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Strong-coupling calculation of the lower critical field in high-T_c superconductors

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Using the strong-coupling version of Usadel's dirty-limit theory, the lower critical field H_{c1} of high-T_c superconductors is calculated. It is shown that the strong-coupling theory predicts a marked plateau in $H_{c1}(T)$ at low temperatures, which is most pronounced for large electron-phonon coupling parameters λ. This effect can be used as a criterion for conventional electron-phonon superconductivity in high-T_c superconductors. The plateau has not been observed in early measurements of H_{c1} in granular samples.

I. INTRODUCTION

The superconducting high-T_c oxide compounds[1-3] have enjoyed immense experimental and theoretical interest during the past months. One of the reasons for the great deal of work invested is associated with some open questions about the microscopic mechanism that leads to superconductivity in these materials. At present there exists no convincing evidence against the Bardeen-Cooper-Schrieffer (BCS) Cooper-pair model, which has proved most successful in conventional superconductors (alternative models have been suggested in Refs. 4-6). However, the question arose[3,7] as to whether or not the attractive electron-electron interaction, which leads to the formation of Cooper pairs is still phonon mediated. This is conceivable, although an unprecedentedly strong electron-phonon coupling would be necessary in order to obtain the high-T_c values.

Several observations on the high-T_c superconductors seems to indicate deviations from the model of a strong attractive electron-phonon mechanism. For example, far-infrared energy-gap measurements[8-11] always yield, instead of a characteristic strong-coupling enhancement, a ratio $2\Delta_0/k_BT_c$ which is below the weak-coupling value of 3.53. Another point concerns the recently observed complete absence of an oxygen isotope effect in $Ba_2YCu_3O_7$ and $Ba_2EuCu_3O_7$.[12] If this observation is confirmed, the present concept of a dominant role of specific oxygen vibrational modes[13] in mediating the high T_c's has to be reconsidered. As alternatives to phonon-mediated BCS superconductivity, purely electronic attractive electron-electron mechanisms have been proposed.[7,14]

On the other hand, a number of recent publications[13,15,16] point out the consistency of specified experimental and theoretical evidence with phonon-mediated BCS superconductivity. For example, tunneling gap measurements in Y-Ba-Cu-O (Ref. 11) and La-Sr-Cu-O (Refs. 16-18) yield a ratio $2\Delta_0/k_BT_c$ of $\sim$4.3-5.8, which indicates an unusually strong electron-phonon coupling. Possible sources for the disagreement between these measurements and the above-mentioned far-infrared data are discussed in Ref. 11. By means of x-ray-absorption studies in La-(Sr,Ba)-Cu-O, Boyce *et al.*[19] maintain that the phonon-coupling mechanism is just compatible with the observed T_c. Also, band-structure and electron-phonon coupling calculations[13,15,20] for the same system show that the electron-phonon mechanism can account for the high T_c's ($T_c \approx 35$ K).

Further conclusions on the superconducting mechanism in the high-T_c oxide compounds can only be drawn from a comparison between experiments on well-characterized samples and results of the theories to be tested. The electron-phonon coupling theory of superconductivity (strong-coupling theory, Eliashberg theory[21,22]) is by now well established. It has been used in the past by a number of authors to successfully describe various properties of conventional (low-T_c) superconductors (e.g., Refs. 23-25). Very recently, Ashauer, Lee, and Rammer[26] applied Eliashberg's theory to the high-T_c superconductors. Using "reasonable" model Eliashberg spectral functions, these authors calculated Δ_0, $H_c(T)$, $H_{c2}(T)$, $c_s(T)$, $\kappa_1(T)$, and $\kappa_2(T)$ and obtained deviations from the weak-coupling BCS theory that can be as high as 100%.

In this paper, we use the strong-coupling theory to calculate the dirty-limit lower critical field of high-T_c superconductors and compare the results with previous weak-coupling calculations[27] and with experiments in the La-Sr-Cu-O (Ref. 28) and Ba-Y-Cu-O (Ref. 29) systems. The lower critical-field H_{c1} is expected to yield more conclusive information than the upper critical-field H_{c2}, which can, because of present limitations in the available magnetic fields, only be measured near T_c. The H_{c1} data lie well inside the experimental reach over the whole temperature range.

