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Polariton and effective-medium theory of magnetic superlattices

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The retarded modes of a superlattice comprising alternating layers of two magnetic media characterized by a gyrotropic permeability tensor are investigated. The transfer matrix and dispersion relation are given in general form; a number of previous results are included as special cases. An effective-medium description, valid for long wavelengths, is derived first by expansion of the general result and second by general continuity arguments. Some numerical illustration of the effective-medium theory is given.

I. INTRODUCTION

The increasing interest in superlattice structures, in which successive layers of two different component media are deposited to form a specimen with a long period Λ in one direction, has led to a need to understand the way in which the elementary excitation spectrum is modified by the introduction of a new Brillouin-zone edge at π/Λ. This paper addresses this question for long-wavelength excitations in a superlattice composed of two alternating magnetic media.

The dominant restoring force for the magnetic excitations in a bulk medium depends upon the wave number k. The different regimes were pointed out by Auld,[1] and a brief discussion is given by Sarmento and Tilley.[2] For short wavelengths $k \sim 1/a$, where a is an interatomic spacing, only the exchange interaction between adjacent spins contributes, and the resulting modes are called spin waves. Some results for the spin-wave spectrum of a magnetic superlattice were given by Albuquerque *et al.*[3]

For smaller k the electromagnetic coupling between spins some distance apart must be taken into account. There is an important intermediate range of k values in which the electromagnetic coupling can be treated within a magnetostatic approximation, so that the magnetic dipole-dipole interaction, without retardation, is added to the exchange interaction. This magnetostatic regime has proved of particular importance for the modes on the surface of a semi-infinite bulk specimen or in a thin film. The original theoretical work is due to Damon and Eshbach,[4] and magnetostatic surface modes on ferromagnets have been extensively investigated by Brillouin scattering; a review is given by Tilley.[5] The magnetostatic modes of a superlattice consisting of alternate layers of a magnetic and a nonmagnetic medium were discussed theoretically by Camley *et al.*[6] and by Grünberg and Mika.[7] Of particular interest is their conclusion concerning the existence of a surface magnetostatic mode of the Damon-Eshbach type on a semi-infinite superlattice. This mode is predicted to exist for $d_{\rm M} > d_{\rm NM}$, but not for $d_{\rm M} < d_{\rm NM}$, where $d_{\rm M}$ and $d_{\rm NM}$ are the thicknesses of the magnetic and nonmagnetic layers.

For small k roughly $k \sim 1/\lambda$, where λ is the free-space wavelength, retardation of the electromagnetic coupling becomes important, and Maxwell's equations in their full form must be employed. Fortunately the exchange coupling is important only in that it determines the equilibrium value of the magnetization, so macroscopic equations are adequate. The resulting modes are usually called magnetic polaritons. Considerable theoretical effort has been devoted to surface and thin-film magnetic polaritons; see the reviews by Sarmento and Tilley[2] and Tilley.[5] Camley and Mills[8] predict that the surface magnetic polariton on a semi-infinite antiferromagnet should be observable by attenuated total reflection (ATR), but no experiments have been reported to date. Indirect evidence for the existence of these modes comes from the reflectivity measurements of Remer *et al.*[9]

Magnetic polaritons in superlattices were first discussed by Barnaś;[10] he restricted attention to a system in which magnetic and nonmagnetic layers alternate, so that he has generalized the results of Camley *et al.*[6] and Grünberg and Mika[7] to include retardation. In this paper we go further by considering a superlattice of alternating magnetic layers. We derive expressions for the magnetic polaritons in the Voigt configuration, in which the plane of propagation is normal to the magnetic field configuration. Each medium is characterized by a gyrotropic permeability tensor, so that the formal results apply equally to antiferromagnets and to ferromagnets. In addition to the general dispersion equation for polaritons, we give a simplified result for the long-wavelength limit $\lambda \gg \Lambda$ when the superlattice is found to behave like an anisotropic bulk medium. The corresponding result for the optics of a nonmagnetic superlattice[11–13] has proved of value in interpreting far-infrared reflectance experiments on $\mathrm{GaAs/Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ superlattices.[14]

II. TRANSFER MATRIX AND DISPERSION EQUATION

The notation to be used is indicated in Fig. 1. Magnetic layers of thickness a and b alternate; the spatial period is $\Lambda = a + b$. A static magnetic field B_0 is applied in the plane of the layers. As indicated in Fig. 1, we choose axes with B_0 along the z axis, and we consider only propagation in the x-y plane, that is, the Voigt

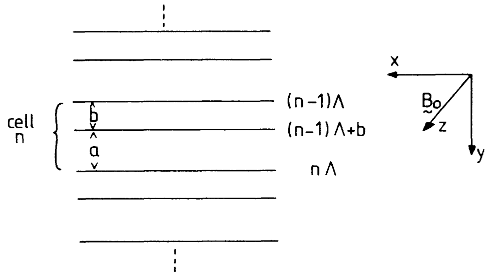

FIG. 1. Notation for magnetic-superlattice calculation.

geometry.

