Effects of the dipole interaction in superfluid ^{3}He-B

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The effects of the dipole interaction on the equilibrium and nonequilibrium properties of ^{3}He-B are calculated using the quasiclassical theory. In zero field, the dipole interaction distorts the $l=1$ gap, generates an $l=3$ gap, and induces a new rotated planar phase next to the normal-state transition. The field dependence of the Leggett angle θ_L is calculated to order $(\gamma H/\Delta)^2$, including the effect of nonquasiparticle corrections. Using the equilibrium properties as input, we calculate the zero-field NMR frequencies in the collisionless regime. A calculation of the field dependence of the longitudinal NMR frequency reveals the existence of a Fermi-liquid oscillation term due to the out-of-phase motion of the normal and superfluid components. Finally, we compare the collisionless and hydrodynamic theories of NMR.

I. INTRODUCTION

Before the first[1] NMR experiments in superfluid ^{3}He, little theoretical interest was paid to the small orienting effects of the dipole interaction. Only after unexplained shifts in NMR frequencies were observed did Leggett[2] realize that the condensed state of ^{3}He enhances the effects of the dipole forces. Though the orientational energy between neighboring ^{3}He nuclei is only 10^{-7} K, the orientational energy of a collection of Cooper pairs, all in the same angular-momentum state, is many times as large.[3] By breaking the spin-orbit symmetry of superfluid ^{3}He, the dipole interaction selects the equilibrium order parameter and shifts the collective-mode frequencies.

In this paper we use the quasiclassical formalism to calculate the effects of the dipole interaction in superfluid ^{3}He-B. In Sec. II we calculate the equilibrium effects, and in Sec. III we calculate the shifts in NMR frequencies in the collisionless, low-temperature regime. This theory describes the longitudinal NMR experiments of Candela *et al.*,[4] which were performed at temperatures $T/T_c \lesssim 0.5$. Finally, in Sec. IV we compare the collisionless and hydrodynamic results for the longitudinal NMR frequency. This paper continues the work of Fishman and Sauls[5] (FS) on the response functions and collective-mode frequencies of the B phase in a strong magnetic field $[\gamma H \leq \Delta(T)]$. As discussed in FS, a magnetic field in the $\hat{\mathbf{z}}$ direction depopulates the $S_z=0$ Cooper-pair state, compressing the energy gap, enhancing the magnetic susceptibility, and altering the frequencies of the collective modes. The results of this paper, like the results of FS, are valid in the weak-coupling limit and include all Fermi-liquid parameters and pairing interactions.

Part of the motivation of FS and the present work is to obtain information about the material parameters that regulate superfluid properties. Reliable determinations of the higher-angular-momentum Fermi-liquid and pairing-interaction parameters test the consistency of the quasiclassical theory and check the reliability of the microscopic theories[6] which predict the effective interactions of liquid ^{3}He. More information about material parameters can be obtained from NMR and collective-mode spectroscopy[7,8] in the superfluid than can be extracted from normal-state measurements. In particular, higher-angular-momentum pairing interactions affect superfluid but not normal-state properties. Like the results of FS for the field dependence of the susceptibility and of the real squashing-mode frequencies, the results of this paper should ultimately provide precise values for the $l=2$ antisymmetric Fermi liquid parameter F_2^a and for the $l=3$ transition temperature T_{c3}, at least at low pressures were strong-coupling corrections are believed to be small.[9] The results of this paper can also be used to determine the renormalized dipolar coupling constant from longitudinal NMR measurements in zero field.

We begin Sec. II by briefly reviewing the quasiclassical equations for equilibrium ^{3}He. A calculation of the zero-field gap reveals that the dipole interaction distorts the $l=1$ gap, generates an $l=3$ gap, and induces a rotated planar phase next to the normal state. The temperature width of this new phase is proportional to the dipole interaction strength g_D. We then calculate the field dependence of the Leggett angle θ_L through order $(\gamma H/\Delta)^2$. All moments of the quasiparticle renormalization factor $R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ are included in these calculations. Because the dipole interaction is short ranged, nonquasiparticle corrections modify the equilibrium properties of the superfluid. These corrections are included first order in the $l=1$ pairing interaction.

Using the equilibrium properties as inputs, we calculate the collisionless NMR frequencies in Sec. III. In zero field only the longitudinal frequency is shifted by the dipole interaction. The magnetic field dependence of the longitudinal NMR frequency is calculated through order $(\gamma H/\Delta)^2$. The out-of-phase motion of the normal and superfluid components generates a Fermi-liquid oscillation term, proportional to $(F_2^a)^2$ and independent of g_D. At $T=0$, when the normal component is absent, and at

$T=T_c$, when the superfluid component is absent, this term vanishes. Unlike the field dependence of the longitudinal resonance, the field dependence of the transverse resonance can be computed perturbatively only at $T=0$, where $\omega=\pm\gamma H$ to zero order in g_D.

Previous calculations[10,11] of the field dependence of the longitudinal NMR frequency followed the method of Leggett and Takagi.[12] In the original hydrodynamic theory of Leggett,[13] the quasiparticles respond adiabatically to the nonequilibrium motion of the spin and the internal degrees of freedom of the order parameter are not excited by the spin resonance, provided that $\omega << \Delta$. Leggett and Takagi worked to extend the range of validity of this theory into the collisionless regime by including the effects of relaxation between the quasiparticles and the spin resonance. They suggested that, at least at $T=0$, when the normal component vanishes, the hydrodynamic theory should accurately describe the spin dynamics of the order parameter. In Sec. IV we follow the formalism of Leggett to calculate the longitudinal NMR frequency, including all material parameters and all moments of the quasiparticle renormalization factor, through order $(\gamma H/\Delta)^2$. We find that at $T=0$ the zero-field collisionless and hydrodynamic frequencies do indeed agree. However, the field dependences of the collisionless and hydrodynamic frequencies differ, even at $T=0$. The description of the NMR provided by the collisionless theory is quite different from the hydrodynamic picture. In Sec. III we find that the collisionless NMR couples to internal degrees of freedom of the order parameter first order in the field. These "off-diagonal" components contribute to the field dependence of the NMR frequencies, even at $T=0$. Because it cannot incorporate the effect of the internal order-parameter modes, the hydrodynamic theory is inappropriate to describe the collisionless dynamics.

A review of the tensor notation used extensively in this paper is given in Appendix A. Other technical details and lengthy results are left to Appendixes B–F. We shall often refer to FS for the derivation of results that are needed as a starting point for this work.

The results of this paper for the field dependence of the Leggett angle [Eq. (52)] and for the longitudinal NMR frequency [Eqs. (73), (80), (81), and (F2)] place new constraints on the material parameters of the B phase. In particular, analysis of NMR results may confirm previous determinations[9] of F_2^a and T_{c3}. Future measurements of the Leggett angle, which contains sizable nonquasiparticle corrections, may yield new information about the quasiparticle energy cutoff ε_c.

II. DIPOLAR EFFECTS IN EQUILIBRIUM

The quasiclassical theory of superconductivity, developed by Eilenberger,[14] Eliashberg,[15] and Larkin and Ovchinnikov,[16] was adapted to superfluid ^{3}He by Rainer and Serene,[17] whose notation[18] we follow, and by Eckern.[19] The quasiclassical theory is formulated in terms of (4×4)-matrix Green's functions, in particle-hole and spin space, that are integrated over the magnitude of the quasiparticle momentum near the Fermi surface. Equilibrium properties are calculated from the Matsubara propagator

$$\hat{g}^{m}(\hat{\mathbf{p}},\mathbf{R};\varepsilon_n)\propto\int_{-\varepsilon_c}^{\varepsilon_c}d\xi_{\mathbf{p}}\,\hat{\tau}_3\hat{G}(\mathbf{p},\mathbf{R};\varepsilon_n)\ , \tag{1}$$

where $\hat{G}$ is the one-particle Matsubara Green's function, $\hat{\tau}_3$ is the Pauli matrix in particle-hole space, $\xi_{\mathbf{p}}=v_F(|\mathbf{p}|-p_F)$ is the quasiparticle energy near the Fermi surface in terms of the Fermi velocity v_F and Fermi momentum p_F, and ε_c is the energy cutoff for the quasiparticle spectrum. The structure of $\hat{g}^{m}$ in particle-hole space is given by

$$\hat{g}^{m}=\begin{bmatrix} g^{m} & f^{m} \\ \bar{f}^{m} & \bar{g}^{m}\end{bmatrix}\ , \tag{2}$$

where $g^{m}(\hat{\mathbf{p}},\mathbf{R};\varepsilon_n)$ is the conventional one-particle Green's function integrated over $\xi_{\mathbf{p}}$ and $f^{m}(\hat{\mathbf{p}},\mathbf{R};\varepsilon_n)$ is the corresponding anomalous propagator. These propagators depend on the quasiparticle momentum $\hat{\mathbf{p}}$, the center of mass $\mathbf{R}$, and the Matsubara frequencies $\varepsilon_n=(2n+1)\pi T$. The time-reversed propagators are $\bar{g}^{m}=[g^{m}(-\hat{\mathbf{p}},\mathbf{R};\varepsilon_n)]^*$ and $\bar{f}^{m}=[f^{m}(-\hat{\mathbf{p}},\mathbf{R};\varepsilon_n)]^*$. The quasiclassical equations for homogeneous ^{3}He consist of the commutation relation

$$[i\varepsilon_n\hat{\tau}_3-\hat{\Delta}-\hat{\sigma},\hat{g}^{m}]=0\ , \tag{3}$$

the normalization condition

$$[\hat{g}^{m}(\hat{\mathbf{p}};\varepsilon_n)]^2=-\pi^2\hat{1}\ , \tag{4}$$

and the self-energy equations for the diagonal ($\hat{\sigma}$) and off-diagonal ($\hat{\Delta}$) self-energies, which are defined diagrammatically in terms of the propagator $\hat{g}^{m}$ and the quasiparticle interactions. The mean-field relations for the self-energies,

$$\hat{\sigma}=\begin{bmatrix}\sigma & 0\\ 0 & \bar{\sigma}\end{bmatrix},\quad \hat{\Delta}=\begin{bmatrix}0 & \Delta\\ \bar{\Delta} & 0\end{bmatrix}\ , \tag{5}$$

are given by

$$\sigma_{\alpha\beta}(\hat{\mathbf{p}})=\nu_{\alpha\beta}(\hat{\mathbf{p}})+\tfrac{1}{2}T\sum_n\int\frac{d\Omega'}{4\pi}[A(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')]_{\alpha\gamma,\beta\rho}g^{m}_{\rho\gamma}(\hat{\mathbf{p}}';\varepsilon_n)\ , \tag{6}$$

$$\Delta_{\alpha\beta}(\hat{\mathbf{p}})=\tfrac{1}{2}T\sum_n\int\frac{d\Omega'}{4\pi}[V(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')_{\alpha\gamma,\beta\rho}+g_DR^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\delta_{\mu\nu}-3\hat{q}_\mu\hat{q}_\nu)\sigma^{\mu}_{\alpha\gamma}\sigma^{\nu}_{\beta\rho}]\times f^{m}_{\rho\gamma}(\hat{\mathbf{p}}';\varepsilon_n)\ , \tag{7}$$

where $\nu_{\alpha\beta}$ represents an external field that couples to the quasiparticles. The time-reversed self-energies $\bar{\sigma}$ and $\bar{\Delta}$ are related to σ and Δ by the same symmetries as the corresponding propagators. The scattering amplitudes $A(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ and pairing interactions $V(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$, which parametrize the self-energy equations, can be decomposed as

$$[A(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')]_{\alpha\gamma,\beta\rho}=A^{s}(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\delta_{\alpha\beta}\delta_{\gamma\rho}+A^{a}(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\boldsymbol{\sigma}_{\alpha\beta}\cdot\boldsymbol{\sigma}_{\gamma\rho}\ , \tag{8}$$

$$[V(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')]_{\alpha\gamma,\beta\rho}=V^s(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\sigma^2_{\alpha\beta}\sigma^2_{\gamma\rho} + V^t(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\sigma^i\sigma^2)_{\alpha\beta}(\sigma^2\sigma^i)_{\gamma\rho} , \quad (9)$$

where σ^i are the Pauli matrices in spin space. In accordance with the exclusion principle, the singlet-pairing interaction $V^s(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ contains only even-l components and the triplet-pairing interaction $V^t(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ contains only odd-l components. We define the components V_l by

$$V^{s(t)}(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')=\sum_l^{\text{even (odd)}}(2l+1)V_lP_l(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}') , \quad (10)$$

while we use Landau's original convention for the scattering amplitudes,

$$A^{s(a)}(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')=\sum_l A_l^{s(a)}P_l(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}') , \quad (11)$$

with $A_l^{s(a)}$ related to the Fermi-liquid parameters $F_l^{s(a)}$ by

$$A_l^{s(a)}=\frac{F_l^{s(a)}}{1+F_l^{s(a)}/(2l+1)} . \quad (12)$$

The propagators g^m and f^m can also be decomposed in spin space:

$$g^m_{\rho\gamma}(\hat{\mathbf{p}};\varepsilon_n)=g(\hat{\mathbf{p}};\varepsilon_n)\delta_{\rho\gamma}+\mathbf{g}(\hat{\mathbf{p}};\varepsilon_n)\cdot\boldsymbol{\sigma}_{\rho\gamma} , \quad (13)$$

$$f^m_{\rho\gamma}(\hat{\mathbf{p}};\varepsilon_n)=[f_0(\hat{\mathbf{p}};\varepsilon_n)\delta_{\rho\beta}+\mathbf{f}(\hat{\mathbf{p}};\varepsilon_n)\cdot\boldsymbol{\sigma}_{\rho\beta}]i\sigma^2_{\beta\gamma} . \quad (14)$$

In the absence of the second pairing term in (7), which is contributed by the dipole interaction, physical results can be expressed in terms of the Landau parameters $F_l^{s,a}$ and the transition temperatures for pairing in the lth partial wave,

$$T_{cl}=1.13\varepsilon_c e^{-1/V_l} . \quad (15)$$

Of course, $T_c\equiv T_{c1}$ is the physical transition temperature. The energy cutoff ε_c, which separates the high-energy, nonquasiparticle regime from the quasiparticle energy spectrum, is chosen so that $\pi T_c \ll \varepsilon_c \ll \varepsilon_F$, but is otherwise undefined by Fermi-liquid theory. When the dipole interaction is neglected, the cutoff ε_c and the pairing interactions V_l can always be eliminated in favor of the physical l-wave transition temperatures.

