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Effects of quasiparticle screening on collective modes: Incommensurate charge-density-wave systems

K. Y. M. Wong* and S. Takada
Department of Physics, University of California, Los Angeles, California 90024
and Institute of Physics, University of Tsukuba, Ibaraki 305, Japan
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By including the Coulomb interaction term in the usual Frölich Hamiltonian, the dispersion of the phase mode in incommensurate charge-density-wave systems is found. In the long-wavelength limit, the spectrum consists of an optic branch and an acoustic branch, with the optic branch increasingly dominant at low temperatures and the acoustic branch dominant at high temperatures. The optic-mode frequency is enhanced because of the small effective mass of the quasiparticles. For increasing wave numbers, the two branches merge together, beyond which the single branch approaches an optic frequency $(\frac{3}{2})^{1/2}\omega_\alpha$, where ω_α is the amplitude-mode frequency. In the acoustic branch, the charge densities of the condensed electrons and the quasiparticles oscillate out of phase, balancing each other, while in the optic branch, both components oscillate in phase.

I. INTRODUCTION

It is well known that order-parameter phase fluctuations in incommensurate charge-density-wave (CDW) systems[1] induce charge-density fluctuations, and hence couple to the long-range Coulomb interaction, shifting the frequency of the acoustic phase mode to an optic frequency at 0 K. In these systems, however, quasiparticles are present at nonzero temperatures, and they are able to screen out the Coulomb interaction and reduce it back to an acoustic mode. Thus the motion of the condensed electrons is always accompanied by a countermotion of the quasiparticles. This picture bears a striking similarity with the situation in superconductors.[2] In the latter case, the so-called Carlson-Goldman mode[3] was observed near T_c. The mode consists of the motion of the condensed electrons, as manifested by the fluctuation in the order-parameter phase, counterbalanced by the motion of quasiparticles, which are responsible for the damping of the wave. The damping is sufficiently small for temperatures near T_c, since the quasiparticle countercurrent is small in this case. Below T_c, the mode becomes increasingly damped.

Nakane and Takada[4] pointed out that the existence of the acoustic mode near T_c is a general feature of an order-parameter phase mode which couples to the long-range Coulomb interaction. In fact, the phase-mode velocity they derived in CDW near T_c is identical to that derived in clean superconductors by Artemenko and Volkov,[5] except that the former is divided by the effective-mass ratio $\mu = m^*/m$, and the latter is divided by 3, the dimensionality of the system. This striking similarity of the two systems motivates us to study and compare the underlying dynamics of the modes in detail. We are also interested in whether the similarity of the two systems is a feature near T_c or, as we should expect, extends to lower temperatures. If this is valid, the phase mode in superconductors, which, so far, has only been experimentally observed near T_c, should be observable at temperatures lower than T_c under favorable conditions. These constitute the purposes of the present study.

The presence of the acoustic mode is well established near T_c and, at least theoretically,[4,6] in the intermediate-temperature range in CDW. (By intermediate temperature, we mean the temperature much lower than T_c so that the gap is essentially developed, viz., $T < \Delta$, but high enough that quasiparticle screening can still play a role. The distinction between intermediate and low temperatures will be clarified in subsequent discussions.) On the other hand, the acoustic mode is expected to disappear at zero temperature, since all the quasiparticles are "frozen out." Only optic modes are present at low temperatures. However, it remains unclear how the acoustic mode dominant at higher temperatures evolves into the optic mode at low temperatures. Another purpose of this paper is to resolve this issue. In this paper, we study clean systems in which the electronic transport relaxation rate τ^{-1} is much less than the gap Δ. So far, the Carlson-Goldman mode in superconductors has been observed near T_c for frequencies up to the order of 10^{10} Hz. However, if the temperature in a sufficiently clean system is low enough and the frequency high enough, we can attain a regime in which the quasiparticles are essentially collisionless, i.e., $\omega > \tau^{-1}$. In this regime, we shall show that the acoustic mode continues to exist in the intermediate-temperature range. In particular, we shall study the structure factor for the phase spectrum, which shows that two peaks in the structure factor coexist in the intermediate-temperature range in the long-wavelength limit. One peak corresponds to the acoustic mode and the other to the optic mode, their relative weights being dependent on temperature. At high temperatures, the acoustic branch is dominant, whereas at low temperatures the optic branch is dominant.

In CDW systems, we start by giving a brief survey of the theoretical developments. In these systems, the phase mode with a velocity of propagation $v_\phi = v_F/\sqrt{\mu}$, where $\mu = m^*/m$ is the effective-mass ratio of the CDW, was first predicted by Lee, Rice, and Anderson.[1] Including long-range Coulomb effects, Lee and Fukuyama[7] derived a plasmalike dispersion for the phase mode, namely

$$\omega^2 = v_\phi^2 q_z^2 + \frac{\omega_{p\phi}^2 q_z^2}{\epsilon_z q_z^2 + q_\perp^2} , \tag{1.1}$$

where $\omega_{p\phi}^2 = 4\pi n e^2/m^*$ and $\epsilon_z = 1 + \omega_{pe}^2/6\Delta^2$, with $\omega_{pe}^2 = 4\pi n e^2/m$. As usual, the chain direction of the CDW is taken to be the z direction. Including quasiparticle screening effects, Kurihara[6] derived a dispersion relation, which is essentially

$$\omega^2 = v_\phi^2 q_z^2 + \frac{\omega_{p\phi}^2 q_z^2}{\epsilon_z q_z^2 + q_0^2} . \tag{1.2}$$

Here $q_0^2 = 4\pi e^2 n_{\text{qp}}/T$ is the square of the Thomas-Fermi screening wave number of the quasiparticle gas where n_{qp} is the number density of excited quasiparticles. He pointed out that the acoustic mode disappears at sufficiently low temperature. Nakane and Takada[4] obtained a more detailed expression, which can be written as

$$\omega^2 = v_\phi^2 q_z^2 + \frac{\omega_{p\phi}^2 q_z^2}{\epsilon_z q_z^2 + q_\perp^2 + \left\langle \dfrac{v_{kz}^2 q_z^2}{v_{kz}^2 q_z^2 - \omega^2} \right\rangle q_0^2} , \tag{1.3}$$

where $v_k = v_F \xi_k / E_k$ is the quasiparticle group velocity and $\langle \cdots \rangle$ denotes the thermodynamic average over all quasiparticles. They pointed out that the reason why the acoustic mode disappears at low temperatures is that the quasiparticles become too few and too slow to screen the Coulomb interaction. The importance of their argument will be confirmed in our paper.

In the present paper we give an extension of the Nakane-Takada expression (1.3). We notice that in the intraband screening term, i.e., that third term of the denominator in (1.3), dissipative effects are absent. In practice, a quasiparticle suffers dissipation by emitting or absorbing phonons. Furthermore, a quasiparticle can dissipate energy directly to single-particle excitations. This process is equivalent to the Landau damping process in which the phase velocity of the driving field matches that of the quasiparticle. The former dominates at low frequencies and the latter at high frequencies. As a result of these dissipative processes, the phase mode is no longer a sharply defined spectrum. Instead, its density of states, or structure factor, should be analyzed. The result, as we shall see, is that in the long-wavelength limit the acoustic and optic peaks exist, although their relative weights change with temperature and the acoustic peak disappears at low temperatures. As the wave number increases, the two peaks merge into a single peak. We can classify the situation as acoustic-dominant or optic-dominant according to whether the optic peak disappears into the acoustic peak or vice versa. Generally speaking, the system is optic-dominant at low temperatures but acoustic-dominant at intermediate temperatures.

Because of the length of the material, the discussion will be presented in two separate papers. We shall concentrate on CDW systems in the present paper and deal with superconductors in the accompanying paper. In Sec. II of this paper, we outline the basic formulation for CDW systems. The dielectric function appropriate for the phase mode is derived in Sec. III. The renormalized phase mode is then analyzed in the absence of dissipation and single-particle excitation in Sec. IV, and its structure factor is discussed in the presence of single-particle excitations in Sec. V. Our conclusion is contained is Sec. VI.

II. BASIC FORMULATION

We start with the usual Fröhlich Hamiltonian[8] augmented by the Coulomb interaction term, namely,

$$H = H_F + H_C , \tag{2.1}$$

where

$$H_F = \sum_{\mathbf{k}} c_{\mathbf{k}}^\dagger (\xi_{\mathbf{k}} \sigma_z + \Delta \sigma_1) c_{\mathbf{k}} + \sum_{j,q} \Omega_{jq} b_{jq}^\dagger b_{jq} + \frac{1}{\sqrt{N}} \sum_{j,\mathbf{k},\mathbf{q}} \gamma_{jq} c_{\mathbf{k}+\mathbf{q}}^\dagger \sigma_j c_{\mathbf{k}} \phi_{jq} , \tag{2.2}$$

$$H_C = \frac{1}{2N} \sum_{\mathbf{q}} v(\mathbf{q}) \rho_{\mathbf{q}} \rho_{-\mathbf{q}} , \tag{2.3}$$

where $v(\mathbf{q}) = 4\pi e^2/q^2$ is the Coulomb potential, $\rho_{\mathbf{q}} = \sum_{\mathbf{q}} c_{\mathbf{k}}^\dagger c_{\mathbf{k}+\mathbf{q}}$ is the electronic density operator, and $j = 0,1,2$ corresponds to phonons, the amplitude mode, and the phase mode, respectively. The phonon frequency and electron-phonon coupling for the three modes are, respectively,

$$\Omega_{0\mathbf{q}} = \Omega_{\mathbf{q}}, \quad \Omega_{1\mathbf{q}} = \Omega_{2\mathbf{q}} = \Omega_{\mathbf{Q}} , \tag{2.4a}$$

$$\gamma_{0\mathbf{q}} = \gamma_{\mathbf{q}}, \quad \gamma_{1\mathbf{q}} = \gamma_{2\mathbf{q}} = \frac{\gamma_{\mathbf{Q}}}{\sqrt{2}} . \tag{2.4b}$$

All other symbols are defined as in Takada, Wong, and Holstein.[8]

Let us introduce the electron and phonon Green's functions,

$$G(\mathbf{k}, i\nu_n) = -\int_0^\beta d\tau\, e^{i\nu_n \tau} \langle c_{\mathbf{k}}(\tau) c_{\mathbf{k}}^\dagger(0) \rangle , \tag{2.5a}$$

$$D_j(\mathbf{q}, i\omega_n) = -\int_0^\beta d\tau\, e^{i\omega_n \tau} \langle \phi_{j\mathbf{q}}(\tau) \phi_{j,-\mathbf{q}}(0) \rangle , \tag{2.5b}$$

where $\nu_n = (2n+1)\pi T$ and $\omega_n = 2n\pi T$ with integer n. Similarly, we introduce the polarization Green's function,

$$\Pi_{ij}(\mathbf{q}, i\omega_n) = -\int_0^\beta d\tau\, e^{i\omega_n \tau} \langle \rho_{i\mathbf{q}}(\tau) \rho_{j,-\mathbf{q}}(0) \rangle , \tag{2.5c}$$

where $\rho_{i\mathbf{q}} = \sum_{\mathbf{k}} c_{\mathbf{k}-\mathbf{q}}^\dagger \tau_i c_{\mathbf{k}}$ is the generalized electronic density operator. To the lowest order. These functions are given as

$$G^0(\mathbf{k}, i\nu_n) = \frac{i\nu_n + \xi_k \sigma_z + \Delta \sigma_1}{(i\nu_n)^2 - E_k^2} , \tag{2.6a}$$

$$D_j^0(\mathbf{q},i\omega_n)=\frac{2\Omega_{j\mathbf{q}}}{(i\omega_n)^2-\Omega_{j\mathbf{q}}^2}\ , \tag{2.6b}$$

$$\Pi_{ij}(\mathbf{q},i\omega_n)=\frac{T}{N}\sum_{\mathbf{k},i\nu_n}\mathrm{Tr}\sigma_i G^0(\mathbf{k},i\nu_n)\sigma_j \times G^0(\mathbf{K}+\mathbf{q},i\nu_n+i\omega_2)\ , \tag{2.6c}$$

where $E_{\mathbf{k}}^2=\xi_{\mathbf{k}}^2+\Delta^2$. In this paper, we focus our attention on the phason Green's function. In the so-called random-phase approximation, the screening effects of the condensed electron are taken into account by the $2k_F$ density-density correlation Π_{22}. As shown by Lee, Rice, and Anderson (LRA),[1] the corresponding Dyson equation is

$$(D_2^{\mathrm{LRA}})^{-1}(\mathbf{q},i\omega_n)=(D_2^0)^{-1}(\mathbf{q},i\omega_n)-\frac{\gamma_Q^2}{2}\Pi_{22}(\mathbf{q},i\omega_n)\ , \tag{2.7}$$

which a solution

$$D_2^{\mathrm{LRA}}(\mathbf{q},i\omega_n)=\frac{2\Omega_Q}{(i\omega_n)^2-v_\phi^2 q_z^2}\ , \tag{2.8}$$

where $v_\phi^2=v_F^2/\mu$, with $\mu=m^*/m$ being the effective-mass ratio of the CDW.

