Susceptibility and correlation functions in amorphous magnets

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We calculate the site-dependent susceptibility and the neutron scattering form factor S_Q of the random-anisotropy model (RAM) in site-dependent mean-field theory with an additional Lagrange parameter. We obtain for S_Q a Lorentzian and a Lorentzian-squared term. The latter vanishes above the characteristic temperature T_c in zero field but persists at low temperature for all fields. The susceptibility remains finite at all temperatures. The spin-correlation length becomes temperature independent below T_c (apart from the temperature dependence of the exchange and anisotropy constants) but depends strongly on an applied field and on additional coherent uniaxial anisotropy. The saturation magnetization decreases in high fields as h^{-2}.

I. INTRODUCTION

Most magnetically disordered system can be classified either as spin glasses, or as random-field or random-anisotropy systems. All these systems are commonly described by the Hamiltonian

$$H = -\tfrac{1}{2}\sum_{i,j} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j - \sum_i \mathbf{h}_i\cdot\mathbf{S}_i - \tfrac{1}{2}D\sum_i(\hat{\mathbf{n}}_i\cdot\mathbf{S}_i)^2 , \quad (1)$$

with the spins (or, more generally, total momenta) $\mathbf{S}_i$. In the case of spin glasses one has competing positive and negative exchange interactions J_{ij}. Most spin glasses are diluted alloys with a nonmagnetic matrix.[1] Spin glasses can be crystalline or amorphous. Random-field systems are characterized by a random distribution of fields $P(\mathbf{h}_i)$ and are equivalent to diluted antiferromagnets in an uniform field.[2] In random-anisotropy systems one has a random distribution $P(\hat{\mathbf{n}}_i)$ of unit vectors $\hat{\mathbf{n}}_i$ which point out into local easy-axis directions. The anisotropy strength D is assumed to be constant. In both the random-field model (RFM) and the random-anisotropy model (RAM) the exchange is assumed to be predominantly ferromagnetic. In some cases additional nonrandom anisotropy (which depends on the lattice structure) is also important.

Most disordered *amorphous* magnetic alloys are either spin glasses or random-anisotropy systems.[3] In some cases no clear distinction between both cases can be made since there are systems with fairly random exchange *and* with random anisotropies. Most random-anisotropy systems are either amorphous rare-earth alloys or amorphous rare-earth–transition-metal alloys such as Y, Nd, Pr, Sm, Gd, Tb, Dy, Ho, Er, or Tm with Fe, Co, Ni, Cu, Ag, or Au.[3,4] Here, the transition-metal ions can be magnetic or nonmagnetic. Some of these systems order magnetically at sufficiently large concentrations. We do not discuss amorphous insulating glasses.[3]

The RAM was investigated initially in mean-field theory (MFT).[5–12] Some authors predicted a ferromagnetic state with coercitivity, remanence,[7] hysteresis loops,[8–10] and other ferromagnetic effects. However, the possibility of a spin-glass-like state was also suggested early.[6,9–12] Computer simulations in three dimensions led to conflicting results. Chi and Alben[8] found that the lowest-lying states are ferromagnetic with finite remanence, whereas Chi and Egami[12] suggested an instability of this ferromagnetic state. Harris and Sung[9] concluded that there are spin-glass-like states at higher energies which are separated by large energy barriers from each other and from the ferromagnetic ("asperomagnetic") ground state with finite spontaneous magnetization but strong spin disorder. This is in contrast to very careful Monte Carlo simulations of Jayaprakash and Kirkpatrick,[13] for large random anisotropies, who concluded that the ground state exhibits significant short-range ferromagnetic order but no long-range magnetic order.

This conclusion is consistent with an estimate of the energy for the formation of magnetic domains. Such an estimate was first made by Imry and Ma[14] for the RFM, but can easily be applied to the RAM.[15] This estimate predicts ferromagnetic order for all ratios D/J and for dimensions $d \geq 4$, where J is the exchange for nearest neighbors. For $d < 4$ the system should break into magnetic domains of size $L \sim (J/D)^{2/(4-d)}$. The flaw in this argument is that it does not take into account the entropy that might be important in disordered systems. A mean-field theory in which the ferromagnetic exchange interactions J_{ij} in (1) are all equal and of infinite range (corresponding to $d \to \infty$) indeed leads to a *ferromagnetic* state for all ratios of anisotropy to exchange.[16,17] This in contrast to a spin glass, where the same type of mean-field theory [as defined by the Sherrington-Kirkpatrick (SK) model[18]] leads to a spin-glass state and where many predictions of this theory compare favorably with experiments.[19]

One possibility for going beyond MFT is to use renormalization-group methods.[15,20–23] In Refs. 20–23 the average over the random axes $\hat{\mathbf{n}}_i$ with $\sum_{\alpha=1}^{n} n_{i\alpha}^2 = 1$ is performed by employing a truncated cumulant expansion of the anisotropy term in (1). This is equivalent to

replacing the distribution $P(\hat{\mathbf{n}}_i)=\text{const}$ by independent Gaussian distributions for all components $n_{i\alpha}$. This expansion is questionable since it leads, in the limit of infinite-range exchange interactions and strong anisotropy, to a spin-glass state, whereas the MFT (which is exact in this limit) predicts a ferromagnetic state. For short-range ferromagnetic exchange the cumulant expansion leads to an effective Hamiltonian which can be analyzed using conventional renormalization-group techniques. One finds below four dimensions an instability of the ferromagnetic fixed point.[20,21] Further investigations[21–23] of this effective Hamiltonian showed that it describes, for $d<4$, most likely a spin glass. However, the upper critical dimension of an ordinary spin glass is 6 instead of 4 and the behavior of this model between $d=4$ and 6 is not yet completely clear.[23]

The RAM can be treated rather rigorously in the limit $n\to\infty$, where n is the number of spin components.[24–28] A $1/n$ expansion yields results consistent with the Imry-Ma argument discussed above[14,15] and sufficiently large additional coherent anisotropy (such as cubic anisotropy) restores in $d=3$ the long-range ferromagnetic order via a first-order phase transition.[24] These calculations have another flaw, pointed out by Bray and Moore:[29] One has in the limit $n\to\infty$ a spin-glass transition at finite temperatures even for a single spin. So this limit is not useful in order to determine a critical dimension. More information about this question yields the large-cell renormalization-group method used in Ref. 29. This method leads for the RAM with $D/J\to\infty$ and $d=2$ to a zero-temperature phase transition with critical exponents consistent with those of a two-dimensional (2D) Ising spin glass. A RAM with infinitely large anisotropy indeed is an Ising system.[13,16,17] The authors of Ref. 29 conclude that it belongs to the universality class of the Ising spin glasses, at least for $d=2$ and probably for all $d\leq 4$.