Associated with the dipole interaction is the dimensionless coupling constant

$$g_D=\frac{4\pi}{3}N(0)(\gamma\hbar)^2 , \quad (16)$$

where γ is the gyromagnetic ratio that determines the Larmor frequency γH in terms of the external field H, and $N(0)$ is the single-spin density of states at the Fermi surface. The momentum exchanged by the dipole interaction is $\mathbf{q}=\hat{\mathbf{p}}-\hat{\mathbf{p}}'$. The signs of the pairing interaction and of the dipolar coupling constant in Eq. (7) are determined by the convention that positive interactions are attractive. As first explained by Leggett,[13] the renormalization factor $R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ describes the modification of the dipole interaction by the quasiparticles. Since the range of the dipole interaction is only a few angstroms, $R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ also includes high-energy, nonquasiparticle corrections that are outside the domain of Landau's Fermi-liquid theory. Because the dipole interaction varies on the energy scale of the Fermi energy ε_F, $R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ exhibits no temperature dependence on the scale of T_c. To estimate the pressure dependence and magnitude of $R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ requires a microscopic model for the quasiparticle interactions. Note that (7) neglects dipole-interaction terms of order g_D^2 and higher. In principle, the dipole interaction will contribute an additional scattering term in the particle-hole channel, modifying (6). In Appendix C we show that this contribution to the diagonal self-energy can be neglected.

Specializing now to the B phase, we include the effect of a magnetic field perturbatively in the parameter

$$\gamma H/\Delta\simeq 0.16(H/\text{kG})/(\Delta/\text{mK}) , \quad (17)$$

which is small for fields as large as 1 kG, except very close to T_c. The propagators and self-energies are expanded in powers of this parameter:

$$\begin{aligned}\hat{g}^m&=\hat{g}^m_0+\hat{g}^m_1+\hat{g}^m_2+\dots ,\\ \hat{\sigma}&=\hat{\sigma}_1+\hat{\sigma}_2+\cdots ,\\ \hat{\Delta}&=\hat{\Delta}^0+\hat{\Delta}^1+\hat{\Delta}^2+\cdots ,\end{aligned} \quad (18)$$

where, for example, $|\hat{g}^m_i|\sim(\gamma H/\Delta)|\hat{g}^m_{i-1}|$. The relations in FS for $\hat{g}^m$ in terms of the triplet order parameter,

$$\boldsymbol{\Delta}(\hat{\mathbf{p}})=-\tfrac{1}{2}\text{tr}[i\sigma^2\boldsymbol{\sigma}\Delta(\hat{\mathbf{p}})] , \quad (19)$$

and the effective field,

$$\mathbf{h}(\hat{\mathbf{p}})=\tfrac{1}{2}\text{tr}[\boldsymbol{\sigma}\sigma(\hat{\mathbf{p}})] , \quad (20)$$

are still valid, since only (3) and (4) are used in their derivation. It is straightforward to show that the singlet projection of $\Delta(\hat{\mathbf{p}})$ can be neglected since $\text{tr}[\sigma^2\Delta(\hat{\mathbf{p}})]$ is of order g_D^2.

A. Equilibrium in zero field

Our first task is to calculate the distortion of the equilibrium order parameter caused by the dipole interaction, which will be needed to calculate the NMR frequencies. We use the result of FS for $\hat{g}^m_0$ to rewrite the mean-field equation for $\boldsymbol{\Delta}^0(\hat{\mathbf{p}})$ as

$$\Delta_i^0(\hat{\mathbf{p}})=T\sum_n\sum_l(2l+1)V_l\int\frac{d\Omega'}{4\pi}N(\hat{\mathbf{p}}')P_l(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\Delta_i^0(\hat{\mathbf{p}}') ,$$
$$-g_DT\sum_n\int\frac{d\Omega'}{4\pi}N(\hat{\mathbf{p}}')R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\delta_{ij}-3\hat{q}_i\hat{q}_j)\Delta_j^0(\hat{\mathbf{p}}') , \quad (21)$$

where the momentum dependence of

$$N(\hat{\mathbf{p}})=\frac{\pi}{[\varepsilon_n^2+|\boldsymbol{\Delta}^0(\hat{\mathbf{p}})|^2]^{1/2}} \quad (22)$$

comes from the dipolar distortion of $\boldsymbol{\Delta}^0(\hat{\mathbf{p}})$.

In the absence of the dipole interaction, the B-phase order parameter is given by $\Delta_i^0(\hat{\mathbf{p}})=\Delta^0R_{ij}(\hat{\mathbf{n}},\theta)\hat{p}_j$, where $R_{ij}(\hat{\mathbf{n}},\theta)$ is the matrix for a rotation around the $\hat{\mathbf{n}}$ axis by the angle θ, both undetermined by the gap equation. When $g_D=0$, $N(\hat{\mathbf{p}})$ is independent of momentum and the logarithmically divergent sum

$$T\sum_n N(\hat{\mathbf{p}})=T\sum_n^{|\varepsilon_n|<\varepsilon_c}\frac{\pi}{[\varepsilon_n^2+(\Delta^0)^2]^{1/2}}=\ln(1.13\varepsilon_c/T_c) \tag{23}$$

must be regulated by the energy cutoff ε_c. Using (15), we see that the $l=1$ pairing interaction in (21) cancels this sum and that the cutoff drops out of the gap equation.

In the presence of the dipole interaction, we must decompose $\Delta_i^0(\hat{\mathbf{p}})$ into $l=1$ and 3 components:

$$\Delta_i^0(\hat{\mathbf{p}})=d_{i,j}^1\hat{p}_j+d_{i,jkl}^3\hat{p}_j\hat{p}_k\hat{p}_l\ . \tag{24}$$

Performing the momentum integral in the dipole term of (21) yields

$$d_{i,j}^1\left[1-3V_1T\sum_n\int\frac{d\Omega}{4\pi}(\hat{p}_j)^2N(\hat{\mathbf{p}})\right]=-\frac{3}{20}\frac{g_D}{V_1}(R_0-\tfrac{1}{3}R_1)(3\delta_{ij}d_{k,k}^1-2d_{i,j}^1+3d_{j,i}^1)\ , \tag{25}$$

$$d_{i,jkl}^3x_3=-\tfrac{7}{2}T\sum_n\int\frac{d\Omega}{4\pi}N(\hat{\mathbf{p}})[5\hat{p}_m\hat{p}_j\hat{p}_k\hat{p}_ld_{i,m}^1-(\hat{p}_j)^2d_{i,j}^1\delta_{kl}-(\hat{p}_k)^2d_{i,k}^1\delta_{jl}-(\hat{p}_l)^2d_{i,l}^1\delta_{jk}]$$
$$-\frac{1}{12}\frac{g_D}{V_1V_3}(R_0+3R_1-10R_2)[\delta_{ij}d_{kl}^{(2,1)}+\delta_{ik}d_{jl}^{(2,1)}+\delta_{il}d_{jk}^{(2,1)}-\tfrac{2}{5}(\delta_{jk}d_{il}^{(2,1)}+\delta_{jl}d_{ik}^{(2,1)}+\delta_{kl}d_{ij}^{(2,1)})]\ , \tag{26}$$

where $d_{kl}^{(2,1)}$ is the traceless and symmetric matrix

$$d_{kl}^{(2,1)}=\tfrac{1}{2}(d_{k,l}^1+d_{l,k}^1)-\tfrac{1}{3}d_{j,j}^1\delta_{kl} \tag{27}$$

and

$$R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')=\sum_l(2l+1)R_lP_l(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\ . \tag{28}$$

All higher-angular-momentum components of the gap vanish to first order in g_D. In the $l=3$ relation (26), the parameter

$$x_3=V_1^{-1}-V_3^{-1}=\ln(T_{c3}/T_{c1})$$

measures the relative importance of the f- and p-wave pairing interactions, with a negative value of x_3 corresponding to an attractive f-wave pairing interaction. Notice that the two projections $\overline{R^2}\equiv R_0-R_1/3$, first defined by Leggett,[13] and $\overline{S^2}\equiv\frac{1}{3}R_0+R_1-\frac{10}{3}R_2$ enter (25) and (26). When $g_D=0$, Eq. (26) requires that the $l=3$ gap vanishes. Even when the $l=3$ pairing interaction is zero, however, the dipole interaction itself induces a finite $l=3$ gap proportional to $g_D\overline{S^2}/V_1$.

The $l=1$ relation (25) separates into three independent equations:

$$\Delta_3^0(1-3V_1J_2)=-\tfrac{9}{10}g_D\overline{R^2}J_2[3\Delta_1^0(\cos\theta_L^0)+2\Delta_3^0]\ , \tag{29a}$$

$$\Delta_1^0(\cos\theta_L^0)[1-\tfrac{3}{2}V_1(J_1-J_2)]$$
$$=-\tfrac{9}{40}g_D\overline{R^2}(J_1-J_2)[7\Delta_1^0(\cos\theta_L^0)+3\Delta_3^0]\ , \tag{29b}$$

$$\Delta_1^0(\sin\theta_L^0)[1-\tfrac{3}{2}V_1(J_1-J_2)]=\tfrac{9}{8}g_D\overline{R^2}(J_1-J_2)\Delta_1^0\sin\theta_L^0\ , \tag{29c}$$

where we have taken

$$d_{i,j}^1=R_{ij}^0(\hat{\mathbf{z}},\theta_L^0)\Delta_i^0\ ,$$

which (25) demands. The rotation axis $\hat{\mathbf{n}}\|\hat{\mathbf{z}}$ is fixed by an infinitesimal magnetic field $\mathbf{H}=H\hat{\mathbf{z}}$. The axial symmetry about the $\hat{\mathbf{z}}$ axis is then used to set $\Delta_1^0=\Delta_2^0$. The functions J_i are weighted integrals over $N(\hat{\mathbf{p}})$:

$$J_1=T\sum_n\int\frac{d\Omega}{4\pi}N(\hat{\mathbf{p}})\ , \tag{30a}$$

$$J_2=T\sum_n\int\frac{d\Omega}{4\pi}(\hat{p}_3)^2N(\hat{\mathbf{p}})\ , \tag{30b}$$

$$J_3=T\sum_n\int\frac{d\Omega}{4\pi}(\hat{p}_3)^4N(\hat{\mathbf{p}})\ , \tag{30c}$$

$$J_4=T\sum_n\int\frac{d\Omega}{4\pi}(\hat{p}_1)^2(\hat{p}_2)^2N(\hat{\mathbf{p}})\ . \tag{30d}$$

If $\Delta_1^0\neq0$ and $\Delta_3^0\neq0$, then (29b) and (29c) imply that, to zero order[20] in g_D,

$$\cos\theta_L^0=-\frac{1}{4}\frac{\Delta_3^0}{\Delta_1^0}\ . \tag{31}$$

Using this result we can rewrite (29a) and (29c) as

$$1-3V_1J_2=-\tfrac{9}{8}g_D\overline{R^2}J_2\ , \tag{32a}$$

$$1-\tfrac{3}{2}V_1(J_1-J_2)=\tfrac{9}{8}g_D\overline{R^2}(J_1-J_2)\ , \tag{32b}$$

which agree with Tewordt and Einzel[21] when $l=3$ correlations are neglected (when $V_3=\overline{S^2}=0$) in the integrals J_1 and J_2.

Decomposing the $l=3$ gap into $J=2$, 3, and 4 components, we use (26) to solve for the gap tensors in terms of the integrals J_i. For the $J=2$ gap we find

$$d_{11}^{(2,3)}=d_{22}^{(2,3)}=-\tfrac{1}{2}d_{33}^{(2,3)}$$
$$=\tfrac{3}{16}x_3^{-1}\Delta_3^0(J_1-18J_2+25J_3)$$
$$+\frac{3}{16}\frac{g_D\overline{S^2}}{V_1V_3}x_3^{-1}\Delta_3^0\ . \tag{33}$$

The $J=3$ gap tensors can be written

$$d_{1,222}^{(3,3)}=-d_{2,111}^{(3,3)}$$
$$=\tfrac{7}{4}(\sin\theta_L^0)\Delta_1^0x_3^{-1}(-2J_1+7J_2-5J_3+10J_4)\ , \tag{34a}$$

$$d^{(3,3)}_{2,122} = -d^{(3,3)}_{1,211} = \tfrac{7}{4}(\sin\theta^0_L)\Delta^0_1 x_3^{-1}(-J_1+J_2+10J_4) , \tag{34b}$$

$$d^{(3,3)}_{1,233} = -d^{(3,3)}_{2,133} = \tfrac{7}{4}(\sin\theta^0_L)\Delta^0_1 x_3^{-1}(J_1-6J_2+5J_3) . \tag{34c}$$

Finally, the $J=4$ tensors are

$$d^{(4,3)}_{2233} = d^{(4,3)}_{1133} = -\tfrac{1}{2}d^{(4,3)}_{3333} = \tfrac{1}{16}\Delta^0_3 x_3^{-1}(-3J_1-30J_2+65J_3) , \tag{35a}$$

$$d^{(4,3)}_{1122} = \tfrac{1}{16}\Delta^0_3 x_3^{-1}(-8J_1+25J_2-25J_3+70J_4) , \tag{35b}$$

$$d^{(4,3)}_{1111} = d^{(4,3)}_{2222} = \tfrac{1}{16}\Delta^0_3 x_3^{-1}(11J_1+5J_2-40J_3-70J_4) . \tag{35c}$$

All other components vanish.

