Theoretical analysis of plasmon, polar phonon, and hot-electron energy relaxation in nondegenerate semiconductors

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Hot-electron energy relaxation due to the Coulomb scattering in nondegenerate polar semiconductors is treated theoretically by analyzing coupled modes between the electron-hole plasma and longitudinal-optical phonon in the self-consistent-field approximation. The ratio of lattice energy in each coupled mode is calculated in terms of the dielectric functions of the phonon and carriers, by which the screening of the phonon field can be evaluated. The coupled mode consists of three branches which arise from mixing of two plasmon modes and the phonon mode. Energy loss of an electron is mainly governed by the scattering by the highest-frequency mode and individual motion of the carriers, where the latter becomes dominant with increasing carrier density. Numerical computations are made for GaAs at 300 K. It is found that the energy-loss rate $W_{e,h}$ due to the carrier individual motion becomes nearly equal to that due to the coupled mode at the carrier density of 10^{17} cm⁻³. For an electron with energy 3000 K, the values of $W_{e,h}$ are 0.28 and 1.9 ergs/s for 10^{17} and 10^{18} cm⁻³, respectively.

I. INTRODUCTION

Scattering of hot electrons by carriers and phonons plays a central role in semiconductor physics and also in semiconductor devices.¹ Recently, much progress in this field has been made experimentally by measuring the time dependence of hot-electron energy distributions. $^{2-4}$ On the other hand, theoretical research on the Coulomb scattering in polar semiconductors is still in an early stage, though unscreened electron-phonon interactions have been thoroughly investigated.^{5,6} The energy relaxation of photoexcited electrons and holes has been widely investigated, $^{7-11}$ where the screening of electronlongitudinal-optical(LO)-phonon interactions is taken into account with an effective interaction potential which is assumed as the unscreened interaction potential divided by the dielectric constant of the electron-hole plasma. The screening may be, however, correctly treated by considering the mode coupling between the plasmon and LO phonon. Coupled-mode analysis has been given for the degenerate plasma in several papers, ^{4,12,13} in which Kim, Das, and Senturia¹² showed for the first time a theoretical treatment of the scattering of electrons by the coupled modes.

In this paper we discuss the energy relaxation of hot electrons in nondegenerate semiconductors due to scatterings by the coupled plasmon-LO-phonon modes and single-particle motions of electrons and holes, since these scatterings are the main causes of the electron relaxation in some semiconductors like GaAs.¹

In Sec. II dispersion relations of nondegenerate electron-hole plasmons are calculated by using the dielectric function in the self-consistent-field approximation, since they have not been given explicitly in the literature. The mode coupling between the plasmons and LO phonons is analyzed in Sec. III with a method similar to one used for degenerate semiconductors.¹⁴ The phonon contribution to each coupled mode is determined in terms of the dielectric functions of the LO phonon and plasmon.

The energy-loss rate of hot electrons is calculated in Sec. IV as a sum of two terms, where one results from excitation of the coupled modes and the other comes from excitation of single-particle motions. It is shown that both rates are functions of the carrier density and for GaAs at 300 K they become comparable in magnitude at a carrier density of 10^{17} cm⁻³.

II. ELECTRON-HOLE PLASMA

In the self-consistent-field approximation,¹⁵ or the random-phase approximation,¹⁶ the dielectric function $\epsilon_c(q,\omega)$ of free carriers in a medium of a high-frequency dielectric constant ϵ_{∞} is given by

$$\epsilon_{c}(q,\omega) = \epsilon_{\infty} + \lim_{\eta \to 0} \left(\frac{4\pi e^{2}}{\hbar q^{2}\Omega} \right) \sum_{\mathbf{k}} f_{\mathbf{k}}(1 - f_{\mathbf{k}+\mathbf{q}}) \left(\frac{1}{\omega + \omega(\mathbf{q},\mathbf{k}) + i\eta} - \frac{1}{\omega - \omega(\mathbf{q},\mathbf{k}) + i\eta} \right) . \tag{1}$$

Here, k and q are wave vectors, ω is the frequency, Ω is the volume of the specimen, f_k is the distribution function, and

$$\omega(\mathbf{q},\mathbf{k}) = (E_{\mathbf{k}+\mathbf{q}} - E_{\mathbf{k}})/\hbar , \qquad (2)$$

where E_k is the energy of an electron.