Considerable progress in determining the spin structure of the RAM has been made by Chudnovsky *et al.*,[30–34] Serota and Lee,[35] and Saslow.[36] These authors calculate the static spin correlations at low temperatures in the framework of a Ginzburg-Landau model with small random anisotropy. The spin correlations turn out to be finite ranged and scale with the correlation length $\xi\sim R_a(A/R_aD)^2$, where $A(\nabla_\alpha M_\beta)^2$ is the exchange energy and R_a is the correlation length for the random axes. This result is consistent with the Imry-Ma estimate for $d=3$ mentioned above. Depending on the external field h, the authors of Refs. 34 and 36 distinguish three different magnetic regions. The state for $h=0$ has zero net magnetization and can be described as a "correlated spin-glass state" with large but finite susceptibility and slowly rotating moments. A relatively small field h aligns the moments to a considerable extent. The authors of Refs. 34 and 36 call this state a "ferromagnet with wandering axis." The spin structure is slightly noncollinear with a correlated tipping of the spin directions. Coey[3] calls this an "asperomagnetic" state. In a large field, finally, the tipping angles should be completely uncorrelated from site to site. Additional coherent (i.e., nonrandom) anisotropy acts roughly like an additional field[31,32,34] and in this sense enhances the tendency towards ferromagnetic order. A finite spin-correlation length at low temperature has also been obtained by Dotsenko and Feigelman[37,38] for a three-dimensional random-anisotropy model with planar spins. These authors find that the static susceptibility diverges at the phase transition with $\tau=(T_c-T)/T_c\to 0$ as $D^{-4}\tau^{-0.7}$ and varies at low temperatures as D^{-4}.

In this paper we calculate the static spin-correlation function and susceptibility of the RAM in site-dependent MFT based on the Hamiltonian (1) with additional coherent anisotropy and a uniform magnetic field. We consider only weak anisotropy. Our results are as follows. Above the characteristic temperature T_c the system is very similar to a ferromagnet. However, the spin-correlation function (which is proportional to the neutron scattering form factor) contains an additional term in the form of a Lorentzian squared which vanishes in zero field. A similar result recently has been obtained by Feigelman and Tsodyks[37] in the limit of strong anisotropy. Below T_c the susceptibility remains finite. One has in zero field the spin-correlation length ξ_D, due to random anisotropy, which agrees with that found in Ref. 34. The form factor consists for all fields (including the case $h=0$) of a Lorentzian and a Lorentzian squared. The latter has been predicted already by Chudnovsky.[33] A Lorentzian plus Lorentzian-squared form factor with undetermined coefficients has also been suggested by Aharony and Pytte.[38] These authors predicted a low-temperature phase of the RAM with zero net magnetization but diverging susceptibility,[39] in contrast to our results. We find that in a finite field ($J\gg h\gg A\xi_D^{-2}$) the correlation length ξ_D has to be replaced by $\xi=(\xi_D^{-2}+\xi_{\rm FM}^{-2})^{-1/2}$, where $\xi_{\rm FM}^{-2}=h/AM$, is the ferromagnetic contribution, and where M is the induced magnetization. For $h\gg J$ the magnetization deviates from the saturation magnetization $M=S$ by a term proportional to h^{-2}. Similar results have been derived in Ref. 34 from a phenomenological model.

Additional coherent uniaxial anisotropy reduces the spin-correlation length perpendicular to the easy axis. One has the correlation length $\xi=(\xi_D^{-2}+\xi_{\rm FM}^{-1}+\xi_c^{-2})^{-1/2}$, where ξ_c is the Bloch-wall thickness of an ideal ferromagnet. If $\xi\ll\xi_c$ Bloch walls no longer can exist and the spin structure is determined by the random anisotropy. In the opposite limit and for sufficiently small domains the system looks like an ideal ferromagnetic with additional small tilting of the spins within the domains.

II. GENERAL THEORY

We mentioned already in the Introduction that a MFT for the RAM based on infinite-range ferromagnetic interactions[16,17] leads to a ferromagnetic state which disagrees with the estimate of Imry and Ma[14,15] and most other calculations. A simple improvement of the MFT is a local (or site-dependent) mean-field theory which has been considered already in earlier work on the RAM.[4–12] Here we go one step further (which turns out to be crucial) and take into account the constraint

$\mathbf{S}_i^2=\sum_{\alpha=1}^{n} S_{i\alpha}^2=1$ by means of a Lagrange parameter λ_i. Our model is then defined by the Hamiltonian

$$\tilde{H}=-\tfrac{1}{2}\sum_{i,j} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j-\sum_i \mathbf{h}_i\cdot\mathbf{S}_i+\tfrac{1}{2}\sum_i \lambda_i(\mathbf{S}_i^2-1)-H_D \ , \tag{2}$$

$$H_D=-\tfrac{1}{2}D\sum_i(\hat{\mathbf{n}}_i\cdot\mathbf{S}_i)^2 \ , \tag{3}$$

with short-range ferromagnetic exchange interactions J_{ij} and a site-dependent nonrandom field $\mathbf{h}_i$. We are interested in the susceptibility χ_{ij}^{α}. One has, from linear-response theory,

$$T\chi_{ij}^{\alpha}=[\langle S_{i\alpha}S_{j\alpha}\rangle]_D-q_{ij}^{\alpha} \ , \tag{4}$$

with the generalized spin-glass order parameter[1]

$$q_{ij}^{\alpha}=[M_{i\alpha}M_{j\alpha}]_D, \quad M_{i\alpha}=\langle S_{i\alpha}\rangle \tag{5}$$

where $\langle\ \rangle$ denotes the thermal average and $[\ ]_D$ the average over a distribution of random axes. Both χ_{ij}^{α} and q_{ij}^{α} depend in a constant field $\mathbf{h}$ only on the distance $|\mathbf{x}_i-\mathbf{x}_i|$ between the sites i and j. In a way similar to spin glasses, the RAM might have low-lying metastable states or a nearly degenerate ground state, and (4) and (5) refer to the thermal equilibrium for a given field $\mathbf{h}$. In a spin glass such a state presumably can be reached only by sufficiently slow field cooling.

