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Magnetization in a quenched random-bond transverse Ising model with competing interactions

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The ferromagnetic-phase stability limit and magnetization of a quenched bond-mixed spin-$\frac{1}{2}$ transverse Ising model in an anisotropic simple square lattice are studied within the framework of an effective-field theory considering both competing and noncompeting interactions. In the bond-disordered case, where bonds J_1 and J_2 occur with probabilities $1-p$ and p, respectively, critical curves are obtained for various α's ($\alpha \equiv J_1/J_2$), and the critical bond concentrations p_c at which the ferromagnetic phase breaks down are determined. As $\alpha \to 0$, the mixed model reduces to the diluted Ising model and the results obtained are compared to those available in the literature. The effect of the transverse field (for different α's) in the thermal behavior of the magnetization as a function of the concentration is also discussed.

I. INTRODUCTION

The Ising model in a transverse field (TIM) is described by the Hamiltonian:

$$\mathcal{H} = -\Omega \sum_i S_i^x - \sum_{i,j} J_{ij} S_i^z S_j^z \ , \tag{1}$$

where S_i^x, S_i^z are components of a spin-$\frac{1}{2}$ operator at site i, Ω represents the transverse field, J_{ij} is the exchange interaction between neighboring sites, and the sums extend over all sites. This model has been used to describe a variety of physical systems. It was originally introduced by de Gennes[1] as a pseudospin model for hydrogen-bonded ferroelectrics[2,3] such as KH_2PO_4. It provides a good description for some real anisotropic magnetic materials in a transverse field, and it also applies to systems showing a cooperative Jahn-Teller transition[4] (namely $DyVO_4$ and $TbVO_4$). An extensive discussion on the applications of this model can be found in the article by Stinchcombe.[5]

The critical behavior of the TIM in one dimension with $J_{ij} = J$, has been already established through exact results;[6] at $T>0$ there is no phase transition, but at $T=0$ the system remains ordered up to a critical value Ω_c. In higher dimensions, the critical behavior of the TIM has been established through series-expansions results.[7–10] These results show that there exists a phase boundary in the ΩT plane limited by the Ising critical point $T=T_c$, $\Omega=0$ and the point $T=0$, $\Omega=\Omega_c$. As one would expect from the similarity of the roles played by the transverse field and the temperature in the transition phenomena, as Ω increases from zero, T_c falls from the Ising critical point and at Ω_c it reaches zero. Below this critical frontier, the S_z components are ordered ($\langle S_z \rangle \neq 0$) while above it the S_z components are disordered ($\langle S_z \rangle = 0$) although there is still a certain degree of order in the system characterized by $\langle S_x \rangle \neq 0$. Furthermore, an interesting exact result[11–15] is that the critical behavior of the TIM in a given dimension (d), as a function of $\Omega(T=0)$, is identical to the critical behavior of the pure Ising model in one higher dimension ($d+1$), as a function of $T(\Omega=0)$. During the last decade, there has been great interest in the problem of quenched disorder in the TIM.[16–19] For the bond-diluted TIM at zero temperature, the percolative behavior is expected[20] to yield a discontinuous jump in the critical field at the percolation concentration p_c. Below p_c there is no long-range order so that $\Omega_c = 0$, whereas at p_c the critical field changes discontinuously to the finite value needed to destroy the order in the chainlike percolating clusters. Although earlier theoretical treatments such as series expansions (SE),[21] coherent-potential-approximation calculations,[22] and experiments[23] could not verify this conjecture, recent real-space renormalization-group (RG) calculations[24,25] for a two-dimensional bond-diluted TIM at $T=0$ have shown the existence of the critical-field discontinuity as a result of the existence of two fixed points at the percolation threshold (see also Ref. 26). At finite T this model has been analyzed in more detail in Ref. 27. Recently, the thermodynamical properties of the diluted TIM has been discussed by an effective-field treatment[28,29] and also by a variational approach.[30] In what concerns the diluted TIM in three-dimensional (3D) systems,[31] complete phase diagrams in the (T,p,Ω) space has been presented by Saxena.[32] However, detailed calculations of the phase diagrams and magnetization are not available for a quenched mixed TIM when competing interactions are allowed.

