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Thermomagnetic transport coefficients: Solitons in an easy-plane magnetic chain

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Using a simple model, we calculate the transport properties of a one-dimensional easy-plane ferromagnet in the presence of an in-plane magnetic field. The model incorporates the combined effects of a magnetic field gradient and a temperature gradient acting on a gas of solitons and spin waves. Suggestions are made for experiments capable of measuring these effects in materials such as $CsNiF_3$ and $(C_6H_{11}NH_3)CuBr_3$. We also discuss their use as another type of probe for solitons.

I. INTRODUCTION

Long-range order cannot exist in one dimension (1D); however, short-range order is possible. For the particular case of one-dimensional magnets,[1,2] the spins interact largely along a specific chain direction. The interaction between the chains is weak and leads to three-dimensional ordering at some low temperature T_c. For $T \gtrsim T_c$, there is a wide range of temperature (depending on the ratio of in-chain and interchain exchange constants) where spins display short-range order along the chains. The ordered regions are separated by domain walls. These walls are dynamic objects and in an easy-plane system, their properties are similar to those of sine-Gordon (SG) solitons. The thermodynamics above T_c is then determined by the linear waves (magnons) and solitons.[3] An extensive series of neutron scattering, susceptibility, specific heat, and spin relaxation measurements lend qualitative, occasionally even quantitative, support to this picture, for materials such as $CsNiF_3$ (Ref. 1) and $(C_6H_{11}NH_3)CuBr_3$ (CHAB) (Ref. 4) and also for antiferromagnets such as $(CD_3)_4NMnCl_3$ (TMMC) (Ref. 2).

The object here is to investigate whether the waves and soliton picture from equilibrium thermodynamics can be extended to transport phenomena, in terms of weakly coupled soliton and magnon ideal gases, subjected to gradients of temperature and applied field. Also, we look into the possibility of analogs of two well-known effects in semiconductors (where the carriers are described semiclassically). In a semiconductor, carriers can be moved by application of either an electric field or a temperature gradient. Application of one of them leads to, under proper circuit conditions, appearance of the other. The equivalent quantities here are the magnetic field gradient and the temperature gradient. The former directly changes the energy of the soliton while the latter affects the thermal population. Thus a field gradient can cause the soliton (or magnon) population to increase at one end of the sample, leading to an effective increase in temperature and vice versa. In as much as an easy-plane ferromagnetic chain can be described by sine-Gordon theory, and that an ideal-gas-like description of the SG model is feasible, it should be possible to study this and other transport effects in a 1D magnetic system.

The calculations described below are in the ideal-gas view of thermodynamics. An obvious improvement of this work will be to address the well-known difficulties (quantum spins and excursions off the easy plane) of the SG description, and include more precisely the interactions between solitons and between solitons and magnons. Here the effects of the magnons upon the solitons are grossly taken into account via an appropriate normalization of the equilibrium soliton distribution function f^0, as taken from Ref. 3 [Currie, Krumhansl, Bishop, and Trullinger (CKBT)]. We calculate the magnon response treating them as a degenerate boson gas while keeping the solitons nondegenerate. We have also assumed a single relaxation time. Again, in semiconductors, the relaxation times for mass or charge current and for the heat current are different. We expect the relaxation times for mass and heat currents of solitons to be different as well. Yet, we expect the calculations reported below to be qualitatively correct. Indeed an experiment would provide invaluable help in constructing transport theory of nonlinear excitations.

II. TRANSPORT FORMALISM

The ferromagnetic Hamiltonian for the spin degrees of freedom of a single chain is taken to be[5]

$$\mathcal{H}=\sum_{n=1}^{N}\left[-J\mathbf{S}_n\cdot\mathbf{S}_{n+1}+A(S_n^z)^2-HS_n^x\right] . \tag{1}$$

Here J is the nearest-neighbor exchange coupling, $A>0$ is the single-ion anisotropy, the $\mathbf{S}_n$ are classical spin vectors, and the applied field H $(=g\mu_B B)$ is in the easy (x,y) plane; g and μ_B are the Landé g factor and Bohr magneton, respectively. Using spherical coordinates

$$\mathbf{S}_n=S(\cos\theta\cos\phi,\ \cos\theta\sin\phi,\ \sin\theta) ,$$

and assuming $H/2AS \ll 1$, a continuum limit approximately produces a sine-Gordon equation of motion for the in-plane angle ϕ:

$$c_0^2\phi_{zz}-\phi_{tt}=\omega_0^2\sin\phi,\quad \theta=(\hbar/2AS)\phi_t \tag{2a}$$

with

$$c_0^2=2AJS^2a^2/\hbar^2,\quad \omega_0^2=2AHS/\hbar^2 , \tag{2b}$$

where a is the lattice spacing and z is the position on the chain. This SG limit has essentially converted the out-of-plane degree of freedom θ to the momentum conjugate to ϕ—this linearization of a nonlinear degree of freedom is a substantial part of the error introduced in approximating the full equations of motion by a SG equation. The SG equation has well-known soliton, breather, and low-amplitude linear modes, and we review some of their properties needed here.[6]

The solitons are traveling-wave rotations of the spins through 2π within the easy plane, with the spin tilting out of the easy plane being proportional to the soliton velocity $v<c_0$. This rotation occurs over a characteristic length d_0, determined by the applied field (for velocities $v \ll c_0$)

$$d_0=c_0/\omega_0=\sqrt{JS/H} . \tag{3}$$

The energy is that of a relativistic particle of rest mass m_0 and rest energy E_0,

$$E=E_0\gamma,\quad \gamma=(1-v^2/c_0^2)^{-1/2} , \tag{4a}$$

$$E_0=8(JHS^3)^{1/2},\quad m_0=E_0/c_0^2 , \tag{4b}$$

but for simplicity we shall use the nonrelativistic limit for γ, $\gamma=1+\frac{1}{2}(v^2/c_0^2)$. The solitons will be treated using classical Maxwell-Boltzmann statistics.

