Universal conductance fluctuations as a probe of chaotic behavior in mesoscopic metallic spin glasses

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(Received 31 March 1987)

We study the conductance fluctuations in mesoscopic metallic spin glasses with Ruderman-Kittel-Kasuya-Yosida (RKKY) interactions at low temperatures. It is found that the conductance of the system is sensitive to the configurations of the spins in the system. As a result, the chaotic nature of spin reorganizations, as temperature is varied, manifests itself in the form of measurable conductance fluctuations. Thermal-conductance fluctuations at a fixed temperature are shown also to be directly related to the corresponding thermal spin fluctuations.

Two seemingly unconnected developments have recently attracted much attention in the condensed-matter physics community. First, it has been demonstrated both theoretically[1,2] and experimentally[3-5] that, when the size of a given disordered metallic system is small and the temperature is low enough so that both $k_BT/\hbar$ and the inelastic scattering rate $1/\tau_{in}$ are less than the inverse of the time it takes electrons to diffuse across the sample, there exist large sample-to-sample fluctuations in the system's electrical conductance. This phenomenon is understood as the result of quantum interference among the many different multiple-elastic-scattering paths which a conduction electron undergoes in traversing the sample. Second, there has been proposed[6-8] a new approach to understand the various properties of spin-glass systems with short-range [or Ruderman-Kittel-Kasuya-Yosida (RKKY)] interactions, based on the scaling properties of a $T=0$ fixed point, as opposed to the solutions of the infinite-range Sherrington-Kirkpatrick (SK) model. The basic claims of this "zero-temperature scaling theory" are the existence of a true phase transition in three dimensions, the chaotic nature of spin reorganizations at different temperatures, and the absence of a replica-symmetry breaking.

In this paper, we show that the universal conductance fluctuations are a powerful probe of the microscopic spin configurations of metallic spin-glass systems. In fact, we shall show that at low enough temperature, as the temperature changes, the equilibrium (or quasiequilibrium) conductance $G(T)$ of a mesoscopic metallic spin-glass sample fluctuates, in the absence of any applied magnetic field, in reflection of the *chaotic* nature[9] of the spin-glass phase. Observation of these fluctuations would provide a critical test of the scaling theory[6-8] of the spin-glass phase. We shall also show that the conductance at a fixed temperature undergoes thermal fluctuations, which are related directly to the corresponding thermal spin fluctuations in the system due to spontaneous spin-droplet formation, which has been shown before to have a quasi-$1/f$-noise form.[8]

We consider a three-dimensional metallic alloy system with magnetic impurities, such as AuFe, of size L. Ideal conducting leads are attached on two opposing sides, which defines the current direction z. The exchange interaction between the localized moments $\{\mathbf{S}_i\}$ and the conduction electrons is given by

$$H_{ex} = -J\sum_i \boldsymbol{\sigma}(\mathbf{r}_i)\cdot\mathbf{S}_i \ , \tag{1}$$

where $\boldsymbol{\sigma}(\mathbf{r})$ is the spin density of the conduction electrons, and J is an exchange constant. It is well known that an interaction of this type induces an effective exchange coupling among the localized moments, by the so-called RKKY mechanism,[10] i.e., the Hamiltonian describing the effective spin-spin interaction can be written

$$H_{eff} = -\sum_{i,j} J_{RKKY}(\mathbf{r}_{i,j})\mathbf{S}_i\cdot\mathbf{S}_j \ , \tag{2}$$

where $J_{RKKY}(r)\approx(J^2/\varepsilon_F)\cos(2k_Fr)/(2k_Fr)^3$. It is well known[6,11,12] that various sources of anisotropy (e.g., dipolar coupling, or Dzyaloshinskii-Moriya anisotropy arising from spin-orbit coupling) induce crossover to Ising spin-glass behavior at sufficiently large length scales $L\gtrsim L_c$. For simplicity of presentation we will assume here that $L_c\lesssim b$, where $b=n_s^{-1/3}$ is the typical spacing between localized moments and n_s is their density. (This condition is more easily achieved for more anisotropic spin glasses, such as AuFe.) Up to logarithmic corrections,[12] the transition temperature is then approximately given by $T_c\approx J^2/k_B\varepsilon_F(k_Fb)^3$. In the case $J<0$, another temperature scale comes in, namely, the Kondo temperature $T_K\approx(\varepsilon_F/k_B)\exp[-\varepsilon_F/(-J)]$. For $T<T_K$, the Kondo effect becomes important. We restrict our attention to the temperature range $T_k<T<T_c$, i.e., we concentrate on the usual spin-glass phase.