To include Coulomb effects, we consider the phase-mode self energy as shown in Fig. 1(a). In this figure, the Coulomb line $\tilde{V}$ represents the Coulomb potential screened by the electrons. Thus $\tilde{V}$ is given in Fig. 1(b) by

$$\tilde{V}(\mathbf{q},i\omega_n)=\frac{V(\mathbf{q})}{\epsilon(\mathbf{q},i\omega_n)}\ , \tag{2.9}$$

with the dielectric function defined by

$$\epsilon(\mathbf{q},i\omega_n)=1-\Pi_{00}(\mathbf{q},i\omega_n)V(\mathbf{q})\ . \tag{2.10}$$

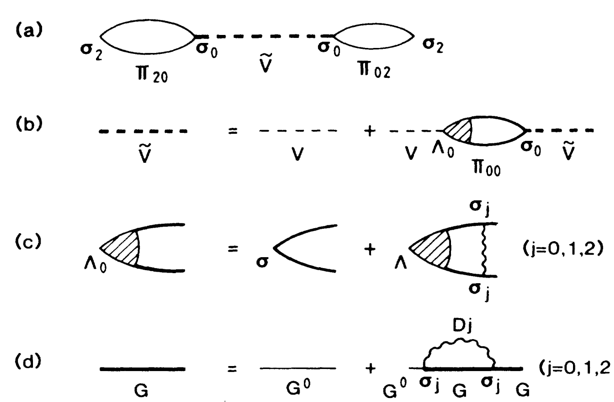

FIG. 1. (a) The phason self-energy due to Coulomb effects; (b) the screened Coulomb potential used in deriving the dielectric function for the phason; (c) the vertex equation; (d) the dressed-electron Green's function leading to the transport equation for the quasiparticles.

Here we would like to emphasize that the dielectric function as defined in (2.10) is only the dielectric function experienced by the phase mode arising from condensed electrons and quasiparticles. It does not include the screening effect of the phase mode itself. On the other hand, if an external charge is placed into the CDW system, the total dielectric function should include the screening of the electrons plus the phase mode.[9]

Thus far, quasiparticle scattering by thermal phonons, amplitude modes and phase modes, have not been considered. This can be taken into account by introducing the so-called ladder approximation into the density-density correlation Π_{00}. In this approximation, Π_{00} is related to the corrected vertex Λ_0 via the equation

$$\Pi_{00}(q)=\frac{I}{N}\sum_k \mathrm{Tr}\Lambda_0(k_-,k_+)G(k_-)\sigma_0 G(k_+)\ , \tag{2.11}$$

where the corrected vertex Λ_0 is defined by the vertex equation shown in Fig. 1(c), namely

$$\Lambda_0(k_{1-},k_{1+})=\sigma_0-\frac{T}{N}\sum_{j,k_2}\gamma_j^2 D_j(k_2-k_1) \times[\sigma_j G(k_{2-})\Lambda_0(k_{2-},k_{2+}) \times G(k_{2+})\sigma_j]\ . \tag{2.12}$$

Here we have used the four-vector notations $q=(\mathbf{q},i\omega_n)$, $k_-=(\mathbf{k}-\mathbf{q}/2,i\nu_n)$, and $k_+=(\mathbf{k}+\mathbf{q}/2,i\nu_n+i\omega_n)$, etc. The vertex equation will be solved in the following section.

Returning to the Coulomb-coupled Green's function, we can now set up the corresponding Dyson equation, which is

$$D_2^{-1}(q)=(D_2^{\mathrm{LRA}})^{-1}(q)-\frac{\gamma_Q^2}{2}\Pi_{20}(q)\tilde{V}(q)\Pi_{02}(q)\ . \tag{2.13}$$

As will be shown in the following section, the ladder-diagram to the phase-density bubble $\Pi_{20}=-\Pi_{02}$ are of secondary importance. It is therefore sufficient to substitute the zeroth-order expression (2.6c) for II_{20} and II_{02} into (2.13). Explicitly, we have

$$\Pi_{20}(q)=-\Pi_{02}(q) =-\frac{iv_F q_z}{\Delta}N(0)\left[1+O\left[\left[\frac{v_F q_z}{\Delta}\right]^2\right]+O\left[\left[\frac{\omega}{\Delta}\right]^2\right]\right]\ . \tag{2.14}$$

Neglecting higher-order terms of $O((v_F q_z/\Delta)^2)$ and $O((\omega/\Delta)^2)$, substitution of (2.9) and (2.14) into (2.13) simplifies it into

$$D_2(\mathbf{q},i\omega_n)=\frac{2\Omega_Q}{(i\omega_n)^2-v_\phi^2 q_z^2-\dfrac{\omega_{p\phi}^2}{\epsilon(\mathbf{q},i\omega_n)}\dfrac{q_z^2}{q^2}}\ , \tag{2.15}$$

where $\omega_{p\phi}^2 = 4\pi e^2 n / m^*$ takes the meaning of the condensate plasma frequency. In deriving this equation, we have made use of the definitions for the dimensionless electron-phonon coupling,

$$\lambda = \frac{2\gamma_Q^2 N(0)}{\Omega_Q} , \tag{2.16a}$$

the effective-mass ratio of the CDW,

$$\mu = \frac{m^*}{m} = \frac{4\Delta^2}{\lambda \Omega_Q^2} , \tag{2.16b}$$

and the single-spin density of states at Fermi level,

$$2N(0) = \frac{n}{m v_F^2} . \tag{2.16c}$$

The pole of the Coulomb-coupled phason Green's function (2.15) gives the phason dispersion on analytic continuation $i\omega_n \to \omega + i\delta$, which is

$$\omega^2 = v_\phi^2 q_z^2 + \frac{\omega_{p\phi}^2}{\epsilon(\mathbf{q}, \omega + i\delta)} \cos^2\theta , \tag{2.17}$$

with $\cos\theta = q_z / q$. This relation shows that when Coulomb effects are included, the phase mode is shifted to a plasma frequency with effective m^* per electron. Furthermore, the resultant "CDW plasma" is immersed in an electronic medium of dielectric function $\epsilon(q, \omega + i\delta)$. In fact, (1.1)–(1.3) correspond to the phason dispersion in different approximations for the dielectric function. We shall therefore focus ourselves on the evaluation of the dielectric function in the ladder approximation.

III. DIELECTRIC FUNCTION FOR THE PHASON

The solution of the vertex equation (2.12), and hence the evaluation of the density-density correlation Π_{00} in (2.11), is given in Appendix A. Here we just outline the steps. The vertex equation (2.12), together with the self-consistent definition of the dressed electron Green's function shown in Fig. 1(d), can be reduced to a transport equation[10,11] of the quasiparticles. In the collision term of the transport equation, we adopt the so-called elastic collision approximation, in which an electron is scattered from a state of k_z to the final state $-k_z$, owing to the fact that the slope v_F of the one-dimensional electron spectrum is essentially greater than the slope c_s (v_ϕ or $\omega_\alpha / 2k_z$) of the phonons (phase modes or amplitude modes). Although the actual calculation is too complex to give a step-by-step interpretation, the essential features of the result can be captured by the following heuristic argument. In the elastic collision approximation, we approximate the collision term by

$$\mathcal{C} = -\frac{g_k - g_{-k}}{2\tau_{\rm qp}} , \tag{3.1}$$

where $\mathcal{C}$ is the collision term, is the off-equilibrium distribution function of momentum k, and $\tau_{\rm qp}$ is the quasiparticle transport relaxation time. The relaxation time is $2\tau_{\rm qp}$ instead of $\tau_{\rm qp}$ because we have included a factor of $1 - \cos\theta = 2$ in the definition of $\tau_{\rm qp}^{-1}$. Therefore, when driven by a potential $\delta\mu$, the transport equation is given by

$$-i\omega g_k + i v_k q_z g_k - i q_z v_k \frac{\partial f_k}{\partial E_k} \delta\mu = -\frac{g_k - g_{-k}}{2\tau_{\rm qp}} . \tag{3.2}$$

Together with the equation for $g_{-k'}$, we have

$$\begin{bmatrix} -i\omega + i v_k q_z + \tau_{\rm qp}^{-1}/2 & -\tau_{\rm qp}^{-1}/2 \\ -\tau_{\rm qp}^{-1}/2 & -i\omega - i v_k q_z + \tau_{\rm qp}^{-1}/2 \end{bmatrix} \times \begin{bmatrix} g_k \\ g_K \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} i v_k q_z \delta\mu \frac{\partial f_k}{\partial \mu} , \tag{3.3}$$

where we have made use of the identity $\partial f_k / \partial E_k = -\partial f_k / \partial \mu$. The solution to (3.3) is

$$g_k = -\left[\frac{v_k q_z + \omega}{v_k^2 q_z^2 - i\omega\tau_{\rm qp}^{-1} - \omega^2} \right] v_k q_z \delta\mu \frac{\partial fk}{\partial \mu} . \tag{3.4}$$

The quasiparticle charge density induced by the potential is therefore

$$\rho_{\rm qp}^{\rm ind} = -\left\langle \frac{v_{kz}^2 q_z^2}{v_{kz}^2 q_z^2 - i\omega\tau_{\rm qp}^{-1} - \omega^2} \right\rangle e \; \delta\mu \frac{\partial n_{\rm qp}}{\partial \mu} , \tag{3.5}$$

where $n_{\rm qp}$ is the total density of quasiparticles. We have also made use of the central symmetry of the system. The corresponding dielectric function is therefore

$$\epsilon(\mathbf{q}, \omega) = 1 + (\epsilon_z - 1)\cos^2\theta + \left\langle \frac{v_{kz}^2 q_0^2}{v_{kz}^2 q_z^2 - i\omega\tau_{\rm qp}^{-1} - \omega^2} \right\rangle \cos^2\theta , \tag{3.6}$$

where, to repeat, $\epsilon_z = 1 + \omega_{pe}^2 / 6\Delta^2$ is the dielectric function due to the interband contribution and $q_0^2 = 4\pi e^2 n_{\rm qp} / T$ is the square of the Thomas-Fermi screening wave number of the quasiparticle gas. Here we have assumed the limit $T \ll \Delta$ and $\omega \ll 2\Delta$, in which quasiparticles can be treated as Maxwell-Boltzmann particles with an effective mass $m_{\rm pq} = \Delta / v_F^2$. Note that this formula gives correct results in various limiting cases: in the quasistatic limit,

$$\epsilon(\mathbf{q}, \omega) = 1 + (\epsilon_z - 1)\cos^2\theta + \frac{q_0^2}{q^2} , \tag{3.7a}$$

whereas in the Drude limit,

$$\epsilon(\mathbf{q}, \omega) = 1 + (\epsilon_z - 1)\cos^2\theta - \frac{4\pi\sigma_{\rm qp}}{i\omega} \cos^2\theta , \tag{3.7b}$$

where $\sigma_{\rm qp} = n_{\rm qp} e^2 \tau_{\rm qp} / m_{\rm qp}$ is the quasiparticle conductivity, and in the plasma limit of quasiparticle inertial screening,

$$\epsilon(\mathbf{q}, \omega) = 1 + (\epsilon_z - 1)\cos^2\theta - \frac{\omega_{\rm qp}^2}{\omega^2} \cos^2\theta , \tag{3.7c}$$

where $\omega_{\rm qp} = (4\pi e^2 n_{\rm qp} / m_{\rm qp})^{1/2}$ can be considered as the quasiparticle plasma frequency.