Assuming the Maxwell distribution for f_k , and approximating $1 - f_{k+q} = 1$, we obtain¹⁷

$$\epsilon_{c}(q,\omega)/\epsilon_{\infty} = \epsilon_{cr}(q,\omega) + i\epsilon_{ci}(q,\omega) , \qquad (3)$$

$$\epsilon_{\rm cr}(q,\omega) = 1 + \sum_{j=e,h} A_j Y_j^{-3} [F(Z_j + Y_j) - F(Z_j - Y_j)] ,$$
(4)

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$$\epsilon_{\rm ci}(q,\omega) = \sum_{j=e,h} (\pi/2)^{1/2} A_j Y_j^{-3} \\ \times [G(Z_j - Y_j) - G(Z_j + Y_j)], \qquad (5)$$

where

$$F(Z) = \exp(-Z^2/2) \int_0^Z \exp(x^2/2) dx , \qquad (6)$$

$$G(Z) = \exp(-Z^2/2)$$
, (7)

and the sum is taken for electrons (j = e) and holes (j = h). In the case of the electrons we have

$$A_e = \pi n e^2 \hbar^2 / 2m_n \epsilon_{\infty} (k_B T_e)^2 , \qquad (8)$$

$$Y_e = \hbar q / 2 (m_n k_B T_e)^{1/2} , \qquad (9)$$

$$Z_e = \hbar \omega / 2k_B T_e Y_e \quad , \tag{10}$$

where *n* is the electron density, m_n is the electron effective mass, T_e is the electron temperature, and k_B is the Boltzmann constant. For the holes, A_h , Y_h , and Z_h are obtained by replacing *n*, m_n , and T_e with the corresponding quantities *p*, m_p , and T_h in Eqs. (8)–(10). In the following n = p is always assumed.

We first calculate the dispersion relation from

$$\epsilon_{\rm cr}(q,\omega) = 0 \tag{11}$$

and then examine the Landau damping term $\epsilon_{\rm ci}(q,\omega)$. Figure 1 shows the electron-hole plasma dispersion relation obtained from Eqs. (4) and (11) with $T_e = T_h = 300$ K, $m_n = 0.067m$, $m_p = 0.45m$, and $\epsilon_{\infty} = 10.9$ for $n = 10^{17}$ and 10^{18} cm⁻³. The main feature of the dispersion relation, which is independent of material constants, is the existence of two branches, the optical and acoustic modes, limited by a critical wave number.

In the long-wavelength limit the frequencies of the two branches are easily found. When q goes to zero with fixed ω , Eq. (4) becomes

$$\epsilon_{\rm cr}(0,\omega) = 1 - \omega_p^2 / \omega^2 \tag{12}$$

with $\omega_p^2 = (4\pi ne^2/\epsilon_\infty)(m_n^{-1} + m_p^{-1})$. Therefore, the optical-mode frequency at q=0 equals the plasma frequency ω_p . On the other hand, if Z_j 's are fixed, Eq. (11) for small q reads

$$1 + \sum_{j} 2A_{j}Y_{j}^{-2}[1 - Z_{j}F(Z_{j})] = 0, \qquad (13)$$

from which the acoustic-mode frequency near q=0 can be obtained. It is noted that the acoustic branch is obtained even when the hole mass becomes infinitely large.

It is found from Eq. (5) that magnitudes of the Landau damping are very much different for different branches. At q=0, ϵ_{ci} becomes infinity for the acoustic branch, while it vanishes for the optical branch. Calculated values of $\epsilon_{\infty} \epsilon_{ci}(q,\omega)$ for $n=10^{17}$ and 10^{18} cm⁻³ are shown in Fig. 2, which indicates that the acoustic plasmon cannot actually be observed. The possibility of observing the acoustic plasmon was discussed by Pines and Schrieffer.¹⁷ The optical branch below a wave number q_c may be considered as the stable mode, since there the damping is small. We assume from Fig. 2 that the

