an anisotropic polycrystal. This theory is based on the well-known separable model for the electron-phonon anisotropy which is expanded to describe the Fermi-velocity anisotropy. It is assumed to be sufficient to describe these anisotropies by the mean-square parameters $\langle a^2 \rangle$ and $\langle b^2 \rangle$, respectively. Comparison with existing N-band models shows that our model only allows formation of Cooper pairs from quasiparticles which belong to the same Fermi-surface sheet. Predictions of this theory are discussed using a model system emphasizing the deviation of the upper critical field of the anisotropic system from an equivalent isotropic one. Finally, a procedure is suggested which should enable experimentalists to make a more detailed analysis of experimental data in terms of anisotropy effects.

I. INTRODUCTION

In real superconductors almost all the important physical properties deviate from the values predicted by isotropic theories, i.e., theories which do not take into account the anisotropic nature of the Fermi surface. The first steps toward a more accurate theory were made by Anderson,[1] and later on by Markovitz and Kadanoff[2] and Clem.[3] They investigated on the basis of the BCS theory the effect of varying impurity concentrations on the critical temperature and on the thermodynamics of anisotropic superconductors by defining a separable pairing potential $V_{\mathbf{k},\mathbf{k}'}=(1+a_{\mathbf{k}})V_{\mathrm{BCS}}(1+a_{\mathbf{k}'})$, $a_{\mathbf{k}}$ is the temperature-independent anisotropy parameter which describes the effect of the anisotropic Fermi surface on the isotropic BCS coupling potential V_{BCS}. $a_{\mathbf{k}}$ has the important feature that its Fermi-surface average $\langle a \rangle = 0$, and as anisotropy effects are rather small it is sufficient to keep only the mean-square anisotropy $\langle a^2 \rangle$ as the important anisotropy parameter.

Niel *et al.*[4] were able to show that this basic concept, extended to the strong-coupling Eliashberg theory, was capable of giving an accurate interpretation of experimental data for indium in terms of a mean-square anisotropy of the electron-phonon interaction, not only for the critical temperature T_c but also for other bulk properties like the thermodynamic critical field $H_c(T)$ in its temperature dependence. (This concept was also applied successfully to describe the variation of the critical temperature of vanadium as a function of impurity content.[5])

It is possible to observe anisotropy effects directly by measuring the angular dependence of the upper critical field H_{c2} of a single crystal. H_{c2} is basically determined by the effective electron-phonon coupling and by the Fermi velocity. Thus any anisotropy of H_{c2} is the result of the influence the anisotropic Fermi surface has on the electron-phonon interaction and on the Fermi velocity. From this it is obvious that only nonlocal microscopic theories will be able to give an accurate description of the upper critical field. Important contributions to this subject have been made by Butler,[6,7] Teichler,[9-10] Takanaka and Nagashima,[11-13] and Youngner and Klemm.[14]

The theoretical work of Butler was concentrated on niobium single crystals and was based on the strong-coupling H_{c2} equations of Eilenberger and Ambegaokar,[15] and on extensive band-structure calculations. His approach was restricted to the clean limit, and used the assumption that the anisotropy of the Fermi surface is the dominant feature. He was able to calculate the angular variation of H_{c2}, and his results agreed well with experimental data reported by Kerchner *et al.*[16]

Teichler's theory is more general and can be applied to all cubic type-II superconductors. The angular dependence of H_{c2} in this theory is expressed in terms of normalized cubic harmonics, giving explicit formulas for the temperature dependence of the expansion coefficients. Seidl *et al.*[17] analyzed experimental data of niobium in terms of Teichler's theory, and found that the temperature dependence of all expansion terms investigated (up to sixth order) agreed reasonably well with the theoretical predictions. Nevertheless, under improved experimental techniques[18] the agreement was less satisfying, and in some cases experimental trends were inconsistent with theoretical predictions. Finally, it is not an easy task to identify the abstract expansion coefficients with special physical significance.

The obvious success of the separable model for the pairing potential in describing the effect of anisotropy on the bulk properties of superconductors, and its easy physical interpretation, tempted the authors of this paper to expand this model to the theoretical description of the upper critical field of polycrystals. Polycrystals do not show an angular variation of H_{c2}; on the other hand H_{c2} as a function of temperature does show significant deviations from isotropic results.[19] We therefore present in Sec. II a straightforward expansion of the recently formulated strong-coupling theory of H_{c2} (Ref. 20) to the case of an anisotropic polycrystal. The theory is compared to other N-band model calculations, and the dirty limit is discussed. In Sec. III numerical results are presented, and an attempt is made to separate the various effects. Finally, in Sec. IV conclusions are drawn.