Equation (3.6), however, does not give the expression in the diffusive limit, or the hydrodynamic limit. This is due to the elastic collision approximation, in which electronic distribution functions of different $|k_z|$ do not mix in the transport equation. The diffusive limit expression, however, can be obtained easily by including inelastic scattering in the transport equation.[12] As derived in Appendix B, the result for the hydrodynamic regime is given by

$$\epsilon(\mathbf{q},\omega)=1+(\epsilon_z-1)\cos^2\theta+\frac{\mathcal{D}q_0^2}{-i\omega+\mathcal{D}q_z^2}\cos^2\theta\ , \qquad (3.7\text{d})$$

where $\mathcal{D}=\langle v_{k_z}^2\rangle\tau$ is the diffusion constant.

Following (2.15), we arrive at the dispersion relation of the phase mode,

$$\omega^2=v_\phi^2q_z^2+\frac{\omega_{p\phi}^2q_z^2}{\epsilon_zq_z^2+q_\perp^2+\left\langle\frac{v_{kz}^2q_z^2}{v_{kz}^2q_z^2-i\omega\tau_{\text{qp}}^{-1}-\omega^2}\right\rangle q_0^2}\ , \qquad (3.8)$$

which is a further development of formulas (1.1)–(1.3).

It is interesting to note that in the long-wavelength regime at microwave frequency, the dissipative term in the denominator of the quasiparticle average in (3.8) is dominant and we get

$$\omega^2=v_\phi^2q_z^2-i\omega\frac{\omega_{p\phi}^2}{4\pi\sigma_{\text{qp}}}=v_\phi^2q_z^2-i\omega\frac{n}{n_{\text{qp}}}\frac{m_{\text{qp}}}{m^*}\tau_{\text{qp}}^{-1}\ , \qquad (3.9)$$

the damping factor being in agreement with the prediction of Takada, Wong, and Holstein,[8] except that the band electron mass is replaced by the quasiparticle effective mass. In the case of the CDW representative material TaS_3, this is of the order $\Delta/v_F^2\sim m_e/500$. (The smallness of the quasiparticle effective mass has far-reaching consequences, which will be discussed below.) Thus the phase mode is damped because the phase motion is coupled to the quasiparticle motion, which is damped by emission and absorption of thermal phonons, amplitude modes, and phase modes.[13]

IV. PHASE-MODE DISPERSION IN THE ABSENCE OF SINGLE-PARTICLE EXCITATIONS

We now examine the regime where $\omega\gg\tau_{\text{qp}}^{-1}$. We assume that quasiparticles are collisionless, i.e., $\omega\gg\tau_{ee}^{-1}$, where τ_{ee} is the electron-electron collision time. (Otherwise, a hydrodynamic description will be necessary.) Equation (3.8) then reduces to the Nakane-Takada result (1.3). The average Doppler factor is then given by

$$D\equiv D_1+iD_2=\left\langle\frac{v_{kz}^2q_z^2}{v_{kz}^2q_z^2-i\omega\delta-\omega^2}\right\rangle\ , \qquad (4.1)$$

where δ is the infinitesimal positive quantity. The real part of (4.1) can be written as

$$D_1=\left[\frac{2}{\pi}\right]^{1/2}\tilde{c}\int_0^\infty dx\, e^{-\tilde{c}^2x^2/2}\frac{x^2}{x^2-1}\ , \qquad (4.2)$$

where $\tilde{c}^2=\omega^2/\langle v_{k_z}^2\rangle q_z^2$. Equation (4.2) can be shown to satisfy the differential equation

$$\frac{\partial D_1}{\partial\tilde{c}}=-(\tilde{c}-\frac{1}{\tilde{c}})D_1-\frac{1}{\tilde{c}}\ . \qquad (4.3)$$

The appropriate boundary conditions for its solution can be seen from two limiting cases. When $c^2\ll\langle v_{kz}^2\rangle$, most particles are at the "hot" end of the Maxwell-Boltzmann distribution and the average Doppler factor approaches 1 from below, but when $c^2\gg\langle v_{kz}^2\rangle$ most particles are at the "cold" end, and the average Doppler factor approaches zero from the negative side. The solution to (4.2) satisfying $D_1(0)=1$ [or, equivalently, $D_1(\infty)=0$] is

$$D_1=1-\tilde{c}e^{-\tilde{c}^2/2}\int_0^{\tilde{c}}e^{v^2/2}dv\ . \qquad (4.4)$$

The imaginary part is given by

$$D_2=\left[\frac{\pi}{2}\right]^{1/2}\tilde{c}e^{-\tilde{c}^2/2}\ . \qquad (4.5)$$

Since only those states with $v_{kz}=\omega/q_z$ contribute to D_2, it represents the contribution from single-particle excitations, in which the z component of the phase velocity of the driving field, given by ω/q_z, matches that of the group velocity of the quasiparticle at state k. In this way, the quasiparticle is able to ride on the driving field and continually absorb energy from it. This is the mechanism of the so-called Landau damping process. A plot of (4.4) and (4.5) is shown in Fig. 2.

In order to study the dispersion in detail, we neglect D_2 initially. Below, we shall discuss the propagating modes, the optic mode and the general dispersion separately.

A. Propagating modes

Our starting point is the Nakane-Takada expression (1.3). In the long-wavelength limit and for $q_\perp=0$ the search for propagating mode solutions is equivalent to solving graphically the pair of equations

$$y=\frac{\omega_{p\phi}^2/q_0^2}{c^2-v_\phi^2}=\frac{e^{\Delta/T}}{\mu}\left[\frac{\Delta/T}{2\pi}\right]^{1/2}\frac{1}{\tilde{c}^2-\tilde{v}_\phi^2}\ , \qquad (4.6\text{a})$$

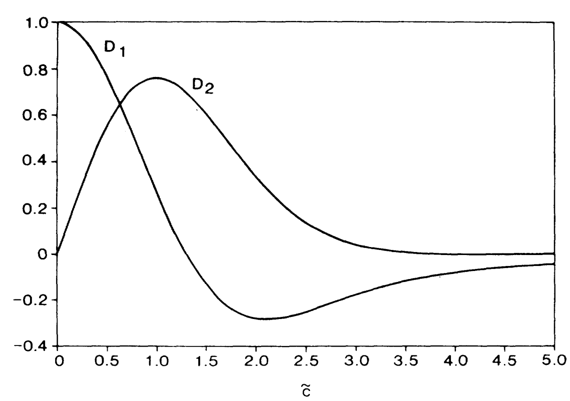

FIG. 2. A plot of (4.4) and (4.5).

$$y = D_1(\tilde{c}) , \tag{4.6b}$$

where $\tilde{c}^2 = c^2/\langle v_{kz}^2 \rangle$ and $\tilde{v}_\phi^2 = v_\phi^2/\langle v_{kz}^2 \rangle$. It is evident from Fig. 2 that, in general, two propagating solutions are possible. Consider first the "high"-temperature limit. In this limit, the slow-mode velocity is given by the condition $D_1(\tilde{c}) = 1$, yielding

$$c^2 = v_\phi^2 + \frac{\omega_{p\phi}^2}{q_0^2} = \left[1 + \frac{e^{\Delta/T}}{(2\pi\Delta/T)^{1/2}} \right] v_\phi^2 , \tag{4.7}$$

agreeing with the renormalized phase-mode velocity of Nakane and Takada. The fast-mode velocity is given by the zero of $D_1(\tilde{c})$, which is $\tilde{c}_0 = 1.31$. This gives

$$c^2 = 1.72(T/\Delta) v_F^2 . \tag{4.8}$$

Next, we consider the situation for all temperatures below T_c. In the case $\mu = 1000$, which is comparable to the effective-mass ratio of CDW in TaS_3, two propagating branches are possible for temperatures higher than $T_1 = 0.17\Delta$, whereas no propagating mode is possible for temperatures lower than T_1, as discussed by Nakane and Takada.[4] Figure 3 shows the temperature dependence of the velocities of propagation. As the temperature is lowered, the slow-mode velocity increases because the quasiparticles become less effective in screening the Coulomb forces due to its smaller density (as reflected by a smaller value of q_0) and its less effective screening motion [as reflected by a smaller value of $D_1(\tilde{c})$].

As the temperature decreases, the slow-mode velocity increases. When T_1 is reached, the velocities of the two modes coincide. Below this temperature, no more propagating modes are possible. This corresponds to the case discussed by Nakane and Takada, when the quasiparticles are too few and too slow to screen the charge induced by the phase motion, and only optic oscillations remain.

The difference of the two branches can best be analyzed by studying the solution to the Boltzmann equation, which gives the off-equilibrium distribution function as

$$g_k = -\frac{v_{kz}}{v_{kz} - c} \left[-\frac{\partial f_k}{\partial E_k} \right] \delta\mu . \tag{4.9}$$

The Doppler factor $v_{kz}/(v_{kz} - c)$ is positive for all quasiparticles with $v_{kz} > c$, and negative for those with $v_{kz} < c$. Consider the case for which the two branches have very different velocities. In the slow branch $c \ll \langle v_{kz}^2 \rangle^{1/2}$, and almost all forward going particles (i.e., particles with $v_{kz} > 0$) have a positive Doppler factor, screening the potential effectively. In the fast branch, $c \ll \langle v_{kz} \rangle^{1/2}$, and almost all forward-going particles have a negative Doppler factor; they counterscreen the potential, leaving the job of screening solely to very fast quasiparticles. The screening is much weaker and a higher propagation velocity results. Table I summarizes the different phase relations of the plasma, fast, and slow branches.

The dispersion relation for a general wave number can be obtained similarly. In the "high"-temperature limit, the dispersion relation of the slow mode for a general wave number can be obtained by putting $D_1 = 1$ into the Nakane-Takada expression (1.3). The result is[14]

$$\omega^2 = \left[v_\phi^2 + \frac{\omega_{p\phi}^2}{\epsilon_z q_z^2 + q_0^2} \right] q_z^1 . \tag{4.10}$$

The limiting cases are then

$$\omega^2 = \left[v_\phi^2 + \frac{\omega_{p\phi}^2}{q_0^2} \right] q_z^2 \quad \text{for } q_z^2 \ll \frac{q_0^2}{\epsilon_z} , \tag{4.11a}$$

$$\omega^2 = \frac{\omega_{p\phi}^2}{\epsilon_z} + v_\phi^2 q_z^2 = \tfrac{3}{2}\omega_\alpha^2 + v_\phi^2 q_z^2 \quad \text{for } q_z^2 \gg \frac{q_0^2}{\epsilon_z} , \tag{4.11b}$$

where $\omega_\alpha^2 = \lambda \Omega_Q^2$ is the square of the amplitude-mode frequency discussed by Lee, Rice, and Anderson.[1] For $q_z \ll q_0/(\epsilon_z)^{1/2}$, the phase mode is acoustic and (4.11a) agrees with (4.7), whereas for $|q_z \gg q_0/(\epsilon_z)^{1/2}$ the phase mode is optic. The crossover wave number $q_0/\sqrt{\epsilon_z}$ is the Thomas-Fermi screening wave number of the quasiparticles in the medium. When $|q_z| \ll q_0(\epsilon_z)^{1/2}$, the phase-mode wavelength is much longer than the quasiparticle screening length, so that no net charge density appears on the average distance of one phase-mode wavelength, and the spectrum remains acoustic. Nevertheless, the charge density induced by the phase fluctuations enhances the stiffness of the phase oscillations, thus giving a much higher phase-mode velocity. On the other hand, when $|q_z| \gg q_0/(\epsilon_z)^{1/2}$, the phase-mode wavelength is much shorter than the quasiparticles' screening length, and so quasiparticles are ineffective in screening the charge fluctuation on the average distance of one

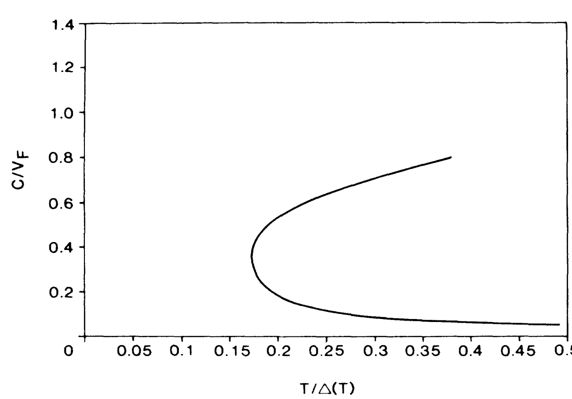

FIG. 3. Temperature dependence of the velocities of propagation for the fast and slow modes. Here $\mu = 1000$.

