Collective excitation spectrum of the $\nu=\frac{2}{5}$ fractionally quantized Hall state

W. P. Su and Y. K. Wu*
Department of Physics, University of Houston, Houston, Texas 77004

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We diagonalize a model Hamiltonian for up to eight particles. The collective excitation spectrum is significantly different from that for $\nu=\frac{1}{3}$. Results for the two-particle distribution function are also presented.

While considerable progress has been made in the $\nu=1/n$ fractionally quantized Hall effect,[1] much less is known about other fractions. In the absence of a complete analytic theory,[2] numerical diagonalization remains a very useful tool.

The following model Hamiltonian[3] describes interacting electrons in the lowest Landau level:

$$H=\sum_{j} A_{j_1j_2j_3j_4}C_{j_1}^{\dagger}C_{j_2}^{\dagger}C_{j_3}C_{j_4} , \tag{1}$$

where $C_j^{\dagger}$ creates an electron in the jth Landau orbital ϕ_j. The A's are matrix elements of the Coulomb interaction. A square geometry together with periodic boundary conditions are assumed. ϕ_j is a momentum eigenstate in the y direction. The total y momentum is a good quantum number.

The operator T which takes ϕ_j into ϕ_{j+1} for every j leaves H invariant. For a simple rational fraction $\nu=p/q$ the qth power of T conserves the total y momentum because of the periodic boundary conditions. Eigenvectors of T^q are x-momentum eigenstates. The low-lying excitations can be classified according to their momentum. The collective excitation spectrum for $\nu=\frac{1}{3}$ was correctly computed this way in the rectangular geometry by Haldane.[4] Good convergence[5] was seen in the data for 4–7 particles. The spectrum exhibits a simple minimum near $kl=1.5$ (the roton minimum).

The corresponding spectrum for $\nu=\frac{2}{5}$ is shown in Fig. 1 for four, six, and eight particles. For the eight-particle case only the lowest eigenstates are given due to computer time limitation. The convergence is not as good as the $\nu=\frac{1}{3}$ case. We have therefore drawn solid and dashed lines to guide the eye. The overall shapes of the two curves are very much alike. While a ten-particle calculation is needed to be definitive, we believe that the solid line is very close to the true collective excitation spectrum.

In contrast to the $\nu=\frac{1}{3}$ case, the dispersion curve here shows two minima, one at about $kl\approx 0.8$ and another one at about $kl\approx 1.6$. The energy gap measured from the second minimum is about $0.024(e^2/l)$. In an earlier publication[6] by one of us, only excited states with zero x momentum were calculated for eight particles.

To gain some insight into the nature of the ground state and low-lying excited states, we have calculated the pair distribution function $g(\mathbf{r})$ for four particles. The qualitative features are similar to those[7] of $\nu=\frac{1}{3}$. The ground state is liquidlike. To get a quantitative measure, we expand $g(r)$ in powers of r^2 for small $r \ll l$ [l is the magnetic length which is set equal to 1 in formula (2)]:

$$g(r)=c_1r^2+c_2r^4+c_3r^6+\cdots . \tag{2}$$

The coefficients c_1, c_2, and c_3 are plotted in Fig. 2 versus excitation energy. A positive correlation between c_1 and the energy is obvious. This suggests that the good short-distance behavior in $g(r)$ is at least part of the energy-lowering mechanism in the ground state.

Girvin, MacDonald, and Platzman[8] have shown that the collective modes at $\nu=\frac{1}{3}$ are density oscillations. The density operator projected to the lowest Landau level is proportional to

$$\bar{\rho}_{\mathbf{k}}=\exp(-\tfrac{1}{4}k^2l^2)\times\sum_{j}\exp[2\pi is(j+t/2)/m]C_{j+t}^{\dagger}C_j , \tag{3}$$

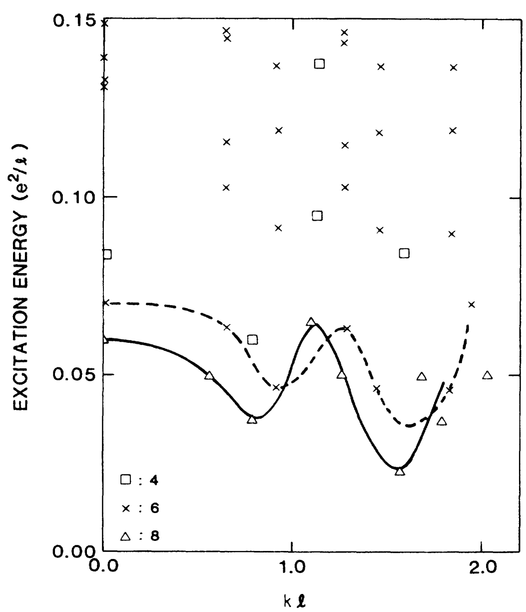

FIG. 1. Low-lying excitation spectrum at $\nu=\frac{2}{5}$. Squares, crosses, and triangles are used for the four-, six-, and eight-electron data, respectively. The solid and dashed curves are guides to the eye.