To proceed further, we express the integrals J_i in terms of the gap components by expanding $N(\hat{\mathbf{p}})$ in the dipole interaction, with the results

$$J_1 = T\sum_n N^0 - \tfrac{1}{6}T\sum_n \frac{(N^0)^3}{\pi^2}[(\Delta^0_3)^2-(\Delta^0_1)^2] , \tag{36a}$$

$$J_2 = \tfrac{1}{3}T\sum_n N^0 - \tfrac{1}{10}T\sum_n \frac{(N^0)^3}{\pi^2}[(\Delta^0_3)^2-(\Delta^0_1)^2] - \tfrac{2}{35}T\sum_n \frac{(N^0)^3}{\pi^2}R^0_{im}d^3_{i,m33}\Delta^0_i , \tag{36b}$$

$$J_3 = \tfrac{1}{5}T\sum_n N^0 - \tfrac{1}{14}T\sum_n \frac{(N^0)^3}{\pi^2}[(\Delta^0_3)^2-(\Delta^0_1)^2] - \tfrac{4}{315}T\sum_n \frac{(N^0)^3}{\pi^2}(3R^0_{im}d^3_{i,m33}\Delta^0_i + 2d^3_{3,333}\Delta^0_3) , \tag{36c}$$

$$J_4 = \tfrac{1}{15}T\sum_n N^0 - \tfrac{1}{210}T\sum_n \frac{(N^0)^3}{\pi^2}[(\Delta^0_3)^2-(\Delta^0_1)^2] - \tfrac{4}{315}T\sum_n \frac{(N^0)^3}{\pi^2}(R^0_{im}d^3_{i,m22}\Delta^0_i + R^0_{i1}d^3_{i,122}\Delta^0_1 + R^0_{i2}d^3_{i,211}\Delta^0_1) , \tag{36d}$$

where

$$N^0 = \frac{\pi}{[\varepsilon_n^2+(\Delta^0_1)^2]^{1/2}} . \tag{37}$$

The final ingredient needed is an expression for the $l=1$ gap distortion $(\Delta^0_1)^2-(\Delta^0_3)^2$, which we obtain from (32) and (36):

$$(\Delta^0_1)^2-(\Delta^0_3)^2 = \frac{45}{8}\frac{g_D\overline{R^2}}{V_1 Y^0_{3/2}}T\sum_n N^0 + \tfrac{9}{7}R^0_{im}d^3_{i,m33}\Delta^0_i , \tag{38}$$

where $Y^0_{m/2} = T\sum_n (N^0)^m/\pi^{m-1}$ are the generalized Yoshida functions, in terms of which the original Yoshida function is given by $y = 1-(\Delta^0_1)^2 Y^0_{3/2}$. Equations (33)–(38) completely determine the integrals J_i and the zero-field gap. Solving this set of equations, we find that the $l=1$ gap distortion is given by

$$(\Delta^0_1)^2-(\Delta^0_3)^2 = \frac{45}{8}\frac{g_D\overline{R^2}}{(V_1)^2 Y^0_{3/2}}[1-\tfrac{3}{5}(\Delta^0_1)^2 Y^0_{3/2}x_3^{-1}] - \frac{9}{28}\frac{g_D\overline{S^2}}{V_1V_3}x_3^{-1}(\Delta^0_1)^2 . \tag{39}$$

The results for the $l=3$ gaps and for $N(\hat{\mathbf{p}})$ are given in Appendix B. If $V_3=\overline{S^2}=0$, then (39) agrees with Tewordt and Einzel.[21]

In the absence of $l=3$ correlations the dipole-interaction constant only appears in the dimensionless factor $g_D\overline{R^2}/(V_1)^2$, which can be considered a new phenomenological parameter, to be determined alongside the other interaction parameters. We shall see shortly that $g_D\overline{R^2}/(V_1)^2$ can be expressed in terms of the temperature width of the dipole-induced planar phase. Therefore, despite appearances, this factor is actually independent of the energy cutoff ε_c. In practice, $g_D\overline{R^2}/(V_1)^2$ can be obtained from longitudinal NMR measurements in zero field, as shown in Sec. III. The last term in (39), which remains finite when $V_3\rightarrow 0$, originates from the $l=3$ correlations induced by the dipole interaction itself. The relation

$$\frac{x_3^{-1}}{V_1V_3} = \frac{x_3^{-1}}{(V_1)^2} - \frac{1}{V_1} \tag{40}$$

can be used to show that the dipole-induced $g_D x_3^{-1}/(V_1V_3)$ term contains a correction of order V_1 smaller than the $g_D/(V_1)^2$ term. Although $V_1 = 1/\ln(1.13\varepsilon_c/T_c)$ vanishes in the $\varepsilon_c/T_c\rightarrow\infty$ limit, the more realistic estimate[22] $\varepsilon_c = 0.07\varepsilon_F$ indicates that $V_1\simeq 0.2$ at 0 bar, so this correction can be sizable.[23] Unlike $g_D\overline{R^2}/(V_1)^2$ and x_3^{-1} (to zero order in g_D), the factor $g_D\overline{S^2}/(V_1V_3)$ cannot be expressed in terms of the physical transition temperatures and does depend on the cutoff ε_c. If $g_D\overline{R^2}/(V_1)^2$ is obtained from zero-field NMR measurements, then $g_D\overline{S^2}/(V_1V_3)$ can be estimated by using a microscopic model for $(V_1\overline{S^2})/(V_3\overline{R^2})$.

It is not surprising that, in the presence of the dipole interaction, the cutoff cannot be eliminated from observable quantities such as the gap distortion. Since the dipole interaction varies on the energy scale $\varepsilon_F \gg \varepsilon_c$, the dipolar constant g_D is renormalized by cutoff-dependent factors. When $V_3=0$, the $l=1$ moment of the dipole interaction is scaled by $(V_1)^2$ and the $l=3$ moment is scaled by V_1. Since it includes high-energy corrections, the renormalization factor $R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ also depends on the energy cutoff. The ratio of cutoff-dependent quantities $\overline{R^2}/(V_1)^2$, however, is independent of ε_c.

One interesting consequence of (39) is that Δ^0_3/Δ^0_1 must vanish as the temperature is increased. At the temperature

$$T^B_c = 1.13\varepsilon_c\exp\left[\frac{-1}{V_1-\tfrac{3}{2}g_D\overline{R^2}}\right] , \tag{41}$$

we find that $\Delta^0_3=0$, $\cos\theta^0_L=0$, and

$$(\Delta^0_1)^2 = \frac{45}{8}\frac{g_D\overline{R^2}}{(V_1)^2}\left[T^B_c\sum_n \frac{\pi}{|\varepsilon_n|^3}\right]^{-1} . \tag{42}$$

To examine the intermediate region between the normal state and the B phase, we must solve (29) with $\Delta_3^0=0$ and $\Delta_1^0\neq 0$. We find that $\cos\theta_L^0=0$ throughout this region and that the $l=1$ gap is given by

$$(\Delta_1^0)^2=-\tfrac{5}{2}\ln\left[\frac{T}{T_c^P}\right]\left[T\sum_n\frac{\pi}{|\varepsilon_n|^3}\right]^{-1}, \quad (43)$$

where T_c^P is the temperature of the normal-state transition:

$$T_c^P=1.13\varepsilon_c\exp\left[\frac{-1}{V_1+\frac{3}{4}g_D\overline{R^2}}\right]. \quad (44)$$

The $l=3$ gap components in this dipole-induced state are listed in Appendix B.

Notice that the gap tensors and the rotation angle θ_L^0 change continuously across T_c^B, which therefore marks a second-order phase transition from the rotated planar state into the B phase. The width of the rotated planar phase is proportional to the renormalized dipole-interaction constant:

$$T_c^P-T_c^B=\tfrac{9}{4}T_c^B\frac{g_D\overline{R^2}}{(V_1)^2}, \quad (45)$$

confirming the assertion made earlier. In the planar phase, the $S_z=0$ Cooper-pair state has been completely depopulated by the dipole interaction, which favors pairing in the $S_z=\pm 1$ states. The existence of the dipole-induced phase was first discussed by Leggett.[3] The effects of higher-order self-energy diagrams, which introduce fluctuations of the order parameter about its mean-field value, were later included by Jones *et al.*,[24] who demonstrated that this phase is, in fact, stable.

B. Equilibrium in a magnetic field

To study the field dependence of the equilibrium gap in (7) requires the off-diagonal Matsubara Green's functions $f_{\rho\gamma}^m=i\mathbf{f}\cdot(\sigma\sigma^2)_{\rho\gamma}$ given in FS. Our starting point is the field-dependent version of (21):

$$\Delta_i(\hat{\mathbf{p}})=T\sum_n\sum_l(2l+1)V_l\int\frac{d\Omega'}{4\pi}P_l(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')f_i(\hat{\mathbf{p}}')$$
$$-g_DT\sum_n\int\frac{d\Omega'}{4\pi}R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\delta_{ij}-3\hat{q}_i\hat{q}_j)f_j(\hat{\mathbf{p}}'). \quad (46)$$

Again, $\Delta_i(\hat{\mathbf{p}})$ is decomposed into $l=1$ and 3 components, with the $l=1$ gap given by

$$d_{i,j}^1=3V_1T\sum_n\int\frac{d\Omega}{4\pi}\hat{p}_jf_i(\hat{\mathbf{p}})-\tfrac{3}{4}g_DT\sum_n\int\frac{d\Omega}{4\pi}f_k(\hat{\mathbf{p}})[-(R_0+3R_1-10R_2)\hat{p}_j\hat{p}_i\hat{p}_k+(-R_0+R_1-2R_2)\hat{p}_j\delta_{ik}$$
$$-2(R_2-R_0)(\hat{p}_i\delta_{jk}+\hat{p}_k\delta_{ij})]. \quad (47)$$

We shall not be concerned with the $l=3$ gap, except to note that when $g_D=0$ we recover the results of FS, provided $\hat{p}_j$ is replaced by $\hat{s}_j=R_{jk}^0\hat{p}_k$, which is the unit vector of the "dipole representation," discussed in Appendix A.

As can be verified from (47), a magnetic field $\mathbf{H}=H\hat{\mathbf{z}}$ fixes the rotation angle $\hat{\mathbf{n}}=\hat{\mathbf{z}}$. Using the rotation symmetry about the $\hat{\mathbf{z}}$ axis, Eq. (47) can be broken into the three independent equations:

$$\Delta_1(\sin\theta_L)(1-V_1K_2)=\tfrac{9}{4}\Delta_1g_D\overline{R^2}K_2\sin\theta_L, \quad (48a)$$

$$\Delta_1(\cos\theta_L)(1-V_1K_1)=-\tfrac{3}{4}g_D[-2(R_2-R_0)\Delta_3K_3-(R_0+3R_1-10R_2)\Delta_3K_4$$
$$+(4R_0-2R_1+2R_2)\Delta_1K_1(\cos\theta_L)+(R_0+3R_1-10R_2)\Delta_1K_5(\cos\theta_L)], \quad (48b)$$

$$\Delta_3(1-V_1K_3)=-\tfrac{3}{2}g_D[(R_0-R_1+2R_2)\Delta_3K_3+(R_0+3R_1-10R_2)\Delta_3K_4$$
$$-2(R_2-R_0)\Delta_1K_1(\cos\theta_L)-(R_0+3R_1-10R_2)\Delta_1K_5(\cos\theta_L)], \quad (48c)$$

where we take $d_{i,j}^1=R_{ij}(\hat{\mathbf{z}},\theta_L)\Delta_i$ and set $\Delta_1=\Delta_2=\Delta_1^0+\Delta_1^2$. The functions K_i are weighted integrals over $f_j(\hat{\mathbf{p}})$:

$$K_1=\frac{T}{\Delta_1(\cos\theta_L)}\sum_n\int\frac{d\Omega}{4\pi}\hat{p}_1f_1(\hat{\mathbf{p}}), \quad (49a)$$

$$K_2=\frac{T}{\Delta_1(\sin\theta_L)}\sum_n\int\frac{d\Omega}{4\pi}\hat{p}_2f_1(\hat{\mathbf{p}}), \quad (49b)$$

$$K_3=\frac{T}{\Delta_3}\sum_n\int\frac{d\Omega}{4\pi}\hat{p}_3f_3(\hat{\mathbf{p}}), \quad (49c)$$

$$K_4=\frac{T}{\Delta_3}\sum_n\int\frac{d\Omega}{4\pi}\hat{p}_3(\hat{p}_1)^2f_3(\hat{\mathbf{p}}), \quad (49d)$$

$$K_5=\frac{T}{\Delta_1(\cos\theta_L)}\sum_n\int\frac{d\Omega}{4\pi}\hat{p}_1(\hat{p}_3)^2f_1(\hat{\mathbf{p}}). \quad (49e)$$

These integrals can easily be evaluated to zero order in g_D. Using the results in Appendix B for the zero-field $l=3$ gap, we then find that

$$K_2-K_1=-\frac{1}{28}\frac{g_D\overline{S^2}}{V_1V_3}$$
$$\times x_3^{-1}(\gamma H)^2\left[1+\frac{F_2^a}{5}\right]^2D^{-2}Y_{3/2}(1-\tfrac{3}{5}A), \quad (50)$$

where

$$D = 1+(\tfrac{2}{3}+\tfrac{1}{3}y)F_0^a+(\tfrac{1}{3}+\tfrac{2}{3}y)\frac{F_2^a}{5}+yF_0^a\frac{F_2^a}{5} \tag{51}$$

and $A \equiv (\Delta_1^0)^2 Y_{3/2}^0 A_2^a/5$. This result is used in (48a) and (48b) to obtain the field dependence of the Leggett angle, to zero order in g_D:

$$\cos\theta_L = -\frac{1}{4}\frac{\Delta_3}{\Delta_1} - \tfrac{1}{112}(\gamma H)^2 \frac{V_1\overline{S^2}}{V_3\overline{R^2}} \times x_3^{-1}\left[1+\frac{F_2^a}{5}\right]^2 D^{-2} Y_{3/2}^0 \left[1-\frac{A}{3}\right] . \tag{52}$$

The $l=1$ gap distortion was previously calculated in FS:

$$\frac{\Delta_3}{\Delta_1} = 1+\frac{1}{8}\left[\frac{\gamma H}{\Delta_1^0}\right]^2 \left[1+\frac{F_2^a}{5}\right]^2 D^{-2}\left[-5+3A+3\frac{(\Delta_1^0)^2 Y_{5/2}^0}{Y_{3/2}^0}+3(\Delta_1^0)^2 Y_{3/2}^0 x_3^{-1}(1-A)\right] . \tag{53}$$

In the absence of $l=3$ correlations we recover the result of Tewordt and Schopohl[25] for $\cos\theta_L$. Two corrections proportional to x_3^{-1} enter $\cos\theta_L$: one contained in the $l=1$ gap distortion (53) and the other contained in the $l=3$ correlations induced by the dipole interaction, which enter the second term in (52). The latter contribution remains finite when $V_3 \to 0$, although it becomes smaller by V_1 than the other field-dependent terms. Close to T_c the cutoff-dependent contribution, proportional to $(V_1\overline{S^2})/(V_3\overline{R^2})$, will be much smaller than the contribution from the $l=1$ gap distortion. In this regime the strong-coupling and dipolar corrections to the Leggett angle, calculated by Fetter[26] and Greaves,[27] respectively, will be more important than the cutoff-dependent correction.

The cutoff-dependent term can have an appreciable effect on the field dependence of $\cos\theta_L$ at low temperatures. Fomin, Pethick, and Serene[28] estimate $R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ by using polarization potentials[29] to describe the screened response of the Fermi liquid at short wavelengths of order $\hbar/p_F$. Using their model, we find that $\overline{S^2}/\overline{R^2} \simeq 0.48$ at 0 bar, decreasing slightly to 0.45 to 10 bars. Although $V_1/V_3 \to 1$ as $\varepsilon_c/T_c \to \infty$, realistic estimates[22] indicate that $V_1/V_3 \simeq 1.5$, so $(V_1\overline{S^2})/(V_3\overline{R^2}) \simeq 0.75$ For reasonable[9] values of x_3^{-1}, the cutoff-dependent term can change the field dependence of $\cos\theta_L$ by several percent. In Fig. 1 we plot $\cos\theta_L$ versus T/T_c for one possible set of input parameters.