II. THEORY

According to Ref. 20 the upper critical field of an isotropic superconductor is determined by the following set of equations:

$$\tilde{\omega}_n=\omega_n+\pi T\sum_{m=-\infty}^{\infty}\lambda(m-n)\mathrm{sgn}(\tilde{\omega}_m)+\pi t^+\mathrm{sgn}(\tilde{\omega}_n)\ , \tag{1}$$

$$\tilde{\Delta}_n=\pi T\sum_{\omega_c}[\lambda(m-n)-\mu^*]\chi_m\tilde{\Delta}_m+\pi t^+\chi_n\tilde{\Delta}_n\ , \tag{2}$$

$$\chi_n=(2/\sqrt{\alpha})\int_0^\infty dq\,\exp(-q^2)\times\tan^{-1}(q\sqrt{\alpha}/|\tilde{\omega}_n|)\ , \tag{3}$$

$$\lambda(n)=2\int_0^\infty d\Omega\frac{\Omega\alpha^2(\Omega)F(\Omega)}{\Omega^2+\omega_n^2}\ , \tag{4}$$

$$\alpha=eH_{c2}(T)v_F^2/2\ , \tag{5}$$

$$t^+=1/(2\pi\tau_{\mathrm{tr}})\ , \tag{6}$$

$$\omega_n=\pi T(2n+1),\quad n=0,\pm1,\pm2,\ldots \tag{7}$$

where $\tilde{\omega}_n$ and $\tilde{\Delta}_n$ describe the renormalized Matsubara frequencies and gaps, respectively, on the imaginary axis. μ^* is the Coulomb interaction pseudopotential, $\alpha^2(\Omega)F(\Omega)$ the electron-phonon interaction spectral function, v_F the Fermi velocity, τ_{tr} the transport relaxation time, and ω_c is the cutoff frequency, usually taken to be an integer multiple of the Debye frequency. At T_c, the transition temperature, $H_{c2}(T)=0$, and Eqs. (1) and (2) reduce to the standard T_c-Eliashberg equations (linearized Eliashberg equations).

An anisotropic version of the linearized Eliashberg equations was discussed by Daams *et al.*[21] using a separable model to describe the anisotropy of the electron-phonon interaction,

$$[\alpha^2(\Omega)F(\Omega)]_{\mathbf{k},\mathbf{k}'}=(1+a_{\mathbf{k}})\alpha^2(\Omega)F(\Omega)(1+a_{\mathbf{k}'})\ . \tag{8}$$

In this model the T_c equations are given by

$$\tilde{\omega}_{\mathbf{k}}(n)=\omega_n+\pi T_c\sum_{m=-\infty}^{\infty}(1+a_{\mathbf{k}})\lambda(m-n)\mathrm{sgn}[\tilde{\omega}_{\mathbf{k}}(m)]+\pi t^+\mathrm{sgn}[\tilde{\omega}_{\mathbf{k}}(n)]\ , \tag{9}$$

$$\tilde{\Delta}_{\mathbf{k}}(n)=\pi T_c\sum_{\omega_c}(1+a_{\mathbf{k}})\lambda(m-n)\left\langle(1+a_{\mathbf{k}'})\frac{\tilde{\Delta}_{\mathbf{k}'}(m)}{|\tilde{\omega}_{\mathbf{k}'}(m)|}\right\rangle'-\pi T_c\sum_{\omega_c}(\mu^*-\delta_{mn}t^+/T_c)\left\langle\frac{\tilde{\Delta}_{\mathbf{k}'}(m)}{|\tilde{\omega}_{\mathbf{k}'}(m)|}\right\rangle'\ , \tag{10}$$

$$\lambda(m-n)=\langle\langle\lambda_{\mathbf{k},\mathbf{k}'}(m-n)\rangle'\rangle\ , \tag{11}$$

where $\langle\cdots\rangle$ denotes the Fermi-surface average, and $\langle a_{\mathbf{k}}\rangle=0$.