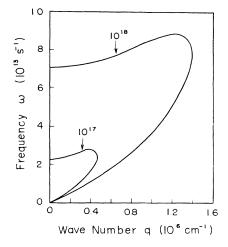


FIG. 1. Dispersion relations of the nondegenerate carrier plasma with $m_n = 0.067m$, $m_p = 0.45m$, $\epsilon_{\infty} = 10.9$, $T_e = T_h = 300$ K, and $n = p = 10^{17}$ and 10^{18} cm⁻³.

values of the damping cutoff wave number q_c are 0.2×10^6 and 0.6×10^6 cm⁻¹ for $n = 10^{17}$ and 10^{18} cm⁻³, respectively, though these values are somewhat ambiguous.

III. COUPLED MODE OF PLASMON AND PHONON

The interaction between electric fields associated with the LO phonon and electron-hole plasma implies a coupling between these modes. Since in the self-consistentfield approximation the polarizabilities of electrons and ions are additive, ^{12,14} the coupled modes are obtained from

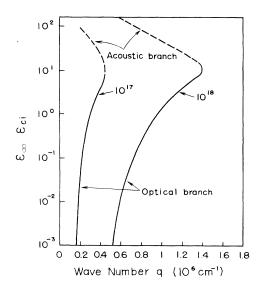


FIG. 2. Landau damping $\epsilon_{\infty}\epsilon_{ci}$, the imaginary part of the dielectric function, of the modes in Fig. 1. Solid and dashed lines indicate, respectively, the optical and acoustic branches.

(14)

$$\epsilon_{Lr}(\omega) + \epsilon_{cr}(q,\omega) = 1$$
,

where

$$\epsilon_{Lr}(\omega) = (\omega_l^2 - \omega^2) / (\omega_t^2 - \omega^2) . \qquad (15)$$

 $\epsilon_L(\omega) = \epsilon_{\infty} \epsilon_{Lr}(\omega)$ is the dielectric function of the lattice ions with neglect of dispersion and damping.^{12,14} In Eq. (15) ω_l and ω_t are the LO-phonon and transverseoptical-phonon frequencies, and $\omega_t^2 = (\epsilon_{\infty}/\epsilon_s)\omega_l^2$, where ϵ_s is the static dielectric constant.

Figure 3 shows the coupled-mode dispersion relations calculated from Eq. (14) combined with Eqs. (4) and (15), where solid and dashed lines represent, respectively, the cases of $n = 10^{18}$ and 10^{17} cm⁻³ with $\hbar \omega_l / k_B = 430$ K, $\epsilon_s = 12.8$, and the same values of other parameters as in Fig. 1. We denote the upper branch by mode 1 and the lower by mode 2 with neglect of the acoustic branches.

According to Eq. (14) the lattice polarization is proportional to $\epsilon_{\rm cr}(q,\omega)$, so that the ratio of the phonon strength *R*, that is, the ratio of lattice energy in the mode, can be defined by

$$R = \epsilon_{\rm cr}^2(q,\omega) / [\epsilon_{Lr}^2(\omega) + \epsilon_{\rm cr}^2(q,\omega)] .$$
(16)

As illustrated in Fig. 4, the nature of the modes changes remarkably when the electron density increases from 10^{17} to 10^{18} cm⁻³. At $n = 10^{17}$ cm⁻³ mode 1 is almost phononlike, while at $n = 10^{18}$ it is plasmonlike below $q = 1.4 \times 10^{6}$ cm⁻¹; it changes to phononlike above $q = 1.4 \times 10^{6}$ cm⁻¹, and becomes completely the phonon mode for $q \ge 1.6 \times 10^{6}$ cm⁻¹.

Qualitatively, the degree of transformation of the mode character in the long-wavelength region can be found by using $\epsilon_{\rm cr}(0,\omega)$ instead of $\epsilon_{\rm cr}(q,\omega)$. Then, the phonon strength ratio is approximated by

$$R = \epsilon_{\rm cr}^2(0,\omega) / [\epsilon_{Lr}^2(\omega) + \epsilon_{\rm cr}^2(0,\omega)] , \qquad (17)$$

where the mode frequencies are derived from

$$\epsilon_{Lr}(\omega) + \epsilon_{cr}(0,\omega) = 1 . \tag{18}$$

Thusly obtained phonon strength ratios for modes 1 and 2 are shown in Fig. 5 as a function of the electron density. Between $n = 10^{17}$ and 10^{18} cm⁻³ R for mode 1 decreases from 0.97 to 0.03, while for mode 2 it increases from 0.03 to 0.23.