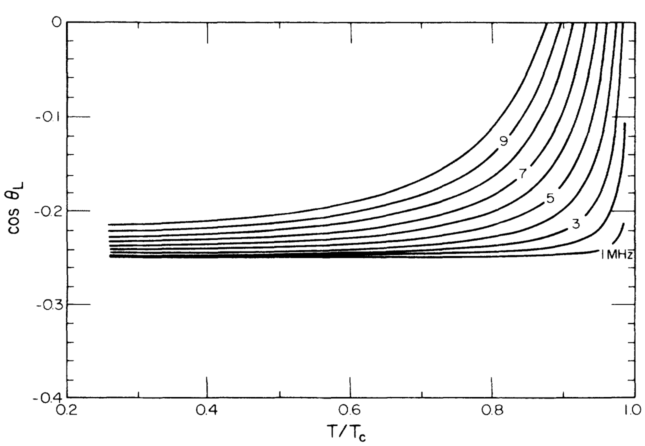

FIG. 1. The field dependence of the Leggett angle at 0 bars using $F_0^a=-0.7$, $F_2^a=0.2$, $x_3^{-1}=-0.6$, $(V_1\overline{S^2})/(V_3\overline{R^2})=0.0$, and $T_c=1.04$ mK.

We pause here to note that the dipolar corrections to the zero-field equilibrium gap will generate corrections to the effective field $\mathbf{h}_1$. If we neglect the dipole-induced $l=4$ component of $\mathbf{h}_1$, which does not contribute to the lowest-order field dependence of the NMR frequencies, then

$$\mathbf{h}_1 = \frac{\gamma}{2}\mathbf{H}[\Theta_1+\Theta_3(\hat{\mathbf{s}}_3)^2]+\frac{\gamma}{2}\Theta_2\hat{\mathbf{s}}\,\mathbf{H}\cdot\hat{\mathbf{s}} , \tag{54}$$

where $\mathbf{H}=H\hat{\mathbf{z}}$. The results for Θ_1, Θ_2, and Θ_3 are given in Appendix C. In the following section we shall use these parameters and the results for the equilibrium gap to calculate the shifts in NMR frequencies.

III. COLLISIONLESS NMR

The importance of the dipole interaction to the spin dynamics of superfluid ^{3}He was first recognized by Leggett,[2] who sought to understand the shifts in the transverse NMR frequencies of the A phase.[1] Leggett's theory of "spontaneously broken spin-orbit symmetry" demonstrated that the dipole forces were enhanced by the condensed state of the superfluid. This theory also predicted[13] a shift in the B-phase longitudinal NMR frequency, which was later observed by Osheroff[30] and others.[31]

The Leggett-Takagi equations[12] are a semiphenomenological description of the spin dynamics of ^{3}He based on equations of motion for the vectors which specify the equilibrium configuration of the order parameter. The theory postulates that, other than the spin resonance, no internal degrees of freedom of the order parameter are excited. This is a good assumption provided that the resonance frequency ω is much smaller than the internal excitation frequencies of the order parameter, which are of order Δ, and that the quasiparticles have sufficient time to respond to the motion of the spin. Thus the Leggett-Takagi (LT) theory should be valid if $\omega \ll \Delta$ and $\omega\tau_{\rm LT} \ll 1$, where $\tau_{\rm LT}$ is the spin-relaxation time.

In Fig. 2 we have plotted $\tau_{\rm LT}$ using the results of Pethick *et al.*,[32] and Einzel and Wölfle.[33] Also plotted are the zero-field longitudinal NMR frequencies[31] for 0 and 16 bars, with temperature dependence estimated from the hydrodynamic theory. At 0 bar the collisionless regime $\omega\tau_{\rm LT} \gg 1$ extends to $T/T_c \simeq 0.55$ and the hydrodynamic

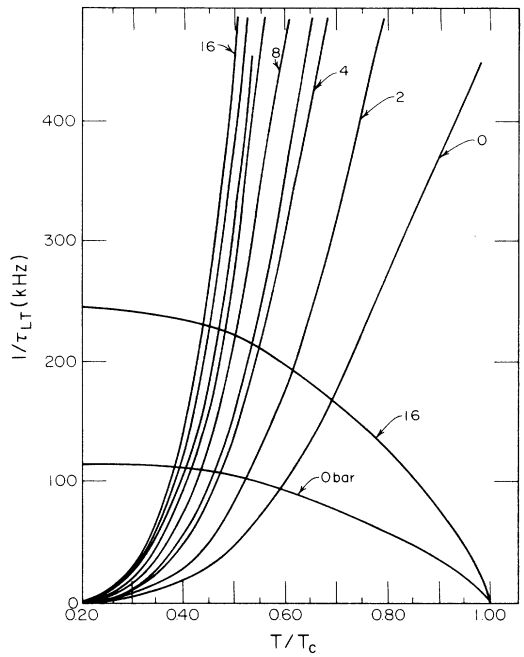

FIG. 2. The inverse Leggett-Takagi relaxation time in kHz and the zero-field longitudinal NMR frequency for 0 and 16 bars vs T/T_c. The temperature dependence of the NMR frequency is calculated with the hydrodynamic formula (85).

regime $\omega\tau_{\rm LT} \ll 1$ begins at $T/T_c \simeq 0.65$. The midway point $\omega\tau_{\rm LT} \simeq 1$ decreases in relative temperature as the pressure increases.

The theory developed in this section treats the low-temperature, collisionless dynamics of the NMR. We find that in zero field the longitudinal frequency is proportional to $g_D \overline{R^2}/(V_1)^2$, but the transverse frequencies remain zero. In a finite field and at finite temperature, a term proportional to $(F_2^a)^2$ but independent of g_D contributes to the longitudinal frequency squared. This term, which arises from the restoring force between the quasiparticles and the spin resonance, vanishes at $T=0$ when the quasiparticle density vanishes. Unfortunately, because the zero-field frequency of the transverse resonance vanishes, the perturbative calculation of the field dependence of the transverse NMR frequencies breaks down, except at $T=0$, when we recover the expected result $\omega = \pm\gamma H$ to zero order in g_D.

A theoretical description of the collisionless modes of ^{3}He-B requires a nonequilibrium generalization of the quasiclassical theory. The linearized transport equation is written

$$[\varepsilon\hat{\tau}_3, \delta\hat{g}] + \tfrac{1}{2}\omega\{\hat{\tau}_3, \delta\hat{g}\} - [\hat{\sigma}^{\,\rm eq}, \delta\hat{g}] - \delta\hat{\sigma}\,\hat{g}^{\,\rm eq}(\varepsilon - \omega/2) + \hat{g}^{\,\rm eq}(\varepsilon+\omega/2)\delta\hat{\sigma} = 0 \,, \tag{55}$$

where the square and curly brackets represent the usual commutator and anticommutator, and $\delta\hat{g}(\hat{\mathbf{p}};\varepsilon,\omega)$ and $\delta\hat{\sigma}(\hat{\mathbf{p}};\omega)$ are small deviations from the equilibrium Keldysh Green's function $\hat{g}^{\,\rm eq}(\hat{\mathbf{p}};\varepsilon)$ and the equilibrium self-energy $\hat{\sigma}^{\,\rm eq}(\hat{\mathbf{p}})$. The Keldysh Green's function and the self-energy are functions of the quasiparticle energy ε and the frequency ω, which is determined by the external field. The equilibrium Keldysh Green's function can be obtained from the Matsubara Green's function using

$$\hat{g}^{\,\rm eq}(\varepsilon) = [\hat{g}^{\,m}(\varepsilon_n = -i\varepsilon + \eta) - \hat{g}^{\,m}(\varepsilon_n = -i\varepsilon - \eta)]\tanh\left[\frac{\varepsilon}{2T}\right]\Bigg|_{\eta\to 0^+} \,. \tag{56}$$

Relations for $\hat{g}^{\,\rm eq}(\varepsilon)$ are given in FS.

The nonequilibrium generalizations of the mean-field equations (6) and (7) are

$$\delta\sigma_{\alpha\beta}(\hat{\mathbf{p}};\omega) = \tfrac{1}{2}\int \frac{d\Omega'}{4\pi}\int\frac{d\varepsilon}{4\pi i}[A(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')]_{\alpha\gamma,\beta\rho}\delta g_{\rho\gamma}(\hat{\mathbf{p}}';\varepsilon,\omega) \,, \tag{57}$$

$$\delta\Delta_{\alpha\beta}(\hat{\mathbf{p}};\omega) = \tfrac{1}{2}\int\frac{d\Omega'}{4\pi} \times \int\frac{d\varepsilon}{4\pi i}\{[V(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')]_{\alpha\gamma,\beta\rho} + g_D R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\delta_{\mu\nu} - 3\hat{q}_\mu\hat{q}_\nu)\sigma^\mu_{\alpha\gamma}\sigma^\nu_{\beta\rho}\} \times \delta f_{\rho\gamma}(\hat{\mathbf{p}}';\varepsilon,\omega) \,. \tag{58}$$

As in Sec. II, the Green's functions and self-energies are decomposed in spin space:

$$\begin{aligned} \delta g_{\alpha\beta}(\hat{\mathbf{p}};\varepsilon,\omega) &= \delta g(\hat{\mathbf{p}};\varepsilon,\omega)\delta_{\alpha\beta} + \delta\mathbf{g}(\hat{\mathbf{p}};\varepsilon,\omega)\cdot\boldsymbol{\sigma}_{\alpha\beta} \,, \\ \delta f_{\alpha\beta}(\hat{\mathbf{p}};\varepsilon,\omega) &= [\delta f_0(\hat{\mathbf{p}};\varepsilon,\omega)\delta_{\alpha\rho} + \delta\mathbf{f}(\hat{\mathbf{p}};\varepsilon,\omega)\cdot\boldsymbol{\sigma}_{\alpha\rho}]i\sigma^2_{\rho\beta} \,, \end{aligned} \tag{59}$$

and

$$\begin{aligned} \delta\sigma_{\alpha\beta}(\hat{\mathbf{p}};\omega) &= e(\hat{\mathbf{p}};\omega)\delta_{\alpha\beta} + \mathbf{e}(\hat{\mathbf{p}};\omega)\cdot\boldsymbol{\sigma}_{\alpha\beta} \,, \\ \delta\Delta_{\alpha\beta}(\hat{\mathbf{p}};\omega) &= [d(\hat{\mathbf{p}};\omega)\delta_{\alpha\rho} + \mathbf{d}(\hat{\mathbf{p}};\omega)\cdot\boldsymbol{\sigma}_{\alpha\rho}]i\sigma^2_{\rho\beta} \,. \end{aligned} \tag{60}$$

The time-reversed quantities are then given by the symmetry relations

$$\begin{aligned} &\delta\bar{g} = \delta g(-\hat{\mathbf{p}};-\varepsilon,\omega), \quad \delta\bar{\mathbf{g}} = \delta\mathbf{g}(-\hat{\mathbf{p}};-\varepsilon,\omega) \,, \\ &\delta\bar{f} = -[\delta f(-\hat{\mathbf{p}};-\varepsilon,-\omega)]^* , \quad \delta\bar{\mathbf{f}} = [\delta\mathbf{f}(-\hat{\mathbf{p}};-\varepsilon,-\omega)]^* \,, \\ &\bar{e} = e(-\hat{\mathbf{p}};\omega), \quad \bar{\mathbf{e}} = \mathbf{e}(-\hat{\mathbf{p}};\omega) \,, \\ &\bar{d} = [d(\hat{\mathbf{p}};-\omega)]^* , \quad \bar{\mathbf{d}} = [\mathbf{d}(\hat{\mathbf{p}};-\omega)]^* \,. \end{aligned} \tag{61}$$

It is convenient to introduce the sum and difference functions $A^{\pm} \equiv A \pm \overline{A}$, which have simple transformation properties. For example, $\mathbf{d}^+$ ($\mathbf{d}^-$) represents the real (imaginary) part of the time-dependent spin-triplet order parameter. Density and spin-density modes of the superfluid correspond to oscillations of $e^{\pm}$ and $\mathbf{e}^{\pm}$, respectively.

The NMR corresponds to an oscillation of the real spin density, $\mathbf{e}^+$, which couples through (55) to the real triplet order parameter $\mathbf{d}^+$. As shown in FS, the homogeneous eigenvalue equation governing the real, triplet modes can be written in the matrix form

$$\underline{d}(\hat{\mathbf{p}}) = \underline{L}(\hat{\mathbf{p}},\hat{\mathbf{p}}';\omega) * \underline{d}(\hat{\mathbf{p}}') \,, \tag{62}$$

where $\underset{\sim}{d}^{\rm tr}=(\mathbf{d}^+,\mathbf{e}^+)$ is a six-component vector and the star product is defined as

$$[\underline{L}(\hat{\mathbf{p}},\hat{\mathbf{p}}';\omega)*\underset{\sim}{d}(\hat{\mathbf{p}}')]_k=\int\frac{d\Omega'}{4\pi}[L(\hat{\mathbf{p}},\hat{\mathbf{p}}';\omega)]_{kj}d_j(\hat{\mathbf{p}}')\ . \quad (63)$$

The matrix $\underline{L}$ can be decomposed in particle-hole space as

$$\underline{L}=\begin{vmatrix} V^{\rm PD}_{\alpha\beta}(\hat{\mathbf{p}},\hat{\mathbf{p}}')L_{1\beta\gamma}(\hat{\mathbf{p}}') & V^{\rm PD}_{\alpha\beta}(\hat{\mathbf{p}},\hat{\mathbf{p}}')L_{2\beta\gamma}(\hat{\mathbf{p}}') \\ F^a(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')L_{3\alpha\gamma}(\hat{\mathbf{p}}') & F^a(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')L_{4\alpha\gamma}(\hat{\mathbf{p}}')\end{vmatrix}\ , \quad (64)$$

where

$$V^{\rm PD}_{ij}(\hat{\mathbf{p}},\hat{\mathbf{p}}')=V^t(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\delta_{ij}-g_DR^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\delta_{ij}-3\hat{q}_i\hat{q}_j) \quad (65)$$

is the pairing interaction modified by the dipole forces and $F^a(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ is the Fermi-liquid exchange interaction. The matrices $L_i(\hat{\mathbf{p}}';\omega)$ are given in FS as functions of the equilibrium order parameter, the effective field, and the frequency ω.