In the present work we study a quenched bond-mixed spin-$\frac{1}{2}$ Ising model in the presence of a transverse field. The model is treated on an anisotropic simple square lattice where nearest-neighbor exchange coupling constants are allowed to take two different values J_1 or J_2 with probabilities $1-p$ and p, respectively. We shall be particularly interested in systems where $\alpha \equiv J_1/J_2$ takes negative values (competing interactions) and when it is zero (bond-diluted case). Two (mutually nonexclusive) main sources of crystalline anisotropy may exist, namely, anisotropic coupling constants or anisotropic bond-occupancy

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probabilities; we are herein particularly concerned with the former. The problem is studied using an effective-field theory which is based on the introduction of a differential operator (in the same spirit as that of Honmura and Kaneyoshi[33]) into a generalized but approximated Callen[34] relation derived by Sá Barreto *et al.*[35] for the TIM. A very recent application of this method was carried out to discuss the surface magnetism of the TIM with a disordered surface.[36,37] Here we calculate the spontaneous magnetization as a function of temperature, transverse field, and bond concentration for some interesting cases. By imposing the condition of vanishing magnetization, we obtain the critical curves associated with the ferromagnetic-phase stability limit.

The outline of this paper is as follows: In Sec. II we briefly present the formalism; in Sec. III we present the phase diagrams, together with the longitudinal and transverse magnetization curves; the overall conclusions are summarized in Sec. IV.

II. MODEL AND FORMALISM

In the model Hamiltonian given by Eq. (1) we consider J_{ij} a random variable governed by the following probability distribution law:

$$P(J_{ij})=(1-p)\delta(J_{ij}-J_1)+p\delta(J_{ij}-J_2) \ , \tag{2}$$

where we assume $0\le p\le 1$, and $\alpha\equiv J_1/J_2\le 0$.

The starting point for the statistics of our spin system is the relation proposed by Sá Baretto *et al.*[35] in which the longitudinal and transverse site magnetizations for the TIM are approximately given by

$$\langle\sigma_i^z\rangle=\left\langle\frac{\sum_j J_{ij}\sigma_j^z}{H_i}\tanh\left[\frac{\beta}{4}H_i\right]\right\rangle \ , \tag{3}$$

$$\langle\sigma_i^x\rangle=\left\langle\frac{2\Omega}{H_i}\tanh\left[\frac{\beta}{4}H_i\right]\right\rangle \ . \tag{4}$$

Here $\langle\cdots\rangle$ indicates the canonical thermal average, $\langle\sigma_i^z\rangle=2\langle S_i^z\rangle$, $\langle\sigma_i^x\rangle=2\langle S_i^x\rangle$, $\beta=1/k_BT$, and

$$H_i=\left[(2\Omega)^2+\left[\sum_j J_{ij}\sigma_j^z\right]^2\right]^{1/2} \ . \tag{5}$$

In the limit $\Omega=0$, $\langle\sigma_i^x\rangle=0$, $H_i=\sum_j J_{ij}\sigma_j^z$, and Eq. (3) reproduce the Callen's[34] identity for the pure Ising model. Expanding the right-hand side of Eqs. (3) and (4) as a formal series in the spin variables and neglecting correlations of H_i, the standard molecular-field-approximation (MFA) results[2] are recovered.

Introducing the differential operator $D=\partial/\partial x$, [defined by $\exp(cD)f(x)=f(x+c)$] we may rewrite Eqs. (3) and (4) as

$$\langle\sigma_i^z\rangle=\left\langle\exp\left[\left[\sum_j J_{ij}\sigma_j^z\right]D\right]\right\rangle f(x)\,|_{x=0} \ , \tag{6}$$

$$\langle\sigma_i^x\rangle=\left\langle\exp\left[\left[\sum_j J_{ij}\sigma_j^z\right]D\right]\right\rangle g(x)\,|_{x=0} \ , \tag{7}$$

where the functions $f(x)$ and $g(x)$ are given by

$$f(x)=\frac{x}{[(2\Omega)^2+x^2]^{1/2}}\tanh\left[\frac{\beta}{4}[(2\Omega)^2+x^2]^{1/2}\right] \ , \tag{8}$$

$$g(x)=\frac{2\Omega}{[(2\Omega)^2+x^2]^{1/2}}\tanh\left[\frac{\beta}{4}[(2\Omega)^2+x^2]^{1/2}\right] \ . \tag{9}$$