TABLE I. Different phase relations of the plasma, fast, and slow modes.

Mode	Condensed electrons	Slow qp	Fast qp
Plasma	+	+	+
Fast	+	+	−
Slow	+	−	−

phase-mode wavelength. The spectrum therefore approaches the optic frequency $(\frac{3}{2}\omega_\alpha^2+v_\phi^2 q_z^2)^{1/2}$ as the wave number increases.

B. Optic mode

The dispersion of the optic branch can be obtained by inserting into (1.3) the appropriate expression of D_1 in the limit $\tilde{c} \gg 1$, namely, $D_1 = -\langle v_{kz}^2 \rangle q_z^2/\omega^2$. This yields

$$\omega^2 = \frac{\omega_{p\phi}^2 + \langle v_{kz}^2 \rangle q_0^2}{\epsilon_z} . \tag{4.12}$$

We can interpret the second term as follows. The quasiparticle spectrum obeys classical statistics and is parabolic at temperatures $T \ll \Delta$, with an effective mass $m_{\rm qp} = \Delta / v_F^2$. Therefore the equipartition theorem applies, so that $\langle v_{kz}^2 \rangle = T/m_{\rm qp}$, and the second term can be shown to be the effective plasma frequency for quasiparticles. Therefore,

$$\omega^2 = \frac{\omega_{p\phi}^2 + \omega_{\rm qp}^2}{\epsilon z} , \tag{4.13}$$

which suggests that the optic mode is actually a mixture of CDW condensate plasma and quasiparticle plasma together with a dielectric screening described by ϵ_z. In this mode, the charge densities induced by the CDW condensate and the quasiparticles move in phase (reflected by a negative D_1), in contrast to the out-of-phase motion in the propagating modes. The quasiparticles, though few in number, nevertheless have an unusually small effective mass, contributing to a significant increase of the optic frequency from its bare CDW value, which is $\omega_{p\phi}^2/\epsilon_z = \frac{3}{2}\omega_\alpha^2$.

As the wave number increases from zero, the power series of the Doppler factor in terms of $\langle v_{kz}^2 \rangle q_z^2/\omega^2$ works for a neighborhood of $q_z = 0$, giving

$$\omega^2 = \frac{\omega_{p\phi}^2 + \omega_{\rm qp}^2}{\epsilon z} + \frac{v_\phi^2 \omega_{p\phi}^2 + \langle v_{kz}^4 \rangle q_0^2}{\omega_{p\phi}^2 + \omega_{\rm qp}^2} q_z^2 . \tag{4.14}$$

As the wave number continues to increase further, the power expansion of the Doppler factor fails and an increasingly significant portion of the forward-going quasiparticles changes its counterscreening role to screening role. The optic frequency drops sharply and, as will be shown below, merges with the fast mode.

C. General dispersion relation

The general dispersion relation can be obtained by generalizing (4.6a) and (4.6b) to the $q_z \neq 0$ case, i.e.,

$$y = \frac{\omega_{p\phi}^2/q_0^2}{c^2 - v_\phi^2} - \frac{\epsilon_z q_z^2}{q_0^2} = \frac{e^{\Delta/T}}{\mu} \left[\frac{\Delta/T}{2\pi} \right]^{1/2} \frac{1}{\tilde{c}^2 - \tilde{v}_\phi^2} - \frac{2}{3} \frac{e^{\Delta/T}}{(2\pi\Delta/T)^{1/2}} - \tilde{q}^2 , \tag{4.15a}$$

$$y = D_1(\tilde{c}) , \tag{4.15b}$$

where $\tilde{q} = v_F q_z / 2\Delta$. The situation for $T > T_1$ is shown in Fig. 4(a). It confirms the picture discussed previously: the existence of the fast and slow modes, the slow mode approaching the bare CDW plasma frequency $(\frac{3}{2})^{1/2}\omega_\alpha$ in the large-wave-number limit, the optic branch enhanced by the quasiparticle contribution, and the hyperbolic dispersion of the optic branch in the neighborhood of $q_z = 0$ before merging with the fast mode.

As the temperature approaches T_1, the velocities of the fast and slow branches approach one another and finally coincide at T_1, as shown in Fig. 4(b). It is interesting to note that at temperatures slightly below T_1 [Fig. 4(c)] we still have three branches for a range of wave numbers, although only the optic branch exists in the long-wavelength limit. These pseudofast and pseudoslow branches indicate that the acoustic branch does not disappear abruptly at T_1, but continuously evolves into the optic branch at low temperatures. At very low temperatures, we get only one branch, the optic branch, at all wave numbers.

V. STRUCTURE FACTOR OF THE PHASE MODE IN THE PRESENCE OF SINGLE-PARTICLE EXCITATIONS

The situation is drastically modified if we include the imaginary part of the average Doppler factor (4.5). Since the fast mode lies in the regime $\tilde{c} \sim 1$ where $D_2 \gg D_1$, the inclusion of D_2 completely suppresses the fast mode. The damping comes from single-particle excitations, and is intrinsically present even if we ignore damping by emission and absorption of thermal phonons, amplitude modes, and phase modes. In this case, it is more appropriate to study the density of states, or the structure factor, of the phase spectrum as defined by

$$\rho_{\bf q}(\omega) = -\frac{1}{\pi}\frac{\omega}{\Omega} {\rm Im} D_2({\bf q}, \omega + i\delta) , \tag{5.1}$$

where ω/Ω is the renormalization factor discussed by Holstein (see Appendix III of Ref. 12). The explicit expression of (5.1) is

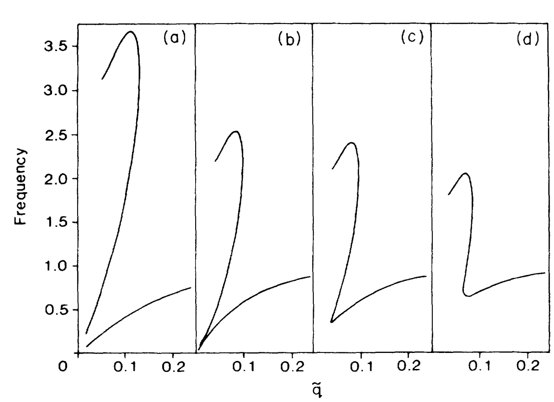

FIG. 4. The dispersion relation of the Nakane-Takada phase mode at different temperatures: $T/\Delta =$ (a) 0.20; (b) 0.173; (c) 0.17; (d) 0.16. The frequency is in units of $(1.5)^{1/2}\omega_\alpha$.

$$\rho_{\mathbf{q}}(\omega) = -\frac{1}{\pi}\mathrm{Im}\frac{2\tilde{\omega}}{\tilde{\omega}^2 - \frac{2}{3}\tilde{q}^2 - \cfrac{1}{1+\cfrac{D}{\frac{2}{3}\frac{e^{\Delta/T}}{(2\pi\Delta/T)^{1/2}}\tilde{q}^2}}}\,, \tag{5.2}$$

where $\tilde{\omega}^2 = 2\omega^2/3\omega_\alpha^2$ and D is the average Dopplere factor given by (4.1). Equation (2.19) can be shown to satisfy the sum rule

$$\int_0^\infty \rho_{\mathbf{q}}(\tilde{\omega})d\tilde{\omega} = 1\,, \tag{5.3}$$

thus justifying its definition. The simplest case to study is $q_z = 0$, where we can directly write

$$\rho_{\mathbf{q}}(\omega) = -\frac{1}{\pi}\mathrm{Im}\frac{2\omega}{\omega^2 - \cfrac{\omega_{p\phi}^2}{\epsilon_z - \cfrac{\omega_{\mathrm{qp}}^2}{(\omega+i\delta)^2}}}\,. \tag{5.4}$$

Peaks are located at $\omega = 0$ and the optic-mode frequency given by (4.13). The weight of each peak is given by

$$n_i = \left| \frac{\partial}{\partial\omega^2}\left[\omega^2 - \frac{\omega_{p\phi}^2}{\epsilon_z - \frac{\omega_{\mathrm{qp}}^2}{\omega^2}}\right]\right|_{\omega_i}^{-1} \quad (i = a, o)\,, \tag{5.5}$$

so that the weight of the acoustic peak is

$$n_a = \frac{\omega_{\mathrm{qp}}^2}{\omega_{p\phi}^2 + \omega_{\mathrm{qp}}^2}\,, \tag{5.6a}$$

whereas for the optic peak,

$$n_o = \frac{\omega_{p\phi}^2}{\omega_{p\phi}^2 + \omega_{\mathrm{qp}}^2}\,. \tag{5.6b}$$

Note that $n_a + n_0 = 1$, satisfying the sum rule, and

$$\frac{n_o}{n_a} = \frac{\omega_{p\phi}^2}{\omega_{\mathrm{qp}}^2} = \frac{e^{\Delta/T}}{\mu}\left[\frac{\Delta/T}{2\pi}\right]^{1/2}, \tag{5.6c}$$

showing that both branches are present at all temperatures, though the acoustic branch dominates at high temperatures while the optic branch dominates at low temperatures.

Figure 5 illustrates the situation for small but nonzero $\tilde{q}$, which numerically results from (4.4), (4.5), and (5.2). For $T > T_1$ the two peaks are given approximately by the Nakane-Takada results (4.6a), (4.6b), and (4.14). As temperature is lowered, the two peaks start to merge together to form one peak. At $T \sim 0.13\Delta$, the optic peak starts to dominate over the acoustic peak. At very low temperatures, the single peak, which is the continuation of the optic peak in the double-peak regime, becomes increasingly sharp.

Figures 6 and 7 illustrate the situation for constant

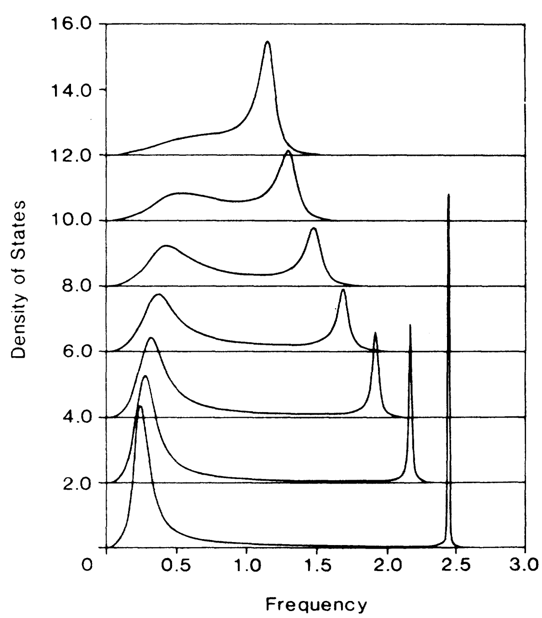

FIG. 5. The density of states, or the structure factor, of the phase mode for different temperatures at $\tilde{q} = 0.05$. Starting from the bottom, the curves correspond to $T/\Delta =$ (a) 0.18; (b) 0.17; (c) 0.16; (d) 0.15; (e) 0.14; (f) 0.13; (g) 0.12. The frequency is in units of $(1.5)^{1/2}\omega_\alpha$.

