RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 36, NUMBER 11 15 OCTOBER 1987-I

New path-integral solution for the density of states of two-dimensional electrons in high magnetic fields

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(Received 20 July 1987)

A new method for calculating the density of states of the two-dimensional electron gas in high magnetic fields with random scatterers is presented. Within the path-integral formalism we go beyond the short-time approximation and introduce a nonlocal harmonic-oscillator correction. Since the path integral of the nonlocal harmonic oscillator in a magnetic field can be solved exactly, a new solution of the density of states is obtained.

I. INTRODUCTION

The density of states (DOS) of the two-dimensional electron gas (2D EG) in high magnetic fields is of central interest for the understanding of many quantum phenomena discovered in these systems. Extensive calculations of the DOS have been performed by Ando and coworkers.[1-3] The result from the self-consistent Born approximation (SCBA) gives an elliptical shape of the DOS. By taking into account the multiple-scattering effect, Ando[4] obtained an asymmetrical shape of the DOS. Gerhardts[5] and Gevorkyan[6] used the path-integral technique by employing cumulant expansions, resulting in a Gaussian DOS. An exact result was obtained by Brezin, Gross, and Itzykson[7] for a short-range correlated impurity distribution. A systematical study of the Landau-level width including screening effects was performed by Lassnig and Gornik.[8]

In this Communication we present a new method for the calculation of the DOS of the 2D EG in high magnetic fields by using the path-integral formalism. The effect of the random scatterers is introduced into the path integral in terms of a potential-correlation function. This is equivalent to including contributions from coherent multiple scattering by clusters of impurities.[5]

The short-time approximation (STA), which corresponds to a constant potential-correlation function, gives an analytical solution for the DOS. In the present calculation we go for the first time beyond the STA by including the first-order correction term. This term is a nonlocal harmonic oscillator. In order to do this, the Gaussian real-space potential-correlation function is expanded up to the nonlocal harmonic-oscillator term; the constant term in this expansion corresponds to the STA. Since the path integral of the nonlocal harmonic oscillator in a magnetic field can be solved exactly, we obtain a new analytical form of the DOS as a product of a Gaussian function times a weighted infinite series of Hermite polynomials.

Depending on system parameters, a Gaussian or an asymmetrical shape of the DOS can be obtained. The critical parameter for the shape of the DOS is the coefficient of the infinite Hermite series. When this coefficient approaches zero, the DOS has a Gaussian shape; when this coefficient approaches one, the DOS becomes asymmetric.

II. THE CALCULATION OF THE DOS

We consider a two-dimensional electron system in a high magnetic field with random scatterers. The path integral of this system for one configuration [R] of the scatterers can be written as

$$K(\mathbf{r}_1,\mathbf{r}_2;t,[\mathbf{R}])=\int D[\mathbf{r}(\tau)]\exp\left[\frac{i}{\hbar}\int_0^t d\tau L_0-\frac{i}{\hbar}\int_0^t d\tau\sum_j V(\mathbf{r}(\tau)-\mathbf{R}_j)\right], \tag{1}$$

where $L_0=m/2(\dot{x}^2+\dot{y}^2)-eB/2(x\dot{y}-y\dot{x})$ is the Lagrangian of the free particle in a magnetic field, and $V(\mathbf{r}(\tau)-\mathbf{R}_j)$ is the potential of a single scatterer at position $\mathbf{R}_j$. For completely randomly distributed scatterers, the distribution of the scatterers can be expressed probabilistically as

$$P[\mathbf{R}]d[\mathbf{R}]=\prod_{N,A\to\infty}\frac{d\mathbf{R}_1,\ldots,d\mathbf{R}_N}{A^N}, \tag{2}$$

where N is the number of the scatterers, and A is the area of the system. Edwards and Gulyaev[9] showed that an average over all configurations of the scatters can be performed exactly resulting in

$$\begin{aligned}K(\mathbf{r}_1,\mathbf{r}_2;t)&=\int d[\mathbf{R}]P[\mathbf{R}]K(\mathbf{r}_1,\mathbf{r}_2;t,[\mathbf{R}])\\&=\int D[\mathbf{r}(\tau)]\exp\left\{\frac{i}{\hbar}\int_0^t d\tau L_0+\rho d\mathbf{R}\left[\exp\left(-\frac{i}{\hbar}\int_0^t d\tau V(\mathbf{r}(\tau)-\mathbf{R})\right)-1\right]\right\},\end{aligned} \tag{3}$$