By using the spin-$\frac{1}{2}$ identity $e^{\lambda\sigma_j}=\cosh\lambda+\sigma_j\sinh\lambda$ Eqs. (6) and (7) become

$$\langle\sigma_i^z\rangle=\left\langle\prod_{j=1}^{4}[\cosh(J_{ij}D)+\sigma_j^z\sinh(J_{ij}D)]\right\rangle f(x)\,|_{x=0} \ , \tag{10}$$

$$\langle\sigma_i^x\rangle=\left\langle\prod_{j=1}^{4}[\cosh(J_{ij}D)+\sigma_j^z\sinh(J_{ij}D)]\right\rangle g(x)\,|_{x=0} \ , \tag{11}$$

where the subscripts run from 1 to 4 corresponding to the four nearest neighbors of the site i. Note that Eqs. (10) and (11) yield a set of relations between the longitudinal magnetization of the ith site and associated multispin correlation functions once the bond configuration $\{J_{ij}\}$ is completely specified.

The main purpose of the present work is to obtain from Eqs. (10) and (11) the phase diagrams and the behavior of the longitudinal as well as the transverse magnetization as function of the parameters α, T, Ω, and p. It is clear that if we try to treat exactly all the spin-spin correlations which appear in Eqs. (10) and (11), and to perform the configurational averages properly, which is still to be done, the problem becomes mathematically untractable. Therefore, some approximations are needed. Ignoring multispin correlations, the present framework shares with MFA the fact that the critical exponents are all Landau type. In this case the topology of the system is taken into account, essentially through its coordination number; nevertheless, we verify that its results are quite superior to the standard MFA.[28,29,36,37] Based on this approximation, Eqs. (10) and (11) can be rewritten as follows:[38]

$$m^z=\{4m^z\langle\sinh(J_{ij}D)\rangle_J[\langle\cosh(J_{ij}D)\rangle_J]^3+4(m^z)^3\langle\cosh(J_{ij}D)\rangle_J[\langle\sinh(J_{ij}D)\rangle_J]^3\}f(x)\,|_{x=0} \ , \tag{12}$$

$$m^x=\{[\langle\cos(J_{ij}D)\rangle_J]^4+6(m^z)^2[\langle\sinh(J_{ij}D)\rangle_J]^2[\langle\cosh(J_{ij}D)\rangle_J]^2+(m^z)^4[\langle\sinh(J_{ij}D)\rangle_J]^4\}g(x)\,|_{x=0} \ , \tag{13}$$

where $m^z = \langle\langle \sigma_i^z \rangle\rangle_J$, $m^x = \langle\langle \sigma_i^x \rangle\rangle_J$, and

$$\langle \sinh(J_{ij}D)\rangle_J = (1-p)\sinh(J_1D) + p\,\sinh(J_2D)\ , \tag{14a}$$

$$\langle \cosh(J_{ij}D)\rangle_J = (1-p)\cosh(J_1D) + p\,\cosh(J_2D)\ . \tag{14b}$$

Equation (12) admits two solutions, namely $m^z \equiv 0$ (nonferromagnetic phase), and a nontrivial one (associated with the ferromagnetic phase) given by

$$m^z = \left[\frac{1-A^+}{A^-}\right]^{1/2}\ , \tag{15}$$

where the coefficients $A^{\pm}$ are obtained by a straightforward calculation [which makes use of the property $\exp(\lambda D)f(x)\,|_{x=0} = f(\lambda)$, $\forall\lambda$] and are given by

$$\begin{aligned} A^{\pm} = {} & (1-p)^4K_1^{\pm} + 4(1-p)^3pK_2^{\pm} \\ & + 6(1-p)^2p^2K_3^{\pm} + 4(1-p)p^3K_4^{\pm} + p^4K_5^{\pm}\ , \end{aligned} \tag{16}$$

where

$$K_1^{\pm} = \tfrac{1}{2}[f(4\alpha J_2) \pm 2f(2\alpha J_2)]\ , \tag{17a}$$

$$\begin{aligned} K_2^{\pm} = {} & \tfrac{1}{2}[f((3\alpha+1)J_2) \pm \tfrac{1}{2}f((3\alpha-1)J_2) \\ & \pm \tfrac{3}{2}f((\alpha+1)J_2)]\ , \end{aligned} \tag{17b}$$