As shown in Fig. 6, the Landau damping $\epsilon_{\infty} \epsilon_{ci}$ for each mode increases abruptly at a wave number corresponding to the cutoff q_c defined in Sec. II, so that above this the mode becomes unstable. The Landau damping for mode 1 is found to be significantly large in a wide range of wave numbers containing a pure phonon mode, since there phonons are frequently absorbed and emitted by the carriers. In this range, therefore, the LO phonons are strongly coupled to carrier individual motions.

IV. ENERGY RELAXATION OF HOT ELECTRONS

Hot electrons injected into the semiconductor lose their energy by interactions with carriers and lattice vibrations. In this paper we consider the long-range Coulomb interaction only, so that in the Born approxi-

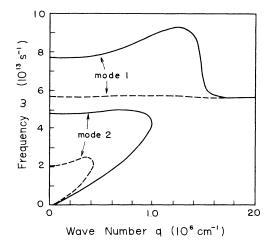


FIG. 3. Dispersion relations of the coupled plasmon and LO-phonon modes for $n = 10^{18}$ (solid lines) and 10^{17} cm⁻³ (dashed lines). Values of parameters are the same as in Fig. 1.

mation the energy-loss rate W of an electron which has momentum $\hbar p$ and energy E_p is given by¹⁶

$$W = \sum_{q} \int d(\hbar\omega) \, \hbar\omega w (q,\omega) \delta(\hbar\omega - E_{p} + E_{p-q}) \,, \qquad (19)$$

where $w(q,\omega)$ is the scattering rate in which the electron loses the energy $\hbar\omega$ with transfer of momentum $\hbar q$:

$$w(q,\omega) = (8\pi e^2/\hbar q^2 \Omega) \{-\operatorname{Im}[1/\epsilon(q,\omega)]\}.$$
(20)

In Eq. (20) $\epsilon(q,\omega)$ is the dielectric function which represents the response of the carriers and ions to the electron. We assume the hot-electron energy is written as $E_p = \hbar^2 p^2 / 2m_n$.

We shall discuss the energy loss of hot electrons by dividing the loss process into two parts: part A, energy loss due to excitation of collective motions of the carriers and ions, and, part B, energy loss due to excitation of single-particle motions of the carriers.

A. Energy loss to collective modes

We start by studying energy loss due to unscreened LO phonons, since it offers a typical example of the treatment of hot-electron energy relaxation caused by

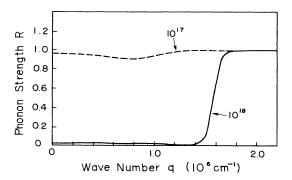


FIG. 4. Phonon strength ratio defined by Eq. (16) of mode 1 indicated in Fig. 3.

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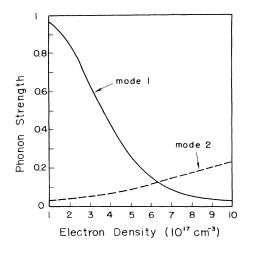


FIG. 5. Phonon strength ratio vs carrier density in the long-wavelength limit, Eq. (17), for mode 1 (solid line) and mode 2 (dashed line).

the collective-mode excitation. For $n = 10^{17}$ cm⁻³ the coupling between the plasmon and phonon is weak, so that the LO phonon can be treated independently of the plasmon.