Our calculations[30] are based on Usadel's dirty-limit version[31,32] of the Eilenberger-Gorkov weak-coupling theory[33] including the strong-coupling corrections as they emerge from Eliashberg's theory.[21] In complete analogy to the weak-coupling case,[27] one obtains a system of coupled differential (diffusionlike) and integral (self-consistency) equations for the impurity-averaged Green's function parametrized by $\Phi_l(r)$, the gap function $\Delta_l(r)$, the renormalization function $Z_l(r)$, and the supercurrent $j(r)$, which are all space dependent. Except for the substitutions

$$\omega_l \rightarrow Z_l(r)\omega_l \ ,$$

$$\psi(r) \rightarrow \Delta_l(r) \ ,$$

the diffusionlike equation is unchanged [$\omega_l = \pi k_BT(2l+1)$ are the Matsubara frequencies]. The weak-coupling gap equation is replaced by

$$\Delta_l(r) = \pi k_B T \sum_n [\lambda(\omega_l - \omega_n) - \mu^*] \sin\Phi_n(r) \ ,$$

and the additional self-consistency equation for the renormalization function reads:

$$Z_l(r)\omega_l = \omega_l + \pi k_B T \sum_n \lambda(\omega_l - \omega_n) \cos\Phi_n(r) \ ,$$

where

$$\lambda(\omega_l - \omega_n) = 2\int_0^\infty d\omega \frac{\alpha^2 F(\omega)\omega}{\omega^2 + (\omega_l - \omega_n)^2} \ ,$$

with $\alpha^2 F(\omega)$ the Eliashberg spectral function. The self-consistency equation for the current $j(r)$ remains unchanged. In analogy to weak-coupling superconductors, a free-energy functional has been derived which yields the diffusion and the self-consistency equations as conditions for stationarity with respect to the above functions. At the stationary point, the functional gives the free energy of the superconductor. Details and further results, especially for conventional (low-T_c) superconductors will be published in a forthcoming paper.[34]

This theory, which is strictly valid only in the dirty limit ($k_B T_c \ll \hbar/\tau_{\rm tr}$, where $\tau_{\rm tr}$ is the transport lifetime), has in its original, weak-coupling form been applied to the mixed state of type-II superconductors by Kramer, Pesch, and Watts-Tobin[27] and by Watts-Tobin, Kramer, and Pesch.[35] These authors obtained, among other quantities, the lower critical-field H_{c1} which is defined as the external magnetic field where the Gibbs free energies of the Meissner state and the mixed state with one isolated flux line are equal. We will present in Sec. II the results of our calculations of the *strong-coupling* lower critical field for high-T_c superconductors.

II. RESULTS AND CONCLUSIONS

Since there exist at present no $\alpha^2 F(\omega)$ measurements, we have to rely on model assumptions.[26] In Fig. 1 three of the five model Eliashberg spectral functions $\alpha^2 F(\omega)$, which will be used are shown. Spectrum F135 (for a definition of our notation, see Table I) is the theoretical Eliashberg function for $La_{1.85}(Ba,Sr)_{0.15}CuO_4$ obtained by Weber.[13] Model spectra B235 and C235 were constructed on the basis of measurements of the generalized

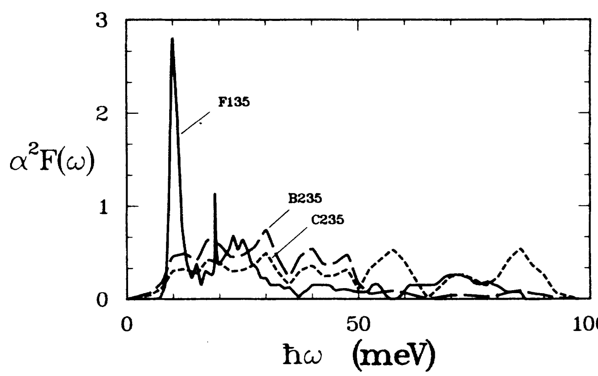

FIG. 1. Model Eliashberg spectral functions (see Table I).