The magnetic properties of each medium are described by a gyrotropic permeability tensor of the form

$$\overleftrightarrow{\mu}^{\alpha}=\begin{pmatrix} \mu_{xx}^{\alpha} & \mu_{xy}^{\alpha} & 0 \\ -\mu_{xy}^{\alpha} & \mu_{xx}^{\alpha} & 0 \\ 0 & 0 & \mu_{zz}^{\alpha} \end{pmatrix}, \quad \alpha=1,2 \ . \tag{1}$$

This form is sufficiently general to describe ferromagnets, antiferromagnets, and ferrimagnets.[2] μ_{xx} and μ_{xy} have poles at the magnetic resonance frequencies, while μ_{zz} does not have strong frequency dispersion. In the absence of damping μ_{xx} is real and μ_{xy} is pure imaginary. In addition to its magnetic permeability, each medium is taken to have an isotropic dielectric constant ϵ.

As is to be expected from the form of Eq. (1), the magnetic polaritons of interest are TE modes with the electric field E along z and the magnetic field vectors in the x-y plane. We therefore concentrate on this polarization, although later for completeness we give the results for TM modes. All magnetic field components in a layer are given by a sum of a forward- and a backward-traveling wave; for example, in cell n

$$H_{ny}^{(1)}=\{ A_n^{(1)}\exp[-\beta_1(y-n\Lambda)]+B_n^{(1)}\exp[\beta_1(y-n\Lambda)]\} \times\exp[i(k_x x-\omega t)] \ . \tag{2}$$

Here the last factor is common to the fields in all layers. Within the layer the wave number k_{1y} can be real or pure imaginary (in the absence of damping), and for later convenience we have written it as

$$k_{1y}=i\beta_1 \ . \tag{3}$$

The components k_{1y} and k_x are related by the bulk wave equation, which has the general form

$$k^2\mathbf{H}-\mathbf{k}(\mathbf{k}\cdot\mathbf{H})-(\epsilon\omega^2/c^2)\overleftrightarrow{\mu}\cdot\mathbf{H}=0 \ . \tag{4}$$

The solvability condition for (4), together with (3), gives

$$\beta_1=(k_x^2-\epsilon_1\mu_v^1\omega^2/c^2)^{1/2} \ , \tag{5}$$

where

$$\mu_v^1=\mu_{xx}^1+(\mu_{xy}^1)^2/\mu_{xx}^1 \tag{6}$$

is the Voigt permeability of medium 1.

The electromagnetic boundary conditions are now used to relate field amplitudes (A_n,B_n) in successive layers. As pointed out by Irving,[15] for magnetic interfaces it is more convenient to work in terms of (H_{nx},B_{ny}), since these components are continuous at successive interfaces. Application of boundary conditions at two interfaces gives the basic result

$$\begin{bmatrix} H_{(n-1)x}^{(1)} \\ B_{(n-1)y}^{(1)} \end{bmatrix}=T\begin{bmatrix} H_{nx}^{(1)} \\ B_{ny}^{(1)} \end{bmatrix} , \tag{7}$$

where the fields are evaluated at $y=(n-2)\Lambda+b$ and $y=(n-1)\Lambda+b$. The transfer matrix T has components

$$T_{11}=[\cosh(\beta_1 a)+\Delta_1^1\sinh(\beta_1 a)] \times[\cosh(\beta_2 b)+\Delta_1^2\sinh(\beta_2 b)] +\Delta_2^1\Delta_3^2\sinh(\beta_1 a)\sinh(\beta_2 b) \ , \tag{8}$$

$$T_{12}=[\cosh(\beta_1 a)+\Delta_1^1\sinh(\beta_1 a)]\Delta_2^2\sinh(\beta_2 b) +[\cosh(\beta_2 b)-\Delta_1^2\sinh(\beta_2 b)]\Delta_2^1\sinh(\beta_1 a) \ , \tag{9}$$