If one introduces a separable model to describe the anisotropy of the Fermi velocity in the form[8]

$$v_{F\mathbf{k}}=\langle v_F\rangle(1+b_{\mathbf{k}}) \tag{12}$$

($\langle v_F\rangle$ is the Fermi-surface average of the Fermi velocity), it is possible to expand Eqs. (1)–(3) to describe an anisotropic system in analogy to Eqs. (9)–(11),

$$\tilde{\omega}_{\mathbf{k}}(n)=\omega_n+\pi T\sum_{m=-\infty}^{\infty}(1+a_{\mathbf{k}})\lambda(m-n)\mathrm{sgn}[\tilde{\omega}_{\mathbf{k}}(m)]+\pi t^+\mathrm{sgn}[\tilde{\omega}_{\mathbf{k}}(n)]\ , \tag{13}$$

$$\tilde{\Delta}_{\mathbf{k}}(n)=\pi T\sum_{\omega_c}(1+a_{\mathbf{k}})\lambda(m-n)\langle(1+a_{\mathbf{k}'})\tilde{\Delta}_{\mathbf{k}'}(m)\chi_{\mathbf{k}'}(m)\rangle'-\pi T\sum_{\omega_c}(\mu^*-\delta_{mn}t^+/T)\langle\tilde{\Delta}_{\mathbf{k}'}(m)\chi_{\mathbf{k}'}(m)\rangle'\ , \tag{14}$$

$$\chi_{\mathbf{k}}(n)=(2/\sqrt{\alpha_{\mathbf{k}}})\int_0^\infty dq\,\exp(-q^2)\times\tan^{-1}[q\sqrt{\alpha_{\mathbf{k}}}/|\tilde{\omega}_{\mathbf{k}}(n)|]\ , \tag{15}$$

$$\alpha_{\mathbf{k}}=eH_{c2}(T)\langle v_F\rangle^2(1+b_{\mathbf{k}})^2/2\ . \tag{16}$$

Equations (13) and (14) imply the ansatz

$$\tilde{\Delta}_{\mathbf{k}}(n)=\tilde{\Delta}_0(n)+a_{\mathbf{k}}\tilde{\Delta}_1(n)\ , \tag{17}$$

with two isotropic gap functions $\tilde{\Delta}_0(n)$ and $\tilde{\Delta}_1(n)$. In following closely the procedure outlined in Ref. 21 it is possible to transform Eqs. (13) and (14) into

$$\hat{\Delta}_0(n)=\pi T\sum_m\left[\lambda(m-n)-\frac{\mu^*(1-E_n)}{1-\pi T\mu^*\sum_m\frac{s_m}{1+\pi t^+s_m}}\frac{r_m+s_mp_m}{r_m+s_m+p_m(s_m+t_m)}\right]\frac{\hat{\Delta}_0(m)}{\hat{\chi}(m)}\ , \tag{18}$$

with

$$r_m=\langle\chi_{\mathbf{k}}(m)\rangle,\quad s_m=\langle a_{\mathbf{k}}\chi_{\mathbf{k}}(m)\rangle,\quad t_m=\langle a_{\mathbf{k}}^2\chi_{\mathbf{k}}(m)\rangle,\quad p_m=(1-\pi t^+r_m)/(1+\pi t^+s_m)\ ,$$

$$E_n=\pi T\sum_m\frac{\lambda(m-n)(s_m+t_m)}{1+\pi t^+s_m}+\frac{\pi t^+s_n}{1+\pi t^+s_n},\quad \hat{\chi}(m)=\frac{p_m}{r_m+s_m+p_m(s_m+t_m)}\ , \tag{19}$$

$$\tilde{\Delta}_0(n)=\hat{\Delta}_0(n)/p_n,\quad \tilde{\Delta}_1(n)=p_n\tilde{\Delta}_0(n)+\frac{\mu^*M}{1+\pi t^+s_n},\quad M=\pi T\sum_m[r_m\tilde{\Delta}_0(m)+s_m\tilde{\Delta}_1(m)]\ .$$

It is immediately apparent that Eq. (18) is formally identical to the isotropic T_c-Eliashberg equation without impurity contributions. It is therefore possible to use the standard algorithm to solve Eq. (18) numerically. Furthermore, it is convenient to use

$$R^2=\sum_n [\langle \bar{\Delta}_{\mathbf{k}}(n)^2\rangle-\langle \bar{\Delta}_{\mathbf{k}}(n)\rangle^2]\Big/\sum_n \langle \bar{\Delta}_{\mathbf{k}}(n)^2\rangle \ , \tag{20}$$

with $\bar{\Delta}_{\mathbf{k}}(n)=\tilde{\Delta}_{\mathbf{k}}(n)/|\tilde{\omega}_{\mathbf{k}}(n)|$ as a measure for the mean-square anisotropy of the gap function.[21]

Having established the basic equations it is essential to analyze some important limits.