$$K_3^{\pm} = \tfrac{1}{2}[f((2\alpha+2)J_2) \pm f(2\alpha J_2) \pm f(2J_2)]\ , \tag{17c}$$

$$\begin{aligned} K_4^{\pm} = {} & \tfrac{1}{2}[f((3+\alpha)J_2) \pm \tfrac{1}{2}f((3-\alpha)J_2) \\ & \pm \tfrac{3}{2}f((\alpha+1)J_2)]\ , \end{aligned} \tag{17d}$$

$$K_5^{\pm} = \tfrac{1}{2}[f(4J_2) \pm 2f(2J_2)]\ . \tag{17e}$$

The critical curves characterizing the ferromagnetic-phase stability limit are determined by $m^z = 0$, hence by

$$A^+ = 1\ . \tag{18}$$

III. RESULTS AND DISCUSSION

In this section we shall present and discuss the results (phase diagrams and magnetizations) using the following convenient notation: $\tau \equiv k_BT/J_2$, $\Gamma \equiv \Omega/J_2$, and $q = 1-p$.

A. Random-bond Ising model

Let us start by presenting the phase diagram associated with the quenched random-bond Ising model ($\Omega = 0$) for both competing and noncompeting cases. Equation (18) defines the critical surface in the (p, T, α) space (see Fig. 1). This zero field behavior has already been reported previously,[38] and we present it here merely to contrast it with the corresponding behavior in the presence of the field.

For finite positive values of α the critical temperature does not vanish at any possible value of p. Therefore, for positive values of α at $T = 0$, the probability that the system has an infinite cluster of spins aligned in the z direction is always different from zero. This result, which is physically expected, arising in the present formalism from the fact that Eq. (18) is not satisfied at $T = 0$ for any value of p in the range, $0 \le p \le 1$.

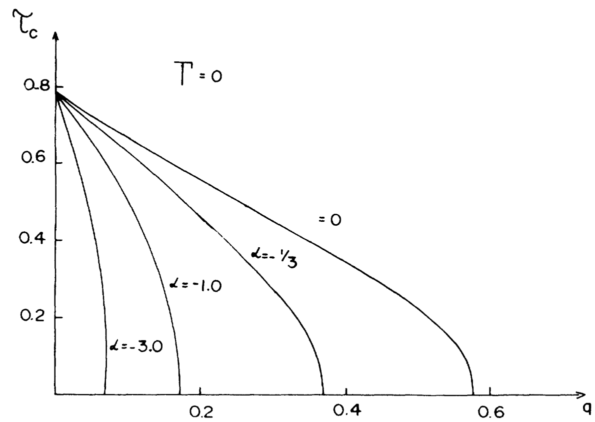

FIG. 1. Concentration dependence of Curie temperature for both diluted ($\alpha = 0$) and quenched bond-mixed (with $\alpha < 0$) Ising model.

In the bond-diluted case ($\alpha = 0$), the critical line displays a percolation concentration $p_c = 0.428$ (in agreement with Matsudaira[39]). Note that other results available in the literature are: $p_c^{\text{exact}} = 0.5$ (Ref. 40), $p_c = \frac{1}{3}$ (Ref. 30), and $p_c^{\text{MFA}} = 0$. A positive aspect of the effective-field approach is that it enables one to capture the exact asymptotic form of the critical line ($\alpha = 0$) simultaneously in both limits $T \to 0$ and $p \to 1$.

The phase diagrams associated with the competing model ($\alpha < 0$) show that the critical temperature vanishes at higher concentrations as α decreases. Thus for $\alpha = -\frac{1}{3}$, $\alpha = -1$, and $\alpha = -3$ we find the respective percolation concentrations: $p_c \cong 0.635$, $p_c \cong \frac{5}{6}$, and $p_c \cong 0.931$. The particular value $p_c \cong \frac{5}{6}$ associated with the isotropic model corresponds to an impurity concentration $q_c \equiv 1 - p_c = \frac{1}{6} \cong 0.167$ which agrees well with the results from the Monte Carlo method[42] ($q_c = 0.15$–0.20), the replica method[43] ($q_c = 0.166$) and the Bethe approximation[44] ($q_c = 0.167$), whereas other methods[45–47] provide a lower value, i.e., $q_c \cong 0.1$. The present framework has been successfully applied to the random-bond Ising model for noncompeting interactions ($\alpha \ge 0$). However, its application to the competing cases ($\alpha < 0$) was considered[38] not reliable in the limit $T \to 0$, for the following reason.

