Reversed-spin excitations of the fractionally quantized Hall effect from finite-size calculations

E. H. Rezayi
Department of Physics and Astronomy, California State University, Los Angeles, California 90032
(Received 27 May 1986; revised manuscript received 17 April 1987)

The fractionally quantized Hall states at $\nu=\frac{1}{3}$ and $\frac{2}{5}$ filling factors have been studied by finite-size numerical calculations allowing for reversed spins. I report on the fractionally charged defect states, including their spin distribution, and the spin-wave type neutral collective mode of the $\frac{1}{3}$ state. The defects of the $\frac{2}{5}$ unpolarized incompressible fluid are found to be spin-$\frac{1}{2}$ objects leading to singlet and triplet neutral collective modes. Possible relevance to experiments are discussed.

The discovery[1] of the fractionally quantized Hall effect (FQHE) has attracted attention to the properties of an interacting two-dimensional electron gas in an intense magnetic field. The bulk of the theoretical efforts have been aimed at understanding the behavior of a pure system of fully *spin-polarized* electrons confined to the lowest Landau level. Laughlin's[2] incompressible fluid picture of the ground state and the fractionally charged defect excitations has been directly confirmed,[3] and explains the observed plateaus with Hall coefficient $R_H=h/\nu e^2$, where ν is the Landau-level filling factor. The neutral collective "quasiexcitonic" excitations of the incompressible fluid have also been discussed.[3,4,5]

Exclusion of turned over spins has been justified by the large Zeeman energies said to be involved in the observed FQHE. However, as first noted by Halperin[6], the Zeeman gap for reversed spin states in GaAs may not be prohibitively large. In fact, with the magnetic field B in teslas and assuming a constant g factor, in degrees Kelvin, this gap is given as $\Delta E_z \simeq 0.35B$. On the other hand, the scale for the excitation energies of the fully polarized neutral system is set by the gap for creating a pair of oppositely charged defects. In the pure case, $\nu=\frac{1}{3}$, theoretical predictions for this is approximately $\Delta \simeq 5\sqrt{B}$. Based on these estimates, it is difficult to rule out spin-reversed states in GaAs for range of fields for which observations have been made.

However, the experimental gaps are a factor of 3–4 smaller; more importantly, the observed magnetic field dependence of the gap in GaAs (Ref. 7) is not in agreement with the theoretical square-root dependence. In particular, beyond a threshold field the gap becomes nonzero, and is seen to increase linearly with the field for up to 18 T, saturating to, evidently, a constant value for larger fields. While disorder is a contributing factor for this behavior, the reversed-spin states are also likely to be important in the low-field regime.

In this paper I report a finite-size study for up to six electrons with reversed spins for $\nu=\frac{1}{3}$ and $\frac{2}{5}$ filling factors in spherical geometry.[8] I only consider states within the lowest Landau level. The aim here has not been to achieve quantitative fit to experimental data; instead, a clear picture of the spin reversed excitations has been obtained. Most of the results are presented with physical spin in mind, though conclusions can equally be drawn for spin-polarized systems having SU(2) internal symmetry, such as valley degeneracy in Si.

On the sphere, each electron carries a total angular momentum $L=K$;[8] $2K$ is an integer representing the total flux in units of the flux quantum $\Phi_0=h/e$. To thread the amount $2K$ of flux through the sphere, the radius is given by $R=l\sqrt{K}$, where l is the magnetic length. The interparticle separation, in this geometry, is taken to be the geometric chord distance between the particles. The only relevant energy in the lowest Landau level is the Coulomb repulsion $e^2/4\pi\epsilon_0 r$. The energies are then quoted in units of $e^2/4\pi\epsilon_0 l$. For the sake of simplicity, the Zeeman energies have been consistently left out in this study. Given the value of the magnetic field, this contribution will be an additive constant proportional to the total spin. In addition to Coulomb interaction, I have also studied a model short-ranged hard-core potential for which analytical wave functions can be found.