A. The dirty limit

This limit is defined for zero-transport relaxation time ($t^+\rightarrow\infty$), and Markovitz and Kadanoff[2] proved in their analysis that in this limit the anisotropy of the electron-phonon interaction (represented by $a_{\mathbf{k}}$) becomes smeared out totally. It is therefore sufficient to keep only $b_{\mathbf{k}}$ and we expand (15),[22]

$$\chi_{\mathbf{k}}(n)\cong\frac{1}{|\tilde{\omega}_{\mathbf{k}}(n)|}-\frac{\alpha_{\mathbf{k}}}{3|\tilde{\omega}_{\mathbf{k}}(n)|^3} \ , \tag{21}$$

and find for Eq. (14),

$$\tilde{\Delta}(n)=\pi T\sum_m [\lambda(m-n)-\mu^*+\delta_{mn}t^+/T]\tilde{\Delta}(m) \times\left\langle\frac{1}{|\tilde{\omega}_{\mathbf{k}}(m)|}-\frac{\alpha_{\mathbf{k}}}{3|\tilde{\omega}_{\mathbf{k}}(m)|^3}\right\rangle . \tag{22}$$

For Eq. (13) we write

$$\tilde{\omega}(n)=\omega_n+\pi T\sum_m \lambda(m-n)\mathrm{sgn}[\tilde{\omega}(m)]+\pi t^+\mathrm{sgn}[\tilde{\omega}(n)] =\tilde{\omega}^0(n)+\pi t^+\mathrm{sgn}[\tilde{\omega}(n)] \ . \tag{23}$$

The remaining Fermi-surface integral in Eq. (22) can easily be performed by introducing the distribution function $P(b)$ according to Clem.[3] $P(b)db$ denotes the probability for $b_{\mathbf{k}}$ to lie in the interval $b\le b_{\mathbf{k}}\le b+db$. $P(b)$ has the obvious properties.

$$\int_{\mathrm{FS}} db\, P(b)=1, \quad \int_{\mathrm{FS}} db\, bP(b)=0 \ , \quad \int_{\mathrm{FS}} db\, b^2P(b)=\langle b^2\rangle \tag{24}$$

(FS denotes the Fermi surface). This gives the final formula which determines the upper critical field of an anisotropic superconductor in the dirty limit,

$$\tilde{\Delta}(n)=\pi T\sum_m [\lambda(m-n)-\mu^*]\tilde{\Delta}(m)/[\,|\tilde{\omega}^0(n)|+\rho(T)] \ , \tag{25}$$

with

$$\rho(T)=eDH_{c2}(T)(1+\langle b^2\rangle) \ , \tag{26}$$

$$D=\langle v_F\rangle^2\tau_{\mathrm{tr}}/3 \ . \tag{27}$$

Equation (25) is identical to the result quoted by Rainer and Bergmann[23] for the isotropic superconductor in the dirty limit. The only difference from their result is the $1+\langle b^2\rangle$ factor appearing in the definition of the "pair-breaking" parameter $\rho(T)$. Therefore the solution $\rho(T)$ will be the same for isotropic as well as anisotropic superconductors. Because of Eq. (26) we find the rather unexpected result that in the dirty limit $H_{c2}(T)$ of the anisotropic system will be smaller by a factor of $1+\langle b^2\rangle$ than the upper critical field of the equivalent isotropic system.

B. Equivalence to existing N-band models

N-band models have been extensively studied as a tool to describe anisotropic features of superconductors.[24–26] It is therefore of interest how the presented model compares to N-band model calculations. First of all we observe that the separable model can be described in its simplest form by

$$P(a)=\delta(-a)/2+\delta(a)/2 \ , \tag{28}$$

thus describing a Fermi surface split into two half-spheres of equal weight with radii $r\pm a$, if r is the radius of the equivalent isotropic Fermi sphere.[4] Using the Fermi-surface harmonics (FSH) notation introduced by Allen,[27,28] Daams[29] observed that this separable model was equivalently described by a restriction to zeroth-order FSH in each of the two subregions of the Fermi surface. This again is the essence of proposed N-band models which divide the Fermi surface into N parts approximating the superconducting parameters in each part by their mean values.