We can easily find the response function $\epsilon_{\rm ph}(q,\omega)$ of the LO phonon to the electron field **D** with the standard perturbation theory.¹⁸ Expressing the lattice polarization P_L in the form¹⁹

$$P_L(\mathbf{r}) = (\hbar/2\gamma\omega_l\Omega)^{1/2} \sum_q (b_q e^{i\mathbf{q}\cdot\mathbf{r}} + b_q^{\dagger} e^{-i\mathbf{q}\cdot\mathbf{r}}) , \qquad (21)$$

where b_q and b_q^{\dagger} are the quantized phonon operators and $\gamma^{-1} = (\omega_l^2/4\pi)(\epsilon_{\infty}^{-1} - \epsilon_s^{-1})$, we obtain from the interaction Hamiltonian $-\int \mathbf{P}_L \mathbf{D} d\mathbf{r}$

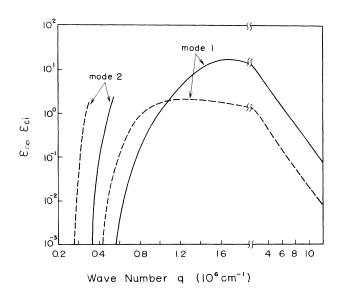


FIG. 6. Landau damping of modes 1 and 2 indicated in Fig. 3 for $n = 10^{17}$ (dashed line) and 10^{18} cm⁻³ (solid line).

$$-\operatorname{Im}[1/\epsilon_{\rm ph}(q,\omega)] = (2\pi^2/\gamma\omega_l)[(N_0+1)\delta(\omega-\omega_l) -N_0\delta(\omega+\omega_l)], \quad (22)$$

where $N_0 = [\exp(\hbar \omega_l / k_B T) - 1]^{-1}$ is the number of thermal phonons. By combining Eqs. (19), (20), and (22) the energy loss rate $W_{\rm ph}$ to the LO phonon is found to be⁵

$$W_{\rm ph} = \hbar \omega_l^2 (1 - \epsilon_{\infty} / \epsilon_s) (E_D / E_p)^{1/2} \\ \times [(N_0 + 1)g (E_p) - N_0 g (E_p + h \omega_l)], \qquad (23)$$

where

$$g(E_p) = \ln \left[\frac{E_p^{1/2} + (E_p - \hbar \omega_l)^{1/2}}{E_p^{1/2} - (E_p - \hbar \omega_l)^{1/2}} \right],$$
(24)

$$E_D = m_n e^4 / 2\hbar^2 \epsilon_{\infty}^2 . \qquad (25)$$

For hot electrons Eq. (23) is approximated by

$$W_{\rm ph} = \hbar \omega_l^2 (1 - \epsilon_{\infty} / \epsilon_s) (E_D / E_p)^{1/2} \ln(4E_p / \hbar \omega_l) , \qquad (26)$$

provided that $E_p >> \hbar \omega_l$.

This result can be obtained by a simple procedure applicable to all the collective modes considered here. The dielectric function $\epsilon_j(\omega)$ of mode *j* near the mode frequency ω_j may be written as¹⁶

$$\epsilon_{i}(\omega) = (\partial \epsilon / \partial \omega)_{i} [(\omega - \omega_{i}) + i\eta], \qquad (27)$$

where $(\partial \epsilon / \partial \omega)_j$ indicates the value of $\partial \epsilon / \partial \omega$ at $\omega = \omega_j$. Then we have

$$-\operatorname{Im}[1/\epsilon_{j}(\omega)] = \pi(\partial\epsilon/\partial\omega)_{j}^{-1}\delta(\omega-\omega_{j}), \qquad (28)$$

so that, substituting this into Eq. (20), we obtain the energy loss rate to mode j in the form

$$W_{j} = a_{j} \hbar \omega_{j}^{2} (E_{D} / E_{p})^{1/2} \ln(q_{ju} / q_{jl}) , \qquad (29)$$

where a_i is defined by

$$(\partial \epsilon / \partial \omega)_j^{-1} = a_j \omega_j / 2 \epsilon_{\infty}$$
 (30)

The upper and lower limits of the wave number, q_{ju} and q_{jl} , concerning the scattering by mode *j* are found from energy conservation to be

$$q_{ju} = p + (p^2 - q_j^2)^{1/2} , \qquad (31)$$

$$q_{jl} = p - (p^2 - q_j^2)^{1/2} , \qquad (32)$$

where q_j is given by $\hbar^2 q_j^2 / 2m_n = \hbar \omega_j$. When the damping cutoff wave number q_{jc} of mode j is smaller than q_{ju} , in Eq. (29) q_{ju} must be replaced by q_{jc} .