The neutron scattering form factor is proportional to the Fourier transform of the spin-correlation function

$$S_{\mathbf{Q}}^{\alpha}=\sum_{i-j} e^{i\mathbf{Q}\cdot(\mathbf{x}_i-\mathbf{x}_j)}(T\chi_{ij}^{\alpha}+q_{ij}^{\alpha})\equiv T\chi_{\mathbf{Q}}^{\alpha}+q_{\mathbf{Q}}^{\alpha} \ . \tag{6}$$

An external field $\mathbf{h}$ produces a homogeneous magnetization $\mathbf{M}_0$ and leads in $q_{\mathbf{Q}}^{\alpha}$ to a term $M_{0\alpha}^2\delta_{\mathbf{Q},0}$. We shall calculate the susceptibility χ_{ij}^{α} and the order parameters q_{ij}^{α} in a slightly different way. The susceptibility is obtained from a partition function in which we keep the full random-axis Hamiltonian H_D, Eq. (3), and treat only the exchange in a generalized local MFT. In the case of q_{ij}^{α} we separate H_D into mean-field and fluctuation contributions. The Lagrangian parameters λ_i in both cases are written as

$$\lambda_i=J_0\lambda+A\bar{\lambda}+\delta\lambda_i \ . \tag{7}$$

Here, $J_0\lambda$ with $J_0=\sum_k J_k$ is the conventional ferromagnetic contribution which leads to a shift of T_c,[40] and $A\bar{\lambda}$ and $\delta\lambda_i$ are contributions which vanish for $D=0$. The constant A will be defined below.

A. Susceptibility

We calculate the susceptibility χ_{ij}^{α} by means of the effective field

$$\mathbf{h}_i^{\text{eff}}=\mathbf{h}_i+\sum_j J_{ij}\mathbf{M}_j-(J_0\lambda+A\bar{\lambda})\mathbf{M}_i \ , \tag{8}$$

and the effective Hamiltonian

$$\tilde{H}_{\text{eff}}=-\sum_i \mathbf{h}_i^{\text{eff}}\cdot\mathbf{S}_i+\tilde{H}_D+\tfrac{1}{2}\sum_{i,j} J_{ij}\mathbf{M}_i\cdot\mathbf{M}_j -\tfrac{1}{2}(J_0\lambda+A\bar{\lambda})\sum_i(M_i^2-1) \ , \tag{9}$$

$$\tilde{H}_D=\tfrac{1}{2}D\sum_i(\hat{\mathbf{n}}_i\cdot\mathbf{S}_i)^2-\tfrac{1}{2}\sum_i\delta\lambda_i(\mathbf{S}_i^2-1) \ . \tag{10}$$

We write $\mathbf{h}_i=\mathbf{h}+\delta\mathbf{h}_i$, $\mathbf{M}_i=\mathbf{M}_{i0}+\delta\mathbf{M}_i$, and $\mathbf{h}_i^{\text{eff}}=\mathbf{h}_{i0}^{\text{eff}}+\delta\mathbf{h}_i^{\text{eff}}$, where the magnetization $\delta\mathbf{M}_i$ is induced by the field $\delta\mathbf{h}_i\to 0$ and where the magnetization $\mathbf{M}_{i0}$ still depends on site i due to the disorder. Expansion of the local partition function

$$Z_i=\text{Tr}\exp\{-\beta[\tilde{H}_{D,i}+(\mathbf{h}_{i0}^{\text{eff}}+\delta\mathbf{h}_i^{\text{eff}})\mathbf{S}_i]\} \tag{11}$$

to second order in $\delta\mathbf{h}_i^{\text{eff}}$ leads to the induced magnetization

$$\delta M_{i\alpha}=\beta\sum_{\gamma}(\langle S_{i\alpha}S_{i\gamma}\rangle_0-M_{i0\alpha}M_{i0\gamma}) \times\left[\delta h_{i\gamma}+\sum_j J_{ij}\delta M_{i\gamma}-(J_0\lambda+A\bar{\lambda})\delta M_{i\gamma}\right] , \tag{12}$$

where $M_{i0\alpha}$ and $\langle S_{i\alpha}S_{i\gamma}\rangle_0$ are averages for $\delta h_i^{\text{eff}}=0$. In what follows we restrict ourselves to small random anisotropy and average the two factors on the right-hand side (rhs) of (12) separately over a distribution of axes $\hat{\mathbf{n}}_i$. This is correct to lowest order in D and leads to

$$[\delta M_{i\alpha}]_D=\beta(p_\alpha-q_\alpha)\left[\delta h_{i\alpha}-\sum_j J_{ij}[\delta M_{j\alpha}]_D -(J_0\lambda+A\bar{\lambda})[\delta M_{i\alpha}]_D\right] , \tag{13}$$

where we have chosen a coordinate system in which $[\langle S_{i\alpha}S_{i\gamma}\rangle_0]_D$ and $[M_{i0\alpha}M_{i0\gamma}]_D$ are diagonal,

$$[\langle S_{i\alpha}S_{i\gamma}\rangle_0]_D=p_\alpha\delta_{\alpha\gamma}, \quad [M_{i0\alpha}M_{i0\gamma}]_D=q_\alpha\delta_{\alpha\gamma} \ . \tag{14}$$

The parameters

$$p_\alpha=[\langle S_{i\alpha}^2\rangle_0]_D, \quad q_\alpha=[M_{i0\alpha}^2]_D \tag{15}$$

enter in this form also into the MFT of spin glasses.[1] If the exchange interactions J_{ij} of the amorphous system differ little from those of the crystalline state, i.e., if $J_{ij}=J_{i-j}$, Eq. (13) can be solved by Fourier transformation. We have the susceptibility

$$\chi_{\mathbf{Q}}^{\alpha}=\frac{\delta[M_{\mathbf{Q}\alpha}]_D}{\delta h_{\mathbf{Q}\alpha}}=\frac{p_\alpha-q_\alpha}{T-(p_\alpha-q_\alpha)(J_{\mathbf{Q}}-J_0\lambda-A\lambda)} \ . \tag{16}$$