The equivalent N-band model H_{c2} equations for a strong-coupling superconductor are given by

$$\tilde{\omega}_i(n)=\omega_n+\pi T\sum_{j,m}[\lambda_{i,j}(m-n)+\delta_{mn}t^+_{i,j}/T] \times\mathrm{sgn}[\tilde{\omega}_j(m)] \ , \tag{29}$$

$$\tilde{\Delta}_i(n)=\pi T\sum_{j,m}[\lambda_{i,j}(m-n)-\mu^*+\delta_{mn}t^+_{i,j}/T] \times\chi_j(m)\tilde{\Delta}_j(m) \ , \tag{30}$$

$$\chi_i(n)=[2/(\alpha_i)^{1/2}]\int_0^\infty dq\,\exp(-q^2)\tan^{-1} \times[q(\alpha_i)^{1/2}/|\tilde{\omega}_i(n)|\,] \ , \quad \alpha_i=eH_{c2}(T)v_{Fi}^2/2 \ , \tag{31}$$

with the band indices i and j. For $T=T_c$ these equations are consistent with those reported by Entel and Peter.[26]

In the FSH notation on the other hand all indexed parameters in Eqs. (29)–(31) are quite generally derived from the following expansion of $\mathbf{k}$-dependent functions to zeroth-order FSH's,

$$f(\mathbf{k};n)=\sum_j f_j(n)F_j^0(\mathbf{k}) \ , \tag{32a}$$

$$g(\mathbf{k},\mathbf{k}';n,m)=\sum_{j,j'} g_{j,j'}(n,m)F_j^0(\mathbf{k})F_{j'}^0(\mathbf{k}') \ , \tag{32b}$$

with j and j' the indices which indicate the surface

sheets on which the FSH $F_j^0(\mathbf{k})$ is defined. The two-band separable model is restricted to FSH's of the form,

$$F_1^0(\mathbf{k})=[N(0)/N_1(0)]^{1/2}\delta_{\mathbf{k},1}\ ,$$
$$F_2^0(\mathbf{k})=[N(0)/N_2(0)]^{1/2}\delta_{\mathbf{k},2}\ , \tag{33}$$

with $N(0)$ the average quasiparticle density of states at the Fermi surface and $N_i(0)$ the equivalent value for sheet i. $\delta_{\mathbf{k},i}$ symbolizes the restriction of allowed $\mathbf{k}$ vectors to sheet i.

According to this analysis, the separable model presented here corresponds to a two-band model in which the quasiparticle-impurity interaction (t^+) (Ref. 30) and the Coulomb pseudopotential (μ^*) (Ref. 31) are assumed to be isotropic. Only the formation of Cooper pairs from quasiparticles belonging to the same FS sheet is considered to be of significance. (This is in agreement with assumptions made by Entel and Peter.[25])

III. NUMERICAL RESULTS

As anisotropy effects are often rather small it seems to be convenient to discuss the results of our calculations in terms of a deviation function:

$$D(t)=100[H_{c2}^a(t)/H_{c2}^i(t)-1],\quad t=T/T_c \tag{34}$$

where $H_{c2}^a(t)$ and $H_{c2}^i(t)$ are the upper critical fields of the anisotropic and the equivalent isotropic system, respectively. Both systems have the same T_c, the same impurity content described by t^+ and $H_{c2}^a(t)$ is calculated by solving Eqs. (13)–(16), while $H_{c2}^i(t)$ is determined from Eqs. (1)–(5). Moreover, by this definition of the deviation function we avoid problems which arise very often from the lack of accurate data for the Fermi velocity $\langle v_F\rangle$ as long as we assume the isotropic and the anisotropic system to have the same value for $\langle v_F\rangle$. (It is obvious that for a given material μ^* has to be calculated independently for the isotropic and the anisotropic system to give the same T_c by solving the standard T_c-Eliashberg equations.)

To present numerical results we use the $\alpha^2(\Omega)F(\Omega)$ data for niobium measured by Arnold *et al.*[32] with a $T_c=9.305$ K for the clean limit ($t^+=0$). (Sauerzopf *et al.*[18] measured a T_c of 9.301 K for a very pure Nb single crystal with an *RRR* of 2080.)

In a first step we want to discuss the influence of the mean-square anisotropy of the Fermi-surface velocity $\langle b^2\rangle$, neglecting all other anisotropy contributions. In this case the gap function is isotropic and all anisotropy effects enter via the "Werthamer function" $\chi_{\mathbf{k}}(n)$ according to Eq. (15). In Fig. 1 the clean limit is studied showing the deviation function $D(t)$ according to Eq. (34) for various values of $\langle b^2\rangle$ keeping $\langle a^2\rangle$ equal to zero. The anisotropic $H_{c2}^a(t)$ is smaller than the isotropic one for all $1\geq t\geq 0.75$, and for all $t<0.75$ $H_{c2}^a(t)$ starts to become larger than $H_{c2}^i(t)$. This general behavior supports an assumption which is very often made in analyzing experimental data: Low-temperature enhancement of $H_{c2}(t)$ over isotropic-model calculations together with a high-temperature tail which lies below the isotropic model are assumed to be an indication of anisotropy