TABLE I. Model Eliashberg functions; in our notation, the letter refers to the $\alpha^2 F(\omega)$ type, and the numbers encode in an obvious way μ^* and T_c.

$\alpha^2F(\omega)$	Type (see Fig. 1)	μ^*	T_c (K)	λ
F135	F	0.13	35	2.3
B235	B	0.2	35	2.0
B291	B	0.2	91	6.1
C235	C	0.2	35	1.7
C291	C	0.2	91	4.0

phonon density of states by means of inelastic neutron scattering in $La_{1.85}Sr_{0.15}CuO_4$ at 6 K.[36] Curve B235 (C235) was obtained from these data by assuming a weak (strong) coupling of the conduction electrons to the oxygen-stretch modes above ≈ 52 meV.[36] For more details see Ref. 26. The heights of the spectra in Fig. 1 were adjusted to give a T_c of 35 K. Two additional model $\alpha^2 F(\omega)$ functions of types B and C will be used below, with their heights adjusted to give a T_c of 91 K (B291, C291, see Table I). They are taken to be representative of the Ba-Y-Cu-O class of superconductors.

Results for the lower critical field of our model high-T_c supercondcutors represented by the Eliashberg functions introduced in the previous paragraph are given in Fig. 2. The diagram pertains to a weak-coupling Ginzburg-Landau (GL) parameter $\kappa^{\rm weak} = 50$, which is in the dirty limit defined by[27]

$$\kappa^{\rm weak} = [3c/2\pi^2 e v_F l][7\zeta(3)/\pi N(0)]^{1/2} \ ,$$

with l the mean free path and v_F and $N(0)$ the (dressed)

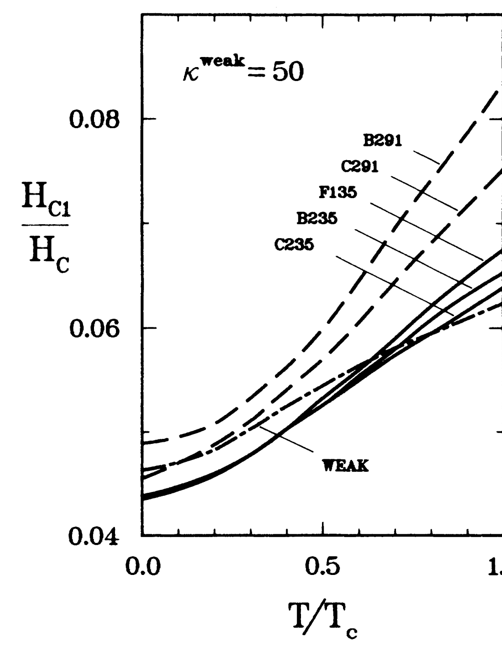

FIG. 2. $H_{c1}(T)/H_c(T)$ for high-T_c model superconductors with Eliashberg functions (B291, etc.) defined in Table I; the weak-coupling curve is given for comparison.

Fermi velocity and the (dressed) electronic density of states at the Fermi energy, respectively. The fact that the curves in Fig. 2 do not coincide at $T=T_c$ indicates that the strong-coupling (directly measured) GL parameter κ differs from κ^{weak} (see also Ref. 37). The GL parameter κ can, e.g., be obtained by extrapolating κ_1 $(=H_{c2}/\sqrt{2}H_c)$ to $T=T_c$. Therefore, $\kappa/\kappa^{\text{weak}}$ is given by the ratio of the H_{c2} to the H_c enhancement parameter at T_c: $\kappa/\kappa^{\text{weak}}=\eta_{H_{c2}}(T_c)/\eta_{H_c}(T_c)$ (see Table I in Ref. 26). We have $\kappa/\kappa^{\text{weak}}=0.92$ for C235, 0.89 for B235, 0.84 for F135, 0.72 for C291, and 0.61 for B 291.