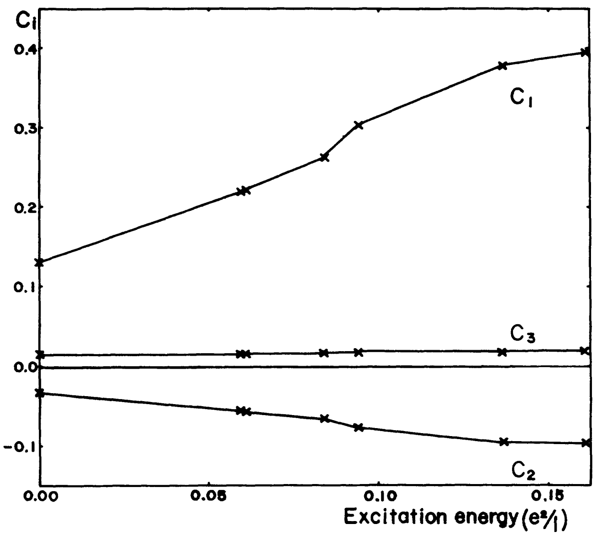

FIG. 2. Coefficients defined in Eq. (2) are plotted vs excitation energy for a few low-lying eigenstates.

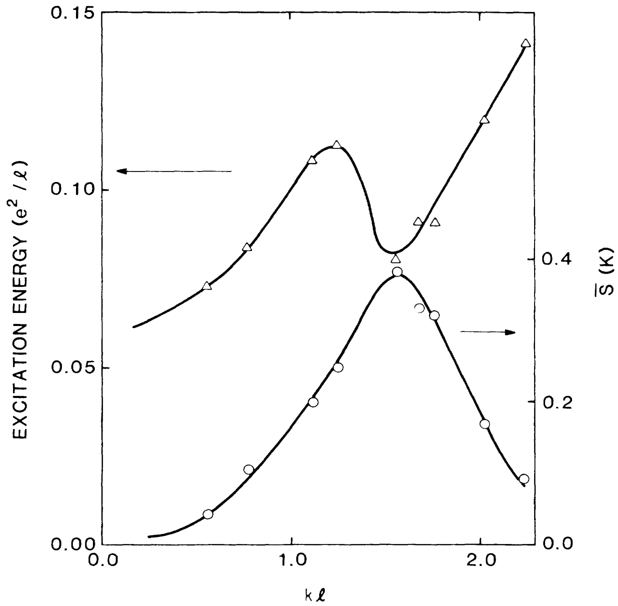

FIG. 3. Collective excitation energy in the single-mode approximation (triangles) and the projected static structure factor (circles). The solid curves are guides to the eye.

where $\mathbf{k}l=\sqrt{2\pi/m}\,(s,t)$. s, t, j, and m are integers. νm is the total number of electrons. Operating on the ground state $\bar{\rho}_\mathbf{k}$ creates an excited state with momentum $\mathbf{k}$. We have verified that for a small system (six particles) the operator $\bar{\rho}$ does reproduce the collective excitation spectrum very well up to $kl=2.0$.

Carrying out the corresponding calculations for $\nu=\frac{2}{5}$ and eight particles we obtained the data points (the triangles) in Figure 3. The solid curve is a guide to the eye. Compared to the exact results in Fig. 1 we see the density-wave description of the collective excitation is good only for very small kl. Although there is a minimum at $kl=1.6$, the energy is too high (almost three times the exact energy). The peak at $kl=1.2$ already merges with the continuum. The local minimum at $kl=0.8$ in Fig. 1 is hardly reflected in the density-wave excitation.

For reference we have also shown the projected static structure factor $\bar{S}(k)$ in Fig. 3. The circles are data points, whereas the solid curve is a guide to the eye. As usual, $\bar{S}(k)$ is the norm of the state created by $\rho_\mathbf{k}$:

$$\bar{S}(k)=\frac{1}{\nu m}\langle 0\,|\,\bar{\rho}^{\dagger}_{\mathbf{k}}\bar{\rho}_{\mathbf{k}}\,|\,0\rangle\ . \tag{4}$$

As in the case of $\nu=\frac{1}{3}$ the minimum at $kl=1.6$ is due to a peak in $\bar{S}(k)$ at the same momentum.

Overall, the single-density-mode approximation is rather inadequate for $\nu=\frac{2}{5}$. Since Girvin *et al.*[8] have shown the backflow correction to be absent for any filling fraction, one probably has to consider higher powers of the density operator.[9]

In summary, we have presented a finite-size study of the neutral collective excitations of the $\nu=\frac{2}{5}$ fractional quantized Hall state. More numerical data are desirable for better convergence. The results so far suggest that the $\nu=\frac{2}{5}$ quantum Hall effect is sufficiently different from that at $\nu=\frac{1}{3}$ that new approaches are needed to understand the former.

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*Permanent address: Department of Physics, Beijing Normal University, People's Republic of China.

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