The central notion of the recent scaling theory[6-8] of short-range (or RKKY) Ising spin glasses based on the

$T=0$ fixed point is that the characteristic free energy δF, which can be thought of as the free energy of a typical droplet of size L, scales as $\delta F \sim \Upsilon(T)(L/b)^{y}$, and the maximum activation barrier scales as $B \sim \Delta(T)(L/b)^{\psi}$. Numerical studies[7] suggest that in three dimensions, $y \approx 0.2$. The value of ψ is not known, although McMillan[6] has argued that $\psi = y$. The amplitude function Υ has values[7,8,13] $\Upsilon(T \to 0) \approx T_c$, and $\Upsilon(T \to T_c) \propto (1 - T/T_c)^{y\nu}$. Similarly,[8] $B(T \to 0) \approx T_c$, and $B(T \to T_c) \propto (1 - T/T_c)^{\psi\nu}$. The fact that in three dimensions (3D) $y \approx 0.2$ is much smaller than $d-1=2$, which is the corresponding exponent for a plane interface, implies that a typical domain wall in a spin glass is a rough, fractal-like interface.[8] In fact, one can write the typical surface area of a droplet as L^{d_s}, with $2 \le d_s \le 3$. Bray and Moore recently showed[9] that as temperature changes from T to $T+\delta T$ below T_c, there exists a "coherence length"

$$L^* \sim b \left(\frac{T_c^2}{T \delta T} \right)^{1/\zeta} , \tag{3}$$

where $\zeta = d_s/2 - y$, such that the equilibrium spin configurations corresponding to the two temperatures are totally uncorrelated on length scales $L > L^*$. Fisher and Huse[8] used the same scheme to obtain the spin autocorrelation function of Ising spin glasses due to the equilibrium thermal fluctuations of the spins, because of the spontaneous formation of spin droplets. Since the system under consideration is Ising-like at large length scales, one obtains, for Heisenberg spin glasses with anisotropy, the Fisher-Huse-like result

$$C(t) \equiv [\langle \mathbf{S}_i(t) \cdot \mathbf{S}_i(0) \rangle_T - \langle \mathbf{S}_i(t) \rangle_T \cdot \langle \mathbf{S}_i(0) \rangle_T]_c$$
$$\approx \frac{q_{\mathrm{EA}}(T) T}{\Upsilon(T)} \left(\frac{\Delta(T)}{T \ln(t/t_0)} \right)^{y/\psi} , \tag{4}$$

where $\langle\ \rangle_T$ stands for the thermodynamic average at temperature T, $[\ ]_c$ stands for an average over the ensemble of spin configurations, t_0 is a microscopic time, and $q_{\mathrm{EA}}(T) \equiv [\langle \mathbf{S}_i \rangle_T^2]_c$ is the Edwards-Anderson order parameter.

The above results are all obtained for a system in true *thermodynamic equilibrium.* In realistic experimental situations, this is rarely achievable, i.e., a system is usually "stuck" in some metastable configuration. This situation can nonetheless still be dealt with in our theoretical framework. For a given waiting time t_w, there is associated a length scale $L_w \approx b(T \ln(t_w/t_0)/\Delta)^{1/\psi}$, such that all the droplets of size smaller than L_w have chances to equilibrate, whereas the ones with size larger than L_w are "frozen" in some metastable states. We term this case a "quasiequilibrium" state. Equation (4) holds only for $t < t_w$.[8] If one now changes temperature by an amount δT, which has an associated length scale L^*, then it is apparent that for $L^* > L_w$, no significant spin reorganization can occur for times $t < t_w$; but for $L^* < L_w$, substantial spin reorganization will take place on time scales $t \gtrsim t^* \approx t_0 \exp[(\Delta/T)(L^*/b)^{\psi}]$, resulting in a new "quasiequilibrium" state with spin arrangements significantly different from the old state on length scales greater than L^*. If one measures the thermal spin noise after a waiting time t_w' at the new temperature, Eq. (4) will still hold, as long as $t < t_w'$.

Returning to the conductance fluctuations, we define as usual the elastic mean free path $l = v_F \tau$ for elastic scattering from static impurities. Inelastic scattering processes, such as electron-electron and electron-phonon interactions are described by an inelastic scattering time τ_{in}. The inelastic diffusion length is then given by $L_{\mathrm{in}} = (D\tau_{\mathrm{in}})^{1/2}$, with $D \equiv v_F l/3$. We assume that the disorder is sufficiently strong that $l \ll L_{\mathrm{in}}, L$. We also assume that the sample size L is somewhat larger than L_{in}, as is true in most experimental situations. [Note that since we are dealing with the *ordered* spin-glass phase ($T < T_c$), the spin-flip process is effectively frozen out, despite the relatively high concentration of magnetic impurities. Thus, it is possible to have a regime with L_{in}/L of order unity at low enough temperatures.]