For small $\mathbf{Q}$ values $J_{\mathbf{Q}}$ can be expanded,

$$J_{\mathbf{Q}}=J_0(1-b^2Q^2), \quad J_0=\sum_{i-j} J_{i-j} \tag{17}$$

and (16) can be written,

$$\chi_{\mathbf{Q}}^{\alpha}=\frac{1}{A(Q^2+\xi_\alpha^{-2})} \ , \tag{18}$$

where

$$\xi_\alpha^{-2} \equiv \frac{T-T_\alpha}{A(p_\alpha - q_\alpha)} + \bar{\lambda} \equiv \xi_{\mathrm{FM},\alpha}^{-2} + \bar{\xi}_D^{-2} , \tag{19}$$

$$T_\alpha = (p_\alpha - q_\alpha)T_0, \quad T_0 = J_0(1-\lambda), \quad A = J_0 b^2 , \tag{20}$$

and where the parameter b is of the order of the average lattice constant.

The susceptibility (18) has the same $\mathbf{Q}$ dependence as an ideal ferromagnet in MFT. However, the correlation length ξ_α still depends through the parameters p_α, q_α, and $\bar{\lambda}$ on the anisotropy $\tilde{H}_D$, Eq. (10). We shall calculate these parameters in Sec. III for various temperature and field regions.

B. Correlation function q_{ij}^α

In the preceding subsection we calculated the response $\delta M_{i\alpha}$ induced by a small field $\delta h_{i\alpha}$. In this subsection we consider the local response $\delta M_{i\alpha}$ due to the anisotropy interaction $\tilde{H}_D$, Eq. (10), which is assumed to be small. For this purpose we add to $\mathbf{h}_i^{\mathrm{eff}}$, Eq. (8), the anisotropy term in local MFT,

$$\bar{\mathbf{h}}_i^{\mathrm{eff}} = \mathbf{h} + \sum_j J_{ij}\mathbf{M}_j + D(\hat{\mathbf{n}}_i \cdot \mathbf{M}_i)\hat{\mathbf{n}}_i - \lambda_i \mathbf{M}_i , \tag{21}$$

with $\delta \mathbf{h}_i = 0$, and replace, in Eq. (10), $\mathbf{S}_i$ by $\mathbf{S}_i - \mathbf{M}_i$. In a finite field $\mathbf{h}$ we have $\mathbf{M}_i = \bar{\mathbf{M}}_i + \delta \mathbf{M}_i$ and $\bar{\mathbf{h}}_i^{\mathrm{eff}} = \bar{\mathbf{h}}_{i0}^{\mathrm{eff}} + \delta \bar{\mathbf{h}}_i^{\mathrm{eff}}$, where $\bar{\mathbf{M}}_i$ is the induced magnetization. Equation (12) now reads, in obvious notation,

$$\delta M_{i\alpha} = \beta(p_\alpha - q_\alpha)\left[D(\hat{\mathbf{n}}_i \cdot \mathbf{M}_i) n_{i\alpha} - \delta\lambda_i M_{i\alpha} + \sum_j [J_{ij} - \delta_{ij}(J_0\lambda + A\bar{\lambda})]\delta M_{j\alpha} \right] , \tag{22}$$

where we replaced the quantities $\langle S_{i\alpha} S_{i\gamma} \rangle_0$ and $M_{i\alpha} M_{i\gamma}$ by their averages over the random axes, and where averages like $\langle \ \rangle_0$ are taken with (9) and (10), but without the MF contribution of $\tilde{H}_D$. Again, this is correct to lowest order in D.[41]

Equation (22) can be solved by means of the Green's function G_{ij}^α as defined by

$$\sum_l \{\delta_{il} - \beta(\bar{p}_\alpha - \bar{q}_\alpha)[J_{il} - \delta_{il}(J_0\lambda + A\bar{\lambda})]\} G_{lj}^\alpha = \beta(\bar{p}_\alpha - \bar{q}_\alpha)\delta_{ij} . \tag{23}$$

One has the formal solution

$$\delta M_{i\alpha} = \sum_l G_{il}^\alpha [D(\hat{\mathbf{n}}_l \cdot \mathbf{M}_l) M_{l\alpha} - \delta\lambda_l M_{l\alpha}] . \tag{24}$$

Equation (23) again is solved by Fourier transformation. The Fourier transform of G_{ij}^α,

$$G_{\mathbf{Q}}^\alpha = \frac{\bar{p}_\alpha - \bar{q}_\alpha}{T - (\bar{p}_\alpha - \bar{q}_\alpha)(J_{\mathbf{Q}} - J_0\lambda - A\bar{\lambda})} , \tag{25}$$

is, apart from the slightly different averages, identical to the susceptibility (16). One has, in the limit $\mathbf{Q} \to 0$,

$$G_{\mathbf{Q}}^\alpha = \frac{1}{A(Q^2 + \bar{\xi}_\alpha^{-2})} , \qquad \bar{\xi}_\alpha^{-2} = \frac{T - \bar{T}_\alpha}{A(\bar{p}_\alpha - \bar{q}_\alpha)} + \bar{\lambda} = \bar{\xi}_{\mathrm{FM},\alpha}^{-2} + \bar{\xi}_D^{-2} \tag{26}$$

with $\bar{T}_\alpha = (\bar{p}_\alpha - \bar{q}_\alpha)T_0$.