The dispersion relation for the linear modes, or magnons in the present context, as a function of wave vector k is

$$\varepsilon_k^2=\varepsilon_0^2+(\hbar c_0 k)^2 , \tag{5a}$$

$$\varepsilon_0=\hbar\omega_0=\frac{\alpha^{1/2}}{8S}E_0,\quad \alpha=\frac{2A}{J} . \tag{5b}$$

The relative anisotropy $2A/J\approx 0.38$ for spin-1 $CsNiF_3$;[7] so typically the energy gap for magnons is much smaller than for the solitons ($\varepsilon_0/E_0\approx 0.07$). The magnons will be treated using Bose-Einstein statistics.

Breather states are bound soliton-antisolition pairs with an internal frequency. Their contributions are neglected in this calculation. The low-energy breather states can be thought of as bound magnon states, dependent on the interaction between magnons, and as such represent a correction term to the soliton-magnon ideal-gas theory. In the simplest equilibrium thermodynamics theory their effects can be neglected. Similarly, we expect that the neglect of breather states in this transport calculation introduces relatively small errors, especially in a parameter regime where solitons are important.

The model used here to calculate the transport properties of this easy-plane ferromagnetic chain consists of a two-component ideal gas of magnons and solitons. Any interaction between magnons and solitons leads to (a) normalization of their energies and (b) a relaxation time τ representing approach to equilibrium. This picture is exact within SG theory which is completely integrable. If we stay close to the field and temperature region where the SG picture is approximately valid, we are allowed to assume that the interaction corrections are negligible. More precisely, we assume that the approach to equilibrium is caused by an extrinsic mechanism, e.g., magnetoelastic coupling leading to scattering of magnons and solitons by phonons. The intrinsic relaxation rate for scattering of solitons by magnons is assumed to be neglible as compared to the extrinsic relaxation ratio. This also means that a Matthiessen's-type rule exists, namely, the soliton and magnon transport currents are additive. In the following the soliton and magnon currents are calculated separately.

The nonequilibrium thermodynamics of both solitons and magnons is described by a linearized Boltzmann equation. In a steady state, the change in distribution function $\delta f(z,p)$ from its equilibrium value $f^0(z,p)$ satisfies (in the relaxation-time approximation),

$$-\frac{\delta f}{\tau}=\frac{\partial E}{\partial p}\frac{\partial f^0}{\partial z}+F\frac{\partial f^0}{\partial p} , \tag{6}$$

where $E(z,p)$ is the energy of the carriers, dependent on position due to the applied temperature and field gradients, and dependent on momentum p. The applied force F represents the effect of the field gradient.

In principle, Eq. (6) should be derived from a microscopic field theory of the Hamiltonian in Eq. (1). Such a derivation is beyond the scope of this paper. One expects that such a calculation will lead to a Boltzmann equation with terms corresponding individually to renormalized solitons and magnons together with relatively small interaction terms, accounting for soliton-soliton, soliton-magnon, and magnon-magnon (or also breather states) interactions. As a lowest-order calculation we presently ignore these interaction terms, except for the modification of the soliton equilibrium distribution by the magnons (see Sec. II A). Then, with this approximation, the soliton and magnons are effectively treated as independent ideal gases. This simplified calculation offers a much clearer view of the kinetic physical processes involved, avoiding the mathematical difficulties, at the expense of some accuracy. These higher-order interaction effects are known to improve agreement between theory and experiment for some equilibrium properties (e.g., specific-heat peaks of $CsNiF_3$ and CHAB), but principally only by rescaling various parameters while leaving the functional form intact.[8] We might expect similar behavior for the transport properties.

The effect of the field gradient will be treated as follows. We assume that the length scale $l_H\approx H/(\partial H/\partial z)$ over which the field changes is very large compared to the solition width d_0. Generally d_0 may be anywhere from a few to tens of lattice spacings, while the field gradient length scale l_H is macroscopic, making this assumption very easily satisfied. Locally, then, the SG solitons are adequate solutions to the equations of motion. But as they move in the slowly changing field, they experience its effect as a mild force towards the region of lower soliton energy (which is towards the lower-field region), being adiabatically modified and exchanging energy with the applied field. Thus the force F_s on a soliton it taken to be

$$F_s = -\frac{\partial E}{\partial z} = -\frac{\partial E}{\partial H}\frac{\partial H}{\partial z} = \tfrac{1}{2}E\left[-\frac{\nabla H}{H}\right] . \tag{7}$$

Similarly, the force on a magnon of wave vector k is taken to be

$$F_k = -\frac{\partial E_k}{\partial z} = \frac{\varepsilon_0^2}{2\varepsilon_k}\left[-\frac{\nabla H}{H}\right] . \tag{8}$$

In the semiconductor context the physically observable transported quantities are the charge current and heat current. For this magnetic chain, the solitons and magnons similarly carry heat current, but the closest analog to the charge current is a magnetization current. In particular, there will be only an x component of magnetization current (parallel to the field); the y and z components average out to zero for both solitons and magnons. Below we treat the different transport properties of solitons and magnons separately.