Since in Eq. (27) thermal excitation of the mode is neglected, in the case of the LO phonon Eq. (29) is expected to be identical with Eq. (22) except for the terms proportional to N_0 . This is confirmed by substituting $\epsilon(\omega) = \epsilon_L(\omega)$ and $\omega_j = \omega_l$ into Eq. (27), where a_j is found to be $1 - \epsilon_{\infty} / \epsilon_s$. Energy loss to the coupled mode 1 or 2 is obtained by setting

$$\boldsymbol{\epsilon}(\omega) = \boldsymbol{\epsilon}_L(\omega) + \boldsymbol{\epsilon}_{\infty} \boldsymbol{\epsilon}_{\rm cr}(0,\omega) , \qquad (33)$$

from which ω_j , a_j , q_{ju} , q_{jl} , and consequently W_j are calculated. For example, for pure plasmon excitation the

energy loss rate becomes

$$W_{\rm pl} = \hbar \omega_p^2 (E_D / E_p)^{1/2} \ln(2pq_c / q_0^2) , \qquad (34)$$

where q_0 is defined by $\hbar^2 q_0^2 / 2m_n = \hbar \omega_p$ and $E_p \gg \hbar \omega_p$ is assumed.

B. Energy loss to single-particle motions

The energy loss rate $W_{e,h}$ with the excitation of single-particle motion of carriers is derived from Eqs. (3) and (20) in the form

$$W_{e,h} = \sum_{q} \int d(\hbar\omega) \hbar\omega (8\pi e^{2} / \epsilon_{\infty} \hbar q^{2} \Omega) \times S \epsilon_{\rm ci} \delta(\hbar\omega - E_{\rm p} + E_{\rm p-q}) , \qquad (35)$$

where

$$S = [\epsilon_{\rm cr}^2(q,\omega) + \epsilon_{\rm ci}^2(q,\omega)]^{-1}$$
(36)

stands for the screening factor due to the electron-hole plasma. If we set $\omega = 0$ and then take the long-wavelength limit, Eq. (36) becomes the well-known static approximation formula^{20,21}

$$S_0 = [1 + 4\pi (n+p)e^2 / \epsilon_\infty k_B T q^2]^{-2} , \qquad (37)$$

provided that $T_e = T_h = T$.

Substituting Eq. (5) into Eq. (35), we have

$$W_{e-h} = W_e + W_h \quad , \tag{38}$$

where W_e and W_h are energy loss rates to electrons and holes, respectively. They are written as

$$W_e = \hbar \omega_n^2 (E_D / \pi k_B T_e)^{1/2} I_e , \qquad (39)$$

$$W_h = \hbar \omega_n^2 (E_A / \pi k_B T_h)^{1/2} I_h , \qquad (40)$$

where $\omega_n^2 = 4\pi n e^2 / \epsilon_{\infty} m_n$, $E_A = m_p e^4 / 2\hbar^2 \epsilon_{\infty}^2$, and numerical factors I_e and I_h are in integral forms:

$$I_{e} = \int_{x_{c}}^{1} dx \int_{Y_{c}}^{\xi x} (\xi x - Y_{e}) Y_{e}^{-2} [G(Z_{e} - Y_{e}) - G(Z_{e} + Y_{e})] S dY_{e} ,$$
(41)

$$I_{h} = \int_{x_{c}}^{1} dx \int_{Y_{c}}^{\xi x} (\xi x - Y_{e}) Y_{e}^{-2} [G(Z_{h} - Y_{h}) - G(Z_{h} + Y_{h})] S dY_{e} .$$

(42)

Here, $x = \cos\theta$ with θ as an angle between p and q, $\xi = \hbar p / (m_n k_B T_e)^{1/2}$, $Y_c = \hbar q_c / 2 (m_n k_B T_e)^{1/2}$, and $x_c = Y_c / \xi$. In Eqs. (41) and (42) we have used the energy conservation

$$\hbar\omega = (\hbar^2 / 2m_n)(2pqx - q^2) , \qquad (43)$$

so that Z_e and Z_h are functions of Y_e .