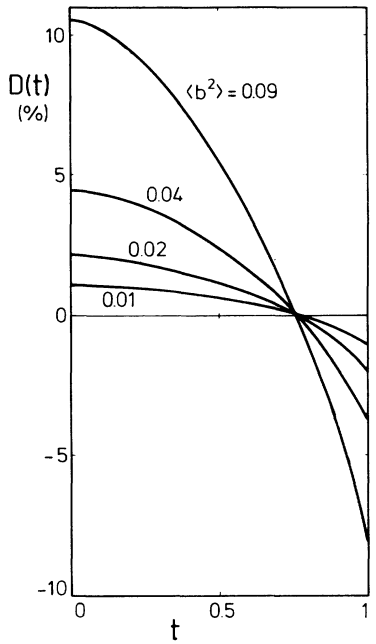

FIG. 1. Influence of the mean-square Fermi-velocity anisotropy $\langle b^2\rangle$ on the upper-critical-field deviation function in the clean limit. The mean-square anisotropy of the electron-phonon interaction $\langle a^2\rangle=0$.

effects in the sample.

Turning to more realistic materials which have normal scattering impurities and, consequently, $t^+>0$, reveals the rather unexpected interaction between Fermi-velocity anisotropy and impurity scattering already indicated by our dirty-limit result. In Fig. 2 we present the

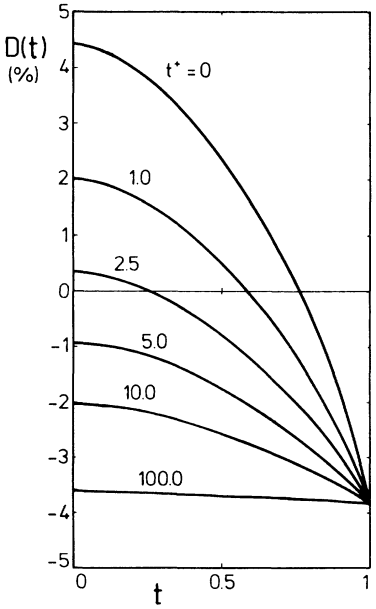

FIG. 2. Influence of the impurity concentration on the upper-critical-field deviation function for an anisotropic superconductor with $\langle a^2\rangle=0$ and $\langle b^2\rangle=0.04$. $t^+=0$ meV corresponds to the clean limit, while $t^+=100$ meV is already well in the dirty limit.

results for a system with $\langle a^2\rangle=0$ and $\langle b^2\rangle=0.04$, and t^+ values ranging from 0 meV (clean limit) to 100 meV (well in the dirty limit). The low-temperature enhancement of $H_{c2}^a(t)$ becomes more and more suppressed as t^+ is increased toward the dirty limit. The result for $t^+=100$ meV is obviously very close to the result of Eq. (25) because the deviation function $D(t)$ is negative and shows almost no variation with temperature. This result is somehow in contrast to experimental data measured by Laa[33] which show even for rather dirty Nb polycrystals a low-temperature enhancement of $H_{c2}^a(t)$. Thus we conclude that for the analysis of polycrystal data Butler's approach[7] of $\langle a^2\rangle=0$ seems not to be appropriate.

In Fig. 3 we want to discuss the influence of $\langle a^2\rangle$ alone, keeping $\langle b^2\rangle=0$. This graph shows the influence of normal scattering impurities on the $H_{c2}^a(t)$ of a system with $\langle a^2\rangle=0.04$. In general one can say that $\langle a^2\rangle\neq 0$, together with $\langle b^2\rangle=0$, results in an enhancement of $H_{c2}^a(t)$ over $H_{c2}^i(t)$ for all temperatures. This enhancement is drastically reduced with increasing impurity concentrations, indicating the well-known effect of smearing out the electron-phonon interaction anisotropy by normal impurity scattering.[2] For $t^+\to\infty$ we find $D(t)=0\%$ for all values of t, thus supporting the argument we used in deriving the dirty-limit formulas in Sec. II.