In presence of spin, the states are classified by four quantum number: L, M, S, and S_z. L is the total rotational quantum number and M its azimuthal component. S and S_z are corresponding quantities for the spin angular momentum. The N-particle Hamiltonian is parametrized by a set of psuedopotential parameters[8] V_m obtained directly from the Coulomb interaction, which is then diagonalized numerically. The model hard-core potential considered here is characterized by two nonzero parameters V_0 and V_1.

As in toroidal geometry,[9] the absolute ground state (AGS) at $\nu=\frac{1}{3}$ is the fully spin-polarized Laughlin's incompressible fluid, even without the Zeeman energy. This is a direct consequence of the relatively strong hard-core component[10] V_0 of the Coulomb interaction. With $V_1, V_2, \ldots,$ fixed at their Coulomb values, a transition to an unpolarized compressible (gap$\rightarrow 0$ as $N\rightarrow\infty$) state as a function of V_0 is observed (Fig. 1). The precise nature of this state is, however, difficult to ascertain on the sphere.

In marked contrast to the neutral state, the AGS of the charged excitations for the Coulomb interactions, at

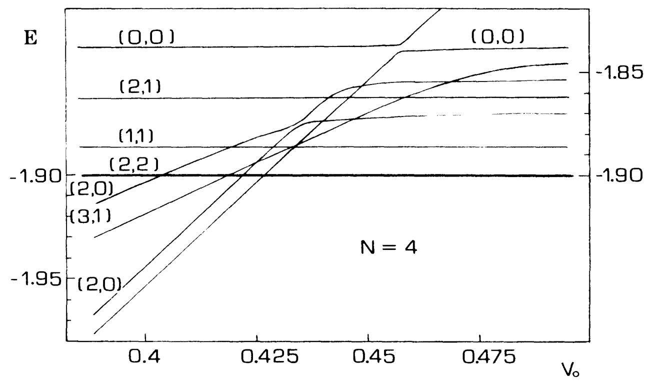

FIG. 1. Dependence of low-lying levels at $\nu=\frac{1}{3}$, $N=4$, on the short-range part of the Coulomb interactions. The thick horizontal line is the AGS energy (polarized). Each level is labeled by (L,S).

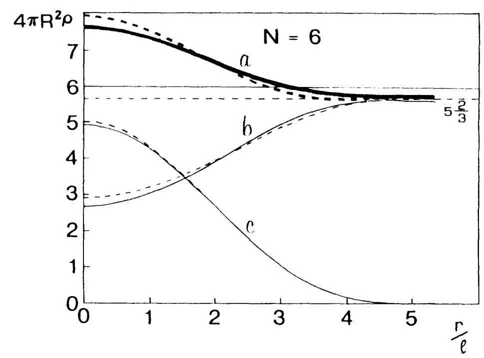

FIG. 2. Electron density $4\pi R^2\rho$ for the quasiparticle $(N=6)$, centered at $r=0$ with one reversed spin as a function of chord distance r/l. a,b,c refer to the total density (thicker line); density of parallel and reversed spins, respectively. Solid lines represent Coulomb, broken model hard-core potential.

flux $2K=2K(\nu=\frac{1}{3})\mp 1$, are found to be unpolarized. In view of the fully polarized state at $\nu=\frac{1}{3}$, this seemingly unexpected result is not a finite-size effect. It is rather a consequence of a near degeneracy $[\Delta E=O(1/N)]$ of a macroscopic number of states with the ground state at $\nu=\frac{1}{3}$. There exists strong evidence, for example, from studies of the filled Landau level, that an entire multiplet of spin-wave excitations (one spin wave, two spin waves, etc.) are, in the absence of Zeeman energy, gapless; the long-wavelength spin-wave modes do not appear to interact appreciably to generate a gap. These states are part of the degenerate manifold responsible for this behavior. Further details will be given elsewhere.[11] In the case of the quasiparticle with one reversed spin, I have found a considerable energy gain (0.043 for $N=6$) relative to the polarized case. This state is accurately described by