A. Solitons

The magnetization current j_m and heat current j_u will be given by integrals of contributions from solitons of all velocities (less than c_0), over the distribution which has been perturbed from equilibrium by the applied driving "forces,"

$$j_m = \int dv\, v\, m(v)\delta f(v) , \tag{9a}$$

$$j_u = \int dv\, v\, u(v)\delta f(v) . \tag{9b}$$

Here $v = \partial E/\partial p$ represents the velocity; we consider δf a function of velocity instead of momentum. The functions $m(v)$ and $u(v)$ represent the effective magnetization and "heat" or internal energy carried by a soliton or antisoliton of velocity v. These functions are determined by requiring that the equilibrium magnetization M and internal energy U due to the solitons, as given in Ref. 3, can also be written as integrals over the equilibrium distribution $f^0(v)$,

$$M = \int dv\, m(v) f^0(v) , \tag{10a}$$

$$U = \int dv\, u(v) f^0(v) . \tag{10b}$$

M and U are given from the soliton and antisoliton equilibrium free energy $F_0^{\rm sol}$,

$$F_0^{\rm sol} = -k_B T n^{\rm tot} , \tag{11a}$$

$$M = -\frac{\partial F_0^{\rm sol}}{\partial H}, \quad U = \frac{\partial}{\partial \beta}(\beta F_0^{\rm sol}) , \tag{11b}$$

where $\beta = (kT)^{-1}$. The total number density of solitons and antisolitons, $n^{\rm tot}$, is

$$n^{\rm tot} = \frac{4}{d_0}\left[\frac{\beta E_0}{2\pi}\right]^{1/2} e^{-\beta E_0} , \tag{12}$$

with d_0 and E_0 as defined in Eqs. (3) and (4). Certainly integration over $f^0(v)$ should give the total number density of solitons and antisolitons:

$$n^{\rm tot} = \int dv\, f^0(v) . \tag{13}$$

Combining the different expressions for M and U, Eqs. (10) and (11), results in consistency conditions involving $m(v)$, $u(v)$, and $f^0(v)$,

$$k_B T \frac{\partial f^0}{\partial H} = m(v) f^0(v) , \tag{14a}$$

$$\frac{\partial f^0}{\partial \beta} = -u(v) f^0(v) . \tag{14b}$$

To completely determine $m(v)$ and $u(v)$ we need to specify $f^0(v)$. Because we assume the solitons obey classical statistics, f^0 can be written as

$$f^0(v) = \frac{A_0}{c_0 d_0} e^{-\beta(E-\mu)} , \tag{15}$$

where the factor $c_0 d_0$ gives the correct dimensions, A_0 is a dimensionless constant, and a chemical potential μ has been introduced in order to write f^0 in a standard form. The constant A_0 is required for proper phase-space counting; the phase-space integral is normalized by Planck's constant, $1/h$, $\int dp/h \rightarrow \int dv(m_0/h)$, and thus A_0 is set by

$$\frac{A_0}{c_0 d_0} = \frac{m_0}{h} . \tag{16}$$

With this assumed form for $f^0(v)$, Eqs. (12) and (13) determine the chemical potential μ necessary to recover the CKBT result for total soliton number density (also assuming nonrelativistic dispersion $E = E_0 + \frac{1}{2}m_0 v^2$),

$$\mu = \frac{1}{\beta}\ln\left[\frac{2\beta E_0}{\pi A_0}\right] , \tag{17}$$

and thus $f^0(v)$ has been specified.

Some comments are in order related to the chemical potential. The quantity μ appears explicitly in Eq. (15) as a chemical potential, and indeed will appear in the Boltzmann equation again as a chemical potential [Eq. (23)]. Effectively the factor $e^{\beta\mu}$ provides the appropriate normalization for f^0, such that we can reproduce the CKBT results for $n^{\rm tot}$, M, U, and so on, while at the same time putting f^0 in a familiar standard form.