For an isotropic distribution, $P(\hat{\mathbf{n}}_i)$ holds $[\delta M_{i\alpha}]_D = 0$, which determines the Lagrange parameter $\delta\lambda_i$. One has, with

$$\sum_\alpha n_{i\alpha}^2 = 1, \quad [n_{i\alpha} n_{i\beta}]_D = \delta_{\alpha\beta}/n \tag{27}$$

to lowest order in D, from (24) (see also Ref. 33),

$$\delta\lambda_i = D(\hat{\mathbf{n}}_i \cdot \mathbf{M}_i)^2 / M_i^2 \tag{28}$$

and

$$\delta M_{i\alpha} = D \sum_l G_{il}^\alpha \hat{\mathbf{n}}_l \cdot \mathbf{M}_l [n_{l\alpha} - M_{l\alpha}(\hat{\mathbf{n}}_l \cdot \mathbf{M}_l)/M_l^2] . \tag{29}$$

We assume for all axes $\hat{\mathbf{n}}_i$ the same constant distribution function

$$P(\hat{\mathbf{n}}_i) = \Omega_n^{-1} = \Gamma(n/2)/2\pi^{n/2} , \tag{30}$$

where Ω_n is the surface of an n-dimensional hypersphere and $\Gamma(z)$ the gamma function. Two axes with the directions $\hat{\mathbf{n}}_i$ and $\hat{\mathbf{n}}_j$ might be correlated over a certain distance as described by the function Γ_{ij}. We need, for the correlation function (5), the averages

$$[n_{i\alpha} n_{i\beta} n_{j\gamma} n_{j\delta}]_D = [n(n+2)]^{-1} \times [\delta_{\alpha\beta}\delta_{\gamma\delta} + \Gamma_{ij}(\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma})] . \tag{31}$$

A simple ansatz for Γ_{ij} is

$$\Gamma_{ij} = \exp(-|\mathbf{x}_i - \mathbf{x}_j|/R_a) , \tag{32}$$

where the characteristic length R_a is of the order of the average lattice constant.

Equation (29) leads, with (31) to lowest order in D, to the correlation function

$$[\delta M_{i\alpha} \delta M_{j\alpha}]_D \equiv \delta q_{ij}^\alpha = D^2(q - q_\alpha)\frac{2}{n(n+2)} \sum_{k,l} G_{ik}^\alpha G_{jl}^\alpha \Gamma_{kl} , \tag{33}$$

with

$$q = \sum_\alpha q_\alpha = \sum_\alpha [M_\alpha^2]_D . \tag{34}$$

For $|\mathbf{x}_i - \mathbf{x}_j| \gg R_a$ the function G_{jl}^α in (33) can be replaced by G_{jk}^α. This leads to

$$\delta q_{ij}^\alpha = \frac{2\Omega}{n(n+2)} D^2(q - q_\alpha) \sum_l G_{il}^\alpha G_{lj}^\alpha , \tag{35}$$

where

$$\Omega = \sum_k \Gamma_k = \Omega_{\mathrm{at}}^{-1} \int dx^n \Gamma(x) = 8\pi\Omega_{\mathrm{at}}^{-1} R_a^3 . \tag{36}$$

Equation (36) holds only for $n = 3$, and Ω_{at} is the atomic volume. It is convenient to introduce the magnetization

density $\delta\mathbf{M}(\mathbf{x}_i)=\Omega_{\rm at}^{-1}\delta\mathbf{M}_i$ and to write (35) for $n=3$ in the form

$$[\delta M_\alpha(\mathbf{x}_l)\delta M_\alpha(\mathbf{x}_2)]_D \equiv \delta q_\alpha(\mathbf{x}_1,\mathbf{x}_2) = \tfrac{16}{15}\pi R_a^3 D^2(q-q_\alpha) \times \int d^3x\, G^\alpha(\mathbf{x}_1-\mathbf{x})G^\alpha(\mathbf{x}_2-\mathbf{x}) . \tag{37}$$

The integral in (37) can be evaluated for large distances, for which (25) can be replaced by (26),

$$\int d^3x\, G^\alpha(\mathbf{x}_1-\mathbf{x})G^\alpha(\mathbf{x}) = (2\pi)^{-3}\int d^3Q\, e^{i\mathbf{Q}\cdot\mathbf{x}_1}(G_{\mathbf{Q}}^\alpha)^2 = A^{-2}(2\pi)^{-3}\int d^3Q\, e^{i\mathbf{Q}\cdot\mathbf{x}_1}(Q^2-\bar\xi_\alpha^{-2})^{-2} = (8\pi A^2)^{-1}\bar\xi_\alpha \exp(-x_1/\xi_\alpha) . \tag{38}$$

This leads, with $\mathbf{M}_i=\overline{\mathbf{M}}+\delta\mathbf{M}_i$, $[\delta\mathbf{M}_i]_D=0$, and $[\overline{M}_{i\alpha}^2]_D\equiv M_{0\alpha}^2$, to

$$[M_\alpha(\mathbf{x}_1)M_\alpha(\mathbf{x}_2)]_D \equiv q_\alpha(\mathbf{x}_1,\mathbf{x}_2) = M_{0\alpha}^2+(\xi_\alpha/2\xi_D) \times (q-q_\alpha)e^{-|\mathbf{x}_1-\mathbf{x}_2|/\bar\xi_\alpha} , \tag{39}$$

where

$$\xi_D \equiv \tfrac{15}{4}R_a^{-3}(A/D)^2 . \tag{40}$$

The result (39) has an interesting property: One has, for $|\mathbf{x}_1-\mathbf{x}_2| \ll \bar\xi_\alpha$,

$$q_\alpha(\mathbf{x}_1,\mathbf{x}_2)=\text{const}-(q-q_\alpha)|\mathbf{x}_1-\mathbf{x}_2|/2\xi_D , \tag{41}$$

i.e., the correlations decay on the scale ξ_D, Eq. (40), which depends on the temperature only through the parameters A and D.

The neutron scattering form factor (6) for small Q values is obtained from (18) and (37) with (38) and (40),

$$S_{\mathbf{Q}}^\alpha = \frac{T}{A}\frac{1}{Q^2+\bar\xi_\alpha^{-2}} + \frac{4\pi\xi_D^{-1}(q-q_\alpha)}{(Q^2+\bar\xi_\alpha^{-2})^2} + M_{0\alpha}^2\delta_{\mathbf{Q},0} . \tag{42}$$

It consists of a Lorentzian and a Lorentzian-squared contribution. This result based on a site-dependent MFT should hold to a good approximation for all temperatures, except near the phase transition. It contains the parameters ξ_α, $\bar\xi_\alpha$, q, and q_α, which will be calculated in the next section. The Lorentzian-squared term is an effect of disorder and vanishes for $\xi_D^{-1}\sim D^2=0$.