$$\Psi=u_1^{N-2}\prod_{\substack{i,j\\1<i<j}}(u_iv_j-u_jv_i)^3\prod_j(u_1v_j-u_jv_1)^2$$

where $(u,v)=(\cos(\theta/2)e^{i\phi/2},\sin(\theta/2)e^{-i\phi/2})$ are spinor coordinates and (u_1,v_1) is the reversed spin. This wave function is an exact eigenstate of the model hard-core potential with vanishing eigenvalue. It is easily seen that $L=S=N/2-1$ for this state. Ψ is closely related to the one suggested in the planar geometry.[12]

Figure 2 shows the electron density for both the exact and the model quasiparticle defect having a single reversed spin for the six-electron system. The reversed spin is seen to be localized at the center of the defect. The exact state appears to be well described by Ψ (overlap for $N=6$ is 98.8%). Figure 3 shows the quasiparticle defect with two reversed spins. Again, we see a centrally localized *droplet* of reversed spins with $L=S=N/2-2$; because of the lower L value this defect is less confined,[13] and its energy was found to be lower than the state with one reversed spin. In general, the ground state $E_0(S)$ for $2K=2K(\frac{1}{3})\pm 1$ in a given spin multiplet S is at $L=S$ with $E_0(S)<E_0(S')$ for two spin multiplets $S<S'$. Thus, at the expense of Zeeman energy, the charged excitations become more uniform terminating in the completely deconfined state $L=S=0$.

The Zeeman energy will severely restrict experimental realization of these states. However, I have found that the neutral combination of the fully *polarized* quasihole $(L=N/2)$ and the spin *reversed* quasiparticle $(L=N/2-1)$ obtains a considerable gain in exchange energy to defeat the Zeeman gap. By a fit of the data to a polynomial in powers of $1/N$, my best estimate of the gap to create such a pair of defects at infinite separation for $N\to\infty$ is 0.075 ∓ 0.007—an amount 0.030 smaller than the polarized gap 0.105[3]. This gain could very well compensate for the Zeeman energy in GaAs, making the pair a strong candidate for a neutral collective mode. Further gain of energy in reversing another spin is rather small and cannot overcome the Zeeman gap in most situations.