$$T_{21}=[\cosh(\beta_2 b)+\Delta_1^2\sinh(\beta_2 b)]\Delta_3^1\sinh(\beta_1 a) +[\cosh(\beta_1 a)-\Delta_1^1\sinh(\beta_1 a)]\Delta_3^2\sinh(\beta_2 b) \ , \tag{10}$$

$$T_{22}=\Delta_3^1\Delta_2^2\sinh(\beta_1 a)\sinh(\beta_2 b) +[\cosh(\beta_1 a)-\Delta_1^1\sinh(\beta_1 a)] \times[\cosh(\beta_2 b)-\Delta_1^2\sinh(\beta_2 b)] \ , \tag{11}$$

where

$$\Delta_1^{\alpha}=k_x\mu_{xy}^{\alpha}/i\beta_{\alpha}\mu_{xx}^{\alpha} \ , \tag{12}$$

$$\Delta_2^{\alpha}=\frac{k_x^2(\mu_{xy}^{\alpha})^2+\beta_{\alpha}^2(\mu_{xx}^{\alpha})^2}{i\mu_0 k_x\beta_{\alpha}\mu_{xx}^{\alpha}[(\mu_{xx}^{\alpha})^2+(\mu_{xy}^{\alpha})^2]} \ , \tag{13}$$

$$\Delta_3^{\alpha}=-\mu_0 k_x[(\mu_{xx}^{\alpha})^2+(\mu_{xy}^{\alpha})^2]/i\beta_{\alpha}\mu_{xx}^{\alpha} \ . \tag{14}$$

It is seen from these expressions that as usual T is unimodular:

$$\det T=1 \ . \tag{15}$$

The derivation of the dispersion relation from (7) is now standard.[3,16] Bloch's theorem gives

$$\begin{bmatrix} H_{nx}^{(1)} \\ B_{ny}^{(1)} \end{bmatrix}=\exp(ik_y\Lambda)\begin{bmatrix} H_{(n-1)x}^{(1)} \\ B_{(n-1)y}^{(1)} \end{bmatrix} , \tag{16}$$

where k_y is the Bloch wave vector. Comparison of Eqs. (7) and (16), with use of (15), then yields

$$\cos(k_y\Lambda)=\tfrac{1}{2}\mathrm{Tr}T \ . \tag{17}$$

Substitution from (8) and (11) leads to the explicit form

$$\cos(k_y\Lambda)=\cos(k_{1y}a)\cos(k_{2y}b) -(\Delta_1^1\Delta_1^2+\tfrac{1}{2}\Delta_2^1\Delta_3^2+\tfrac{1}{2}\Delta_3^1\Delta_2^2) \times\sin(k_{1y}a)\sin(k_{2y}b) \ , \tag{18}$$

where we have reinstated $k_{\alpha y}$ in place of β_{α}.

Equations (7) and (18) are the central results of this paper; it will be shown in the next section that they contain a number of previously known results as special cases. The case of TM polarization, with **H** along z, the external field direction, is not expected to be of any great significance, since μ_{zz} is at most weakly dispersive. For completeness, however, we have investigated this case. The dispersion relation is

$$\cos(k_y\Lambda)=\cos(k_{1y}a)\cos(k_{2y}b) -\tfrac{1}{2}(k_{1y}\epsilon_2/k_{2y}\epsilon_1+k_{2y}\epsilon_1/k_{1y}\epsilon_2) \times\sin(k_{1y}a)\sin(k_{2y}b) , \tag{19}$$

where now

$$k_{\alpha y}=(\epsilon_\alpha\mu_{zz}^{\alpha}\omega^2/c^2-k_x^2)^{1/2} . \tag{20}$$

III. SPECIAL CASES

Equations (7), (18), and (19) are quite general. If both media are nonmagnetic, $\vec{\mu}=I$, they describe optical propagation in a superlattice of alternating dielectric layers, for which the formal results are given by Yeh *et al.*[16] Our results do indeed reduce to theirs in this limit. In particular, Eq. (18) becomes

$$\cos(k_y\Lambda)=\cos(k_{1y}a)\cos(k_{2y}b) -\tfrac{1}{2}(k_{1y}/k_{2y}+k_{2y}/k_{1y}) \times\sin(k_{1y}a)\sin(k_{2y}b) \tag{21}$$

with

$$k_{\alpha y}=(\epsilon_\alpha\omega^2/c^2-k_x^2)^{1/2} \tag{22}$$

as derived by Yeh *et al.*[16] Equation (19), for TM propagation, is already in the form quoted by Yeh *et al.*;[16] for nonmagnetic media $\mu_{zz}^{\alpha}=1$ in (20). If one medium is taken as magnetic and the other as nonmagnetic we recover Barnaś's results.[10] In particular, (18) reduces to his Eq. (23).