Let us consider the system being in quasiequilibrium at temperature T, after a waiting time t_w. To see how the conductance of the sample couples to the configuration of the spins, we introduce a mean-field approximation. Specifically, we replace the spin variables in Eq. (1) by the corresponding thermal average values, and regard the exchange Hamiltonian as an effective scattering potential for the conduction electrons, in addition to the static impurity scatterings, i.e.,

$$H' = -J \sum_i \sigma(\mathbf{r}_i) \cdot \langle \mathbf{S}_i \rangle_T . \tag{5}$$

The justification for this approximation is that at temperatures much below T_c, the spins are in an ordered phase. Thus as far as conductance is concerned, one can regard the spins as fixed vectors. Let us denote $G(T)$ as the conductance of the system at temperature T. Now if we change to a new temperature, $T+\delta T$, and wait for a time $t_w' \gtrsim t^*$, the system will approach a new quasiequilibrium state with a conductance $G(T+\delta T)$. To see how the new conductance correlates with the old one, we define a correlation function

$$A(\delta T) = [\{G(T) - G(T+\delta T)\}^2]_c . \tag{6}$$

Following Al'tshuler and Spivak,[14] this correlation function can be readily computed to the lowest order in the disorder parameter $1/k_F l$, with the result[15]

$$A(\delta T) = \frac{48}{8} \frac{e^4}{h^2} \frac{1}{L^4} \sum_q \left(\frac{1}{q^4} - \frac{1}{[q^2 + \alpha(\delta T)/l^2]^2} \right) , \tag{7}$$

with

$$\alpha(\delta T) = \frac{n_s J^2}{2 n_i u^2} [(\langle \mathbf{S}_i \rangle_T - \langle \mathbf{S}_i \rangle_{T+\delta T})^2]_c , \tag{8}$$

where $q^2 = (\pi/L)^2(m_z^2 + m_x^2 + m_y^2) + 1/L_{\mathrm{in}}^2$, $m_z = 1, 2, \ldots$, m_x, $m_y = 0, 1, 2, \ldots$, n_i is the density of elastic scatters, and u is the total average elastic-scattering potential per scatterer. Thus, we see that *the conductance correlation function provides direct information on the way spins are correlated at different temperatures.* For $\delta T = 0$, we have $\alpha(0) = 0$, which leads to $A(0) = 0$. From the scaling theory,[9] spins do not change appreciably for $t < t_w$, as long as $L^* > L_w$, or equivalently, $\delta T < \delta T_w^*$

$=(T_c^2/T)(L_w/b)^{-\zeta}$. Whereas for $\delta T > \delta T_w^*$ and a waiting time $t_w' \gg t^*$, spins at two different temperatures differ completely on scales longer than L^*, and one has approximately $\alpha(\delta T) \approx (n_s J^2/n_i u^2) q_{\rm EA}$. So if

$$\frac{n_s J^2}{n_i u^2} q_{\rm EA} \sim \frac{T_c k_F l}{\varepsilon_F} < l^2/L_{\rm in}^2 ,$$

we find that

$$A(\delta T > \delta T_w^*) \equiv \delta G_T^2 \approx \frac{e^4}{h^2} \frac{k_F L_{\rm in}^3}{lL} \frac{T_c}{\varepsilon_F} .$$

If $k_F l T_c/\varepsilon_F \geq l^2/L_{\rm in}^2$, we have the saturation regime $\delta G_T^2 \approx 2(e^4/h^2) L_{\rm in}/L$. Therefore we conclude that when temperature is changed in a mesoscopic metallic spin-glass sample, the system's conductance changes randomly, in reflection of the chaotic nature of the spin-glass phase, *provided the temperature change δT exceeds δT_w^**. This dependence of δT_w^* on t_w provides important information about the length scale of equilibration in the system. This established the conductance fluctuation as a unique probe of the quasiequilibrium spin configurations in metallic spin-glass systems.

Similarly, we can study how the quasiequilibrium thermal noise in conductance relates to the corresponding noise in the spins. Defining a temporal conductance correlation function as

$$B(t) \equiv [\langle \{G(t) - G(0)\}^2 \rangle_T]_c , \tag{9}$$

a generalization of Eq. (7) gives

$$B(t) = 6 \frac{e^4}{h^2} \frac{1}{L^4} \sum_q \left(\frac{1}{q^4} - \frac{1}{[q^2 + \beta(t)/l^2]^2} \right) , \tag{10}$$

where

$$\beta(t) = \frac{n_s J^2}{2 n_i u^2} [\langle \{S(t) - S(0)\}^2 \rangle_T]_c$$
$$= \frac{n_s J^2}{n_i u^2} [1 - q_{\rm EA} - C(t)] .$$