Figure 2 shows that H_{c1}/H_c is enhanced over its weak-coupling value by up to $\sim 30\%$ at T_c. At $T=0$, the strong-coupling influence on this quantity is much smaller (deviations are below $\sim \pm 8\%$). It is remarkable that the strong-coupling effect in H_{c1}/H_c is in good correspondence to the strong-coupling effect in H_{c2}/H_c *for all temperatures.*[34] An analogous diagram for a different κ^{weak} can be obtained with good accuracy by scaling the ordinate in Fig. 2 according to the κ dependence in the weak-coupling limit (Table I in Ref. 27). No comparison with experiments can at present be made in Fig. 2, because there exist no experimental data for the temperature dependence of H_c.

The temperature dependence of H_{c1} has recently been measured by Batlogg *et al.*,[38] Renker *et al.*[28] (La-Sr-Cu-O), and Cava *et al.*[29] (Ba-Y-Cu-O). These experiments, together with the theoretical results for our model Eliashberg spectra, are presented in Figs. 3 and 4. The two diagrams display the dimensionless quantities $H_{c1}(T)/[T_cH_{c1}'(T_c)]$ (theory and experiment) and, for comparison, $H_c(T)/[T_cH_c'(T_c)]$ (theory). It can be seen that the strong-coupling effect leads to a reduction of the plotted quantities at low temperatures relative to the weak-coupling curves, which is more pronounced for larger values of the electron-phonon coupling constant λ. We emphasize that the formation of the marked plateau in H_{c1} is a characteristic strong-coupling feature. This behavior is in distinct contradiction to the experimental data, especially for the $\sim$90 K superconductor (Fig. 4). It is also seen that the influence of the detailed $\alpha^2F(\omega)$ structure on the plotted quantities is very weak. At these

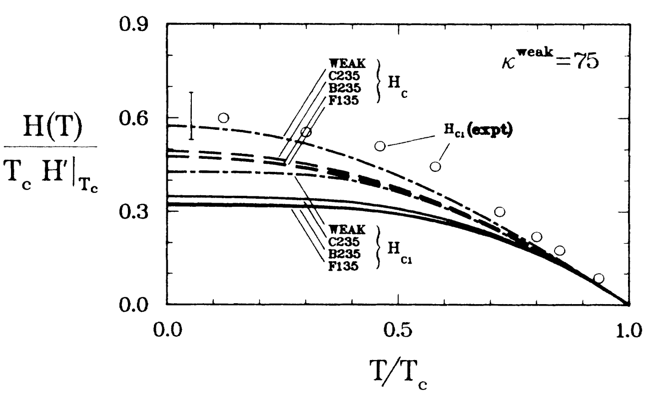

FIG. 3. Temperature dependence of H_c and H_{c1} (normalized to the slopes at T_c) for model Eliashberg functions defined in Table I ($T_c=35$ K). Experimental results (Ref. 28) for La-Sr-Cu-O are included.

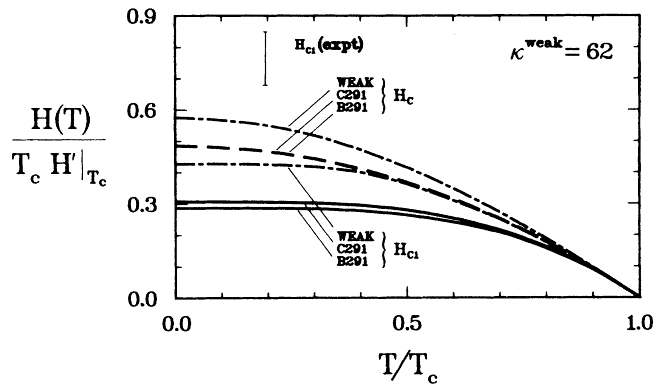

FIG. 4. Temperature dependence of H_c and H_{c1} (normalized to the slopes at T_c) for model Eliashberg functions defined in Table I ($T_c=91$ K). Experimental results (Ref. 29) for Ba-Y-Cu-O are included.

high values of κ^{weak}, the H_{c1} curves are essentially independent of κ^{weak}. GL parameters between, e.g., 50 and 100 lead to deviations from the given curves that are smaller than 1%.