The eigenvalue equation (62) can be solved perturbatively in the magnetic field by expanding $\underline{L}$, $\underset{\sim}{d}$, and ω in powers of $(\gamma H/\Delta^0)$:

$$\begin{aligned} &\underline{L}(\omega)=\underline{L}^{(0)}+\underline{L}^{(1)}+\underline{L}^{(2)}+\cdots\ ,\\ &\underset{\sim}{d}=\underset{\sim}{d}_0+\underset{\sim}{d}_1+\underset{\sim}{d}_2+\cdots\ ,\\ &\omega=\omega_0+\omega_1+\omega_2+\cdots\ ,\end{aligned} \quad (66)$$

where, for example, $|\omega_i|\sim(\gamma H/\Delta^0)|\omega_{i-1}|$. To obtain the frequencies ω_i it is convenient to introduce the transposed eigenvalue equation

$$\underset{\sim}{b}(\hat{\mathbf{p}})=\underset{\sim}{b}(\hat{\mathbf{p}}')*\underline{L}(\hat{\mathbf{p}}',\hat{\mathbf{p}};\bar{\omega}) \quad (67)$$

for the eigenfrequency $\bar{\omega}$ and the transposed eigenfunction $\underset{\sim}{b}(\hat{\mathbf{p}})$, which does not equal $[\underset{\sim}{d}(\hat{\mathbf{p}})]^{\rm tr}$ because $\underline{L}$ is not Hermitian. If $\underset{\sim}{b}$ and $\underset{\sim}{d}$ have the same spin and orbital symmetries, it can be proved by induction that $\bar{\omega}=\omega$ to all orders in field. The zero-field transposed relation can then be used to show that the eigenfrequencies ω_1 and ω_2 are given by

$$\omega_1=-\frac{N_1}{D_\omega},\quad \omega_2=-\frac{N_2}{D_\omega}\ , \quad (68)$$

with

$$N_1=\underset{\sim}{b}_0*\underline{L}^{(1)}(\omega_0)*\underset{\sim}{d}_0\ , \quad (69a)$$

$$\begin{aligned} N_2=&\underset{\sim}{b}_0*\underline{L}^{(2)}(\omega_0)*\underset{\sim}{d}_0+\underset{\sim}{b}_0*\underline{L}^{(1)}(\omega_0)*\underset{\sim}{d}_1\\ &+\omega_1\underset{\sim}{b}_0*\frac{\partial\underline{L}^{(1)}}{\partial\omega_0}*\underset{\sim}{d}_0+\frac{\omega_1^2}{2}\underset{\sim}{b}_0*\frac{\partial^2\underline{L}^{(0)}}{\partial\omega_0^2}*\underset{\sim}{d}_0\\ &+\omega_1\underset{\sim}{b}_0*\frac{\partial\underline{L}^{(0)}}{\partial\omega_0}*\underset{\sim}{d}_1\ ,\end{aligned} \quad (69b)$$

$$D_\omega=\underset{\sim}{b}_0*\frac{\partial\underline{L}^{(0)}}{\partial\omega_0}*\underset{\sim}{d}_0\ . \quad (69c)$$

As expected in perturbation theory, the second-order frequency ω_2 involves both the zero- and first-order "wave functions" $\underset{\sim}{d}_0$ and $\underset{\sim}{d}_1$. In subsection C we shall see that the expansion of N_1 and N_2 about ω_0 in (69a) and (69b) is valid, in general, only if ω_0 is nonzero.

A. Zero-field NMR

The NMR corresponds to an oscillation of the magnetization, given by the average of the real spin density over the Fermi surface $\delta\mathbf{M}\sim\int d\Omega\,\mathbf{e}^+$. Since only the $l=0$ component of $\mathbf{e}^+$ contributes to this integral, the NMR involves only the $J=1$ component of the real spin density. In the dipole representation, which we adopt for the remainder of this section, the $J=1$ component of the real spin density couples to the $J=1$ component of the real triplet order parameter through Eq. (62). In the momentum representation, on the other hand, the NMR couples to $J=0$, 1, and 2 components of the real triplet order parameter.

Solving the zero-order equation $\underset{\sim}{d}_0=\underline{L}^{(0)}(\omega_0)*\underset{\sim}{d}_0$ for the $J=1$ mode, we find that the order parameter $d^+_{0j}(\hat{\mathbf{s}})=B^{(1,1)}_{ju}\hat{s}_u$ is described by the antisymmetric $l=1$ tensor $B^{(1,1)}_{uv}$. The spin density can be written as a superposition of $l=0$ and 2 tensors:

$$e^+_{0j}(\hat{\mathbf{s}})=E^{(1,0)}_j+\tfrac{3}{2}\hat{s}_jE^{(1,2)}_k\hat{s}_k-\tfrac{1}{2}E^{(1,2)}_j\ , \quad (70)$$

with components related to $B^{(1,1)}_{uj}$ by

$$E^{(1,0)}_k=-\frac{i\omega_0}{6\Delta^0}\lambda F^a_0\varepsilon_{kju}B^{(1,1)}_{uj}U^{-1}\ , \quad (71a)$$

$$E^{(1,2)}_k=\frac{i\omega_0}{6\Delta^0}\frac{\lambda F^a_2}{5}\varepsilon_{kju}B^{(1,1)}_{uj}U^{-1}\ , \quad (71b)$$

where

$$U=1+\tfrac{2}{3}\lambda F^a_0+\frac{1}{3}\frac{\lambda F^a_2}{5} \quad (72)$$

and λ is the lambda function evaluated at zero frequency (see Appendix D and Fig 7).

We use (70)–(72) to obtain the zero-field frequency ω_0 from the zero-order eigenvalue equation. The longitudinal $m_J=0$ solution is obtained by taking $B^{(1,1)}_{13}=B^{(1,1)}_{23}=0$ and $B^{(1,1)}_{12}\neq0$, with the simple result

$$\left[\frac{\omega_0}{\Delta^0}\right]^2_{m_J=0}=\frac{27}{4\lambda}\frac{g_D\overline{R^2}}{(V_1)^2}U\ , \quad (73)$$

independent of x_3^{-1} and $(V_1\overline{S^2})/(V_3\overline{R^2})$ (see Fig. 3). The transverse $m_J=\pm1$ solutions are obtained by taking $B^{(1,1)}_{12}=0$ and $B^{(1,1)}_{13},B^{(1,1)}_{23}\neq0$, with the results

$$\left[\frac{\omega_0}{\Delta^0}\right]^2_{m_J=\pm1}=0\ , \quad (74)$$

unaffected by the dipole interaction. Equation (73) agrees with Tewordt *et al.*[34] when $F^a_2=0$ and their spin-fluctuation parameter $\bar{I}$ is identified with $-F^a_0$. Using (39) we find that if $F^a_0=-1$ and all other material parameters are neglected, then at $T=0$ the zero-field gap distortion and longitudinal NMR frequency are related by

$$(\Delta^0_1)^2-(\Delta^0_3)^2=\tfrac{5}{2}(\omega_0)^2_{m_J=0}\ , \quad (75)$$

in agreement with Tewordt and Einzel.[21]

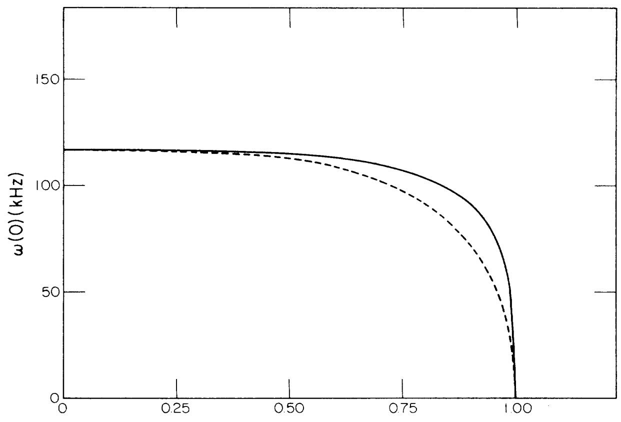

FIG. 3. The collisionless (solid line) and hydrodynamic (dashed line) zero-field longitudinal frequency at 0 bars calculated with the same parameters as Fig. 1.

The transpose relation $\underset{\sim}{b}_0=\underset{\sim}{b}_0 * \underline{L}^{(0)}(\bar{\omega}_0)$ is also easily solved. The $J=1$ order parameter is again given by an $l=1$ tensor $\bar{B}_{uv}^{(1,1)}=[B_{uv}^{(1,1)}]^*$ and the spin density by a superposition of $l=0$ and 2 tensors:

$$\bar{E}_k^0=-\bar{E}_k^{(1,2)}=-\frac{i\bar{\omega}_0}{6\Delta^0}\lambda V_1 \varepsilon_{kju}\bar{B}_{uj}^{(1,1)}U^{-1} \ . \tag{76}$$

We then verify that $\bar{\omega}_0=\omega_0$, as assumed in (68) and (69).

B. Field dependence of longitudinal NMR

To calculate the field dependence of the $m_J=0$ mode requires N_1, N_2, and D_ω, which are defined in (69). Only the zero-order "wave function" is needed to calculate D_ω to lowest order in g_D:

$$D_\omega=\frac{\omega_0}{6(\Delta^0)^2}\lambda V_1 \bar{B}_{ju}^{(1,1)}B_{ju}^{(1,1)}U^{-1} \ . \tag{77}$$

It is also easy to show that $N_1=0$, and hence, as expected, that $\omega_1=0$.

The second-order field dependence of the longitudinal NMR is more difficult to calculate because it involves both the first- and zero-order "wave functions." From the first-order eigenvalue equation, we find that the order parameter d_{1j}^+ is given by a superposition of $l=1$ and 3, $J=2$ components:

$$d_{1j}^+(\hat{s})=C_{ju}^{(2,1)}\hat{s}_u+\tfrac{5}{3}\hat{s}_j C_{uv}^{(2,3)}\hat{s}_u\hat{s}_v-\tfrac{2}{3}C_{ju}^{(2,3)}\hat{s}_u \ , \tag{78}$$

with tensors $C_{ju}^{(2,1)}$ and $C_{ju}^{(2,3)}$ written in terms of $B_{ju}^{(1,1)}$ in Appendix E. The spin density $e_{1j}^+=D_{j,uv}^2\hat{s}_u\hat{s}_v$ contains only a $J=2$ component, also given in Appendix E. Comparison with the work of Sauls and Serene[35] reveals that this first-order "wave function" corresponds to the $m_J=0$ magnetic substate of the real squashing modes, which have the resonant frequencies $(\frac{8}{5})^{1/2}\Delta^0$ in the absence of interaction effects. Similarly, the first-order "wave function" of the $m_J=0$ real squashing mode, calculated in FS, includes a contribution from the longitudinal NMR. In both cases the first-order "wave function" contains contributions from order-parameter modes with different symmetry than the zero-order "wave function." Since the frequencies of the longitudinal NMR and the real squashing modes are quite different, both sets of "off-diagonal" excitations are off resonance. We shall see that these excitations play a fundamental role in the collisionless dynamics.

Using these results, we find that the contributions to the second-order numerator N_2 are either zero order or first order in g_D. The zero-order part $N_2^{(0)}$ can be written

$$N_2^{(0)}=\tfrac{1}{60}V_1\left[\frac{\gamma H}{\Delta^0}\right]^2\bar{B}_{uv}^{(1,1)}B_{uv}^{(1,1)}(\lambda-1)\left[\frac{F_2^a}{5}\right]^2\left[1+\frac{F_2^a}{5}\right]\left[1+\frac{\lambda F_2^a}{5}\right]^{-1}(DU)^{-2}\left[1-\lambda-y\left[1+\frac{\lambda F_2^a}{5}\right]\right]^2 , \tag{79}$$

which vanishes at $T=0$ and $T=T_c$. To see the significance of this term, we parametrize ω^2 by

$$\omega^2=\omega_0^2\left[1-\Gamma\left[\frac{\gamma H}{2\Delta^0(0)}\right]^2\right]+\tfrac{1}{4}\beta(\gamma H)^2 , \tag{80}$$

where $\Delta^0(0)=1.764k_BT_c$ is the zero-temperature gap. Equations (77) and (79) imply that

$$\beta=\frac{4}{5}\frac{1-\lambda}{\lambda}\left[\frac{F_2^a}{5}\right]^2U^{-1}D^{-2}\frac{1+F_2^a/5}{1+\lambda F_2^a/5}\times\left[1-\lambda-y\left[1+\frac{\lambda F_2^a}{5}\right]\right]^2 , \tag{81}$$

which is positive for all temperatures. This term corresponds to a nondipolar oscillation of the longitudinal magnetization, which arises from the out-of-phase motion of the normal quasiparticles and the spin resonance. The torque exerted by the quasiparticle molecular field on the nuclear spins produces a correction to the longitudinal frequency, which vanishes at zero temperature. Formally, this term remains finite when the dipole interaction is turned off. However, since $\underline{L}^{(n)}(\omega)$ contains singular terms proportional to $1/\omega^m$ $(m\le n)$, the perturbative expansion about ω_0 breaks down unless $\omega_2/\omega_0 \ll 1$, or

$$\tfrac{1}{2}[\beta(\Delta^0(0))^2+\Gamma\omega_0^2]\left[\frac{\gamma H}{2\Delta^0(0)}\right]^2 \ll \omega_0^2 \ . \tag{82}$$

Therefore, in the formal limit $g_D\to 0$, the expression for β remains valid only for vanishingly small values of the field and of the temperature such that (82) holds. Since β is very flat for low temperatures and reaches a maximum at $T/T_c \simeq 0.95$ (see Fig. 4), its contribution to the NMR frequency in the collisionless regime will usually be small.