If μ is taken to represent a real effective chemical potential, then it is interesting to consider its effects on quantum degeneracy. Typically quantum degeneracy is expected to become important when μ passes through zero, thereby implying that each soliton is confined to an area approaching h or less in phase space. Equation (17) then gives a corresponding degeneracy temperature T_q defined by

$$k_B T_q = 4\hbar\omega_0 = \frac{\alpha^{1/2}}{2S} E_0 . \tag{18}$$

One finds $k_B T_q \approx \frac{3}{10} E_0$ for either $CsNiF_3$, or CHAB, i.e., rather larger than expected when compared to E_0. However, one of the assumptions of the classical ideal-gas soliton thermodynamics[3] is that $k_BT \ll E_0$, and we see that there appears to be a limited range of temperatures over which the classical approach will be valid. Usually one would attempt to correct this situation by considering the quantum corrections for the statistical mechanics of the SG equation. This viewpoint will not be adopted here. Instead, we recall that the classical sine-Gordon thermodynamics does remarkably well in describing equilibrium experimental data for both $CsNiF_3$ and CHAB, even for temperatures well below the predicted T_q (for instance, as low as $k_BT_q \approx \frac{1}{4}E_0$). In view of such experimental evidence available, it seems reasonable to attempt to use the same *classical* SG thermodynamics also to describe *transport* in these easy-plane ferromagnets, and for the present to ignore any difficulties which may be implied by the relatively high T_q. And, of course, it is not clear whether we can really treat μ as a true chemical potential anyway. Indeed, the locations of the quantum and classical regimes for these materials, for both equilibrium and nonequilibrium problems, is an issue yet to be resolved.

Then, with $f^0(v)$ as already specified, the consistency conditions (14) determine $m(v)$ and $u(v)$ as

$$m(v) = -\frac{\partial}{\partial H}(E-\mu) + \frac{1}{2}\frac{k_BT}{H} = -\frac{\frac{1}{2}E - k_BT}{H}, \tag{19a}$$

$$u(v) = \frac{\partial}{\partial \beta}[\beta(E-\mu)] = E - k_BT. \tag{19b}$$

It can be easily verified that these reproduce the known low-temperature limit $\beta E_0 > 1$ equilibrium quantities,

$$M^{\text{sol}} = -\tfrac{1}{2}n^{\text{tot}}(E_0 - \tfrac{3}{2}k_BT)/H, \tag{20a}$$

$$U^{\text{sol}} = n^{\text{tot}}(E_0 - \tfrac{1}{2}k_BT). \tag{20b}$$

It should be mentioned that $m(v)$ and $u(v)$ include the leading-order effects of the linear modes acting on the solitons, as obtained in equilibrium. The number density used here includes the self-energy effects of scattering events of the linear modes with solitons. To see this in a different manner, consider the magnetization pulse carried by a single unperturbed moving SG soliton. The x component of the soliton profile is a pulse deviating from the aligned ground state, with characteristic width d_0/γ,

$$S^x = S[1 - 2\,\text{sech}^2(\gamma z/d_0)]. \tag{21}$$

Relative to the ground state, the total x magnetization carried is

$$\tilde{m}^x = \int_{-\infty}^{\infty} dz(S^x - S) = -4Sd_0/\gamma. \tag{22}$$

This result shows that for faster-moving solitons, which get narrower due to the relativistic contraction, the absolute value of the magnetization carried decreases with increasing velocity. This is in contrast to the previous result for $m(v)$, Eq. (19a), including temperature and linear modes acting on the soliton, where $|m(v)|$ increases with increasing velocity (for $\beta E_0 > 2$). This reflects the effective soliton mass increase induced by the linear modes. In any case integration of $\tilde{m}^x$ over all velocities cannot give the known equilibrium magnetization. This observation originally led to the present self-consistent method for determining $m(v)$ and $u(v)$.

To completely specify the currents, we rewrite δf in terms of the applied temperature and field gradients, and the force F,

$$-\frac{\delta f}{\tau} = v\left[\frac{\partial f^0}{\partial E}\right]\left[(E-\mu)\left[-\frac{\nabla T}{T}\right] - \nabla\mu + F\right]. \tag{23}$$

Since μ is a function of T and H, we can eliminate $\nabla\mu$ in favor of ∇T and ∇H,

$$\nabla\mu = (k_BT - \mu)(-\nabla T/T) - k_BT(-\nabla H/H). \tag{24}$$

Thus the soliton transport results are determined only by ∇T and ∇H. Again, using the nonrelativistic energy relationship, we obtain

$$j_u = K_{uT}^{\text{sol}}(-\nabla T/T) + K_{uH}^{\text{sol}}(-\nabla H/H), \tag{25a}$$

$$j_m = K_{mT}^{\text{sol}}(-\nabla T/T) + K_{mH}^{\text{sol}}(-\nabla H/H), \tag{25b}$$

with transport coefficients

$$K_{uT}^{\text{sol}} = a_1 I_0[(\beta E_0)^2 + (\beta E_0) + \tfrac{7}{4}], \quad K_{uH}^{\text{sol}} = a_1 I_0[(\beta E_0)^2 + 4\beta E_0 + \tfrac{13}{4}], \tag{26a}$$

$$K_{mT}^{\text{sol}} = a_2 I_0[(\beta E_0)^2 + \tfrac{5}{4}], \quad K_{mH}^{\text{sol}} = a_2 I_0[(\beta E_0)^2 + 3\beta E_0 - \tfrac{1}{4}], \tag{26b}$$

where

$$a_1 = \frac{\tau c_0}{\beta^2}, \quad a_2 = -\frac{\tau c_0}{\beta^2}\frac{1}{2H}, \tag{27a}$$

$$I_0 = \left[\frac{\alpha}{8\pi S^2}\right]^{1/2}(\beta E_0)^{1/2}e^{-\beta E_0}. \tag{27b}$$

Aside from some prefactors these results depend only on βE_0. There is no Onsager symmetry relationship relating K_{mT}^{sol} and K_{uH}^{sol}, due to the fact that the field gradient creates *velocity-dependent forces* (recall $F \sim E$), thereby eliminating any Onsager symmetry. K_{mT}^{sol} and K_{mH}^{sol} are both negative since the soliton magnetization is always a deviation from the aligned ground-state configuration.