The results for a realistic system ($\langle a^2\rangle=\langle b^2\rangle=0.04$, $0\le t^+\le 100$ meV) are presented in Fig. 4. It is now obviously possible to discuss two cases: (i) $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$ have the same sign in same sheets, and (ii) they have opposite signs in same sheets. Case (ii) corresponds to the situation observed by Crabtree *et al.*[31] in niobium: ". . . that (the mass enhancement factor) λ is large where v_F is small and vice versa." From our result it becomes immediately apparent that only for case (ii) (all curves labeled by $-$) can an enhancement of $H_{c2}^a(t)$ over $H_{c2}^i(t)$ be expected. The depressing influence of $\langle b^2\rangle$ is in this case partly compensated by the electron-phonon interaction anisotropy. If we compare the result for $D(t)$ in Fig. 2 for $t^+=5$ meV with that of Fig. 4, we still see an enhanced $H_{c2}^a(t)$ over a rather large region of $0\le t\le 0.9$. From this we conclude that our model seems to give a realistic description of features found by experiment.

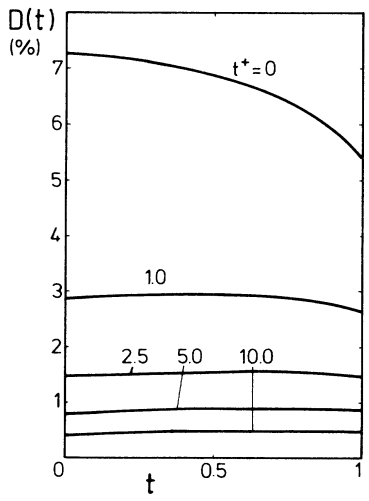

FIG. 3. Influence of the impurity concentration on the upper-critical-field deviation function for an anisotropic superconductor with $\langle a^2\rangle=0.04$ and $\langle b^2\rangle=0$. This figure demonstrates how the anisotropy effect becomes smeared out by normal scattering impurities.

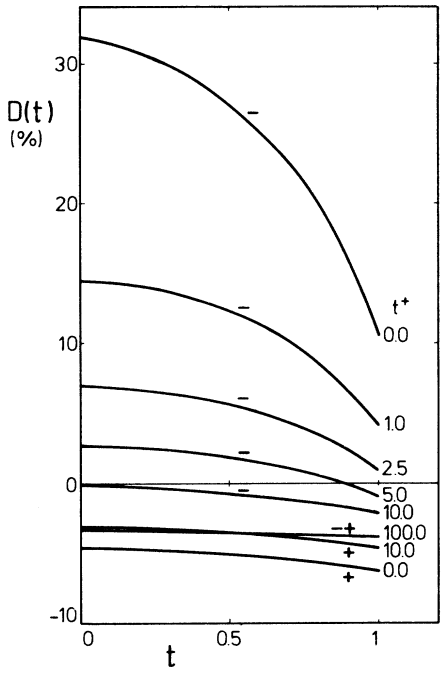

FIG. 4. Influence of the impurity concentration on the upper-critical-field deviation function for an anisotropic superconductor with $\langle a^2\rangle=\langle b^2\rangle=0.04$. The curves labeled with $-$ correspond to the case where $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$ have opposite sign on the same Fermi-surface sheet. $+$ indicates the same signs for $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$.

So far we have only discussed systems with equal weight for the two Fermi-surface regions according to Eq. (28). It is of course also possible to define different weights for the two regions, thus changing the distribution function, keeping in mind its basic properties (24). The influence of different weights in a separable model on the thermodynamics of anisotropic superconductors was already studied by Daams[29] and found to be of no significance. Figure 5 discusses the effect of different weights on $H_{c2}^a(t)$ in the clean limit of a system with $\langle a^2\rangle=\langle b^2\rangle=0.04$. Obviously the case of equal weights for both regions represents an extreme case showing the most pronounced temperature dependence of $D(t)$. Any deviation from this extreme configuration results in a more "isotropic" $D(t)$ still showing a significant enhancement of $H_{c2}^a(t)$ over $H_{c2}^i(t)$ which becomes less and less temperature dependent.

Finally, in Fig. 6 the root-mean-square anisotropy $R(t)$ [Eq. (20)] of the gap function is presented as a function of the impurity content. We compare systems with $\langle a^2\rangle=\langle b^2\rangle=0.04$ (solid line for $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$ having

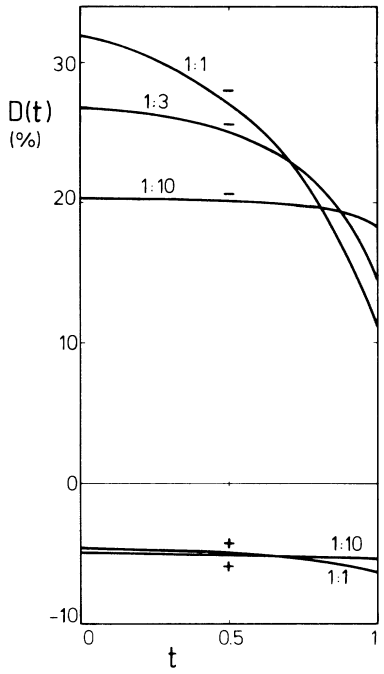

FIG. 5. Influence of different weights for the two model Fermi-surface sheets on the upper-critical-field deviation function for an anisotropic superconductor with $\langle a^2\rangle=\langle b^2\rangle=0.04$ in the clean limit. The labels indicate the weight ratios. (1:1 corresponds to equal weights.)