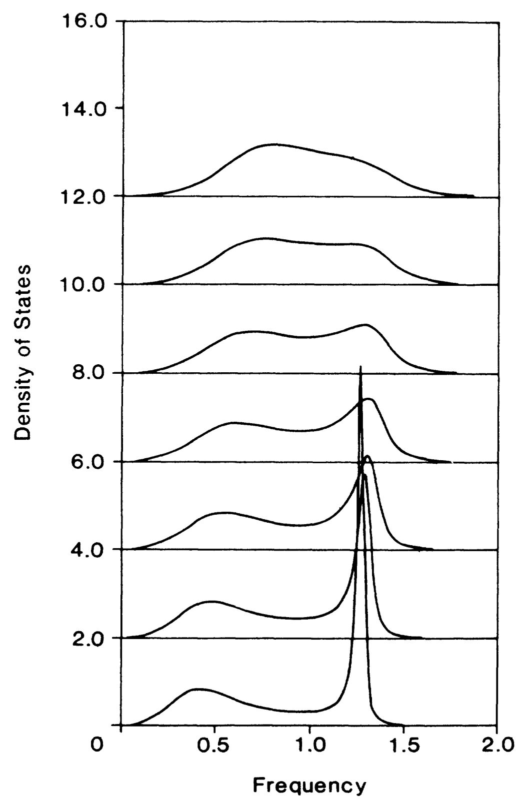

FIG. 6. The density of states, or the structure factor, of the phase mode for different wave numbers at $T/\Delta = 0.13$. Starting from the bottom, the curves correspond to $\tilde{q} =$ (a) 0.04; (b) 0.045; (c) 0.05; (d) 0.055; (e) 0.06; (f) 0.065; (g) 0.07. The frequency is in units of $(1.5)^{1/2}\omega_\alpha$.

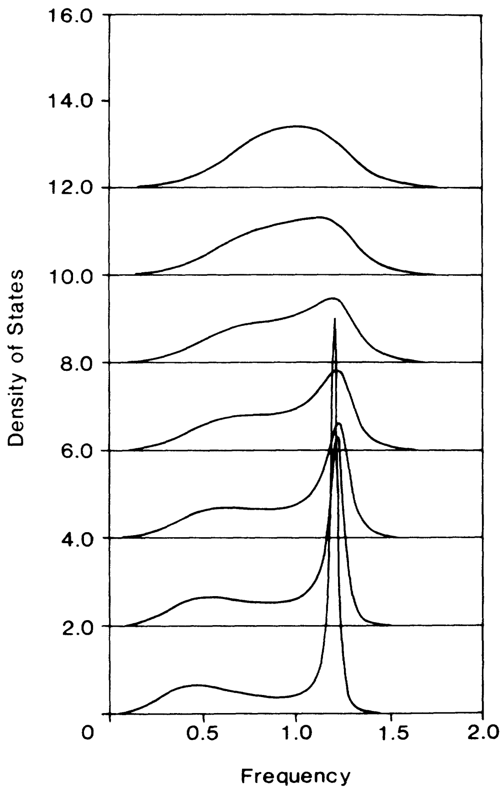

FIG. 7. The density of states, or the structure factor, of the phase mode for different wave numbers at $T/\Delta = 0.125$. Starting from the bottom, the curves correspond to $\tilde{q} =$ (a) 0.04; (b) 0.045; (c) 0.05; (d) 0.055; (e) 0.06; (g) 0.07. The frequency is in units of $(1.5)^{1/2}\omega_\alpha$.

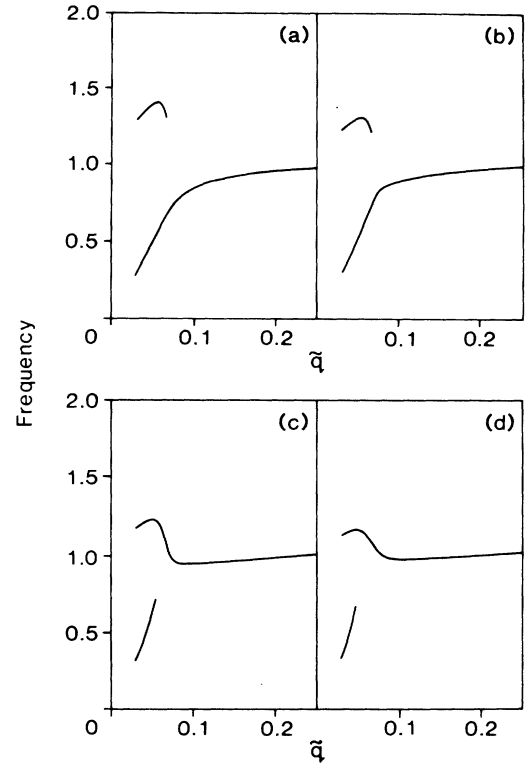

FIG. 8. The peak positions for different temperatures. $T/\Delta =$ (a) 0.135; (b) 0.13; (c) 0.125; (d) 0.12. The frequency is in units of $(1.5)^{1/2}\omega_\alpha$.

temperature. For $T \gtrsim 0.13\Delta$, the optic peak disappears at increasing wave number, whereas for $T \lesssim 0.13\Delta$, the acoustic peak disappears.

Figure 8 shows the peak positions for different temperatures. As before, we note that the acoustic-mode velocity increases with lowering temperatures because of the weakening quasiparticle screening. Also, the optic mode in the long-wavelength limit is enhanced by the presence of the light quasiparticles. On the other hand, the single peak in the large-wave-number limit approaches the optic frequency $(\frac{3}{2})^{1/2}\omega_\alpha$, unenhanced by the quasiparticle term. This is because the quasiparticles are ineffective in screening the Coulomb interaction, and, moreover, the electric field is zero on the average distance of the quasiparticle screening length, so that quasiparticles become unresponsive to the driving field.

In addition, we note that the single peak is continuous with the acoustic peak in the double-peak regime for $T \gtrsim 0.13\Delta$ and with the optic peak for $T \lesssim 0.13\Delta$. The wave number $\tilde{q}$ at which the spectrum crosses over from the double-peak to single-peak regime decreases with temperature. We can see this by noting that as $\tilde{q}$ increases from zero the crossover point occurs at $\tilde{c} \sim 1$ when the hyperbola (4.5a) touches D_1 at one point, and so $\tilde{q} \sim$ the optic frequency, which decreases with lowering temperature as the quasiparticles become fewer. We therefore are able to map out the single-peak and double-peak regimes in the temperature and wave-number space, as schematically shown in Fig. 9. The boundary is given by $\partial\rho/\partial\omega = \partial^2\rho/\partial\tilde{\omega}^2 = 0$. The point P on the boundary is given by $\partial\rho/\partial\tilde{\omega} = \partial^2\rho/\partial\tilde{\omega}^2 = \partial^3\rho/\partial\tilde{\omega}^3 = 0$, corresponding to $T = 0.128\Delta$. Above this temperature, the acoustic peak is dominant, whereas below it, the optic peak is dominant.

Recent experiments show that a small fraction of the Fermi surface remains gapless at 0 K for such CDW sys-

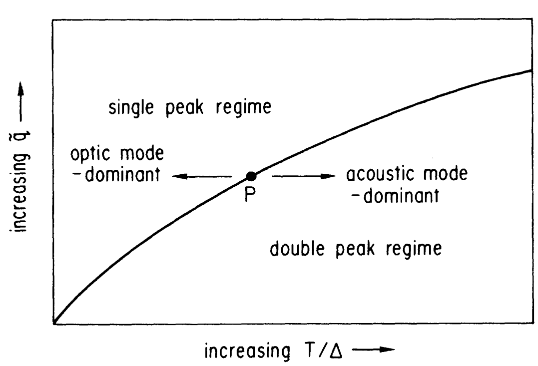

FIG. 9. Single-peak and double-peak regimes in the temperature and wave-number space.

tems as KCP.[15] In this case, the dispersion relation is modified by

$$\omega^2 = v_\phi^2 q_z^2 + \frac{\omega_{p\phi}^2 q_z^2}{\epsilon_z q_z^2 + q_1^2 + \left\langle \frac{v_{kz}^2 q_z^2}{v_{kz}^2 q_z^2 - i\omega\delta - \omega^2} \right\rangle q_0^2 + q_e^2} , \tag{5.7}$$

where $q_e^2 = 8\pi e^2 N_1$, $2N_1$ being the density of states of the Fermi surface remaining gapless. The spectrum is modified, rendering two acoustic peaks as shown in Fig. 10. This modification of the phase-mode spectrum should be considered in experimental analyses.

To conclude this section, we remark that the phase-mode spectrum has been derived in the limit $\omega \gg \tau_{\mathrm{qp}}^{-1}$. Thus the analysis applies to infrared frequencies. Infrared-absorption and neutron scattering experiments are therefore useful tools to test the theory.

VI. CONCLUSION

We have described the dynamics in which order-parameter phase fluctuations couple with charge-density fluctuations. We find that the spectrum consists of two

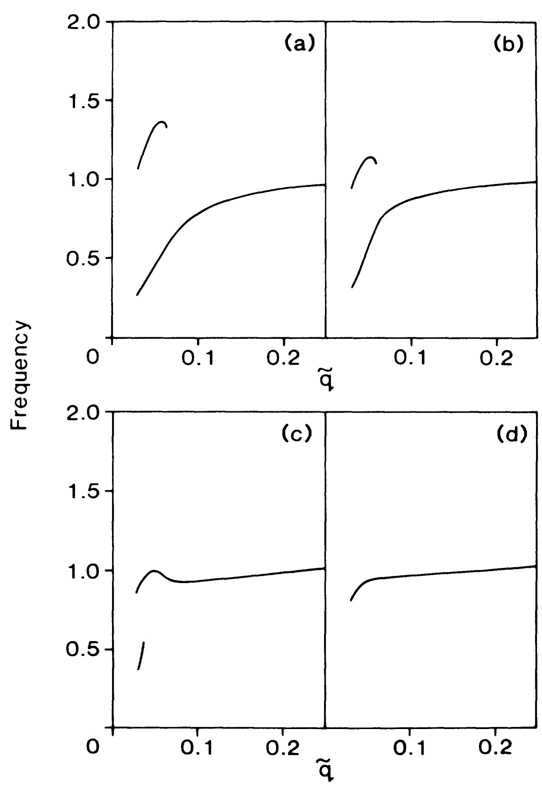

FIG. 10. The peak positions for different temperatures when a fraction of the Fermi surface remains gapless at 0 K. Here $N_1/N_0 = 5\times10^{-4}$. The temperatures are $T/\Delta=$ (a) 0.14; (b) 0.13; (c) 0.12; (d) 0.11. The frequency is in units of $(1.5)^{1/2}\omega_\alpha$.

branches, one acoustic and the other optic. The structure factor is found to satisfy the sum rule. The optic branch is increasingly dominant at low temperatures while the acoustic branch is dominant at intermediate temperatures. In the acoustic branch, the condensate current is balanced by a counterflow of quasiparticle current, whereas in the optic branch, both components oscillate in phase with their own effective masses characterizing their dynamics. The acoustic-mode velocity increases with lowering temperature because of weakening quasiparticle screening. The square of the optic frequency in the long-wavelengh limit is the sum of the condensate plasma frequency ($\omega_{p\phi}^2 = 4\pi e^2 n/m^*$) plus the quasiparticle plasma frequency ($\omega_{\mathrm{qp}}^2 4\pi e^2 n_{\mathrm{qp}}$) divided by ϵ_z, and because of the small effective mass of the quasiparticles the optic frequency is enhanced significantly. When the wave number increases, the two branches merge together, and the single peak is continuous with the acoustic peak at high temperature and the optical peak at low temperatures. In the single-peak regime, the mode frequency approaches $(\frac{3}{2})^{1/2}\omega_\alpha$, because the quasiparticles are ineffective in screening and responding to the driving field.

The situation in superconductors is very similar. In fact, as will be shown in the accompanying paper, the acoustic mode, known as the Carlson-Goldman mode, has the same dispersion relation as that in CDW systems, except that the CDW effective mass is replaced by the electronic mass, and the dimensionality is changed to 3.