IV. LONG-WAVELENGTH LIMIT

Ferromagnetic resonance frequencies are typically in the microwave frequency region, and antiferromagnetic resonance frequencies in the far infrared. Thus in the regions of high frequency dispersion, which are those of most interest, the free-space wavelength is much greater than the superlattice period Λ. This means that, perhaps with the exception of very narrow frequency intervals close to resonance, the wave numbers k appearing in the dispersion equations are small compared with Λ^{-1}. It is therefore possible to carry out systematic Taylor expansions to order $k^2\Lambda^2$. As mentioned in Sec. I, the resulting approximations have proved very useful in the description of the reststrahl region of semiconductor superlattices.

The expansions are straightforward, and we simply quote the result; (18) reduces to

$$k_y^2(a+b)^2+k_x^2\left[a^2+b^2-\left[\frac{2\mu_{xy}^1\mu_{xy}^2}{\mu_{xx}^1\mu_{xx}^2}-\frac{(\mu_{xx}^2)^2+(\mu_{xy}^2)^2}{\mu_{xx}^1\mu_{xx}^2}-\frac{(\mu_{xx}^1)^2+(\mu_{xy}^1)^2}{\mu_{xx}^1\mu_{xx}^2}\right]ab\right]=\frac{\omega^2}{c^2}[(\mu_v^1a+\mu_v^2b)(\epsilon_1a+\epsilon_2b)] . \tag{23}$$

Equation (23) describes propagation in the superlattice, treated as an effective homogeneous anisotropic medium. It is not easy to appreciate its significance as it stands, so we now give an alternative derivation which is a development of the method used for a dielectric superlattice by Agranovich and Kravtsov.[12]

We consider an rf magnetic field in the x-y plane of the superlattice of Fig. 1; the wavelength is taken much longer than the superlattice period so that the **B** and **H** fields are the same in successive layers of component 1 and in successive layers of component 2. The boundary conditions are that H_x and B_y are continuous across interfaces, so these components are everywhere equal to their average values $\overline{H}_x$ and $\overline{B}_y$. In medium 1 the field components are related by

$$B_x/\mu_0=\mu_{xx}^1\overline{H}_x+\mu_{xy}^1H_y , \tag{24}$$

$$\overline{B}_y/\mu_0=-\mu_{xy}^1\overline{H}_x+\mu_{xx}^1H_y , \tag{25}$$

so that

$$B_x/\mu_0=\mu_v^1\overline{H}_x+(\mu_{xy}^1/\mu_{xx}^1)\overline{B}_y/\mu_0 , \tag{26}$$

$$H_y=(\mu_{xy}^1/\mu_{xx}^1)\overline{H}_x+(1/\mu_{xx}^1)\overline{B}_y/\mu_0 . \tag{27}$$

Similar equations hold in component 2, and combined with (26) and (27) give expressions for the average fields $\overline{B}_x=(aB_x^1+bB_x^2)/(a+b)$ and $\overline{H}_y$:

$$(a+b)\overline{B}_x/\mu_0=(a\mu_v^1+b\mu_v^2)\overline{H}_x +(a\mu_{xy}^1/\mu_{xx}^1+b\mu_{xy}^2/\mu_{xx}^2)\overline{B}_y/\mu_0 , \tag{28}$$

$$(a+b)\overline{H}_y=(a\mu_{xy}^1/\mu_{xx}^1+b\mu_{xy}^2/\mu_{xx}^2)\overline{H}_x +(a/\mu_{xx}^1+b/\mu_{xx}^2)\overline{B}_y/\mu_0 . \tag{29}$$