Several conclusions may be inferred from these results. However, it should be noted that the present experimental H_{c1} data contain considerable uncertainties; in particular, the slope at T_c could not be obtained with sufficient accuracy for a conclusive direct comparison with the theory. These problems are perhaps associated with the inhomogeneity of the presently available polycrystalline samples. In our opinion, H_{c1} measurements on single crystals with dimensions much larger than the magnetic penetration depth are most desirable. Using London's theory, Clem and Kogan[39] have estimated the influence of the granular sample structure on the magnetization. Unfortunately, no results on the temperature dependence of the critical fields have been reported.

The calculation presented here pertains to homogeneous, isotropic samples. Since a dirty-limit approximation is used, anisotropy effects brought about by the layered structure of the superconducting perovskites should not play an essential role. In case the material is not in the dirty limit, the detailed band structure (anisotropic Fermi surface) has to be taken into account. Preliminary studies based on the simple model Fermi-surface anisotropy introduced in Ref. 40 show, however, that the temperature dependence of H_{c1} is not affected, even for a (rotationally invariant) extreme uniaxial Fermi-surface deformation (the magnetic field was chosen to be parallel to the symmetry axis).

In summary, we have calculated the lower critical field of high-T_c superconductors using the strong-coupling version of Usadel's dirty-limit theory. To our knowledge, these are the first strong-coupling calculations of H_{c1} for type-II superconductors. The theory predicts a pronounced plateau in $H_{c1}(T)$ at low temperatures, which extends to quite high reduced temperatures if the electron-phonon coupling parameter λ is large. This effect can be used as a criterion for conventional electron-phonon superconductivity in the high-T_c oxides. Present superconducting quantum interference device (SQUID) magnetometer experiments do not show the predicted plateau. We suggest $H_{c1}(T)$ measurements in homogeneous, single-crystal samples, using also different experimental techniques.[41]