As for the plasma branch in superconductors, care must be taken of the mode counting. In CDW, the collective modes are the coupled modes of the ions and electrons, while those are the coupled modes of only electrons in a superconductor. Therefore the number of collective modes is different in CDW systems and in superconductors. In CDW systems, there exist two optic collective modes, one the condensate mode with $\omega^2 = \frac{3}{2}\omega_\alpha^2$ and the other the electron plasma mode with $\omega^2 = 4\pi e^2 n/m$. In superconductors, however, the only optic collective mode is the electronic plasma mode. Furthermore, a sum rule can be proved in superconductors, giving the dispersion $\omega^2 = 4\pi e^2 n/m$, where m is the bare electronic mass. The distinction between the band mass and quasiparticle effective mass is irrelevant here. This is natural, since the plasma mode is a collective translational mode describable by classical dynamics.

We also have to emphasize the requirement of a large CDW effective mass in the derivation of the above dispersion relations. In fact, the optic-mode frequency is given by $\omega^2 = \omega_{p\phi}^2/\epsilon_z = 6\Delta^2/\mu$ $(=\frac{3}{2}\omega_\alpha^2)$, and it requires $\mu \gg \frac{3}{2}$ so that the CDW condensate plasma frequency will remain much less than 2Δ. In fact, we have assumed $\omega \ll 2\Delta$ for the derivation. Otherwise, the expression for ϵ_z has to be modified. In superconductors, however, the effective-mass ratio is 1, and the situation is so modified that the optic mode at $\sqrt{6}\Delta$ does not exist. Since our assumption $\omega \ll 2\Delta$ breaks down, the correct treatment gives the plasma mode with $\omega^2 = 4\pi n e^2/m$.

For the major part of the paper, we have been interested in the collisionless regime, but a generalization to the dissipative regime (i.e., the regime in which the system can dissipate energy into a reservoir of phonons, phase modes, and amplitude modes) has been described in Sec. III. A generalization to the hydrodynamic regime can also be worked out. With these generalizations, we can extend our theory to various regimes, and this will also enable us to study the problem of acoustic attenuation.

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APPENDIX A: DIELECTRIC FUNCTION EXPERIENCED BY THE PHASONS

1. Derivation of the transport equation

The transport equation can be most conveniently derived by introducing matrix direct products, direct sums, and direct differences, which are defined, respectively, as

$$(A\otimes B)_{ij,mn}=A_{im}B_{jn} \ , \tag{A1a}$$

$$A\oplus B=A\otimes I+I\otimes B \ , \tag{A1b}$$

$$A\ominus B=A\otimes I-I\otimes B \ , \tag{A1c}$$

and satisfy the identity

$$AB\otimes CD=(A\otimes C)(B\otimes D) \ . \tag{A2}$$

Here I is the identity matrix. Treating the vertex Λ_0 as a column vector, (2.12) can then be written in the form

$$\Lambda_0(k_{1-},k_{1+})=\sigma_0-\frac{T}{N}\sum_{j,k_2}\gamma_j^2D_j(k_2-k_1)(\sigma_j\otimes\sigma_j^t)\times[G(k_{2+})\otimes G(k_{2-})]\times\Lambda_0(k_{2-},k_{2+}) \ , \tag{A3}$$

where the superscript t denotes the transpose of a matrix. Introduce the distribution function $\Phi_0(k_{2-},k_{2+})$ defined by the matrix equation

$$i\Lambda_0(k_{2-},k_{2+})=[G^{-1}(k_{2+})\ominus G^{-1}(k_{2-})]\Phi_0(k_{2-},k_{2+}) \ , \tag{A4}$$

and (A3) is transformed into

$$-i(G^{-1}(k_{1+})\ominus G^{-1}(k_{1-}))\Phi_0(k_{1-},k_{1+})=\sigma_0-\frac{T}{N}\sum_{j,k_2}\gamma_j^2D_j(k_2-k_1)(\sigma_j\otimes\sigma_j^t)i(G(k_{2+})\ominus G(K_{2-}))\Phi_0(k_{2-},k_{2+}) \ . \tag{A5}$$

Now the Dyson equation in Fig. 1(d) is

$$G^{-1}(k_1)=(G^0)^{-1}(k_1)-\Sigma(k_1) \ , \tag{A6}$$

where

$$\Sigma(k_1)=-\frac{T}{N}\sum_{j,k_2}\gamma_j^2D_j(k_2-k_1)\sigma_jG(k_2)\sigma_j \ . \tag{A7}$$

This simplifies the vertex equation (A4) into

$$-i((G^0)^{-1}(k_{1+})\ominus(G^0)^{-1}(k_{1-}))\Phi_0(k_{1-},k_{1+})=\sigma_0-\frac{T}{N}\sum_{j,k_2}i\gamma_j^2D_j(k_2-k_1)(\sigma_j\otimes\sigma_j^t)[G(k_{2+})\ominus G(k_{2-})]\times[\Phi_0(k_{2-},k_{2+})-(\sigma_j\otimes\sigma_j^t)\Phi_0(k_{1-},k_{1+})] \ . \tag{A8}$$

Equation (A8) now resembles a transport equation. The left-hand side describes the spatial and temporal evolution of the distribution function. The first term on the right-hand side is the driving term, and the second term the collision term. Performing the analytic continuation and the contour integration as usual, we obtain, for the case $i\omega_{1-}\to z_1-\omega/2-i\delta$, $i\omega_{1+}\to z_1+\omega/2+i\delta$,

$$\mathcal{C}=-\frac{1}{N}\sum_{j,\mathbf{k}_2}\int_{-\infty}^{\infty}\frac{d_{z_2}}{2\pi}[n_B(z_3)+f(z_2)]\gamma_j^2[D_j^R(z_3)-D_j^A(z_3)](\sigma_j\otimes\sigma_j^t)[G^R(k_{2+})\ominus G^A(k_{2-})]\times[\Phi_0{}^{AR}(k_{2-},k_{2+})-(\sigma_j\otimes\sigma_j^t)\Phi_0^{AR}(k_{1-},k_{1+})] \ , \tag{A9}$$

where $z_3=z_2-z_1$. Following Holstein,[10] we take the approximation

$$D_j^R(z_3)-D_j^A(z_3)=-2\pi i\sum_{m_3=\pm1}\frac{\Omega_j}{m_3\omega_{j3}}\delta(z_3-m_3\omega_{j3})\ , \tag{A10}$$

and also

$$G^R(k_{2+})\ominus G^A(k_{2-})=P\frac{\xi_2}{z_2^2-E_2^2}\sigma_3\ominus\sigma_3+P\frac{\Delta}{z_2^2-E_2^2}\sigma_1\ominus\sigma_1-(z_2I\oplus I+\xi_2\sigma_3\oplus\sigma_3+\Delta\sigma_1\oplus\sigma_1)i\pi\sum_{m_2=\pm1}\frac{\delta(z_2-m_2E_2)}{2m_2E_2}\ , \tag{A11}$$

where $\omega_{j3}\equiv\omega_j(\mathbf{q}_3)\equiv\omega_j(\mathbf{k}_2-\mathbf{k}_1)$. Noticing that the rest of the integrand in (A9) are slowly varying functions of z_2 when compared with $P/(z_2^2-E_2^2)$, we ignore the first two terms of (A11). This simplifies (A9) into

$$\mathcal{C}=-\frac{1}{N}\sum_{j,\mathbf{k}_2}\int_{-\infty}^{\infty}\frac{dz_2}{2\pi}[n_B(z_3)+f(z_2)]\gamma_j^2\frac{\Omega_j}{m_3\omega_{j3}}\delta(z_3-m_3\omega_{j3})\frac{\delta(z_2-m_2E_2)}{2m_2E_2}(\sigma_j\otimes\sigma_j^t)$$
$$\times(z_2I\oplus I+\xi_2\sigma_3\oplus\sigma_3+\Delta\sigma_1\oplus\sigma_1)[\Phi_0^{AR}(k_{2-},k_{2+})-(\sigma_j\otimes\sigma_j^t)\Phi_0^{AR}(k_{1-},k_{1+})]\ . \tag{A12}$$

We are especially interested in the case $z_1=m_1E_1$ with $m_1=\pm1$. In this case, the major contribution comes from $m_2=m_1$, rendering the collision term into

$$\mathcal{C}=-\frac{1}{N}\sum_{j,\mathbf{k}_2}\{[n_B(\omega_{je})+f(E_2)]\delta(E_2-E_1-\omega_{j3})+[n_B(\omega_{j3})+1-f(E_2)]\delta(E_2-E_1+\omega_{j3})\}$$
$$\times\gamma_j^2\frac{\Omega_j}{\omega_{j3}}(\sigma_j\otimes\sigma_j^t)\left[I\oplus I+\frac{\xi_1}{m_1E_1}\sigma_3\oplus\sigma_3+\frac{\Delta}{m_1E_1}\sigma_i\oplus\sigma_1\right][\Phi_0(\mathbf{k}_2)-(\sigma_j\otimes\sigma_1^+)\Phi_0(\mathbf{k}_1)]\ , \tag{A13}$$

where $\Phi_0(\mathbf{k}_1)$ is the shorthand form of $\Phi_2^{AR}(q_{1-},\, m_1E_1-\omega/2,\, m_1E_1+\omega/2)$.

It is now time to introduce the elastic collision approximation, in which

$$k_{2z}\approx-k_{1z},\quad E_2\approx E_1\ . \tag{A14}$$

This simplifies (A13) into

$$\mathcal{C}=\sum_j\frac{\tau_j^{-1}(k_{1z})}{8}(\sigma_j\otimes\sigma_j^t)\left[I\oplus I-\frac{\xi_1}{m_1E_1}\sigma_3\oplus\sigma_3+\frac{\Delta}{m_1E_1}\sigma_1\oplus\sigma_1\right][\Phi_0(-k_{1z})-(\sigma_j\otimes\sigma_j^t)\Phi_0(k_{1z})]\ , \tag{A15}$$

where $\tau^{-1}(k_{1z})$ is the transport relaxation rate as predicted by Fermi's golden rule,

$$\tau_j^{-1}(k_{1z})=\frac{4\pi}{N}\sum_{q3}\gamma_j^2\frac{\Omega_j}{\omega_{j3}}\{[n_B(\omega_{j3})+f(E_2)]\delta(E_2-E_1-\omega_{j3})+[n_B(\omega_{j3})+1-f(E_2)]\delta(E_2-E_1+\omega_{j3})\}\ . \tag{A16}$$

The last factor of 2 comes from the usual 1-cosθ term in transport calculations. The complete transport equation is therefore

$$-i((G^0)^{-1}(k_{1+})\ominus(G^0)^{-1}(k_{1-}))\Phi_0(k_{1-},k_{1+})=\sigma_0+\sum_j\frac{\tau_j^{-1}(k_{1z})}{8}(\sigma_j\otimes\sigma_j^t)\left(I\oplus I-\frac{\xi_1}{m_1E_1}\sigma_3\oplus\sigma_3+\frac{\Delta}{m_1E_1}\sigma_1\oplus\sigma_1\right)$$
$$\times[\Phi_0(-K_{1z})-(\sigma_j\otimes\sigma_j^t)\Phi_0(k_{1z})]\ . \tag{A17}$$

2. Solution of the transport equation

Since the equation involves the unknowns $\Phi_0(\pm k_{1z})$, we divide Φ_0 into the even and odd components satisfying

$$\Phi_0(k_{1z})=\Phi_g(k_{1z})+\Phi_u(k_{1z})\ , \tag{A18a}$$

$$\Phi_g(-k_{1z})=\Phi_g(k_{1z})\ , \tag{A18b}$$

$$\Phi_u(-k_{1z})=-\Phi_u(k_{1z})\ . \tag{A18c}$$

Furthermore, we introduce the scalars ϕ_k $(k=0,1,2,3)$ by

$$\Phi_0(k_{1z})=\phi_0+\phi_1\sigma_1+\phi_2\sigma_2+\phi_3\sigma_3\ . \tag{A19}$$