The combination of the above two defects with

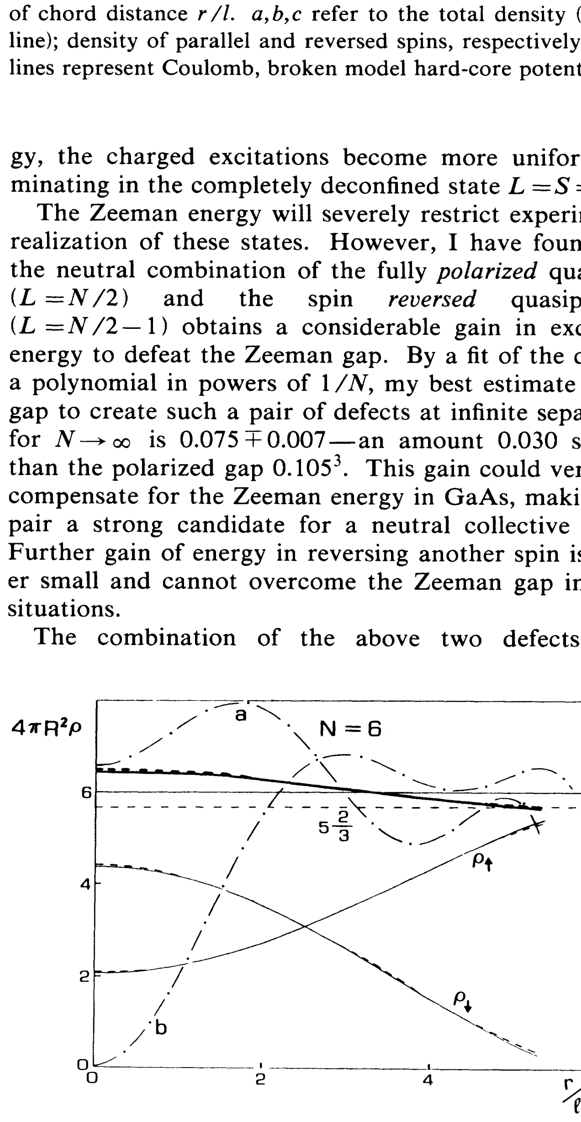

FIG. 3. Density profile of the quasiparticle centered at $r=0$ for $N=6$ and $S=1$ (two reversed spins). The thick line is the total density. $\rho_{\uparrow(\downarrow)}$ is the density for parallel (reversed) electrons. Also shown are the defects of the hard-core interaction (broken lines). The dashed-dot lines are the polarized defects,[3] included for comparison.

$L=N/2$ and $L=N/2-1$ leads to total angular momentum of the composite object $L=1,\ldots,N-1$. A set of isolated levels with the correct quantum numbers, providing a clear evidence of a collective mode for the neutral system at $\nu=\frac{1}{3}$, is shown in Fig. 4. The angular momentum L has been transcribed to wave number k by $k=L/R$. The behavior at long wavelengths, suggesting a gapless mode, is consistent with Goldstone's theorem.[14] For real spins, the Zeeman energy will result in a gap. The parallel to a Goldstone mode, however, is valid, even though a true massless excitation does not exist. The collective state for small k can, therefore, be regarded as an ordinary spin-wave of the polarized incompressible fluid. In this regime, the single-mode approximation (SMA), recently applied to multicomponent systems,[15] is in excellent agreement with the numerical results. At large k, the spin-wave description is no longer valid. Instead, there is a well separated pair of spin-polarized quasihole and a spin reversed quasiparticle, at a relative distance of $D=kl^2/\nu$, with the reversed spin pinned at the center of the quasiparticle; see Fig. 2. The two-spin wave collective mode occurs at a lower energy, but not sufficiently so to defeat the Zeeman gap in present day samples.

Another state with possibility of reversed spins worth considering is at $\nu=\frac{2}{5}$. The AGS in this filling is a spin singlet.[6,9] However, due to the large Zeeman energy required to reverse half of the electron spins, this state may be less relevant to GaAs (Ref. 9) but will be of interest for systems with two-component fermions, such as twofold valley-degenerate Si.

Halperin[6] has proposed a Jastrow-type model wave function for this state. On the sphere it can be written as

$$\Psi_H=\prod_{i,j}(u_iv_j-u_jv_i)^3\prod_{i<j}(u_i'v_j'-u_j'v_i')^3 \times\prod_{i,j}(u_iv_j'-u_j'v_i)^2\,,$$

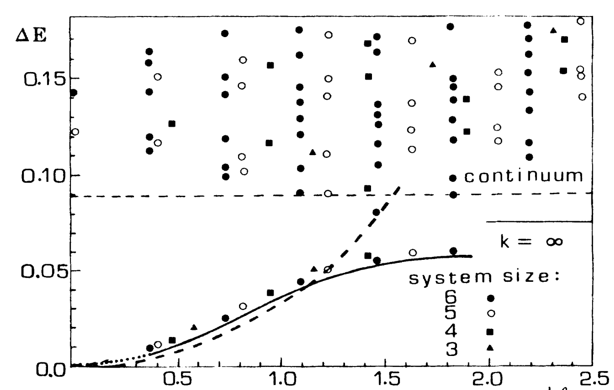

FIG. 4. Low-lying excitation with spin $S=N/2-1$ (one reversed spin) vs wave number k for various size systems showing a collective mode. The broken line (denoting the "continuum") and the solid curve are guides to the eye. The dotted line is extension of the latter to zero k. The broken curve is the SMA result for a two-component system.

where primed variables denote the coordinates of the reversed spins. Ψ_H can be shown to be an exact eigenstate of the short range hard-core potential mentioned previously. For the six-electron system the overlap between he exact and the Halperin's wave function is 98.5%. The variational energy per particle, -0.4919, is very close to the exact value, -0.4925. Figure 5(a) shows the pair correlations in the exact ground state, as well as in Halperin's model wave function. As in the $\frac{1}{3}$ case,[3] the exact ground state is well described by the hard-core model potential.