where ρ is the density of the scatterers per unit area. In the limits of high density $\rho\to\infty$ and weak scatterers $V\to 0$ so that ρV^2 remains finite, Eq. (3) can be simplified to

$$K(\mathbf{r}_1,\mathbf{r}_2;t)=\int D[\mathbf{r}(\tau)]\exp\left\{\frac{i}{\hbar}\int_0^t d\tau L_0-\frac{1}{2\hbar^2}\int_0^t d\tau\int_0^t d\sigma\, W(\mathbf{r}(\tau)-\mathbf{r}(\sigma))\right\}, \tag{4}$$

where

$$W(\mathbf{r}(\tau)-\mathbf{r}(\sigma))=\rho\int d\mathbf{R}\, V(\mathbf{r}(\tau)-\mathbf{R})V(\mathbf{r}(\sigma)-\mathbf{R})$$

is the correlation function.

In Eq. (4) the perturbation due to the random scatterers has been reformulated in the path integral through the potential-correlation function. This potential-correlation function is the measure of the fluctuating potential at different points in the system. Since this function is introduced in the path integral as a potential term, it causes problems for solving the path integral because it is a nonlocal potential. This nonlocal potential corresponds to a non-Markov process and has a self-attracting effect on the electrons.

One way to solve this path integral analytically is to approximate the potential-correlation function by a constant (STA). However, since the potential-correlation function usually decays, one should go beyond the STA. In order to do this, we assume the correlation function to have a Gaussian form

$$W(\mathbf{r}(\tau)-\mathbf{r}(\sigma))=\rho\eta^2\exp\left\{-\frac{1}{L^2}[\mathbf{r}(\tau)-\mathbf{r}(\sigma)]^2\right\}, \tag{5}$$

where ρ is the density of the scatterers per unit area, η determines the scatterers strength, and L is a correlation length. Then we expand the Gaussian correlation function to be

$$W(\mathbf{r}(\tau)-\mathbf{r}(\sigma))\approx\rho\eta^2\left\{1-\frac{1}{L^2}[\mathbf{r}(\tau)-\mathbf{r}(\sigma)]^2+\cdots\right\}. \tag{6}$$

The first two terms in this expansion correspond to the STA and the nonlocal harmonic oscillator, respectively. For convenience, we rewrite Eq. (4) in the form

$$K(\mathbf{r}_1,\mathbf{r}_2;t)=\exp\left\{-\frac{\rho\eta^2t^2}{2\hbar^2}\right\}\int D[\mathbf{r}(\tau)]\exp\left\{\frac{i}{\hbar}\int_0^t d\tau L_0-\frac{imv^2}{4\hbar t}\int_0^t d\tau\int_0^t d\sigma[\mathbf{r}(\tau)-\mathbf{r}(\sigma)]^2\right\}, \tag{7}$$

where $v^2=2i\rho\eta^2t/m\hbar L^2$ is the frequency of the nonlocal harmonic oscillator.

For the path integral of the nonlocal harmonic oscillator in a magnetic field, Sa-yakanit *et al.*[10] already obtained the exact solution. The density matrix from this solution has the form

$$K(t)=\exp\left\{-\frac{\rho\eta^2t^2}{2\hbar^2}\right\}\left\{\frac{mv^2t}{4\pi i\hbar[\cos(\omega t/2)-\cos(\Omega t)]}\right\}, \tag{8}$$

where $\Omega=(\omega/2)(1+4v^2/\omega^2)^{1/2}$, and $\omega=eB/m$ is the cyclotron frequency. From this equation one can see that the density matrix damps down very fast when the time t approaches infinity. In order to understand this behavior of the density matrix one has to keep in mind that its absolute square is the probability to find the electron at the starting point after a period of time t. For a long period of time the chance that the particle will come back to the starting point is very low because the particle can wander randomly in the system. Therefore, the density-matrix approaches zero for the long-time propagation.