Multiplying (A17) by $\sigma_i\otimes I$ $(i=0,1,2,3)$ and taking the trace, we arrive at four equations:

$$-i\omega\phi_0+i\xi\phi_3=-\frac{\tau_0^{-1}}{2}\left[\phi_{u0}-m_1\frac{\xi_1}{E_1}\phi_{u3}+m_1\frac{\Delta}{E_1}\phi_{u1}\right]-\frac{\tau_1^{-1}}{2}\left[\phi_{u0}+m_1\frac{\xi_1}{E_1}\phi_{g3}+m_1\frac{\Delta}{E_1}\phi_{u1}\right)-\frac{\tau_2^{-1}}{2}\left[\phi_{u0}+m_1\frac{\xi_1}{E_1}\phi_{g3}-m_1\frac{\Delta}{E_1}\phi_{g1}\right]+1\,, \tag{A20a}$$

$$-i\omega\phi_1+2\xi_1\phi_3=-\frac{\tau_0^{-1}}{2}\left[\phi_{u1}+m_1\frac{\Delta}{E_1}\phi_{u0}\right]-\frac{\tau_1^{-1}}{2}\left[\phi_{u1}+m_1\frac{\Delta}{E_1}\phi_{u0}\right]-\frac{\tau_2^{-1}}{2}\left[\phi_{g1}-m_1\frac{\Delta}{E_1}\phi_{u0}\right]\,, \tag{A20b}$$

$$-i\omega\phi_2-2\xi_1\phi_1+2\Delta\phi_3=-\frac{\tau_0^{-1}}{2}\phi_{u2}-\frac{\tau_1^{-1}}{2}\phi_{g2}-\frac{\tau_2^{-1}}{2}\phi_{u2}\,, \tag{A20c}$$

$$-i\omega\phi_3+i\xi\phi_0-2\Delta\phi_2=-\frac{\tau_0^{-1}}{2}\left[\phi_{u3}-m_1\frac{\xi_1}{E_1}\phi_{u0}\right]-\frac{\tau_1^{-1}}{2}\left[\phi_{g3}+m_1\frac{\xi_1}{E_1}\phi_{u0}\right]-\frac{\tau_2^{-1}}{2}\left[\phi_{g3}+m_1\frac{\xi_1}{E_1}\phi_{u0}\right]\,, \tag{A20d}$$

where ξ stands for v_Fq_z. The key to solving this eight-variable equation lies in the following approximation. An estimate of the transport relaxation rate shows that τ_j^{-1} $(j=0,1,2)$ is of the order T. Thus we can take $\tau_j^{-1}\ll\Delta$. In this case, (A20c) can be replaced by

$$\phi_3=\frac{\xi_1}{\Delta}\phi_1\,. \tag{A21}$$

Eliminating ϕ_2 from (A20b) and (A20d), and together with (A20a), we have two equations for ϕ_0 and ϕ_1:

$$-i\omega\phi_0+i\xi\frac{\xi_1}{E_1}\left[\frac{E_1}{\Delta}\phi_1\right]=-\frac{\tau_0^{-1}}{2}\left[\phi_{u0}-m_1\left[\frac{\xi_1}{E_1}\right]^2\frac{E_1}{\Delta}\phi_{g1}+m_1\frac{\Delta}{E_1}\phi_{u1}\right]-\frac{\tau_1^{-1}}{2}\left[\phi_{u0}+m_1\frac{E_1}{\Delta}\phi_{u1}\right]-\frac{\tau_2^{-1}}{2}\left[\phi_{u0}+m_1\left[\frac{\xi_1}{E_1}\right]^2\frac{E_1}{\Delta}\phi_{u1}-m_1\frac{\Delta}{E_1}\phi_{g1}\right]+1\,, \tag{A22a}$$

$$-i\omega\left[\frac{E_1}{\Delta}\phi_1\right]+i\xi\frac{\xi_1}{E_1}\phi_0=-\frac{\tau_0^{-1}}{2}\left\{\frac{\Delta}{E_1}\phi_{u1}+\left[\frac{\xi_1}{E_1}\right]^2\frac{E_1}{\Delta}\phi_{g1}+m_1\left[\left[\frac{\Delta}{E_1}\right]^2-\left[\frac{\xi_1}{E_1}\right]^2\right]\phi_{u0}\right\}-\frac{\tau_1^{-1}}{2}\left[m_1\phi_{u0}+\frac{E_1}{\Delta}\phi_{u1}\right]-\frac{\tau_2^{-1}}{2}\left\{\frac{\Delta}{E_1}\phi_{g1}+\left[\frac{\xi_1}{E_1}\right]^2\frac{E_1}{\Delta}\phi_{u1}-m_1\left[\left[\frac{\Delta}{E_1}\right]^2-\left[\frac{\xi_1}{E_1}\right]^2\right]\phi_{u0}\right\}\,. \tag{A22b}$$

Adding the two equations, and introducing $\theta=\phi_0+m_1E_1\phi_1/\Delta$, we get

$$-i\omega\theta_g+i\xi\frac{\xi_1}{E_1}\theta_u=1\,, \tag{A23a}$$

$$-i\omega\theta_u+i\xi\frac{\xi_1}{E_1}\theta_g=-\tau_{\text{qp}}^{-1}\theta_u\,, \tag{A23b}$$

where

$$\tau_{\text{qp}}^{-1}=\tau_0^{-1}\left[\frac{\Delta}{E_1}\right]^2+\tau_1^{-1}+\tau_2^{-1}\left[\frac{\xi_1}{E_1}\right]^2\,. \tag{A24}$$

Equation (A24) is exactly the expression for quasiparticle transport relaxation rate appropriate in the elastic collision approximation. To see this, we introduce the quasiparticle and quasihole operators α_k and β_k according to

$$\alpha_k=u_kc_{k+k_F}+v_kc_{k-k_F}\ \ \text{(particle)}\,, \tag{A25a}$$

$$\beta_k=-v_kc_{k+k_F}^\dagger+u_kc_{k-k_F}\ \ \text{(hole)}\,, \tag{A25b}$$

where

$$\begin{bmatrix}u_k^2\\ v_k^2\end{bmatrix}=\frac{1}{2}\left[1\pm\frac{\xi_k}{E_k}\right]\,. \tag{A26}$$

Neglecting the interband terms, we obtain, for the electron-phonon interaction Hamiltonian,

$$H_{e\phi}=\frac{1}{\sqrt{N}}\sum_{\mathbf{k}}V_{\mathbf{k+q,k}}^{(0)}(\alpha_{\mathbf{k+q}}^\dagger\alpha_{\mathbf{k}}+\beta_{\mathbf{k+q}}^\dagger\beta_{\mathbf{k}})\phi_{0\mathbf{q}}+V_{\mathbf{k+q,k}}^{(1)}(\alpha_{\mathbf{k+q}}^\dagger\alpha_{\mathbf{k}}-\beta_{\mathbf{k+q}}^\dagger\beta_{\mathbf{k}})\phi_{1\mathbf{q}}+V_{\mathbf{k+q,k}}^{(2)}(\alpha_{\mathbf{k+q}}^\dagger\alpha_{\mathbf{k}}+\beta_{\mathbf{k+q}}^\dagger\beta_{\mathbf{k}})\phi_{2\mathbf{q}}\,, \tag{A27}$$

where

$$V_{\mathbf{k}+\mathbf{q},\mathbf{k}}=\gamma_0(u_{\mathbf{k}+\mathbf{q}}u_{\mathbf{k}}+v_{\mathbf{k}+\mathbf{q}}v_{\mathbf{k}}) , \quad (A28a)$$

$$V^{(1)}_{\mathbf{k}+\mathbf{q},\mathbf{k}}=\gamma_1(u_{\mathbf{k}+\mathbf{q}}v_{\mathbf{k}}+v_{\mathbf{k}+\mathbf{q}}u_{\mathbf{k}}) , \quad (A28b)$$

$$V^{(2)}_{\mathbf{k}+\mathbf{q},\mathbf{k}}=-i\gamma_2(u_{\mathbf{k}+\mathbf{q}}v_{\mathbf{k}}-v_{\mathbf{k}+\mathbf{q}}u_{\mathbf{k}}) . \quad (A28c)$$

Upon substitution of (A26), and the introduction of the elastic collision approximation (A14), we obtain

$$| V^{(0)}_{\mathbf{k}+\mathbf{q},\mathbf{k}} |^2=\gamma_0^2\left[\frac{\Delta}{E_k}\right]^2 , \quad (A29a)$$

$$| V^{(1)}_{\mathbf{k}+\mathbf{q},\mathbf{k}} |^2=\gamma_1^2 , \quad (A29b)$$

$$| V^{(2)}_{\mathbf{k}+\mathbf{q},\mathbf{k}} |^2=\gamma_2^2\left[\frac{\xi_k}{E_k}\right]^2 . \quad (A29c)$$

Hence the factors $(\Delta/E_1)^2$, 1, and $(\xi_1/E_k)^2$ preceding τ_0^{-1}, τ_1^{-1}, and τ_2^{-1} arise from the transformation of the electronic coordinates to the quasiparticle coordinates.

Equations (A23a) and (A23b) are easily solved, yielding

$$\theta_g=\frac{\tau_{\rm qp}^{-1}-i\omega}{\xi^2\left[\frac{\xi_1}{E_1}\right]^2-i\omega\tau_{\rm qp}^{-1}-\omega^2} , \quad (A30a)$$

$$\theta_u=\frac{-i\xi\frac{\xi_1}{E_1}}{\xi^2\left[\frac{\xi_1}{E_1}\right]^2-i\omega\tau_{\rm qp}^{-1}-\omega^2} . \quad (A30b)$$

As far as Π_{00} is concerned, we shall see that the value of θ_g is already sufficient. Once θ_g and θ_u are solved, other variables can be solved easily if desired.

3. Correlation function Π_{00}

Having solved the vertex equation, we proceed to evaluate the polarization Π_{00}. As usual, the correlation function consists of the advanced-retarded and retarded-retarded components.[10] We focus on the advanced-retarded component first. Writing (2.11) in matrix direct-product form,

$$\Pi_{00}(q)=\frac{T}{N}\sum_{k_1}{\rm Tr}G(k_{1+})\otimes G(k_{1-})\Lambda_0(k_{1-},k_{1+}) . \quad (A31)$$

Substituting (A4) into (A31), we get

$$\Pi_{00}(q)=\frac{T}{N}\sum_{k_1}{\rm Tr}\, i(G(k_{1+})\ominus G(k_{1-}))\Phi_0(k_{1-},k_{1+}) . \quad (A32)$$

Performing the contour integration as usual and upon analytic continuation, we obtain

$$\Pi_{00}^{AR}(\mathbf{q},\omega+i\delta)=-\frac{i\omega}{2N}\sum_{m_1,\mathbf{k}_1}\left[-\frac{\partial f_1}{\partial E_1}\right](I\oplus I-\frac{\xi_1}{m_1e_1}\sigma_3\oplus\sigma_3+\frac{\Delta}{m_1E_1}\sigma_1\oplus\sigma_1)\Phi_0(\mathbf{k}_1,m_1E_1) , \quad (A33)$$

where the approximation (A11) has been used. Using (A21), we can easily show that

$$\Pi_{00}^{AR}(\mathbf{q},\omega+i\delta)=-\frac{i\omega}{N}\sum_{\mathbf{k}_1}2\left[-\frac{\partial f_1}{\partial E_1}\right]\theta_g=\frac{1}{N}\sum_{k_1}2\left[-\frac{\partial f_1}{\partial E_1}\right]\frac{-i\omega\tau_{\rm qp}^{-1}-\omega^2}{\xi^2\left[\frac{\xi_1}{E_1}\right]^2-i\omega\tau_{\rm qp}^{-1}-\omega^2} . \quad (A34)$$

Next we evaluate the retarded-retarded component of Π_{00}. This is equal to

$$\Pi_{00}^{RR}(\mathbf{q},\omega+1\delta)=-\frac{1}{N}\sum_{\mathbf{k}_1}\int_{-\infty}^{\infty}\frac{dz_1}{2\pi i}[f(z_{1-})+f(z_{1+})]i\,{\rm Im}\,{\rm Tr}G(z_{1+}+i\delta)\Lambda_0(z_{1-}+i\delta,z_{1+}+i\delta)G(z_{1-}+i\delta) , \quad (A35)$$

where $z_{1\pm}=z_1\pm\omega/2$. It can be shown that

$$\Lambda_0(z_{1-}+i\delta,z_{1+}+i\delta)=\sigma_0+O\left[\frac{\tau^{-1}}{\Delta}\right] . \quad (A36)$$