For the unsaturated case

$$\beta(t) \leq \frac{n_s J^2}{n_i u^2} (1 - q_{\rm EA}) \sim \frac{k_F l T}{\varepsilon_F} < l^2/L_{\rm in}^2 ,$$

which is most likely to be true in experimental situations, Eq. (10) can be expanded to yield for the overall temporal conductance fluctuation $\delta G_t^2 \approx e^4/h^2 (k_F L_{\rm in}^3/lL) T/\varepsilon_F$, and for the conductance autocorrelation function

$$D(t) \equiv [\langle G(t) G(0) \rangle_T - \langle G(t) \rangle_T \langle G(0) \rangle_T]_c$$
$$\approx \frac{e^4}{h^2} \frac{k_F L_{\rm in}^3}{lL} \frac{T}{\varepsilon_F} C(t) .$$

Note that δG_t^2 is smaller than the corresponding fluctuations with changing temperatures δG_T^2 by a factor T/T_c, which allows one to experimentally separate these two effects. Note also that as the spin autocorrelation function $C(t)$ gives rise to a quasi-$1/f$ form in its Fourier transform,[8] the same is rue for the function $D(t)$. Thus, we see again that *the conductance thermal noise provides a direct measure of the corresponding noise in the spins.* A schematic picture of our predictions is given in Fig. 1.

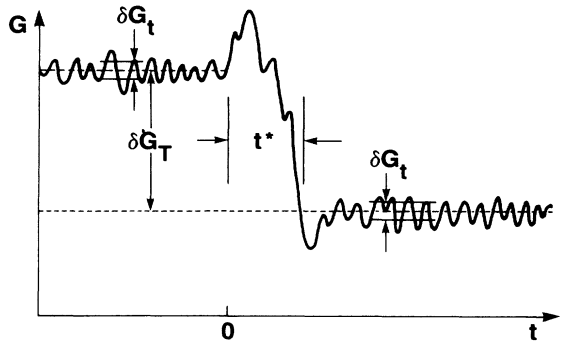

FIG. 1. A schematic picture of the conductance fluctuations in a metallic spin-glass system. The system is first prepared in a quasiequilibrium state after waiting time t_w. It should exhibit thermal fluctuations about a mean conductance with a noise magnitude δG_t. A temperature change δT is applied at time $t = 0$. We assume δT exceeds the correlation temperature δT_w^*. On time scale $t^* \approx t_0 \exp[(\Delta/T)(L^*/b)^\psi]$, where $L^* \approx b(T_c^2/T\delta T)^{1/\zeta}$, massive spin reorganizations occur. On time scales $t^* \ll t < t_w$, the system approaches to a new quasiequilibrium state with a different average conductance, undergoing the same temporal fluctuations as in the beginning state. The typical difference between the two average conductances is of order $\delta G_T > \delta G_t$, in reflection of the chaotic nature of the spin-glass phase.

All of the above discussions have been restricted to the regime $L_{\rm in} < L_T \equiv (\hbar D/k_B T)^{1/2}$. If the opposite is true, an additional thermal averaging factor $(L_T/L_{\rm in})^2$ should be included in all the results. We have so far neglected the possibility of impurity migration, via either quantum tunneling or thermal activation, which has been demonstrated both theoretically[16] and experimentally[17] to give rise to conductance fluctuations, and which can also be shown to induce changes in the RKKY exchange interactions.[18] The justification for this is that these migration events are relatively rare at low tempertures in mesoscopic samples,[17] and the sensitivity of the conductance to these migrations is relatively weak in three dimensions.[16] However, they may explain the rounding of the spin-glass transition observed in recent experimental studies.[19]

We now provide some rough numerical estimates for an experimentally realizable sample (*Au*Fe). Let us suppose the sample size is $L \approx 1$ μm. The values of the other parameters are assumed as follows: $n_s \approx 6 \times 10^{20}$ cm^{-3} (which leads to $b \approx 12$ Å), $\varepsilon_F \approx 6$ eV, $T_c \approx 10$ K, $l \approx 100$ Å, $L_{\rm in} \approx 2000$ Å, $\zeta \approx 1$, $\psi \approx 0.2$, and $t_0 \approx 10^{-12}$ s. At a temperature $T \approx 1$ K and with a waiting time $t_w = 1$ h at each temperature, we then estimate $\delta T_w^* \approx 0.2$ K, $\delta G_t \approx 0.1 e^2/h$, and δG_T has the saturated value $\approx 0.2 e^2/h$. We feel that these numbers are well within the reach of current experimental tools. The major source of uncertainty in the above estimates is the value of the exponent ψ, which clearly deserved further study.

We are grateful to J. Banavar, D. A. Huse, D. Prober, and W. Skocpol for useful discussions. We also acknowledge partial support from the National Science Foundation under Grant No. DMR-82-17965.

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