In Fig. 7 calculated values of W_e and W_h are shown for $n = 10^{18}$ and 10^{17} cm⁻³ by the solid and dashed lines as a function of the electron energy E_p normalized by $k_B T_e$, where the values of parameters are the same as those in Figs. 1 and 3. It is found that W_h is an order of

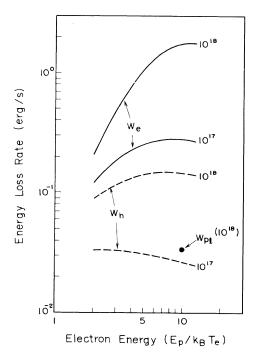


FIG. 7. Energy dependence of energy loss rates W_e and W_h due to electron and hole individual motions for $n = 10^{17}$ and 10^{18} cm⁻³ with the same parameter values as in Fig. 1. The energy loss rate due to the plasmon at $n = 10^{18}$ cm⁻³ is indicat-

ed by a solid circle.

magnitude smaller than W_e . A value of $W_{\rm pl}$ obtained from Eq. (34) at $n = 10^{18}$ and $E_p / k_B T_e = 10$ is also shown by a solid circle, which indicates that the energy loss to mode 1 in the stable plasmon range is much smaller than the energy loss to the individual carrier motions.

Energy dependence of W_{e-h} derived from Fig. 7 is shown in Fig. 8, where for comparison $W_{\rm ph}$ calculated with Eq. (23) is indicated by the dashed line. At $n = 10^{17}$ cm⁻³ mode 1 is almost phononlike, so that in this case $W_1 = W_{\rm ph}$ and, as seen in Fig. 8, it is nearly equal to W_{e-h} . At $n = 10^{18}$ cm⁻³ the hot-electron energy relaxation is found to be governed by W_{e-h} . For this carrier density, energy loss to the LO phonon is reduced from $W_{\rm ph}$ due to the screening effect, which causes the disappearance of the phonon in the small-wave-number range of mode 1. Energy loss to mode 2 can be neglected since the frequency is small and the stable range in wave number is very limited.

It is remarked that in the static approximation the screening of the Coulomb interaction between carriers seems to be overestimated. To show this, I_e is compared in Fig. 9 with I_{e0} defined by

$$I_{e0} = \int_{0}^{1} dx \int_{0}^{\xi x} (\xi x - Y_{e}) Y_{e}^{-2} [G(Z_{e} - Y_{e}) - G(Z_{e} + Y_{e})] S_{0} dY_{e}$$
(44)

for $n = 10^{17}$ cm⁻³ and the same parameter values as before.

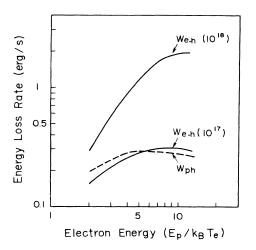


FIG. 8. Energy dependence of energy loss rates $W_{e-h} = W_e + W_h$ (solid lines) and W_{ph} (dashed line) with $T = T_e = T_h = 300$ K and the same parameter values as in Fig 1, where W_{ph} is due to the unscreened LO phonon.

V. SUMMARY

Dispersion relations of the plasmon and coupled modes between the plasmon and LO phonon in nondegenerate semiconductors have been calculated with the self-consistent-field approximation. For the plasmon we find the optical and acoustic modes with wave numbers smaller than a critical value, which depends on the carrier density and temperature. The Landau damping is large except for the optical branch with long wavelengths, so that only optical plasmons which have wave numbers below the damping cutoff q_c are considered to be stable. For GaAs at 300 K assumed values of q_c are 0.2×10^6 and 0.6×10^6 cm⁻¹ for $n = 10^{17}$ and 10^{18} cm⁻³, respectively, which are fairly smaller than the critical wave-number values.

The coupled modes are composed of three branches which arise from mixing of two plasmon modes and the LO-phonon mode. In these three branches the lowestfrequency acoustic mode is neglected because of the

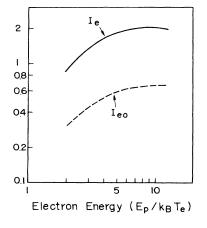


FIG. 9. Numerical values of the integrals I_e (solid line), Eq. (41), and I_{e0} (dashed line), Eq. (44).

strong Landau damping. Of the remaining two branches the higher-frequency branch is denoted by mode 1 and the lower by mode 2. For small carrier densities, where the plasma frequency is much smaller than the LOphonon frequency, mode 1 is equivalent to the LO phonon and mode 2 is the optical plasmon. Accordingly, mode 1 extends through the whole first Brillouin zone, while mode 2 is limited to a small wave-number range. These features do not change with increasing carrier density, where mixing of the plasmon and LO phonon occurs.