B. Magnons

The general approach used for magnons is the same as for the solitons, with some minor differences. They will be described by Bose-Einstein statistics, with zero chemical potential, and the phase-space integrals will be over wave vectors k instead of velocity. Again the presence of velocity-dependent forces implies a lack of Onsager symmetry. The magnetization and heat carried by a magnon of wave vector k are

$$m(k) = -\frac{\varepsilon_0^2}{2H\varepsilon_k}, \tag{28a}$$

$$u(k) = \varepsilon_k. \tag{28b}$$

The equilibrium distribution is assumed to be

$$f_k^0 = \frac{1}{e^{\beta\varepsilon_k} - 1} \; ; \tag{29}$$

then we obtain the following magnon transport coefficients,

$$K_{uT}^{\text{mag}} = a_1 (\beta\varepsilon_0)^{-1} s_2(\beta\varepsilon_0), \quad K_{uH}^{\text{mag}} = a_1 \tfrac{1}{2} \beta\varepsilon_0 s_0(\beta\varepsilon_0) \; , \tag{30a}$$

$$K_{mT}^{\text{mag}} = a_2 \beta\varepsilon_0 s_0(\beta\varepsilon_0), \quad K_{mH}^{\text{mag}} = a_2 \tfrac{1}{2} (\beta\varepsilon_0)^3 s_{-2}(\beta\varepsilon_0) \; , \tag{30b}$$

with the functions $s_n(x_0)$ defined by the integral

$$s_n(x_0) = \frac{1}{4\pi} \int_{x_0}^{\infty} dx \, x^{n-1} (x^2 - x_0^2)^{1/2} / \sinh^2(\tfrac{1}{2}x) \; . \tag{30c}$$

III. RESULTS

Some typical results for these transport coefficients versus field at fixed temperature and versus temperature at fixed field are shown in Fig. 1, for parameters appropriate to $CsNiF_3$.[7] Note that to obtain the overall scale of these curves we would need reasonable estimates of the relaxation times τ_{sol} and τ_{mag}. Also note that these are results for a single chain, and must be multiplied by the chain number density per unit area to obtain conductivities for the bulk medium. As mentioned in the Introduction, the total currents will be given as the sum of soliton and magnon contributions—the relative contributions being directly proportional to the respective relaxation times.

It would be useful to identify a quantity which might serve as a signature of soliton flux. In equilibrium experiments attempting to identify soliton contributions, the typical approach has been to make measurements with and without the applied easy-plane field, thereby observing cases with and without SG solitons present, and attributing the differences mostly to the solitons and partly to higher-order magnon processes. For example, doing this for the specific heat[9,10] predicts a peak in the specific heat versus field, whose position and height are proportional to T^2 and T, respectively, assuming SG solitons are responsible. We can attempt the same approach here. If the field is set to zero no SG solitons are present. Then only magnons contribute to the currents, and if we also have $\nabla H = 0$, then the magnetization current is identically zero. In this case the heat current simplifies to

$$j_u = \frac{\tau_{\text{mag}} c_0}{\beta^2} s_2(0) \left[-\frac{\nabla T}{T} \right] , \tag{31}$$

where $s_2(0) = 1.0472$. Now if a uniform field is turned on (still with $\nabla H = 0$), the soliton contributions are added, such that the net change in the total thermal conductivity, ΔK_{uT}, is given by

$$\Delta K_{uT} = \left[\frac{c_0}{\beta^2} \right] \left[\tau_{\text{mag}} [s_2(\beta\varepsilon_0) - s_2(0)] + \frac{\tau_{\text{sol}}}{2S} \left[\frac{\alpha}{2\pi} \right]^{1/2} e^{-\beta E_0} (\beta E_0)^{1/2} \times [(\beta E_0)^2 + \beta E_0 + \tfrac{7}{4}] \right] \tag{32}$$

and recall that $\varepsilon_0 = (\alpha / 8S) E_0$. In Fig. 2 we show the respective magnon and soliton contributions to ΔK_{uT}, and the total due to both, assuming equal relaxation times. In a case where $\tau_{\text{sol}} \gg \tau_{\text{mag}}$, we can ignore the magnon contribution. The soliton contribution to ΔK_{uT}