The fractionally charged excitations of this state occur at odd N. Figure 5(b) shows the low-lying levels of the five-electron system at flux $2K=10$. An isolated multiplet at $S=\frac{1}{2}$ and $L=1.0$, corresponding to the *spin*-$\frac{1}{2}$ quasihole excitation, is found to be the absolute ground state, in agreement with Haldane's analysis.[16] Figure 5(c) shows the charge density for the quasihole defect. According to hierarchical theories,[8,17,18] the charge of this excitation is $e^*=e/5$. The asymptotic behavior in Fig. 5(c) is consistent with this. Once again we find good agreement between the exact excited state and the defect of the model potential, shown as a dashed curve.

As expected from combination of two spin-$\frac{1}{2}$ objects, a triplet and a singlet collective mode excitation of the neutral system has been seen. Figure 5(d) shows the low-lying states of the $S=1$ multiplet. Though the system size is not sufficiently large to obtain as transparent a picture as in the $\frac{1}{3}$ case, the evidence for a collective mode with a *finite* gap is clear. The large-k behavior is consistent with the creation energy of a quasihole-quasiparticle pair of the five-electron system at infinite

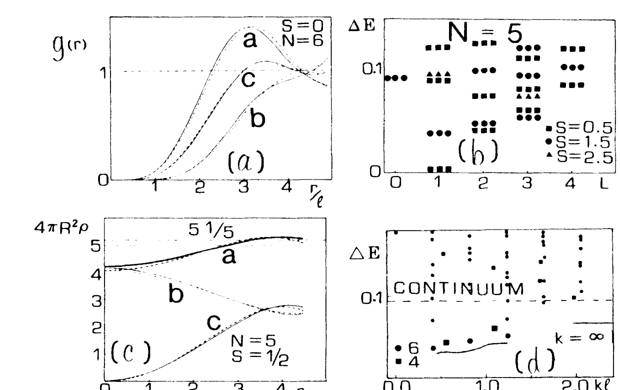

FIG. 5. (a) Pair correlations for the unpolarized $\nu=\frac{2}{5}(N=6)$ ground state: (curve a) antiparallel spins g_{+-}, (curve b) parallel spins g_{++}, and (curve c) total g. Dashed curves are correlations of the hard-core interactions (b) The low-lying levels for the $\frac{2}{5}+$ a quasihole ($N=5$). (c) The charge profile for this defect centered at $r=0$: (curve a) is the total density; (curve b) and (curve c) are densities for reversed and parallel spins, respectively. (d) The excitation spectrum for $N=4,6$ of the $\nu=\frac{2}{5}$ state with $S=1$ vs wave number. A set of low-lying levels (enlarged symbols) having a finite gap can be identified as a triplet collective mode. The solid and dashed curves are guides to the eye.

separation [$\simeq 0.067$ horizontal line in Fig. 5(d)].