III. HIGH TEMPERATURES

For $T>T_c$ and $h=0$ both the local magnetization $\mathbf{M}_i$ and the spin-glass parameter $\mathbf{q}=(\ldots,q_\alpha,\ldots)$ vanish and the form factor (42) reduces to the first term on the rhs. For $D=0$ the spin-correlation length ξ_α, Eq. (19), reduces to that of an ideal ferromagnet in the Ornstein-Zernicke approximation. For $D\neq 0$ one has in (19) essentially the additional term $\bar\xi_D^{-2}$. We will show that at least at low temperatures $\bar\xi_D=\xi_D$, Eq. (40). Hence $\bar\xi_D^{-2}$ is of the order $(D/A)^4\ll 1$ and can be ignored sufficiently far above T_c, but would lead at T_c to a finite correlation length ξ_α. However, near T_c and for $h=0$ the site-dependent MFT breaks down and we do not have a rigorous proof that $\bar\xi_D=\xi_D$ holds at all temperatures. The critical temperature T_c is defined by $\xi_{\rm FM}\to\infty$, which leads, with $T=T_\alpha$, $p_\alpha=S^2/3$, and $q_\alpha=0$, to $T_c=S^2T_0/3$, independently of the random anisotropy [see Eq. (39) of Ref. 52].

In the ideal classical ferromagnet the magnetization is given by the Langevin function

$$\left.\begin{aligned} M &= S(\coth H-1/H) \quad (43a)\\ H &= \beta S(h+T_0M) \quad (43b)\end{aligned}\right\} \quad (D=0)$$

which leads, for the components p_α and q_α parallel and perpendicular to M, to

$$p_1=p_\parallel=S^2-2SM/H,\quad p_2=p_3=p_\perp=SM/H , \tag{44}$$

$$q_\perp=0,\quad q_\parallel=M^2 . \tag{45}$$

One has, to lowest order in M, from (19),

$$\xi_{\rm FM\parallel}^{-2}=\frac{T-T_c+\frac{3}{5}T_0M^2}{\frac{1}{3}AS^2} , \tag{46}$$

$$\xi_{\rm FM\perp}^{-2}=\frac{T-T_c+\frac{1}{5}T_0M^2}{\frac{1}{3}AS^2} . \tag{47}$$

The transverse correlation function $\xi_{\rm FM\perp}$ diverges for $h=0$ with $M=M_s$ (M_s is the spontaneous magnetization) for all temperatures $T\le T_\perp$, where

$$T_\perp=T_c-\tfrac{1}{5}T_0M_s^2 . \tag{48}$$

Such a critical line $T=T_\perp=(p_\perp-q_\perp)T_0$ with $\xi_{\rm FM}^{-1}=0$ and $M_s^2\to q$ should exist also for $D\neq 0$ even if $\xi_\perp^{-2}$, Eq. (19), remains finite. In a finite field one has, for $D=0$ and $T\le T_\perp$,

$$\xi_{\rm FM}^{-2}=\frac{3}{5}\frac{T_0}{AS^2}(M^2-M_s^2) , \tag{49}$$

with $q_\perp=0$ and $q=M^2$. For $D\neq 0$ one has, from (33), $q_\perp=[\delta M_{i1}^2]_D\neq 0$, leading to an additional term in $T_\perp$, Eq. (20), which only vanishes for $h=0$ and $T\ge T_c$. In addition, the (uniform) spontaneous magnetization M_s vanishes for $D\neq 0$.

The Lorentzian-squared term in the form factor (42) vanishes for $h=0$, $T\ge T_c$, where $q=q_\alpha=0$, and becomes, for low fields with $M\sim h$, proportional to h^2. To lowest order in the anisotropy and for a field in direction $\alpha=1$, one has $q_1=M^2$ and $q_2=q_3=0$ or $q-q_\alpha=q_1(1-\delta_{\alpha 1})$, and the longitudinal component of the Lorentzian-squared term vanishes. It becomes nonzero only to order $(D/A)^4$ with

$$q-q_1=q_\perp\simeq M^2\bar\xi_1/\xi_D\simeq M^2\xi_{\rm FM\parallel}/\xi_D\sim D^2$$

from (39) and (40), with $\bar\xi_{\rm FM}\approx\xi_{\rm FM}$. A field-dependent Lorentzian-squared term in the form factor has been ob-

tained also by Feigelmann and Tsodyks[37] for $D \to \infty$ and $T \geq T_c$. In this limit the transverse and longitudinal components of S_Q^α become of the same order of magnitude. A similar term can be derived[42] in the RFM. In this model the factor h^2 has to be replaced by the average $[h_i^2]_{\rm av}$ over the random fields.

IV. LOW TEMPERATURES

The low-temperature limit of the RAM has been discussed quite extensively by Chudnovsky *et al.*[34] We follow their notation and distinguish three regions depending on the applied field.

In zero field the net (or uniform) magnetization is zero. The system is macroscopically isotropic with $\bar{\xi}_\alpha = \bar{\xi}$ and $M_{0\alpha}^2 = 0$ in (39). Summation of (39) over all α leads to

$$[\mathbf{M}(\mathbf{x}_1)\cdot\mathbf{M}(\mathbf{x}_2)]_D = q(\mathbf{x}_1,\mathbf{x}_2) = qe^{-|\mathbf{x}_1-\mathbf{x}_2|/\xi_D} , \quad (50)$$

where

$$\bar{\xi} = \xi_D \quad (51)$$

from the limit $\mathbf{x}_1 \to \mathbf{x}_2$. This result agrees, for $T=0$ and $q=M^2$, with Eq. (2.16) of Ref. 34. For weak anisotropy the correlation length ξ_D, Eq. (40), becomes large and the correlations of the magnetization (43) decay only very slowly. The fluctuations of the magnetization due to the random anisotropy add up to large-angle differences only at large distances of order ξ_D and lead to a destruction of the coherent ferromagnetic state. The system behaves over small distances like a ferromagnet and over large distances like a spin glass with huge ferromagnetic clusters the moments of which point in random directions. This is different in an "ideal" spin glass (with a symmetric bond distribution), which has only few short-range ferromagnetic correlations. The form factor (42) becomes, with (50), $\xi_\alpha = \bar{\xi}_\alpha$, and $q_\alpha = q/3$,

$$S_{\rm Q}^\alpha = \frac{T}{A}\frac{1}{Q^2+\xi_D^{-2}} + \frac{8\pi\xi_D^{-1}q}{3(Q^2+\xi_D^{-2})^2} . \quad (52)$$

A similar form factor (with undetermined coefficients) has been suggested (but not proven) by Aharony and Pytte.[38] The Lorentzian-squared term already has been derived by Chudnovsky,[33] who, unfortunately, replaced $[M_\alpha(\mathbf{x}_1)M_\alpha(\mathbf{x}_2)]_D$ by $[S_\alpha(\mathbf{x}_1)S_\alpha(\mathbf{x}_2)]_D$.