It is straightforward but tedious to evaluate the first-order part $N_2^{(1)}$, which requires the dipolar corrections to the functions $Y_{n/2}^0$ and λ, given in Appendix D, as well as the dipolar corrections to the spin density, given in Appendix E. Contributions to $N_2^{(1)}$ which involve dipolar corrections to $\Delta^2(\hat{\mathbf{p}})$ can be expressed in terms of the integrals K_1 and K_2, which were evaluated in Sec. II. We disregard contributions proportional to $g_D(\lambda-1)(F_2^a)^2$, which produce dipolar corrections to β and which involve g_D^2 corrections to the modified pairing interaction. The evaluation of $N_2^{(1)}$ is simplified if we ignore the explicit dipolar contributions contained in the modified pairing interaction (65), which produce negligible corrections of order V_1. The final expressions for $N_2^{(1)}$ and Γ, given in Appendix F, include cutoff-dependent corrections proportional to $g_D \overline{S^2}/(V_1 V_3)$. Since these terms do not significantly change Γ, the collisionless longitudinal frequency depends only weakly on the cutoff. At $T=0$, $N_2^{(0)}=0$ and $N_2^{(1)}$ simplifies, so that

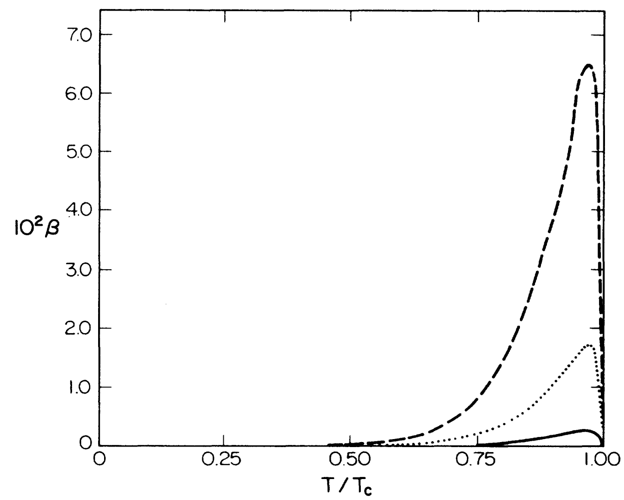

FIG. 4. The quadratic coefficient $\beta(T)$ for $F_0^a=-0.7$ and (1) $F_2^a=0.20$, $x_3^{-1}=-0.60$ (solid curve), (2) $F_2^a=0.50$, $x_3^{-1}=-0.66$ (dotted curve), and (3) $F_2^a=0.95$, $x_3^{-1}=-0.75$ (dashed curve). These three sets give the same real squashing-mode frequency in zero field (Ref. 34) if $T_c=1.04$ mK.

$$\frac{\omega^2(H)}{\omega^2(0)}=1-\frac{1}{2}\left[\frac{\gamma H}{\Delta^0}\right]^2\left[1+\frac{F_2^a}{5}\right]^2 D(0)^{-3}\left\{-\tfrac{1}{5}+\tfrac{1}{3}(1-x_3^{-1})\left[3+\frac{A_2^a}{5}-F_0^a\left[1-\frac{A_2^a}{5}\right]\right]+\tfrac{4}{21}x_3^{-1}D(0)\left[1-\frac{A_2^a}{5}\right]\right\}, \tag{83}$$

where the $(V_1\overline{S^2})/(V_3\overline{R^2})$ terms are neglected. Equation (83) indicates that at low temperatures a magnetic field suppresses the longitudinal NMR frequency.

Since the Fermi-liquid oscillation term β enhances the longitudinal frequency, the β and Γ contributions will exactly cancel at a sufficiently large temperature T^* (see Fig. 6). At this temperature ω_2 will vanish and the field dependence of the longitudinal frequency will become fourth order. Since T^* is a strong function of F_2^a, as seen in Fig. 6, lack of evidence for this field cancellation within the collisionless regime can be used to place stringent limits on $|F_2^a|$. For example, because this effect has not

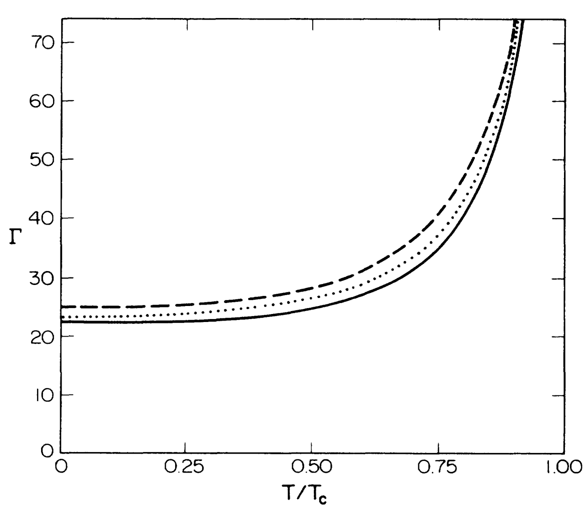

FIG. 5. The quadratic coefficient $\Gamma(T)$ for the same three cases as Fig. 4, with $(V_1\overline{S^2})/(V_3\overline{R^2})=0.0$.

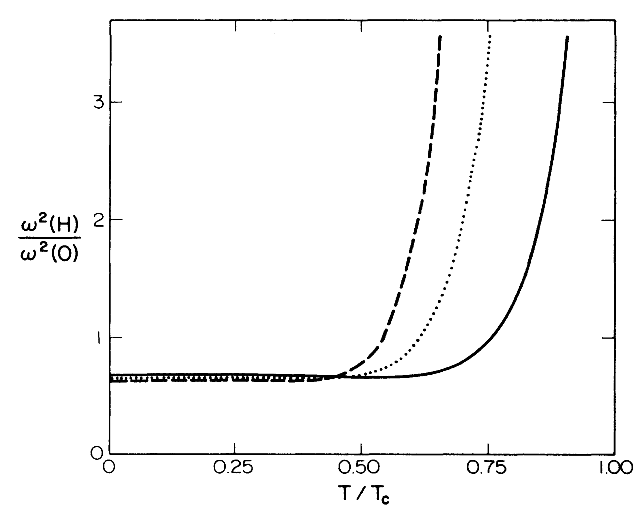

FIG. 6. The collisionless result for $\omega^2(H)/\omega^2(0)$ at 0 bar for the same three cases as Fig. 4 with $(V_1\overline{S^2})/(V_3\overline{R^2})=0.0$, $T_c=1.04$ mK, and $H=2.75$ kG (9 MHz).

been observed in the longitudinal NMR experiments of Candela *et al*,[4] we can conclude that $T^*/T_c \geq 0.6$ and $|F_2^a| \leq 0.8$ at 0 bar. The parameter dependence of β and Γ is shown in Figs. 4 and 5.

C. Field dependence of transverse NMR

The perturbation theory used to obtain the longitudinal NMR frequency fails for the transverse resonance because of the singular terms, proportional to H^n/ω^m ($m \leq n$), in the nth-order effective Hamiltonian $\underline{L}^{(n)}(\omega)$. Since the transverse frequency is proportional to H, all orders of the effective Hamiltonian will contribute to the numerators N_1 and N_2, invalidating (69a) and (69b). Unlike the longitudinal resonance, which is not affected by the singular terms provided that $\omega_2/\omega_0 \ll 1$, the transverse resonance cannot, in general, be treated perturbatively.

At least to second order in field, however, the coefficients of the singular terms vanish at zero temperature. Assuming this holds to all orders, we can solve for the first-order frequency ω_1 from the quadratic equation $N_2(\omega_1) = -\omega_2 D_\omega = 0$ using (69b) at $T=0$. To simplify this calculation, we work to zero order in g_D, in which limit spin-orbit symmetry requires that $\omega = \pm\gamma H$, unaltered by interaction effects.

The results for the first-order "wave function" at $T=0$ are given in Appendix E. We find that the order parameter d_{ij}^+ vanishes, but that the spin density $e_{1j}^+ = D_{j,uv}^2 \hat{s}_u \hat{s}_v$ contains $J=1$ and 2 components. Comparison with the work of Sauls and Serene[35] reveals that the $J=2$ first-order "wave function" of the $m_J = \pm 1$ NMR corresponds to the $m_J = \pm 1$ magnetic substates of the real squashing modes. Analogously, the first-order "wave function" of the $m_J = \pm 1$ real squashing modes, calculated in FS, contains contributions from the transverse NMR. Of course, the real squashing modes cannot be excited through transverse NMR because a sufficiently high field $\gamma H \simeq (\frac{8}{5})^{1/2}\Delta^0$ would destroy the B phase. Using the first-order "wave function," we find that $\omega_1 = m_J \gamma H$ to zero order in g_D, independent of Fermi-liquid corrections and pairing interactions, as required by spin conservation.

IV. HYDRODYNAMIC NMR

The original hydrodynamic theory[13] of NMR was designed for the high-temperature regime where $\omega\tau_{LT} \ll 1$. Leggett and Takagi[12] modified this theory in order to treat the low-temperature, collisionless regime where $\omega\tau_{LT} \gg 1$. They suggest that as long as $\omega \ll \Delta$ no internal degrees of freedom of the order parameter will be excited and a description of the spin dynamics based on equations of motion for the spin $\mathbf{S}$ and the $\hat{\mathbf{n}}$ vector should be valid. At $T=0$, when the normal component of the superfluid vanishes, the authors expected this hydrodynamic description to become exact. In order to treat finite temperatures, Leggett and Takagi introduced the effect of spin relaxation to first order in $1/\omega\tau_{LT}$.

In this section we obtain generalized results for the hydrodynamic longitudinal NMR frequency, including higher pairing interactions and higher moments of the quasiparticle renormalization factor. We do not consider the finite-temperature refinements of Leggett and Takagi, since we are primarily interested in comparing the collisionless and hydrodynamic results at $T=0$. We shall see that in zero field the hydrodynamic calculation does yield the correct zero-temperature frequency. However, the field dependence of the longitudinal NMR in the collisionless and hydrodynamic limits is different, even at $T=0$.

Our starting point is the general expression of Leggett[13] for the longitudinal NMR frequency squared:

$$\omega^2 = 6\gamma^2\chi^{-1}N(0)\frac{g_D}{(V_1)^2}\int\int\frac{d\Omega'}{4\pi}\frac{d\Omega}{4\pi}R^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\{\hat{\mathbf{q}}\cdot\Delta(\hat{\mathbf{p}})\,\hat{\mathbf{q}}\cdot\Delta(\hat{\mathbf{p}}') - \hat{\mathbf{q}}\cdot\Delta(\hat{\mathbf{p}})\,\hat{q}_3\Delta_3(\hat{\mathbf{p}}') - [\hat{\mathbf{q}}\times\Delta(\hat{\mathbf{p}})]_3[\hat{\mathbf{q}}\times\Delta(\hat{\mathbf{p}}')]_3\}\ , \tag{84}$$

where $\chi(H) = \partial M/\partial H$ is the thermodynamic susceptibility. Cutoff-dependent corrections of order V_1 are neglected in (84). From this expression it is clear that dipolar corrections to the equilibrium gap and to the effective field do not contribute to the longitudinal NMR frequency in the hydrodynamic regime.

In zero field, Eq. (84) can be readily solved to yield the relation first found[13] by Leggett:

$$\left[\frac{\omega_0}{\Delta^0}\right]^2 = \frac{27}{4}\frac{g_D\overline{R^2}}{(V_1)^2}D\ . \tag{85}$$

At $T=0$, $D(0)=U(0)$ and $\lambda=1$, so the zero-field hydrodynamic and collisionless NMR frequencies agree. Because of the λ function in the denominator of (73), the collisionless frequency is slightly larger than the hydrodynamic frequency at finite temperature, as shown in Fig. 3. At $T=T_c$, of course, both frequencies vanish.

In a finite field, the longitudinal frequency contains contributions from the $l=1$ and 3 gaps proportional to $\overline{R^2}$ and $\overline{S^2}$. Using the results of FS for the $l=3$ gap and the results of Sec. II for the field dependence of the Leggett angle, (52), we find

$$\omega^2 = \tfrac{3}{10}\gamma^2\chi^{-1}N(0)\frac{g_D\overline{R^2}}{(V_1)^2}\left[8\Delta_1^2 - \tfrac{1}{2}\Delta_3^2 - \tfrac{1}{28}\frac{\overline{S^2}}{\overline{R^2}}(\Delta_1^0)^2 Y_{3/2}x_3^{-1}D^{-2}\left[1+\frac{F_2^a}{5}\right]^2(\gamma H)^2(6-5A)\right]\ . \tag{86}$$

When $V_3 = \overline{S^2} = 0$, this relation agrees with Schopohl[10] and, at $T=0$, with Pleiner and Brand.[11] Because the quasiparticles respond adiabatically to the spin resonance, no analogue of the Fermi-liquid oscillation term occurs in the hydrodynamic limit. Comparing (86) with the collisionless result, we find the zero-temperature equality

$$\left.\frac{\omega^2(H)}{\omega^2(0)}\right|_c = \left.\frac{\omega^2(H)}{\omega^2(0)}\right|_h \frac{\sin^2\theta_L(0)}{\sin^2\theta_L(H)} - \tfrac{2}{21}U(0)^{-2}\left[1+\frac{F_2^a}{5}\right]x_3^{-1}\left[\frac{\gamma H}{\Delta^0}\right]^2 \times \left[1-\frac{1}{12}\frac{\overline{S^2}}{\overline{R^2}}\left[(1+\tfrac{32}{63}x_3^{-1})(1-\tfrac{4}{9}x_3^{-1})^{-1}+\tfrac{13}{5}-\tfrac{3}{5}x_3^{-1}\right]\right], \tag{87}$$

to lowest order in V_1. When $x_3^{-1}=0$, Eq. (87) reduces to an intriguing relation between the collisionless and hydrodynamic frequencies at $T=0$.

It should have been expected that the second-order field dependence of the longitudinal NMR frequency is different in the two regimes. First order in field, the longitudinal NMR couples to the $m_J=0$ magnetic substate of the real squashing modes, which have the noninteracting frequencies $(\frac{8}{5})^{1/2}\Delta^0$. Even at zero temperature the off-resonance real squashing mode

$$d_{1j}^+ \sim [\omega_0 H/(\Delta^0)^2]d_{0j}^+$$

remains finite. The hydrodynamic theory assumes that the order-parameter modes with resonant frequencies of order Δ^0 are not excited by the NMR with the much smaller frequency ω_0. However, in the collisionless theory, the small "off-diagonal" excitation d_{ij}^+ contributes a large correction to the field dependence of the longitudinal NMR. Since it contains both $J=2$ and 1 components, the collisionless NMR in a finite field can no longer be simply described by oscillations of the spin $\mathbf{S}$ and the $\hat{\mathbf{n}}$ vector. Only in zero field and at zero temperature, when internal modes are unexcited and quasiparticles are absent, do the collisionless and hydrodynamic results agree.

The "off-diagonal" modes of the spin density and order parameter are essential to the collisionless dynamics. We have already seen that the first-order, $J=2$ spin density of the transverse NMR is finite at $T=0$. If these spin-density modes were neglected, the transverse frequency would differ from the Larmor frequency γH, and spin-orbit symmetry would be violated. The symmetries of ^{3}He-B are preserved in the collisionless regime only by the complex interaction of internal modes and external forces. The hydrodynamic description of NMR in terms of macroscopic variables that characterize the equilibrium order parameter is not appropriate in this limit.