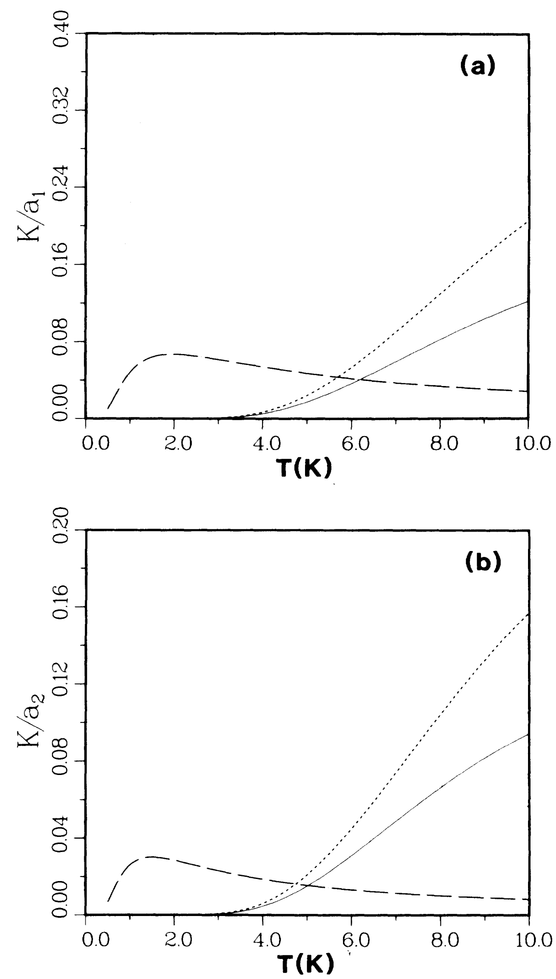

FIG. 1. (a) Calculated transport coefficients K_{uT} and K_{uH} normalized by $a_1 = \tau c_0/(\beta^2)$, for $CsNiF_3$ parameters $J = 23.6K$, $A = 4.5K$, $g = 2.4$, vs temperature T in K. The curves correspond to K_{uT}^{sol} (solid), K_{uH}^{sol} (dashed), K_{uT}^{mag} (dotted), and K_{uH}^{mag} (dashed-dotted); (b) Transport coefficients K_{mT} and K_{mH}, normalized by $a_2 = -a_1/2H$, for $CsNiF_3$ parameters. The curves correspond to K_{mT}^{sol} (solid), K_{mH}^{sol} (dashed), K_{mT}^{mag} (dotted), and K_{mH}^{mag} (dashed-dotted).

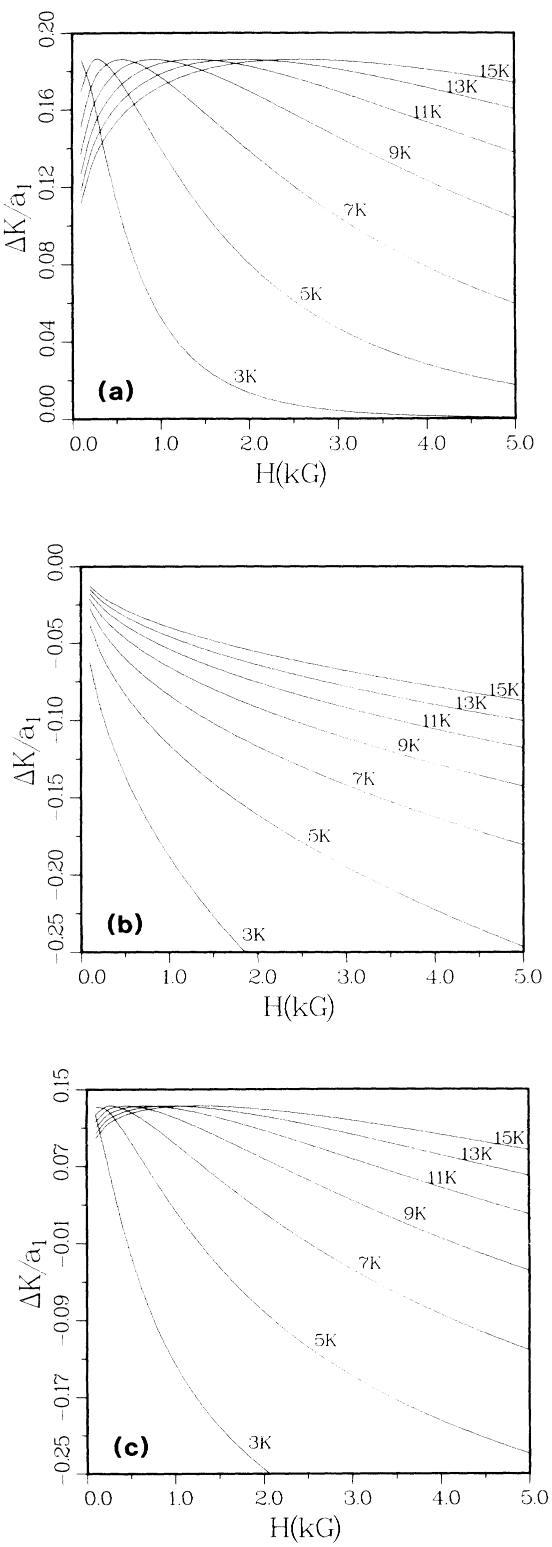

FIG. 2. The change in thermal conductivity $\Delta K_{uT}=K_{uT}(H)-K_{uT}(0)$, using $CsNiF_3$ parameters as in Fig. 1, and normalized by a_1 vs the applied field H for a series of temperatures. The soliton contribution is shown in (a), the magnon contribution is shown in (b), and the total is shown in (c) for $\tau_{sol}=\tau_{mag}$.

taken alone has a peak at a field given by $(\beta E_0)_{peak}=1.6654$. This in turn then predicts that the field at which the peak occurs is proportional to T^2; similarly so will be the height of the peak. Peaks can still occur for $\tau_{sol}=\tau_{mag}$, as in Fig. 2(c), but the positions are shifted from the pure soliton peaks to lower fields. This is shown in Fig. 3, where the peak position H_{peak} versus T^2 is plotted for $\tau_{sol}/\tau_{mag}=\infty$, 10.0, and 1.0, for $3\leq T\leq 15$ K. Perhaps this effect would provide an experimental means for approximately determining the ratio of the two relaxation times, provided it is not too different from 1. Also note that even for $\tau_{sol}=\tau_{mag}$, the height of the peaks in ΔK_{uT} is still very closely proportional to T^2 [Fig. 2(c)].