Line shapes in the neutron scattering cross section that could be fitted by the sum of a Lorentzian and Lorentzian squared have been observed[43–48] in amorphous $Tb_{75}Fe_{25}$, Tb_2Fe_{98}, $TbFe_2$, $Fe_{91}Zr_9$, and $NdFe_2$. Some of these alloys show a fairly temperature-independent correlation length ξ (well below T_c). This agrees with $\xi = \xi_D$, Eq. (51), if the exchange constant A and the anisotropy constant D are temperatures independent. Sometimes a sharp increase of ξ near T_c is observed which cannot be explained by our theory. Above T_c one observed in all systems a sharp decrease of $\xi(T)$ with increasing temperature which can be explained by $\xi_{\rm FM}$, Eq. (19), with (20).

In an external field $J_0 \gg h \gg AS\xi_D^{-2}$ the spin system is partly polarized and $\xi_{\rm FM} \neq 0$ even below T_c. We calculate $\xi_{\rm FM}$ for $T \to 0$ and $D \to 0$. One has from (43)–(45), apart from exponentially small terms in H,

$$p_\perp = \frac{TM}{h+T_0M} \quad (53)$$

and the field-induced part

$$\delta p_\perp = p_\perp(h) - p_\perp(0) = -\frac{Th}{T_0(h+T_0M)} , \quad (54)$$

which leads with (19), (20), and $q_\perp \approx 0$ (see below) to

$$\xi_{\rm FM\perp}^{-2} = -\frac{T_0\delta p_\perp}{Ap_\perp} = \frac{h}{AM} , \quad (55)$$

and to

$$\xi_\perp = \bar{\xi}_\perp = \left[\frac{h}{AM} + \xi_D^{-2}\right]^{-1/2} . \quad (56)$$

This decrease of the correlation length has been observed in Refs. 43, 44, and 48, and $\xi_{\rm FM\perp}$ agrees with the "characteristic length" $R_F^\perp$ introduced by Chudnovsky *et al.*[34] from energy considerations. For the transverse magnetization correlation, (39), holds with $q \approx q_\parallel$, $M_\alpha = M\delta_{\alpha 1}$, and $q - q_\alpha = q$ for $\alpha = 1,2,3$,

$$[M_\perp(\mathbf{x}_1)M_\perp(\mathbf{x}_2)]_D = q(\xi_\perp/\xi_D)\exp(-|\mathbf{x}_1-\mathbf{x}_2|/\xi_\perp) \quad (57)$$

and

$$q_\perp = [M_\perp^2]_D = (q/\xi_D)\left[\frac{h}{AM} + \xi_D^{-2}\right]^{-1/2} . \quad (58)$$

Equation (58) agrees with Eq. (3.12) of Ref. 34 in the limit $T \to 0$ with $q = S^2$ if one ignores the term ξ_D^{-2}. The authors of Ref. 34 call this state a "ferromagnet with wandering axis": One has an induced uniform magnetization and, in addition, fairly long-ranged correlations of transverse magnetization. Similar correlations of smaller amplitude exist also for the longitudinal magnetization.

Finally, we consider the *region of very high fields* ($h \gg J_0S$) and, specifically, deviations from the saturation magnetization. We ignore in this limit the exchange interactions. This leads with (16) to

$$\chi_{\rm Q}^\alpha = \beta(p_\alpha - q_\alpha) , \quad (59)$$

and with (35), $\sum_l G_{il}^\perp G_{lj}^\perp = \delta_{ij}(\chi^\perp)^2$, (36), and (40) to

$$[M_\perp^2]_D = q_\perp = \frac{8\pi A^2}{\xi_D}\frac{S^4}{(h+T_0M)^2} . \quad (60)$$

One has from (60) the change of the longitudinal magnetization

$$[\delta M_1]_D \approx [S^2 - M_\perp^2]_D/2S = q_\perp/2S = \frac{4\pi A^2}{\xi_D}\frac{S^3}{(h+T_0M)^2} . \quad (61)$$

A similar h^{-2} dependence has been predicted by Callen *et al.*,[7] Chudnovsky,[31] and Chudnovsky and Serota.[32]

V. EFFECT OF COHERENT ANISOTROPY

Amorphous magnets contain in many cases a certain amount of coherent anisotropy due to internal strains, magnetoelastic effects, etc. In the simplest case this anisotropy is unidirectional,

$$H_c = -\tfrac{1}{2} D_c \sum_i (\hat{\mathbf{n}}_c \cdot \mathbf{S}_i)^2 = -\tfrac{1}{2} D_c \sum_i (S_i^z)^2 , \tag{62}$$

with $\hat{\mathbf{n}}_c = (0,0,1)$. In site-dependent MFT this adds to the mean field (21) the term $D_c(\hat{\mathbf{n}}_c \cdot \mathbf{M}_i)\hat{\mathbf{n}}_c$. This term enters into the susceptibility (16) or (25) and into the averages p_α, q_α, $\bar{p}_\alpha$, and $\bar{q}_\alpha$ in the same way as the external field. Equation (43b) has now to be replaced by

$$\beta S[\bar{h}_0^{\text{eff}}] \equiv H = \beta S \mid \mathbf{h} + T_0 \mathbf{M} + D_c(\hat{\mathbf{n}}_c \cdot \mathbf{M})\mathbf{n}_c \mid . \tag{63}$$