ACKNOWLEDGMENTS

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APPENDIX A

This appendix reviews the tensor notation. A spin scalar $T(\hat{\mathbf{p}})$ with orbital angular momentum l can be written

$$T(\hat{\mathbf{p}})=t^l_{u_1u_2\cdots u_l}\hat{p}_{u_1}\hat{p}_{u_2}\cdots\hat{p}_{u_l}\,. \tag{A1}$$

The tensor $t^l_{u_1u_2\cdots u_l}$ is by construction traceless and symmetric in the indices $\{u_1,u_2,\ldots,u_l\}$. Since $T(\hat{\mathbf{p}})$ is a spin scalar, the total angular momentum J equals l. A spin vector $\mathbf{V}(\hat{\mathbf{p}})$ with orbital angular momentum l can be written

$$V_i(\hat{\mathbf{p}})=v^l_{i,u_1u_2\cdots u_l}\hat{p}_{u_1}\hat{p}_{u_2}\cdots\hat{p}_{u_l}\,. \tag{A2}$$

The tensor $v^l_{i,u_1u_2\cdots u_l}$ is also symmetric and traceless in the indices $\{u_1,u_2,\ldots,u_l\}$. Since $\mathbf{V}(\hat{\mathbf{p}})$ is a spin vector, J can equal $l-1$, l, and $l+1$.

Because $\hat{\mathbf{p}}\cdot\mathbf{V}(\hat{\mathbf{p}})$ is a spin scalar that contains components with orbital angular momentum $l\pm1$, we can write

$$\hat{\mathbf{p}}\cdot\mathbf{V}(\hat{\mathbf{p}})=v^{(l+1,l)}_{u_1u_2\cdots u_{l+1}}\hat{p}_{u_1}\hat{p}_{u_2}\cdots\hat{p}_{u_{l+1}} + v^{(l-1,l)}_{u_1u_2\cdots u_{l-1}}\hat{p}_{u_1}\hat{p}_{u_2}\cdots\hat{p}_{u_{l-1}}\,, \tag{A3}$$

where $v^{(l\pm1,l)}_{u_1u_2\cdots u_{l\pm1}}$ are symmetric and traceless in the $l\pm1$ indices. These tensors can be defined in terms of $v^l_{i,u_1u_2\cdots u_l}$ by using (A2). The $J=l+1$ tensor can be constructed by symmetrizing the spin tensor and removing the trace:

$$v^{(l+1,l)}_{u_1u_2\cdots u_{l+1}}=\frac{1}{l+1}(v^l_{u_1,u_2\cdots u_{l+1}}+\cdots+v^l_{u_{l+1},u_1\cdots u_l})-\frac{2}{(l+1)(2l+1)}(\delta_{u_1u_2}v^l_{j,ju_3\cdots u_{l+1}}+\cdots+\delta_{u_lu_{l+1}}v^l_{j,ju_1\cdots u_{l-1}}). \tag{A4}$$

The $J=l-1$ tensor is obtained by summing over two indices of the spin vector:

$$v_{u_1u_2\cdots u_{l-1}}^{(l-1,l)} = \frac{l}{2l+1} v_{j,ju_1\cdots u_{l-1}}^{l} . \tag{A5}$$

The $J=l$ tensor is then constructed by subtracting off the $J=l\pm1$ contributions from the spin vector.

The simplest nontrivial example is the $l=1$ spin vector $D_i(\hat{\mathbf{p}})=d_{i,u}^{1}\hat{p}_u$, which can be decomposed into $J=0$, 1, and 2 components:

$$d_{i,u}^{1} = d_{iu}^{(2,1)} + d_{i,u}^{(1,1)} + \delta_{iu} d^{(0,1)} , \tag{A6}$$

where

$$d_{iu}^{(2,1)} = \tfrac{1}{2}(d_{i,u}^{1}+d_{u,i}^{1}) - \tfrac{1}{3}\delta_{iu} d_{k,k}^{1} , \tag{A7a}$$

$$d_{i,u}^{(1,1)} = \tfrac{1}{2}(d_{i,u}^{1}-d_{u,i}^{1}) , \tag{A7b}$$

$$d^{(0,1)} = \tfrac{1}{3} d_{k,k}^{1} . \tag{A7c}$$

Using these relations, we see that the B-phase gap $\Delta_i^0(\hat{\mathbf{p}})=\Delta^0 R_{ij}^0 \hat{p}_j$ contains $J=0$, 1, and 2 components.

Tensors in the "dipole representation" take $\hat{s}_j = R_{jk}^0 \hat{p}_k$ as the unit vector. The tensor associated with a spin vector (or scalar) is different in the two representations:

$$\begin{aligned} V_i &= v_{i,u_1u_2\cdots u_l}^{l} \hat{p}_{u_1}\hat{p}_{u_2}\cdots \hat{p}_{u_l} \\ &= w_{i,u_1u_2\cdots u_l}^{l} \hat{s}_{u_1}\hat{s}_{u_2}\cdots \hat{s}_{u_l} , \end{aligned} \tag{A8}$$

where $v_{i,u_1u_2\cdots u_l}^{l}$ and $w_{i,u_1u_2\cdots u_l}^{l}$ are tensors in the momentum and dipole representations related by

$$v_{i,u_1u_2\cdots u_l}^{l} = w_{i,a_1a_2\cdots a_l}^{l} R_{a_1u_1}^0 R_{a_2u_2}^0 \cdots R_{a_lu_l}^0 , \tag{A9}$$

which mixes up different J components. For example, the gap in the dipole representation, given by $\Delta_i^0(\hat{\mathbf{s}})=\Delta^0\hat{s}_i$, has only a $J=0$ component.

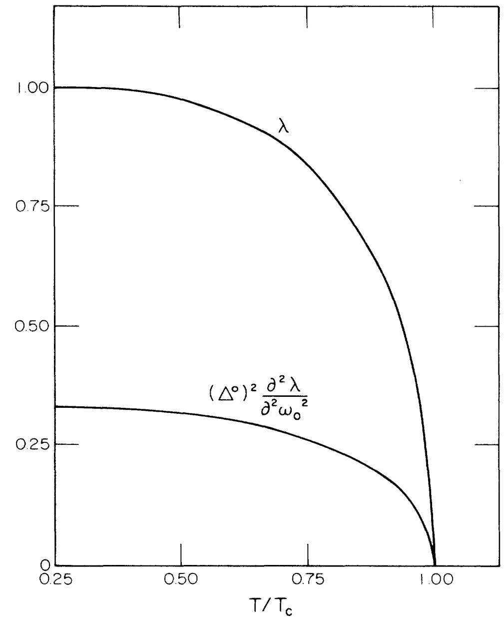

FIG. 7. The λ function and its frequency derivative evaluated at $g_D=0$ (see Appendix D).

APPENDIX B

In this appendix we collect the equilibrium results in zero field. The calculations of Sec. II A yield the $l=1$ gap distortion, given by (38), and the $l=3$ gap tensors given below:

$$d_{11}^{(2,3)} = d_{22}^{(2,3)} = -\tfrac{1}{2} d_{33}^{(2,3)} = \frac{9}{56}\frac{g_D\overline{R^2}}{(V_1)^2}\Delta_3^0 x_3^{-1} + \frac{3}{16}\frac{g_D\overline{S^2}}{V_1V_3}\Delta_3^0 x_3^{-1}[1-\tfrac{121}{441}(\Delta_1^0)^2 Y_{3/2}^0 x_3^{-1}][1-\tfrac{4}{9}(\Delta_1^0)^2 Y_{3/2}^0 x_3^{-1}]^{-1} , \tag{B1}$$

$$\begin{aligned} d_{1,222}^{(3,3)} &= -d_{2,111}^{(3,3)} = -\tfrac{3}{4} d_{1,233}^{(3,3)} = \tfrac{3}{4} d_{2,133}^{(3,3)} = -3 d_{2,122}^{(3,3)} = 3 d_{1,211}^{(3,3)} \\ &= \tfrac{9}{16}(\sin\theta_L^0)\frac{g_D\overline{R^2}}{(V_1)^2}\Delta_1^0 x_3^{-1} - \tfrac{5}{112}(\sin\theta_L^0)\frac{g_D\overline{S^2}}{V_1V_3}(\Delta_1^0)^3 Y_{3/2}^0 x_3^{-2}[1-\tfrac{4}{9}(\Delta_1^0)^2 Y_{3/2}^0 x_3^{-1}]^{-1} , \end{aligned} \tag{B2}$$

$$\begin{aligned} d_{1133}^{(4,3)} &= d_{2233}^{(4,3)} = -\tfrac{1}{2} d_{3333}^{(4,3)} = -\tfrac{4}{3} d_{1111}^{(4,3)} = -\tfrac{4}{3} d_{2222}^{(4,3)} = -4 d_{1122}^{(4,3)} \\ &= \frac{45}{112}\frac{g_D\overline{R^2}}{(V_1)^2}\Delta_3^0 x_3^{-1} + \frac{65}{2352}\frac{g_D\overline{S^2}}{V_1V_3} x_3^{-2}(\Delta_1^0)^2\Delta_3^0 Y_{3/2}^0[1-\tfrac{4}{9}(\Delta_1^0)^2 Y_{3/2}^0 x_3^{-1}]^{-1} , \end{aligned} \tag{B3}$$

while all other components vanish. These results for the $l=3$ gap tensors are also valid in the rotated planar phase, where $\Delta_3^0=0$ and $\sin\theta_L^0=1$. Therefore the $l=3$ gap of the planar phase contains only $J=3$ components.

In Sec. III we require the $l=3$, B-phase gap tensors in the dipole representation, which are

$$e_{11}^{(2,3)} = e_{22}^{(2,3)} = -\tfrac{1}{2} e_{33}^{(2,3)} = \frac{9}{16}\frac{g_D\overline{R^2}}{(V_1)^2}\Delta_3^0 x_3^{-1} + \frac{3}{56}\frac{g_D\overline{S^2}}{(V_1V_3)} x_3^{-1}\Delta_3^0 , \tag{B4}$$

$$e_{1,233}^{(3,3)} = -e_{2,133}^{(3,3)} = -\tfrac{4}{3} e_{1,222}^{(3,3)} = \tfrac{4}{3} e_{2,111}^{(3,3)} = -4 e_{1,211}^{(3,3)} = 4 e_{2,122}^{(3,3)} = \frac{1}{4}\frac{g_D\overline{S^2}}{V_1V_3} x_3^{-1}\Delta_3^0\sin\theta_L^0 , \tag{B5}$$

$$e_{1133}^{(4,3)} = e_{2233}^{(4,3)} = -\tfrac{1}{2} e_{3333}^{(4,3)} = -\tfrac{4}{3} e_{1111}^{(4,3)} = -\tfrac{4}{3} e_{2222}^{(4,3)} = -4 e_{1122}^{(4,3)}$$

$$=\frac{15}{112}\frac{g_D\overline{S^2}}{V_1V_3}x_3^{-1}\Delta_3^0[1-\tfrac{4}{9}(\Delta_1^0)^2Y_{3/2}^0x_3^{-1}]^{-1}\ , \tag{B6}$$

correct to first order in g_D.

Using the equilibrium gap tensors to calculate the integrals J_i defined by (30), we obtain the following expression for $N(\hat{\mathbf{p}})$, valid to first order in g_D:

$$N(\hat{\mathbf{p}})=N^0-\frac{(N^0)^3}{\pi^2}(\Delta_1^0)^2(\alpha_g+\beta_g\hat{p}_3^2+\gamma_g\hat{p}_3^4)\ , \tag{B7}$$

where

$$\alpha_g=\frac{9}{16}\frac{g_D\overline{R^2}}{(V_1)^2}x_3^{-1}-\frac{3}{64}\frac{g_D\overline{S^2}}{V_1V_3}x_3^{-1}[1+\tfrac{32}{63}(\Delta_1^0)^2Y_{3/2}^0x_3^{-1}][1-\tfrac{4}{9}(\Delta_1^0)^2Y_{3/2}^0x_3^{-1}]^{-1}\ ,$$

$$\beta_g=-\frac{45}{16}\frac{g_D\overline{R^2}}{(V_1)^2(\Delta_1^0)^2Y_{3/2}}+\frac{225}{224}\frac{g_D\overline{S^2}}{V_1V_3}x_3^{-1}[1-\tfrac{4}{9}(\Delta_1^0)^2Y_{3/2}^0x_3^{-1}]^{-1}\ , \tag{B8}$$

$$\gamma_g=-\frac{75}{64}\frac{g_D\overline{S^2}}{V_1V_3}x_3^{-1}[1-\tfrac{4}{9}(\Delta_1^0)^2Y_{3/2}^0x_3^{-1}]^{-1}\ .$$

We also note that

$$T\sum_n^{|\varepsilon_n|<\varepsilon_c}N^0=\frac{1}{V_1}-\frac{21}{16}\frac{g_D\overline{R^2}}{(V_1)^2}[1-\tfrac{3}{7}(\Delta_1^0)^2Y_{3/2}^0x_3^{-1}]+\frac{3}{56}\frac{g_D\overline{S^2}}{V_1V_3}x_3^{-1}(\Delta_1^0)^2Y_{3/2}^0\ , \tag{B9}$$

where the cutoff is introduced to regulate the otherwise logarithmically divergent integral.

APPENDIX C

In this appendix we calculate the dipolar corrections to the effective field. The mean-field relation for $\mathbf{h}(\hat{\mathbf{p}})$, including the contribution of the dipole forces to scattering in the particle-hole channel, can be written

$$-\mathbf{h}\cdot\sigma_{\alpha\gamma}=-\frac{\gamma\mathbf{H}\cdot\sigma_{\alpha\gamma}}{2(1+F_0^a)}+\tfrac{1}{2}T\sum_n\int\frac{d\Omega'}{4\pi}\{[A(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')]_{\alpha\beta,\gamma\rho}+\tfrac{1}{2}g_DQ^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')(\delta_{\mu\nu}-3\hat{q}_\mu\hat{q}_\nu)\sigma_{\alpha\rho}^\mu\sigma_{\beta\gamma}^\nu\}[g(\hat{\mathbf{p}}')]_{\rho\beta}\ , \tag{C1}$$

where $Q^2(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ is a quasiparticle renormalization factor. The contribution of the dipole interaction in the particle-hole channel is well defined and independent of the cutoff ε_c, since the sum over ε_n in (C1) is convergent. Thus the dipolar corrections generated by this term are $(V_1)^2$ times smaller than the gap-distortion corrections contained in $g(\hat{\mathbf{p}})$, which are of order $g_D/(V_1)^2$. To first order in V_1, therefore, we are justified in ignoring the dipole interaction in the particle-hole channel.