Reorganization gives relations in terms of the effective medium permeability tensor

$$\begin{bmatrix}\overline{B}_x\\ \overline{B}_y\end{bmatrix}=\mu_0\begin{bmatrix}\bar{\mu}_{xx} & \bar{\mu}_{xy}\\ -\bar{\mu}_{xy} & \bar{\mu}_{yy}\end{bmatrix}\begin{bmatrix}\overline{H}_x\\ \overline{H}_y\end{bmatrix} \tag{30}$$

with

$$\bar{\mu}_{xx}=\frac{(a+b)^2\mu_{xx}^1\mu_{xx}^2+ab[(\mu_{xx}^1-\mu_{xx}^2)^2+(\mu_{xy}^1-\mu_{xy}^2)^2]}{(a+b)(a\mu_{xx}^2+b\mu_{xx}^1)} , \tag{31}$$

$$\bar{\mu}_{xy}=\frac{a\mu_{xy}^1\mu_{xx}^2+b\mu_{xy}^2\mu_{xx}^1}{a\mu_{xx}^2+b\mu_{xx}^1} , \tag{32}$$

$$\bar{\mu}_{yy} = \frac{(a+b)\mu_{xx}^1 \mu_{xx}^2}{a\mu_{xx}^2 + b\mu_{xx}^1} . \tag{33}$$

It is straightforward to find the dispersion equation for wave propagation in a medium with the permeability tensor (30). For a TE mode, the relevant component of the dielectric tensor in the effective-medium description is ϵ_{xx}

$$\epsilon_{xx} = (\epsilon_1 a + \epsilon_2 b)/(a+b) . \tag{34}$$

The dispersion equation is

$$\epsilon_{xx}\omega^2/c^2 = \frac{k_x^2}{\bar{\mu}_{yy} + \bar{\mu}_{xy}^2/\bar{\mu}_{xx}} + \frac{k_y^2}{\bar{\mu}_{xx} + \bar{\mu}_{xy}^2/\bar{\mu}_{yy}} . \tag{35}$$

Substitution of (31) to (33) shows that this is identical to (23). Thus, as stated, the latter is simply the propagation equation for the effective medium described by (30) and (34).

Most previous work on magnetic superlattices, for example, Refs. 6, 7, and 10, have considered the case when one component is nonmagnetic. The effective-medium description of such a superlattice is simply obtained from (31) to (33) as the special case $\mu_{xx}^2 = 1$, $\mu_{xy}^2 = 0$. The explicit form of (23), which we quote for later reference, is

$$k_y^2(a+b)^2 + k_x^2[a^2+b^2+(\mu_v^1 + 1/\mu_{xx}^1)ab] = \frac{\omega^2}{c^2}(\mu_v^1 a + b)(\epsilon_1 a + \epsilon_2 b) . \tag{36}$$

As an example of the application of this formalism, we give in Figs. 2 and 3 bulk dispersion curves in the long-wavelength limit for a YIG/YAG superlattice (where YIG represents yttrium iron garnet and YAG represents yttrium aluminum garnet) in an applied magnetic field $H_0 = 3500$ Oe. Values of the saturation magnetization M_0, gyromagnetic ratio γ, and dielectric constant ϵ for these materials are given in Table I.

Figure 2 shows the special case of propagation normal to the layers $k_x = 0$. It can be seen from (23) that in that case k_y diverges at frequencies for which either μ_v^1 or μ_v^2 has a pole. The pole in μ_v is at

$$\omega_v = \gamma H_0^{1/2}(H_0 + M_0)^{1/2} . \tag{37}$$

For the numerical values given in Table I, these frequencies are 0.376 cm^{-1} (YAG) and 0.392 cm^{-1} (YIG). The corresponding divergences are seen in Fig. 2. Similarly, (23) shows that the zeros of k_y occur at the zeros of $a\mu_v^1 + b\mu_v^1$, namely 0.382 and 0.461 cm^{-1} for the example chosen.

TABLE I. Parameters of YIG and YAG used for dispersion curves from Refs. 17 and 18.

	M_0 (300 K) (Oe)	γ $(\text{Oe s})^{-1}$	ϵ
YIG	1750	1.76×10^7	15.0
YAG	1200	1.76×10^7	14.8

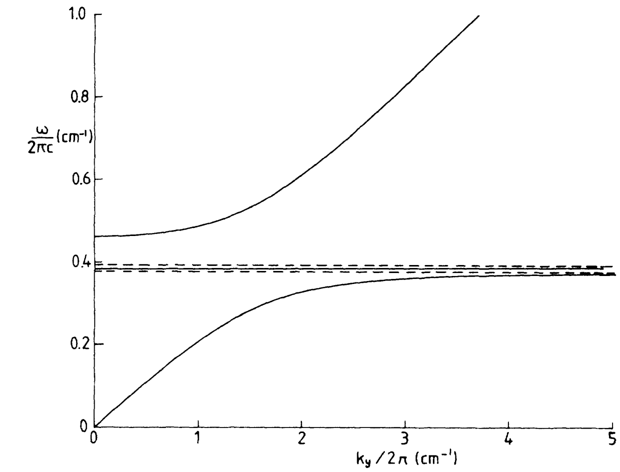

FIG. 2. Long-wavelength bulk dispersion curves for an infinite YIG/YAG superlattice with $a = b$ and propagation normal to the layers.