We consider only the case where the field and the magnetization are parallel to the easy axis with

$$H = \beta S[h + M(T_0 + D_c)] . \tag{64}$$

One has at low temperatures, with $\coth H - H^{-1} \simeq 1 - H^{-1}$,

$$M = M_\parallel = S - \frac{T}{h + M(T_0 + D_c)} , \tag{65}$$

and, from (15),

$$p_\perp = \frac{SM}{H} = \frac{TM}{h + M(T_0 + D_c)}, \quad q_\perp \approx 0 \tag{66}$$

which generalizes (53). However, for $D_c \neq 0$ the transverse susceptibility of an ideal ferromagnet no longer diverges for $Q = 0$ since it costs a finite energy in order to rotate the spins out of the easy-axis directions. Hence (66) has to be replaced by

$$\delta p_\perp = p_\perp(h + D_c M) - p_\perp(0) = -\frac{T(h + D_c M)}{T_0[h + M(T_0 + D_c)]} , \tag{67}$$

which leads to

$$\xi_{\text{FM}\perp}^{-2} = \frac{h + D_c M}{AM} \equiv \frac{h}{AM} + \xi_c^{-2} , \tag{68}$$

with the Bloch-wall width $\xi_c = (A/D_c)^{1/2}$ of an ideal ferromagnet. The correlation length $\xi_a \approx \bar{\xi}_\alpha$, (19) or (26), with $\alpha = 2,3$, now consists of three contributions. One has

$$\xi_\perp^{-2} = \xi_D^{-2} + \xi_c^{-2} + h/AM , \tag{69}$$

with the random-anisotropy term (40), the field term h/AM, and the coherent anisotropy term ξ_c^{-2}. This leads, instead of (58), to

$$q_\perp = \frac{q}{\xi_D} \left[\frac{h}{AM} + \xi_D^{-2} + \xi_c^{-2} \right]^{-1/2} , \tag{70}$$

and to the change of the longitudinal magnetization $[\delta M_\parallel]_D = q_\perp / 2S$. Similar results again have been derived in Refs. 32 and 34.

Equation (70) has a simple interpretation: For $\xi_c \ll \xi_D$ the system is able to form Bloch walls. If, in addition, the size of the magnetic domains is large compared to ξ_D, the system is very similar to an ideal ferromagnet with coherent anisotropy. The domain size itself depends on form and size of the sample. In the opposite limit $\xi_c \gg \xi_D$ the fluctuations due to the random anisotropy destroy the Bloch walls and therewith the domain structure. In this sense sufficiently strong coherent anisotropy can restore the ferromagnetic state.

However, the meaning of the correlation length ξ_D and the Bloch-wall thickness ξ_c is rather different: In the Bloch wall of an ideal ferromagnet the spins remain perfectly correlated and rotate coherently, whereas random anisotropy leads to fluctuations which destroy the coherence over a distance ξ_D.

VI. CONCLUSIONS

Our theory predicts the breakdown of long-range ferromagnetic order in spin systems with ferromagnetic exchange and random anisotropy, in agreement with the results of many other authors. The magnetic state for $T < T_c$ is characterized by strong ferromagnetic correlations with the correlation length $\xi = \xi_D$ proportional to D^{-2}. One has fluctuations of the magnetization which in zero field lead to a gradual misalignment of the spins. In a finite field these fluctuations are superimposed by a constant magnetization. They lead, in the neutron-scattering form factor S_{Q}^α, Eq. (6), to an additional Lorentzian-squared contribution q_0^α which has been observed in many amorphous alloys.[43–48] This term vanishes above the critical temperature T_c in zero field. In a finite field and/or above T_c the correlation length ξ_D has to be replaced by $\xi = (\xi_D^{-2} + \xi_{\text{FM}}^{-2})^{1/2}$, where the "ferromagnetic" contribution ξ_{FM} for weak anisotropy can be replaced by its value at $D = 0$. The parameter ξ_{FM} diverges at $T = T_c$, whereas ξ and the susceptibility remain finite at all temperatures. However, our theory, which is based on a site-dependent mean-field approximation (or, more exactly, spherical approximation), does not hold in the critical region around T_c.

Throughout the paper we consider only weak anisotropy. In this case the susceptibility at $T > T_c$ differs only slightly from that of an ideal ferromagnet. Below T_c the zero-field susceptibility for $Q = 0$ becomes proportional to A^3/D^4 and has only the temperature dependence of the exchange parameter A and the anisotropy constant D. The Lorentzian and the Lorentzian-squared terms in S_Q^α in this temperature region are determined by the correlation length $\xi_D \sim (A/D)^2$. An additional field $h \gg AS\xi_D^{-2}$ leads to $\xi_{\text{FM}\perp}^{-2} = h/AM$ (M is the induced magnetization) and hence to a reduction of ξ. This reduction is enforced by additional coherent anisotropy which acts on the transverse correlation length $\xi_\perp$, similar to a magnetic field. One has $\xi_\perp^{-2} = \xi_D^{-2} + \xi_c^{-2} + h/AM$, where ξ_c is the thickness of a Bloch wall of an ideal ferromagnet.

The spin structure of the RAM has some similarity with that of a spin glass and similar properties indeed have been observed in both types of systems, including

similar critical exponents at the characteristic temperature and a de Almeida–Thouless (AT) line[49] in the field-temperature plane which is defined by the onset of irreversibility effects.[50] In the MFT of spin glasses the AT line is derived either by introducing replicas[49] or in dynamic theory.[51] Both theories are based on the SK model[8] and it is still a matter of debate whether or not a static AT line persists in a model with short-range interactions. A RAM with infinite-range exchange interactions can be solved without invoking replicas. Hence one has no solution for the order parameter, which is based on replica symmetry breaking as in the case of spin glasses. It remains an open question whether the observed AT line[50] is a dynamic effect or can only be derived in a theory which goes beyond our approach.

Recently, an AT line has been derived in the RAM (Refs. 17 and 52) and in ideal ferromagnets,[53] both with infinite-range ferromagnetic exchange interactions and additional cubic (coherent) anisotropy. In both cases the AT line is the consequence of symmetry breaking which leads to a second-order phase transition and to the change of the number of the order-parameter components. In both cases this order parameter is the spontaneous magnetization $\mathbf{M}_s$. In a more realistic RAM the order parameter presumably is the spin-glass parameter $\mathbf{q}$, Eq. (15), all components of which are nonzero below T_c, even in zero field. Hence there is no obvious symmetry breaking in the field-temperature plane and the observed lines most likely can be explained only within a dynamic theory.

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