To conclude, I note that the exchange energy gain in reversing a single spin in the quasiparticle state of the $\frac{1}{3}$ filling ($\simeq 0.03$) appears to be sufficient to compensate the loss in Zeeman energy in GaAs for magnetic fields up to a critical field $B_c = 18$ T. At this field strength, the total, theoretical, cost in energy, for creating a widely separated quasiparticle quasihole of the polarized system and a pair having a spin reversed quasiparticle becomes equal. For B near B_c we expect two activation energies corresponding to the fully polarized and the spin-reversed excitation. The data of Ref. 7 is not inconsistent with this overall picture. However, it is difficult to reconcile the magnitude of the low field gaps[7] with the Zeeman energy for reversing a spin. If the spin-reversed states are the lowest energy excitations for $B < B_c$, then the gap as a function of the field will exhibit a weak cusplike singularity at B_c. To observe this will require continuous adjustment of the field in the neighborhood of B_c at a fixed filling factor. This may be possible in back-gate biased samples where the carrier density can be varied. However, as a result, the layer thickness and hence the form of the interaction will also vary. A realistic calculation of the gap must include effects of disorder, and admixture of states from higher Landau levels as well as finite layer thickness. All of these are known to reduce the pure spin-polarized gap. We expect a similar trend for the spin-reversed case. Finally, the $\nu = \frac{2}{5}$ state in systems with SU(2) symmetry appears to possess all the needed ingredients for FQHE. It should be pointed out that, for the $\nu = \frac{1}{3}$ case, in a two-component system without symmetry-breaking fields, the Laughlin quasiparticle or hole states do not occur as low-lying excitations. Therefore, FQHE does not exist in such systems. This is in line with the observation[15] that there is no discontinuity in the chemical potential at $\nu = \frac{1}{3}$.

In summary, I found that the spin-reversed excitations are qualitatively similar to the polarized ones, but in general have lower energies, which may in some cases overcome the Zeeman gap. A more definitive assessment of the importance of spin-reversed states in GaAs than given here must await a full understanding of the nature of the observed gaps; the origin of small gap values and other features reported in Ref. 7 have not yet been completely understood. It would prove very interesting if a direct observation of the spin-wave spectrum became possible.

Note added. After this work was completed I received a copy of unpublished work discussing the importance of the reversed-spin quasiparticle in GaAs based on a numerical study in toroidal geometry.[19]

ACKNOWLEDGMENTS

I wish to thank F. D. M. Haldane for a number of useful discussions. I also thank A. H. MacDonald for providing the SMA data, and for extensive conversations on finite-size effects. I am grateful to the Nuclear Physics group at California State University—Los Angles for making their computer available to me.

[1]D. C. Tsui, H. L. Störmer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).

[2]R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[3]F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. **54**, 237 (1985).

[4]S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. Lett. **54**, 581 (1985).

[5]R. B. Laughlin, Physica (The Hague) **126B**, 254 (1984).

[6]B. I. Halperin, Helv. Phys. Acta **56**, 75 (1983).

[7]G. S. Boebinger, A. M. Chang, H. L. Störmer, and D. C. Tsui, Phys. Rev. Lett. **55**, 1606 (1985).

[8]F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983).

[9]F. C. Zhang and T. Chakraborty, Phys. Rev. B **30**, 7320 (1984); **29**, 7032 (1984).

[10]In the four-electron system $V_0 = 0.92$ is quite large in comparison to $V_1 = 0.48$ and $V_2 = 0.36$.

[11]E. H. Rezayi (unpublished).

[12]R. Morf and B. I. Halperin, Phys. Rev. B **33**, 2221 (1986); it can be obtained by the transformation proposed by G. Fano, F. Ortolani, and E. Colombo, *ibid.* **34**, 2670 (1986). Although not apparent, Ψ belongs to the correct symmetry type of the permutation group.

[13]L, or more precisely M, is related to first moment of the charge distribution.

[14]J. Goldstone, Nuovo Cimento **19**, 154 (1961).

[15]M. Rasolt, F. Perrot, and A. H. MacDonald, Phys. Rev. Lett. **55**, 433 (1985).

[16]F. D. M. Haldane in *Quantum Hall effect,* edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1986).

[17]R. B. Laughlin, Surf. Sci. **142**, 163 (1984).

[18]B. I. Halperin, Phys. Rev. Lett. **52**, 1583 (1984).

[19]T. Chakraborty, P. Pietiläinen, and F. C. Zhang, Phys. Rev. Lett. **57**, 130 (1986).