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[1] J. G. Bednorz and K. A. Müller, Z. Phys. B **64**, 189 (1986).
[2] J. G. Bednorz, M. Takashige, and K. A. Müller, Europhys. Lett. **3**, 379 (1987).
[3] M. K. Wu, J. R. Ashburn, C. J. Torng, P. H. Hor, R. L. Meng, L. Gao, Z. J. Huang, Y. Q. Wang, and C. W. Chu, Phys. Rev. Lett. **58**, 908 (1987).
[4] A. Alexandrov and J. Ranninger, Phys. Rev. B **23**, 1796 (1981).
[5] P. Prelovšek, T. M. Rice, and F. C. Zhang, J. Phys. C **20**, L229 (1987).
[6] H. Kamimura, Jpn. J. Appl. Phys. (to be published).
[7] P. W. Anderson, Science **235**, 1196 (1987).
[8] D. A. Bonn, J. E. Greedan, C. V. Stager, and T. Timusk, Solid State Commun. **62**, 383 (1987).
[9] U. Walter, M. S. Sherwin, A. Stacy, P. L. Richards, and A. Zettl, Phys. Rev. B **35**, 5327 (1987).
[10] P. E. Sulewski, A. J. Sievers, S. E. Russek, H. D. Hallen, D. K. Lathrop, and R. A. Buhrman, Phys. Rev. B **35**, 5330 (1987).
[11] J. R. Kirtley, R. T. Collins, Z. Schlesinger, W. J. Gallagher, R. L. Sandstrom, T. R. Dinger, and D. A. Chance, Phys. Rev. B **35**, 8846 (1987).
[12] B. Batlogg, R. J. Cava, A. Jayaraman, R. B. van Dover, G. A. Kourouklis, S. Sunshine, D. W. Murphy, L. W. Rupp, H. S. Chen, A. White, K. T. Short, A. M. Mujsec, and E. A. Rietman, Phys. Rev. Lett. **58**, 2333 (1987); L. C. Bourne, M. F. Crommie, A. Zettl, Hans-Conrad zur Loye, S. W. Keller, K. L. Leary, Angelica M. Stacy, K. J. Chang, Marvin L. Cohen, and Donald E. Morris, *ibid.* **58**, 2337 (1987).
[13] W. Weber, Phys. Rev. Lett. **58**, 1371 (1987); **58**, 215(E) (1987).
[14] J. Ashkenazi, C. G. Kuper, and R. Tyk (unpublished).
[15] W. E. Pickett, H. Krakauer, D. A. Papaconstantopoulos, and L. L. Boyer, Phys. Rev. B **35**, 7252 (1987).
[16] M. E. Hawley, K. E. Gray, D. W. Capone II, and D. G. Hinks, Phys. Rev. B **35**, 7224 (1987).
[17] J. Moreland, A. F. Clark, H. C. Ku, and R. N. Shelton, Cryogenics **27**, 227 (1987).
[18] J. R. Kirtley, C. C. Tsuei, Sung I. Park. C. C. Chi, J. Rozen, and M. W. Shafer, Phys. Rev. B **35**, 7216 (1987).
[19] J. B. Boyce, F. Bridges, T. Claeson, T. H. Geballe, C. W. Chu, and J. M. Tarascon, Phys. Rev. B **35**, 7203 (1987).
[20] L. F. Mattheiss, Phys. Rev. Lett. **58**, 1028 (1987).
[21] G. M. Eliashberg, Zh. Eksp. Teor. Fiz. **38**, 966 (1960) [Sov. Phys. JETP **11**, 696 (1960)].
[22] D. Scalapino, in *Superconductivity,* edited by R. D. Parks (Dekker, New York, 1969), p. 449.
[23] D. Rainer and G. Berrgmann, J. Low Temp. Phys. **14**, 501 (1974).
[24] J. M. Daams and J. P. Carbotte, J. Low Temp. Phys. **43**, 263 (1981).
[25] L. Niel, N. Giesinger, H. W. Weber, and E. Schachinger, Phys. Rev. B **32**, 2976 (1985).
[26] B. Ashauer, W. Lee, and J. Rammer, Z. Phys. B **67**, 147 (1987).
[27] L. Kramer, W. Pesch, and R. J. Watts-Tobin, J. Low Temp. Phys. **14**, 29 (1974).
[28] B. Renker, I. Apfelstedt, H. Küpfer, C. Politis, H. Rietschel, W. Schauer, H. Wühl, U. Gottwick, H. Kneissel, U. Rauchschwalbe, H. Spille, and F. Steglich, Z. Phys. B **67**, 1 (1987).
[29] R. J. Cava, B. Batlogg, R. B. van Dover, D. W. Murphy, S. Sunshine, T. Siegrist, J. P. Remeika, E. A. Rietman, S. Zahurak, and G. P. Espinosa, Phys. Rev. Lett. **58**, 1676 (1987).
[30] Parts of the numerical calculations were done by using computer routines written by the authors of Ref. 27.
[31] K. D. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
[32] K. D. Usadel, Phys. Rev. B **4**, 99 (1971).
[33] G. Eilenberger, Z. Phys. **214**, 195 (1968).
[34] J. Rammer (to be published).
[35] R. Watts-Tobin, L. Kramer, and W. Pesch, J. Low Temp. Phys. **17**, 71 (1974).
[36] B. Renker, F. Gompf, E. Gering, N. Nücker, B. Ewert, W. Reinchhardt, and H. Rietschel, Z. Phys. B **67**, 15 (1987).
[37] D. Rainer and K. D. Usadel, Phys. Rev. B **9**, 2409 (1974).
[38] B. Batlogg, A. P. Ramirez, R. J. Cava, R. B. van Dover, and E. A. Rietman, Phys. Rev. B **35**, 5340 (1987).
[39] J. R. Clem and V. G. Kogan, in LT18–The XVIII International Conference on Low Temperature Physics, Kyoto, Japan, August 1987 (unpublished).
[40] J. Rammer, W. Pesch, and L. Kramer, Z. Phys. B **68**, 49 (1987).
[41] Lower critical-field measurements by means of a novel technique are underway [P. Equinazi (private communication)].