Figure 3 is for propagation parallel to the layers $k_y = 0$. Inspection of (23) shows that as in Fig. 2 there are two divergences of k_x. The zeros of k_x occur at the frequencies for which $a\mu_v^1 + b\mu_v^2 = 0$, as do the zeros of k_y in Fig. 2. There is, however, no simple argument to locate the frequencies at which poles of k_x are found, because of the rather complicated form of $\bar{\mu}_{xx}$ in (31), or equivalently the rather complicated coefficient of k_x^2 in (23). For the example chosen, our numerical work shows that the poles of k_x in Fig. 3 occur at about 0.375 and 0.394 cm^{-1}, close to the poles of k_y in Fig. 2, but not the same.

We believe that the close similarity between Figs. 2 and 3 is an accidental consequence of our choice of materials, and probably stems from the fact that we have used parameters for two garnets, both with gyromagnetic ratio $g = 2$. Even for the simple case of the magnetic/nonmagnetic superlattice, (36) shows that

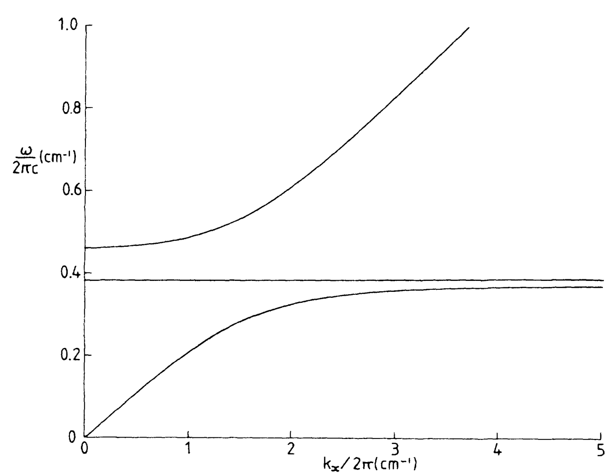

FIG. 3. As in Fig. 2, for propagation parallel to the layers.

whereas the poles and zeros of k_y are determined by those of μ_v, the poles and zeros of k_x are strongly influenced by those of μ_{xx} as well as μ_v. Thus as in the dielectric case[11] the building of the superlattice may be expected to produce strong and distinctive anisotropies.

V. CONCLUSIONS

The principal results of this paper are the transfer matrix and dispersion equation for TE modes (7) and (18), the dispersion equation for TM modes (19), and the simplifications of the dispersion equations in the long-wavelength limit (30) to (33). All the results apply to bulk (infinite) superlattices. The numerical results presented have been restricted to simple illustration in one particular case.

There is scope for considerably more discussion of these results than has been given here. The general consequence of the existence of the long period Λ is that Brillouin-zone edges appear at $k_y = n\pi/\Lambda$. The bulk dispersion equations therefore have the property that stop bands where k_y is complex, $k_y = n\pi/\Lambda + i\eta$, may appear at the Brillouin-zone edges. One implication is that for frequencies within the stop bands surface modes may appear on a semi-infinite superlattice; this is illustrated for the dielectric case by Yeh *et al.*[16] Surface polaritons may also arise in frequency intervals where the Voigt susceptibility is negative; these surface polaritons were first discussed for a semi-infinite homogeneous magnetic medium by Hartstein *et al.*[19] The relation between surface polaritons and stop-band-related surface modes has not been fully discussed even for dielectric superlattices, and certainly needs investigation for the magnetic case.

We believe that the results for the long-wavelength limit presented in Sec. IV should prove very useful, for example, in device design using ferromagnetic superlattices. In antiferromagnetic superlattices, there is likely to be considerable interest in surface polaritons and guided waves in semi-infinite and thin-film specimens. The description given by the long-wavelength expressions facilitates discussion of these modes and the associated ATR spectrum.

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