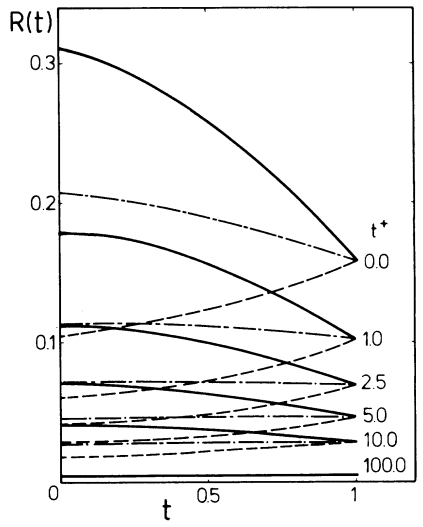

FIG. 6. Anisotropy of the gap function for an anisotropic superconductor as a function of the impurity concentration. Solid lines: $\langle a^2\rangle=\langle b^2\rangle=0.04$, a_k and b_k have opposite sign. Dashed lines: $\langle a^2\rangle=\langle b^2\rangle=0.04$, a_k and b_k have same sign. Dashed-dotted lines: $\langle a^2\rangle=0.04$, $\langle b^2\rangle=0$. For $t^+=100$ meV the three curves lie almost on top of each other.

opposite signs, dashed lines for a_k and b_k having same signs) to a system with $\langle a^2\rangle=0.04$ and $\langle b^2\rangle=0$ (dashed-dotted lines). The two Fermi-surface sheets are assumed to have the same weight. This result agrees completely with deviation functions shown in Fig. 4 and Fig. 3 for the respective systems. A larger deviation from the isotropic system is always coupled to a larger anisotropy of the gap function. It becomes also transparent that $\langle b^2\rangle$ and the sign of a_k compared to that of b_k on the same FS sheet affects largely the gap-function anisotropy. This influence weakens with increasing impurity content and the successive smearing out of the electron-phonon coupling anisotropy. Close to the dirty limit ($t^+=100$ meV) the gap-function anisotropy becomes almost negligible.

We would like to note in passing that a similar increase of the gap-function anisotropy with increasing magnetic effects (in that case—increasing concentration of paramagnetic impurities) was reported by Daams *et al.*[21] It was pointed out that this increase was obviously necessary to establish superconductivity beyond the isotropic limits. As H_{c2} is increasing with decreasing temperature the argument given there can be expanded to the case studied here.

IV. CONCLUSION

We presented in this paper a theory of the upper critical field of anisotropic polycrystals including strong-coupling effects, isotropic impurity scattering, and mean-square anisotropies of the electron-phonon interaction and of the Fermi velocity. The special features of this model allowed a formal separation and discussion of the influence of both anisotropies on the upper critical field. A very interesting result was found for the dirty limit with the anisotropic H_{c2} to be always smaller than the H_{c2} of the equivalent isotropic system.

It should be possible to use the above theory to develop a more detailed analysis of anisotropy effects in polycrystals. First of all it is possible to find rather reliable values for $\langle a^2\rangle$ by studying the T_c depression with increasing impurity content.[4,5] The deviation function $D(t)$ of the upper critical field for rather dirty samples gives a first estimate of $\langle b^2\rangle$, as $\langle a^2\rangle$ is then almost negligible. Finally, the $D(t)$ for the cleanest sample gives an excellent hint about the different weights to be used for the two-model Fermi-surface regions (Fig. 5). Having found this information, only a little freedom is left to fit the system parameters in a way that all physical properties (T_c degradation, thermodynamics, and upper critical field) of all samples can be theoretically reproduced by one set of parameters μ^*, t^+, $\langle a^2\rangle$, $\langle b^2\rangle$, $\langle v_F\rangle$, and $\alpha^2(\Omega)F(\Omega)$.

In light of the results presented here we do not believe that it is sufficient to take only the Fermi-velocity anisotropy into account, as the electron-phonon coupling anisotropy seems to have substantial effect on the upper critical field of a real superconductor.

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