Effect of the electron-hole plasma on electron scattering by the LO phonon has been considered to be the screening of the interaction potential.⁷⁻¹¹ As discussed in Secs. III and IV, however, the effect is reduced to changes in lattice energy and scattering strength of the coupled modes. The scattering strength of each mode is given by $a_i \omega_i$ in Eq. (30), which is obtained by a method similar to the previous work.¹² The phonon-strength ratio is defined by Eq. (16) in terms of the dielectric functions of the phonons and carriers. This definition seems reasonable within the framework of the self-consistentfield approximation, though it is different from the previous ones.^{12,14} For GaAs at 300 K mode 1 is almost phononlike for $n = 10^{17}$ cm⁻³. With increasing carrier density the phonon-strength ratio of mode 1 in the longwavelength range decreases and at $n = 10^{18}$ cm⁻³ it reaches nearly zero for wave numbers less than 1.4×10^6 cm^{-1} . For this carrier density the phonon-strength ratio of mode 2 is 0.23.

The energy loss rate of a hot electron has been considered as a sum of three terms W_1 , W_2 , and W_{e-h} , where W_1 and W_2 are energy loss rates due to the coupled modes 1 and 2, and W_{e-h} is the loss rate due to single-particle motions of the electrons and holes. In these three processes W_2 is always negligible as compared with W_1 because of the small frequency and limited wave number. Numerical computations of W_1 and W_{e-h} have been made for GaAs at carrier and lattice temperatures of 300 K. We have found that at $n = 10^{17}$ $cm^{-3} W_{e-h}$ is comparable in magnitude with W_1 , where W_1 is equal to $W_{\rm ph}$, which is the energy loss rate due to the unscreened LO phonon. At $n = 10^{18}$ cm⁻³ W_1 results from two different contributions. For $q < q_c = 0.6 \times 10^6$ cm⁻¹ W_1 is represented as the plasmon excitation loss rate $W_{\rm pl}$ and for $q > 1.6 \times 10^6$ cm^{-1} it is the phonon excitation loss. As shown in Figs. 7 and 8, $W_{\rm pl}$ is negligibly smaller than W_{e-h} or $W_{\rm ph}$. W_1 for $q > 1.6 \times 10^6$ cm⁻¹ must be also smaller than $W_{\rm ph}$, since the wave number is limited. Consequently, at $n = 10^{18}$ cm⁻³ W_{e-h} dominates the energy loss of hot electrons. For an electron with energy 3000 K values of W_{e-h} are 0.28 and 1.9 ergs/s for $n = 10^{17}$ and 10^{18} cm⁻³, respectively.

In addition to the screening the carriers affect the lifetime of the phonons. As seen in Fig. 6, at $n = 10^{18}$ cm⁻³ the phonons with wave numbers below about 10^7 cm⁻¹ are strongly coupled with carrier individual motions, so that in this wave-number range to distinguish W_1 from $W_{e\cdot h}$ seems somewhat meaningless.

In conclusion we have shown a theoretical treatment

of hot-electron energy relaxation due to the Coulomb scattering in nondegenerate semiconductors by analyzing the coupling between the plasmon and LO phonon. Screening of the phonon field by the electron-hole plasma causes the decrease or even disappearance of the phonon contribution to the coupled modes in a longwavelength region. Coupling between the carrier individual motion and LO phonon results in strong damping of the phonons with transfer of energy to the carrier motion. These screening and damping effects appear in

different wavelength regions. Hot-electron energy relaxation is governed by the LO-phonon excitation process for smaller carrier densities and by the carrier individual motion excitation process for larger carrier densities. For GaAs at 300 K these two processes become nearly equal at $n = 10^{17}$ cm⁻³.

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