The cyclotron frequency ω is perturbed by the frequency v of the nonlocal harmonic oscillator. Keeping only the lowest-order perturbation, we take the limit $v/\omega\rightarrow 0$, and the density matrix in Eq. (8) becomes

$$\begin{aligned}K(t)&=\frac{m\omega}{4\pi i\hbar(1-v^2/\omega^2)}\frac{1}{\sin(\omega t/2)}\exp\left\{-\frac{\rho\eta^2t^2}{2\hbar^2}\right\}\\&=\frac{m\omega}{2\pi\hbar(1-v^2/\omega^2)}\sum_{n=0}^{\infty}\exp\left\{-\frac{\rho\eta^2t^2}{2\hbar^2}-i(n+\tfrac{1}{2})\omega t\right\}.\end{aligned} \tag{9}$$

Thus, the DOS can be calculated by using

$$N(E)=\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}dt\, K(t)\exp(iEt/\hbar). \tag{10}$$

By analytical calculation, the solution of Eq. (10) has the form

$$N(E)=\frac{1}{2\pi l^2\Gamma\sqrt{\pi}}\sum_{n=0}^{\infty}\exp\left[-\left(\frac{E-(n+\frac{1}{2})\hbar\omega}{\Gamma}\right)^2\right]\times\sum_{k=0}^{\infty}\zeta^kH_k\left(\frac{(n+\frac{1}{2})\hbar\omega-E}{\Gamma}\right), \tag{11}$$

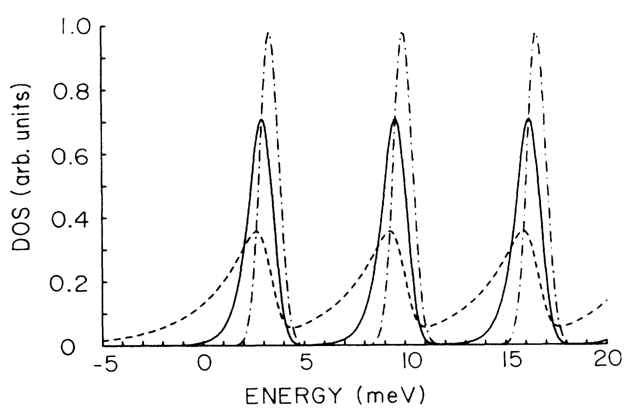

FIG. 1. Density of states for $m=0.07m_0$, $B=4.61T$ ($\hbar\omega=7.64$ meV), and $\Gamma/\hbar\omega=0.1$. The dash-dotted line represents the result for $1/L=0.1$, the solid line corresponds to $1/L=3$, and the dashed line to $1/L=6$.

where

$$\zeta=\left(\frac{\Gamma l^2}{\hbar\omega L^2}\right)\ll 1,\ \Gamma^2=2\rho\eta^2,\ l^2=\hbar/eB,\ \omega=eB/m\ ,$$

$H_k(X)$ is the Hermite polynomial of order k. The coefficient of the infinite series ζ determines the asymmetrical shape of the DOS. From this calculation the broadening parameter Γ depends only on the strength and the density of the scatterers. Therefore, every Landau level has the same behavior.

III. DISCUSSION

The DOS is plotted for various parameters in Fig. 1, evaluating Eq. (10) numerically. The quantity $\Gamma/\hbar\omega$ is kept constant while $1/L$ is varied. The critical parameter determining the shape of the DOS is the coefficient of the infinite Hermite series ζ. For $\zeta\rightarrow 0$ the DOS has a Gaussian shape; for $\zeta\rightarrow 1$ the DOS becomes asymmetric and the maximum shifts to lower energies. In comparison to Ando's work,[4] we have found that both results have a similar behavior. However, our result has no problems with singularities and can be expressed in closed form.

In summary, we have presented a new method for the calculation of the DOS of the 2D EG in high magnetic fields with random scatterers by using the path integral formalism. For the first time, we have performed the calculation by going beyond the STA to the first-order correction term of the real-space potential-correlation function. This correction term is a nonlocal harmonic oscillator. Since the path integral of the nonlocal harmonic oscillator in a magnetic field can be solved exactly, we have obtained a new analytical form of the DOS as the product of the Gaussian DOS times an infinite Hermite series.

ACKNOWLEDGMENTS

We would like to thank Dr. K. K. Thornber and Professor R. R. Gerhardts for helpful and critical discussions. We are indebted to Professor V. Sa-yakanit for showing us his result before publication. This work was partially supported by the Stiftung Volkswagenwerk.

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