Another quantity which could be measured is an effective thermopower-like coefficient, i.e., the ratio of temperature gradient generated to a given applied field gradient, assuming either the magnetization current or heat current can experimentally be set to zero. Some typical results for $(\nabla T/T)/(\nabla H/H)$, combining soliton and magnon contributions, are shown in Figs. 4 and 5, for various lifetime ratios τ_{sol}/τ_{mag}. Once again it would be extremely useful to assign an approximate value to this ratio.

IV. DISCUSSION

We can contrast this calculation with others relating to nonequilibrium 1D soliton dynamics. Other theoretical studies have included the following: (i) numerical calculation of spin dynamics for the easy-plane ferromagnet, in combined dc and ac arbitrary driving fields, and including damping;[11] (ii) similar numerical studies of the dynamics of the damped driven SG system;[12] (iii) Fokker-Planck and other treatments of the overdamped SG system;[13] and (iv) other less closely related problems involving transport, such as in polyacetylene. The first two of these have involved spatially uniform driving, and

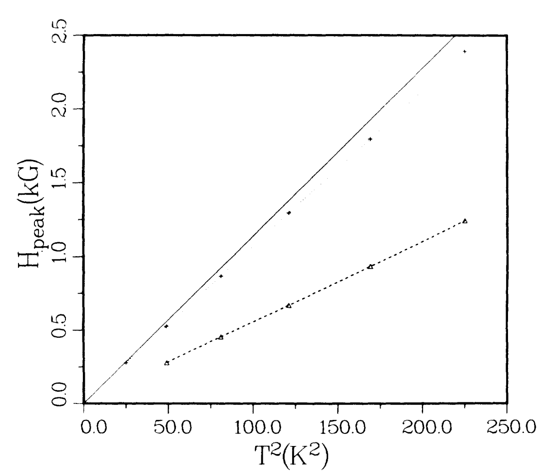

FIG. 3. Positions of the peaks in the total ΔK_{uT}, vs T^2, for $\tau_{sol}/\tau_{mag}=\infty$ (solid), 10.0 (dotted), and 1.0 (dashed), for $CsNiF_3$ parameters.

studying transitions of the system as a whole to chaotic dynamics, rather than transport. They have emphasized as a general concept the idea that spatially coherent structures present in nonchaotic regimes compete with the tendencies of the individual particles of the systems to move onto chaotic trajectories, and thus transitions to chaos occur at different control parameter values for the coupled system compared to the individual particles. The Fokker-Planck studies of SG systems have not stressed any particular physical context such as the easy-plane ferromagnet, and generally have treated only the large damping limit.

This calculation is an attempt to estimate the leading order behavior of soliton transport for the easy-plane ferromagnet, and it should be emphasized that it has a number of limitations. First of all, the continuing controversy over classical mechanics versus quantum mechanics[14] for an equilibrium description of low-spin systems must be just as relevant for nonequilibrium properties. Classical statistical mechanics of the SG model is in fair agreement with some experiments for easy-plane ferromagnets and antiferromagnets, but inconsistencies persist. If the system could be described entirely using classical mechanics, one should then use the solitons of the classical easy-plane ferromagnetic Hamiltonian, including the complete effects of the out-of-plane degree of freedom. But these solitons are known to have dynamics strongly different from SG solitons, including an instability field,[15] instability with respect to collisions, and $E(v)$ not single valued.[16] In particular any effects of an instability have not been seen in experiments. Also, classical transfer-matrix calcula-