Thus

$$\Pi_{00}^{RR}(\mathbf{q},\omega+i\delta)=-\frac{1}{N}\sum_{k_1}\int_{-\infty}^{\infty}\frac{dz_1}{2\pi i}[f(z_{1-})+f(z_{1+})]{\rm Im}\frac{2(z_{1+}z_{1-}+\xi_{1+}\xi_{1-}+\Delta^2)}{(z_{1+}^2-E_{1+}^2+iz_{1+}\delta)(z_{1-}^2-E_{1-}^2+iz_{1-}\delta)} . \quad (A37)$$

Making use of the following approximation,

$$\frac{1}{(z_{1+}^2-E_{1+}^2+iz_{1+}\delta)(z_{1-}^2-E_{1-}^2+iz-\delta)}$$

$$=\sum_{m_1=\pm1}\frac{\pi}{4E_1^2}\left[\frac{\delta(z_1-m_1E_1)}{m_1E_1}+\delta'(z_1-m_1E_1)\right]-\frac{\pi z^2}{32E_1^2}\left[\frac{3\delta}{m_1E_1}+3\delta'+m_1E_1\delta''\right]$$
$$+\frac{\pi\xi^2\xi_1^2}{32E_1^6}\left[\frac{5\delta}{m_1E_1}+5\delta'+3m_1E_1\delta''+\tfrac{1}{3}E_1^2\delta'''\right], \quad \text{(A38)}$$

we finally arrive at the expression

$$\Pi_{00}^{RR}(\mathbf{q},\omega+i\delta)=-\frac{1}{N}\sum_{\mathbf{k}_1}2\left[-\frac{\partial f_1}{\partial E_1}\right]+\frac{\xi^2}{4E_1^2}\left[\frac{1-2f_1}{E_1}\left[\frac{\Delta}{E_1}\right]^2+2\frac{\partial f_1}{\partial E_1}\left[\frac{\Delta}{E_1}\right]^2-E_1\frac{\partial^2 f_1}{\partial E_1^2}\left[\frac{\Delta}{E_1}\right]^2+\frac{\xi_1^2}{3}\frac{\partial^3 f_1}{\partial E_1^3}\right]. \quad \text{(A39)}$$

Among the terms proportional to ξ^2, the first term is dominant. Thus we have

$$\Pi_{00}^{RR}(\mathbf{q},\omega+i\delta)=-\frac{1}{N}\sum_{\mathbf{k}_1}2\left[-\frac{\partial f_1}{\partial E_1}\right]-N(0)\frac{\xi^2}{3\Delta^2}. \quad \text{(A40)}$$

Combining (A34) and (A40), we have

$$\Pi_{00}(\mathbf{q},\omega+i\delta)=-N(0)\frac{\xi^2}{3\Delta^2}-\frac{1}{N}\sum_{\mathbf{k}_1}2\left[-\frac{\partial f_1}{\partial E_1}\right]\frac{\xi^2\left[\dfrac{\xi_1}{E_1}\right]^2}{\xi^2\left[\dfrac{\xi_1}{E_1}\right]^2-i\omega\tau_{qp}^{-1}-\omega^2}. \quad \text{(A41)}$$

The first term of (A41) is the contribution of the condensed electrons, and the last term the quasiparticles. In the latter, note that the quasiparticle group velocity can be written as

$$v_{kz}=v_F\frac{\xi_k}{E_k}, \quad \text{(A42)}$$

and $(-\partial f_1/\partial E_1)\approx f(E_1)/T$, so that (A41) can be rewritten as

$$\Pi_{00}(\mathbf{q},\omega+i\delta)=-N(0)\frac{v_F^2q_z^2}{3\Delta^2}-\left\langle\frac{v_{kz}^2q_z^2}{v_{kz}^2q_z^2-i\omega\tau_{qp}^{-1}-\omega^2}\right\rangle\frac{n_{qp}}{T}, \quad \text{(A43)}$$

where n_{qp} is the total density of quasiparticles, and $\langle\cdots\rangle$ represents the average over all quasiparticles. From (2.10), we finally get the expression for the dielectric function,

$$\epsilon(\mathbf{q},\omega)=1+(\epsilon_z-1)\cos^2\theta+\left\langle\frac{v_{kz}^2q_z^2}{v_{kz}^2q_z^2-i\omega\tau_{qp}^{-1}-\omega^2}\right\rangle\frac{q_0^2}{q^2}, \quad \text{(A44)}$$

where $\epsilon_z=1+\omega_{pe}^2/6\Delta^2$ and $q_0^2=4\pi e^2 n_{qp}/T$.

Other correlation functions can be obtained similarly. They are

$$\Pi_{20}(\mathbf{q},\omega+i\delta)=-\Pi_{02}(\mathbf{q},\omega+i\delta)=\frac{i\xi}{\Delta}N(0)-\frac{1}{N}\sum_{k_1}2\left[-\frac{\partial f_1}{\partial E_1}\right]\frac{\Delta^2}{2E_1^2}\frac{[\xi^2-i\omega(\tau_2^{-1}\tau_0^{-1})]\left[\dfrac{\xi_1}{E_1}\right]^2}{\xi^2\left[\dfrac{\xi_1}{E_1}\right]^2-i\omega\tau_{qp}^{-1}-\omega^2}, \quad \text{(A45a)}$$

$$\Pi_{22}(\mathbf{q},\omega+i\delta)=-\frac{2N(0)}{\lambda}+\frac{N(0)}{2}\frac{\xi^2}{\Delta^2}-\frac{N(0)}{2}\frac{\omega^2}{\Delta^2}$$
$$-\frac{1}{N}\sum_{\mathbf{k}_1}2\left[-\frac{\partial f_1}{\partial E_1}\right]\left\{\frac{3}{8}\frac{\xi^2\Delta^2}{E_1^4}+\frac{i\omega(\tau_0^{-1}+\tau_1^{-1})[-i\omega(\tau_1^{-1}+\tau_2^{-1})-\omega^2+\xi^2]}{4E_1^2\left[\xi^2\left[\dfrac{\xi_1}{E_1}\right]^2-i\omega\tau_{qp}^{-1}-\omega^2\right]}\right.$$

$$+\frac{\omega^2\Delta^2}{4E_1^4}\frac{\xi^2-i\omega(\tau_2^{-1}-\tau_0^1)}{\xi^2\left[\frac{\xi_1}{E_1}\right]^2-i\omega\tau_{\rm qp}^{-1}-\omega^2}-\frac{i\omega(\tau_0^{-1}+3\tau_1^{-1}+\tau_2^{-1}}{16E_1^2}\Bigg\}$$

$$+\frac{1}{N}\sum_{\mathbf{k}_1}2\frac{\partial^2 f_1}{\partial E_1^2}\left[\frac{\xi^2\xi_1^2}{8E_1^3}+\frac{\omega^2}{4E_1}\right]. \tag{A45b}$$

It is evident that in these expressions, however, the quasiparticle corrections are of secondary importance.

APPENDIX B: DIELECTRIC FUNCTION IN THE DIFFUSIVE LIMIT

In the hydrodynamic regime, the inelastic scattering rate τ^{-1} satisfies $\omega\tau<1$, and so the dynamics of the quasiparticles have to be studied by the hydrodynamic equations. Starting from the transport equation (3.2) and replacing the collision term on the right-hand side by the inelastic collision term, we can easily write down the conservation equations for the quasiparticles. Assuming isothermal conditions, the quasiparticle distribution relaxes towards a local equilibrium of chemical potential $\mu_{\rm loc}$. The transport equation can then be written as

$$-i\omega g_k+iv_{kz}q_zg_k-iq_zv_{kz}\frac{\partial f_k}{\partial E_k}\delta\mu=-\tau^{-1}\left[g_k+\frac{\partial f_k}{\partial E_k}\mu_{\rm loc}\right], \tag{B1}$$

which yields

$$g_k=\frac{\partial f_k}{\partial E_k}\frac{iq_zv_{kz}\tau\delta\mu-\mu_{\rm loc}}{1-i\omega\tau+iq_zv_{kz}\tau}, \tag{B2a}$$

$$\mathcal{C}=\left[-\frac{\partial f_k}{\partial E_k}\right]\frac{iq_zv_{kz}\delta\mu+(-i\omega+iq_zv_{kz})\mu_{\rm loc}}{1-i\omega\tau+iq_zv_{kz}\tau}. \tag{B2b}$$

The local equilibrium chemical potential can be obtained by invoking the requirement for the conservation of particle numbers, i.e., setting the sum of the collision rate over the phase space to zero. This gives, for $\omega\tau, v_{kz}q_z\tau\ll 1$,

$$\frac{1}{N}\sum_{\mathbf{k}}\left[-\frac{\partial f_k}{\partial E_k}\right]\{v_{kz}^2q_z^2\tau\delta\mu+[-i\omega+(\omega^2+v_{kz}^2q_z^2)\tau]\mu_{\rm loc}\}=0. \tag{B3}$$

For $T\ll\Delta$, we can approximate the situation by classical statistics, so that $-\partial f_k/\partial E_k\approx f_k/T$, and we obtain

$$\mu_{\rm loc}=-\frac{\mathcal{D}q_z^2}{-i\omega+\mathcal{D}q_z^2}\delta\mu, \tag{B4}$$

where $\mathcal{D}=\langle v_{kz}^2\rangle\tau$ is the diffusion constant. The total induced charge density, via (B2a), is

$$\rho_{\rm qp}^{\rm ind}=-\frac{n_{\rm qp}}{T}\frac{\mathcal{D}q_z^2}{-i\omega+\mathcal{D}q_z^2}\delta\mu. \tag{B5}$$

This yields the following expression for the dielectric function,

$$\epsilon(\mathbf{q},\omega)=1+(\epsilon_z-1)\cos^2\theta+\frac{\mathcal{D}q_0^2}{-i\omega+\mathcal{D}q_z^2}\cos^2\theta. \tag{B6}$$

If we equate the last term with the dielectric contribution of quasiparticle conductivity, we obtain

$$\sigma_{\rm qp}(\mathbf{q},\omega)=\frac{-i\omega\sigma_{\rm dc}}{-i\omega+\mathcal{D}q_z^2}\cos^2\theta, \tag{B7}$$

where $\sigma_{\rm dc}$ is the dc conductivity of the quasiparticles, and can be shown to satisfy the Einstein relation[16]

$$\sigma_{\rm dc}=e^2\frac{\partial n_{\rm qp}}{\partial\mu}\mathcal{D} \tag{B8}$$

We have to emphasize that the above treatment applies to the isothermal situation only. For the adiabatic condition, the local equilibrium corresponds to a local temperature in addition to the local chemical potential. The conservation equations then include the conservation of energy in addition to that of particle numbers. The corresponding expression for the dielectric function can be derived in a similar way.

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*Present address: Department of Physics, Imperial College, London SW7 2BZ, United Kingdom.

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[13]However, care must be taken to interpret the experimental measurement of the damping factor. See, for example, S. Sridhar, D. Reagor, and G. Grüner, Phys. Rev. Lett. **55**, 1196 (1985). Their measured quantity corresponds to the intrinsic damping of the CDW, and does not include the quasiparticle dissipation, since quasiparticle effects are already included in the dielectric function relating the external to the internal electric field. In fact, the damping factor predicted by (3.9) is larger than that measured by Sridhar, Reagor, and Grüner. The issue of avoiding this double counting will be discussed in a future paper.

[14]A more detailed calculation [K. Y. M. Wong and T. Holstein, (unpublished)] shows that the correct expression for the phase-mode dispersion should be

$$\omega^2=\frac{\omega_\alpha^2\xi^2\frac{\sinh^{-1}\xi}{\xi(1+\xi^2)^{1/2}}}{1-\frac{\sinh^{-1}\xi}{\xi(1+\xi^2)^{1/2}}+\frac{n_{\rm qp}}{2N(0)T}}\ ,$$

where $\xi=v_F q_z/2\Delta$. Equation (4.9B) is therefore incorrect. Nevertheless, it suffices to interpret the physical picture.

[15]L. K. Hansen (unpublished).

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