The dipolar corrections to $\mathbf{h}$ contained in $g(\hat{\mathbf{p}})$ can be calculated in a straightforward manner. The parameters Θ_1, Θ_2, and Θ_3 introduced in (54) are given by the set of coupled equations:

$$\Theta_1=\frac{1}{1+F_0^a}+(\Theta_1+\Theta_2)\frac{A_0^a}{3}[(\Delta_1^0)^2Y_{3/2}^0-3(\Delta_1^0)^4Y_{5/2}^0(\alpha_g+\tfrac{3}{5}\beta_g+\tfrac{3}{7}\gamma_g)$$
$$+2(\Delta_3^0-\Delta_1^0)\Delta_1^0Y_{3/2}^0(1-\tfrac{1}{5}A)+\tfrac{2}{5}e_{33}^{(2,3)}\Delta_1^0Y_{3/2}^0A]-\tfrac{1}{3}\Theta_2-\tfrac{1}{3}(1-\tfrac{3}{5}A)\Theta_3\ , \tag{C2}$$

$$\Theta_2=(\Theta_1+\Theta_2)\frac{A_2^a}{5}[(\Delta_1^0)^2Y_{3/2}^0-3(\Delta_1^0)^4Y_{5/2}^0(\alpha_g+\tfrac{3}{7}\beta_g+\tfrac{5}{21}\gamma_g)$$
$$+\frac{\Delta_3^0-\Delta_1^0}{\Delta_1^0}(1-\tfrac{4}{7}A)+\tfrac{1}{7}e_{33}^{(2,3)}\Delta_1^0Y_{3/2}^0(1+4A)+\tfrac{6}{7}e_{2233}^{(4,3)}\Delta_1^0Y_{3/2}^0(2-\tfrac{5}{9}A)]+\tfrac{3}{7}\Theta_3A\ , \tag{C3}$$

$$\Theta_3=(1-\tfrac{3}{7}A)^{-1}(\Theta_1+\Theta_2)\frac{A_2^a}{5}[-3(\Delta_1^0)^4Y_{5/2}^0(\tfrac{3}{7}\beta_g+\tfrac{10}{21}\gamma_g)+(\Delta_3^0-\Delta_1^0)\Delta_1^0Y_{3/2}^0(1+\tfrac{3}{7}A)$$
$$+\tfrac{2}{7}e_{33}^{(2,3)}\Delta_1^0Y_{3/2}^0(4-\tfrac{3}{2}A)-\tfrac{15}{7}e_{2233}^{(4,3)}\Delta_1^0Y_{3/2}^0(2-\tfrac{5}{9}A)]\ . \tag{C4}$$

The tensors $e_{33}^{(2,3)}$ and $e_{2233}^{(4,3)}$ describe the $l=3$ zero-field gaps in the dipole representation and α_g, β_g, and γ_g are the coefficients in the expansion of $N(\hat{\mathbf{p}})$ (see Appendix B). In the limit $g_D \to 0$, we recover $\Theta_1 = D^{-1}(1+yF_2^a/5)$, $\Theta_2 = D^{-1}(1-y)F_2^a/5$, and $\Theta_3=0$.

APPENDIX D

The results of Appendix B can be used to expand the functions $Y_{m/2}$ and λ about $g_D=0$. For the generalized Yoshida functions, we find $(m=3,5,\ldots)$

$$Y_{m/2} \equiv T\sum_n \frac{[N(\hat{\mathbf{p}})]^m}{\pi^{m-1}} = Y_{m/2}^0 - mY_{(m+2)/2}^0(\alpha_g + \beta_g \hat{p}_3^2 + \gamma_g \hat{p}_3^4) , \tag{D1}$$

where $Y_{m/2}^0 = T\sum_n (N^0)^m/\pi^{m-1}$. For the λ function we find

$$\lambda(\omega_0) = \int_1^\infty dx \frac{\tanh[\beta\,|\,\Delta^0(\hat{\mathbf{p}})\,|\,x/2]}{x^2-[\omega_0/2\,|\,\Delta^0(\hat{\mathbf{p}})\,|\,]^2}\frac{1}{(x^2-1)^{1/2}} = \lambda_0 + \frac{\omega_0^2}{2}\frac{\partial^2\lambda}{\partial\omega_0^2} + (\alpha_g + \beta_g\hat{p}_3^2 + \gamma_g\hat{p}_3^4)(\lambda_0 - 1 + y) , \tag{D2}$$

where λ_0 is the λ function evaluated with $g_D=0$. Note that dipolar corrections enter λ through both ω_0 and $|\,\Delta^0(\hat{\mathbf{p}})\,|$. In Fig. 7 we plot λ_0 and $(\Delta^0)^2(\partial^2\lambda/\partial\omega_0^2)$ versus T/T_c.

APPENDIX E

The components of the longitudinal order parameter $d_{1j}^{+} = C_{j,u}^1 \hat{s}_u + C_{j,uvw}^3 \hat{s}_u \hat{s}_v \hat{s}_w$ are given by

$$C_{uv}^{(2,1)} = \frac{3i\omega_0\gamma H}{32\lambda(\Delta^0)^2}\frac{1+F_2^a/5}{1+\lambda F_2^a/5}(DU)^{-1}(1-\lambda x_3^{-1})\left[1+\lambda-y+2\lambda\frac{\lambda F_2^a}{5}+y\left[\frac{\lambda F_2^a}{5}\right]^2\right]A_{uv} , \tag{E1}$$

$$C_{uv}^{(2,3)} = \frac{3i\omega_0\gamma H}{32\lambda(\Delta^0)^2}\frac{1+F_2^a/5}{1+\lambda F_2^a/5}(DU)^{-1}\lambda x_3^{-1}\left[1+\lambda-y+2\lambda\frac{\lambda F_2^a}{5}+y\left[\frac{\lambda F_2^a}{5}\right]^2\right]A_{uv} , \tag{E2}$$

where

$$A_{uv} = (\delta_{u3}\varepsilon_{vab}B_{ba}^{(1,1)} + \delta_{v3}\varepsilon_{uab}B_{ba}^{(1,1)} - \tfrac{2}{3}\delta_{uv}\varepsilon_{3ab}B_{ba}^{(1,1)}) . \tag{E3}$$

The longitudinal spin density $e_{1j}^{+} = D_{j,uv}^2 \hat{s}_u \hat{s}_v$ contains only the $J=2$ components $\check{D}_{11} = \check{D}_{22} = -\tfrac{1}{2}\check{D}_{33}$, where

$$\check{D}_{uv} = \tfrac{1}{2}(\varepsilon_{ujk}D_{k,jv}^{(2,2)} + \varepsilon_{vjk}D_{k,ju}^{(2,2)}) \tag{E4}$$

and, through order g_D,

$$\begin{aligned}
\check{D}_{33} = \varepsilon_{3uv}B_{vu}^{(1,1)}\frac{F_2^a}{5}\frac{\gamma H}{2\Delta_1^0}\left[1+\frac{\lambda_0 F_2^a}{5}\right]^{-1}\Bigg\{&-\frac{3}{16}\left[\frac{\omega_0}{\Delta^0}\right]^2 U^{-1}(\Theta_1+\Theta_2)\left[1+\frac{\lambda_0F_2^a}{5}\right]^{-1} \\
&\times(1-\tfrac{5}{3}\lambda_0 x_3^{-1})\left[\lambda_0+1-y+2\lambda_0\frac{\lambda_0F_2^a}{5}+y\left[\frac{\lambda_0F_2^a}{5}\right]^2\right]-\tfrac{1}{2}(\Theta_1+\Theta_2)(1-y) \\
&+\tfrac{1}{10}(1-y)(\Theta_1+\Theta_2)F_2^a\left[1+\frac{\lambda_0F_2^a}{5}\right]^{-1}\left[\frac{\omega_0^2}{2}\frac{\partial^2\lambda}{\partial\omega_0^2}+(\alpha_g+\tfrac{3}{7}\beta_g+\tfrac{5}{21}\gamma_g)(\lambda_0-1+y)\right] \\
&-\frac{\omega_0^2}{8(\Delta^0)^2}(\Theta_1+\Theta_2)(\lambda_0+1-y)U^{-1}-\tfrac{3}{14}(1-y)\Theta_3-\tfrac{1}{2}(\Theta_1+\tfrac{3}{7}\Theta_2)(1-y)\frac{\Delta_3^0-\Delta_1^0}{\Delta_1^0} \\
&+\tfrac{3}{2}(\Theta_1+\Theta_2)(\Delta_1^0)^4Y_{5/2}^0(\alpha_g+\tfrac{3}{7}\beta_g+\tfrac{5}{21}\gamma_g)+\tfrac{3}{14}\Delta_1^0Y_{3/2}^0e_{33}^{(2,3)}(\Theta_1-\tfrac{1}{3}\Theta_2) \\
&-\tfrac{3}{7}\Delta_1^0Y_{3/2}^0e_{2233}^{(4,3)}(\Theta_1+\tfrac{4}{9}\Theta_2)+U^{-1}\left[\tfrac{1}{6}\Theta_2\lambda_0F_0^a+\tfrac{1}{4}(\Theta_1+\tfrac{1}{3}\Theta_2)\frac{\lambda_0F_2^a}{5}\right] \\
&\times\Bigg[2(1-\lambda_0)-\omega_0^2\frac{\partial^2\lambda}{\partial\omega_0^2}+2(\alpha_g+\tfrac{3}{7}\beta_g+\tfrac{5}{21}\gamma_g)(\lambda_0+1-y)-\tfrac{12}{7}\lambda_0\frac{\Delta_3^0-\Delta_1^0}{\Delta_1^0} \\
&-\frac{4}{7}\frac{\lambda_0}{\Delta^0}e_{33}^{(2,3)}-\frac{32}{21}\frac{\lambda_0}{\Delta^0}e_{2233}^{(4,3)}\Bigg]\Bigg\} .
\end{aligned} \tag{E5}$$

where $e^{(2,3)}_{33}$ and $e^{(4,3)}_{2233}$ are the zero-field gap tensors given in Appendix B.

At $T=0$ the transverse spin density $e^{+}_{1j}=D^{1}_{j,uv}\hat{s}_u\hat{s}_v$ contains the $J=1$ and 2 components,

$$D_i^{(1,0)}=\tfrac{1}{3}F_0^a D(0)^{-1}\frac{\gamma H}{\Delta^0}B_{i3}^{(1,1)} , \tag{E6}$$

$$D_i^{(1,2)}=\frac{1}{6}\frac{F_2^a}{5}D(0)^{-1}\frac{\gamma H}{\Delta^0}B_{i3}^{(1,1)} , \tag{E7}$$

$$\check{D}_{kj}=-\frac{3}{16}\frac{F_2^a}{5}D(0)^{-1}\frac{\gamma H}{\Delta^0}(\delta_{k3}\varepsilon_{jab}B_{ba}^{(1,1)}+\delta_{j3}\varepsilon_{kab}B_{ba}^{(1,1)}) , \tag{E8}$$

while the transverse order parameter d_{ij}^{+} vanishes.

APPENDIX F

The result for $N_2^{(1)}$ is given by

$$\begin{aligned}
N_2^{(1)}=V_1&\left[\frac{\gamma H}{2\Delta^0}\right]^2\bar{B}_{uv}^{(1,1)}B_{uv}^{(1,1)}\frac{g_D\overline{R^2}}{(V_1)^2}\left[1+\frac{F_2^a}{5}\right]^2 D^{-2}\\
&\times\left\{\frac{9}{40}\frac{(\Delta^0)^2}{\lambda}\frac{\partial^2\lambda}{\partial\omega_0^2}\left[\frac{yF_2^a}{5}\right]^2 U^{-1}\right.\\
&+\frac{27}{16\lambda}U^{-1}\left[-\tfrac{1}{6}A(\lambda+1-y)(1-\lambda x_3^{-1})+\frac{1}{10}\left[\lambda+1-y-\lambda\frac{(\Delta_1^0)^2Y_{5/2}}{Y_{3/2}}+\frac{1}{\lambda}(\lambda+1-y)^2(1-\tfrac{5}{3}\lambda x_3^{-1})\right]\right.\\
&\qquad-\tfrac{2}{9}\lambda^2F_0^a(1-A)[1-(1-y)x_3^{-1}]\\
&\qquad+\frac{\lambda F_2^a}{5}\left[\tfrac{5}{9}\lambda+\tfrac{7}{30}(1-y)-\frac{(1-y)^2}{10\lambda}-\tfrac{19}{45}A-\tfrac{2}{15}(1-y)A\right.\\
&\qquad\qquad\left.-\tfrac{1}{18}\lambda x_3^{-1}[7(1-y)+3\lambda](1-A)-\tfrac{1}{5}\lambda\frac{(\Delta_1^0)^2Y_{5/2}}{Y_{3/2}}-\tfrac{1}{6}[\lambda^2-(1-y)^2)x_3^{-1}\right]\\
&\qquad\left.+\tfrac{1}{10}y\left[\frac{F_2^a}{5}\right]^2(1-A)-\tfrac{1}{6}\left[\frac{yF_2^a}{5}\right]^2x_3^{-1}\right]+\frac{y^3}{1-y}U^{-2}\left[\frac{F_2^a}{5}\right]^2\\
&\times\left[-\tfrac{9}{112}[1-\tfrac{7}{15}(1-y)x_3^{-1}]+\tfrac{1}{210}(1-y)x_3^{-1}\frac{V_1\overline{S^2}}{V_3\overline{R^2}}[1-\tfrac{4}{9}(\Delta_1^0)^2Y_{3/2}x_3^{-1}]^{-1}[\tfrac{41}{28}-\tfrac{1}{3}(1-y)x_3^{-1}]\right]\\
&+x_3^{-1}\left[\tfrac{3}{14}(1-y)(1-A)+\frac{1}{56}\frac{V_1\overline{S^2}}{V_3\overline{R^2}}(1-y)[1-\tfrac{4}{9}(\Delta_1^0)Y_{3/2}x_3^{-1}]^{-1}[\tfrac{1}{5}A-\tfrac{20}{21}(1-y)x_3^{-1}+\tfrac{272}{315}(1-y)Ax_3^{-1}]\right.\\
&\qquad\left.\left.\left.-\frac{1}{56}\frac{V_1\overline{S^2}}{V_3\overline{R^2}}\lambda\left[1-\tfrac{3}{5}A-\frac{3}{5}\frac{(\Delta_1^0)^2Y_{5/2}}{Y_{3/2}}-\tfrac{3}{5}(1-y)x_3^{-1}(1-A)\right]\right]\right]\right\} ,
\end{aligned} \tag{F1}$$

valid to lowest order in V_1, as explained in the text. The parameter Γ introduced in (80) is defined by

$$\Gamma=\frac{8N_2^{(1)}}{\omega_0D_\omega}\left[\frac{\Delta^0(0)}{\gamma H}\right]^2 , \tag{F2}$$

where D_ω is given by (77).

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