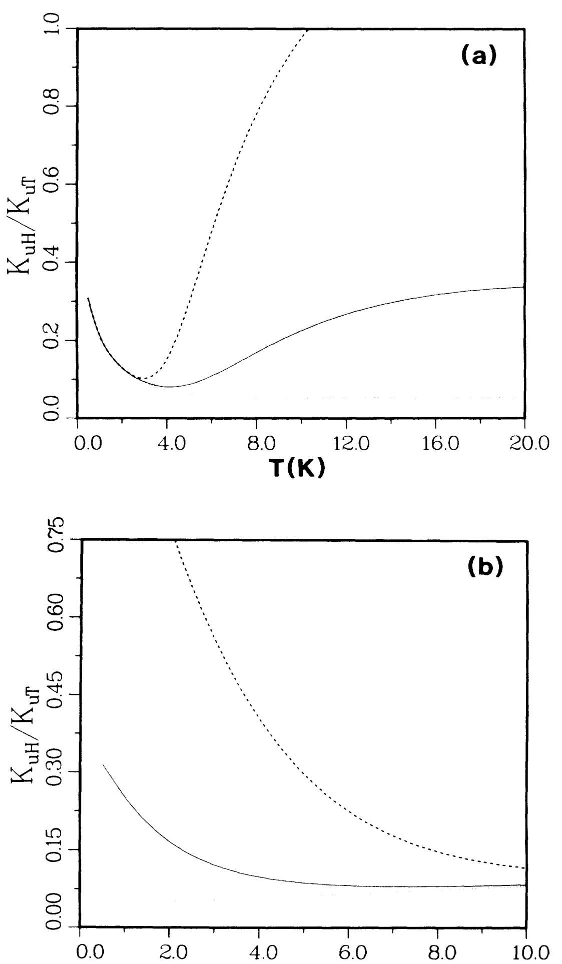

FIG. 4. Ratios of total K_{uH}/K_{uT}, equivalent to $-(\nabla T/T)/(\nabla H/H)$ at zero heat current, for $CsNiF_3$ parameters, and lifetime ratios $\tau_{sol}/\tau_{mag}=0.1$ (dotted), 1.0 (solid), and 10.0 (dashed). Part (a) shows the temperature dependence at $H=5.0$ kG; part (b) shows the field dependence (H in kG) at $T=5.0$ K.

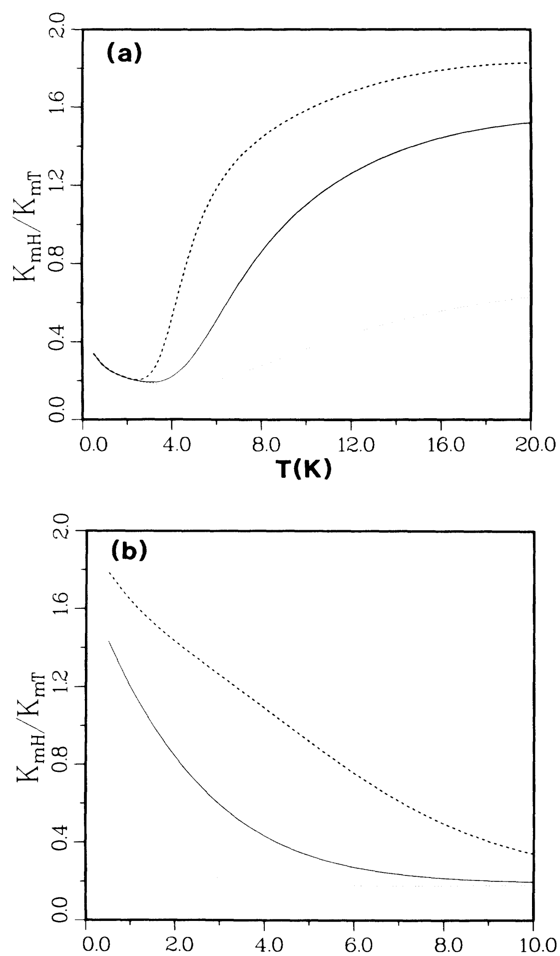

FIG. 5. Ratios of total K_{mH}/K_{mT}, equivalent to $(-\nabla T/T)/(\nabla H/H)$ at zero magnetization current, for $CsNiF_3$ parameters, and lifetime ratios $\tau_{sol}/\tau_{mag}=0.1$ (dotted), 1.0 (solid), and 10.0 (dashed). Part (a) shows the temperature depedence at $H=5.0$ kG; part (b) shows the field dependence (H in kG) at $T=5.0$ K.

tions[17] using the easy-plane ferromagnetic Hamiltonian greatly overestimate the peak heights in the specific heat, when compared to classical SG theory or experiment. Recent "numerically exact" quantum transfer-matrix calculations[18] for a spin-$\frac{1}{2}$ easy-plane ferromagnet also give lower values for the specific-heat peak heights, but not entirely consistent with experimental data[4,17] for CHAB. However, the various classical transfer-matrix calculations,[17,19] including and excluding the out-of-plane degree of freedom (i.e., easy-plane magnet versus SG) when compared with the various quantum calculations available,[18,20] have allowed a reasonable explanation of how classical SG theory can be applicable. The consensus seems to be that quantum effects strongly restrict the spins to the easy plane, thereby competing with (or eliminating) the tendency for out-of-plane motion, and allowing classical SG theory to be valid even for a fundamentally strongly quantum ($S=\frac{1}{2}$) system. For the nonequilibrium calculation presented here this is the viewpoint we must assume. However, at some later stage it may be instructive to investigate transport for the distorted classical solitons of the full easy-plane ferromagnetic Hamiltonian, to extend the contrast between it and the SG system.

Lacking specific knowledge about the dynamics of the relaxation processes, we have used only the relaxation time approximate solution to the linearized Boltzmann equation. Obviously, more detailed information would help in estimating $\tau_{\rm sol}$ and $\tau_{\rm mag}$, which would then set an absolute scale to our results. Finally, we also have ignored contributions due to breather states and other higher-order interaction terms. In spite of these difficulties, there may be some range of adequately low field and temperature over which these results are applicable. Clearly, a nonequilibrium transport experiment will help to determine the relative importance of carrier interactions, and also whether a nonequilibrium soliton picture is reasonable, for materials such as $CsNiF_3$ and CHAB.

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