From Eqs. (16)–(18) it is straightforward to verify analytically that as $T \to 0$ all the critical lines concerning α in the range $-\frac{1}{3} < \alpha < 0$ fall on the same percolation concentration $p_c \cong 0.600$. A similar situation occurs for α in the ranges $-\frac{1}{3} > \alpha > -1$ with $p_c \simeq 0.659$, $-1 > \alpha > -3$ with $p_c \simeq 0.909$ and $\alpha < -3$ with $p_c \simeq 0.945$. The physical reason why such a large number of critical lines share single points at $T = 0$ is still somewhat obscure. For some values of α this result leads to reentrant phases in the low-temperature regime.[48] As an example, we have plotted in Fig. 2 the longitudinal magnetization as a function of temperature for the particular case $\alpha = -0.1$ and $p = 0.59$ to illustrate the existence of a nonmagnetic phase

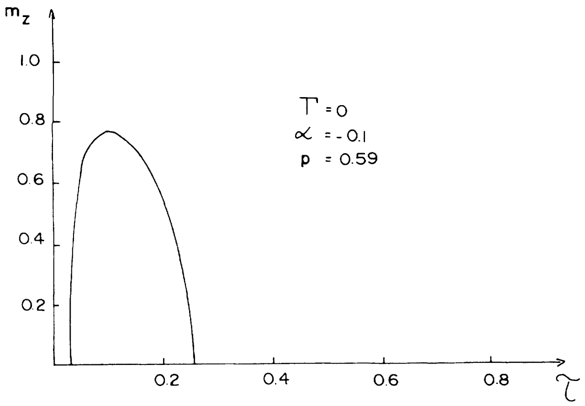

FIG. 2. Longitudinal magnetization curve for the quenched bond-mixed Ising model with $\alpha = -0.1$ and $p = 0.59$.

at low temperature. Although this sort of behavior was considered to be an artifact of the present approximation[38] it may well represent a real effect.

B. Bond-diluted TIM

Let us first investigate the behavior of the TIM in the case $\alpha=0$. In the limit of zero transverse field we obtain from relation $K_5^+=1$ the critical temperature of the pure ($p=1$) Ising model $\tau_c=3.088$ which may be compared with $\tau_c^{\text{exact}}=2.2692$ (Ref. 49), $\tau_c^{\text{RG}}=1.639$ (Ref. 27), $\tau_c^{\text{MFA}}=4$. Note that the present value for τ_c agrees with those obtained in Refs. 50–52. The behavior of the critical temperature τ_c as a function of impurity concentration q is depicted in Fig. 3(i) for several values of the transverse field Γ. For $\Gamma=0$ we obtain the critical value $q_c=0.57$. The effect of the transverse field then is to shift both the transition temperature and the critical impurity concentration to a lower value. For small values of Γ and q we find that the critical lines display nearly identical slopes, thus agreeing with RG calculations.[27]

Figure 3(ii) shows the behavior of the critical temperature as a function of the transverse field for several values of the impurity concentration. Within the present formalism we see that for $\Gamma \sim \Gamma_c$ the temperature decreases rapidly with the field Γ. This result is a consequence of the approximated formulas (3) and (4) which treat the transverse field in a mean-field fashion and also a consequence of neglecting all the many-spin correlations. However, the quantitative results are superior than the RG results.[27] At $q=0$ we obtain the critical transverse field $\Gamma_c=1.37$ which is to be compared with $\Gamma_c^{\text{SE}}=1.52$ (Ref. 8), $\Gamma_c^{\text{RG}}=0.77$ (Ref. 27), and $\Gamma_c^{\text{MFA}}=2.0$.

The critical field as a function of impurity concentration at various temperatures is shown in Fig. (iii). We see that our results do not display the discontinuity in the critical transverse field at $p=p_c$ as discussed earlier. This is most certainly due to our mean-field-type of approach.

Let us now discuss the magnetization results which are determined from Eqs. (12) and (13). The longitudinal magnetization m^z is defined by Eq. (12) which has a trivial solution $m_z=0$. Figure 4 shows the behavior of the

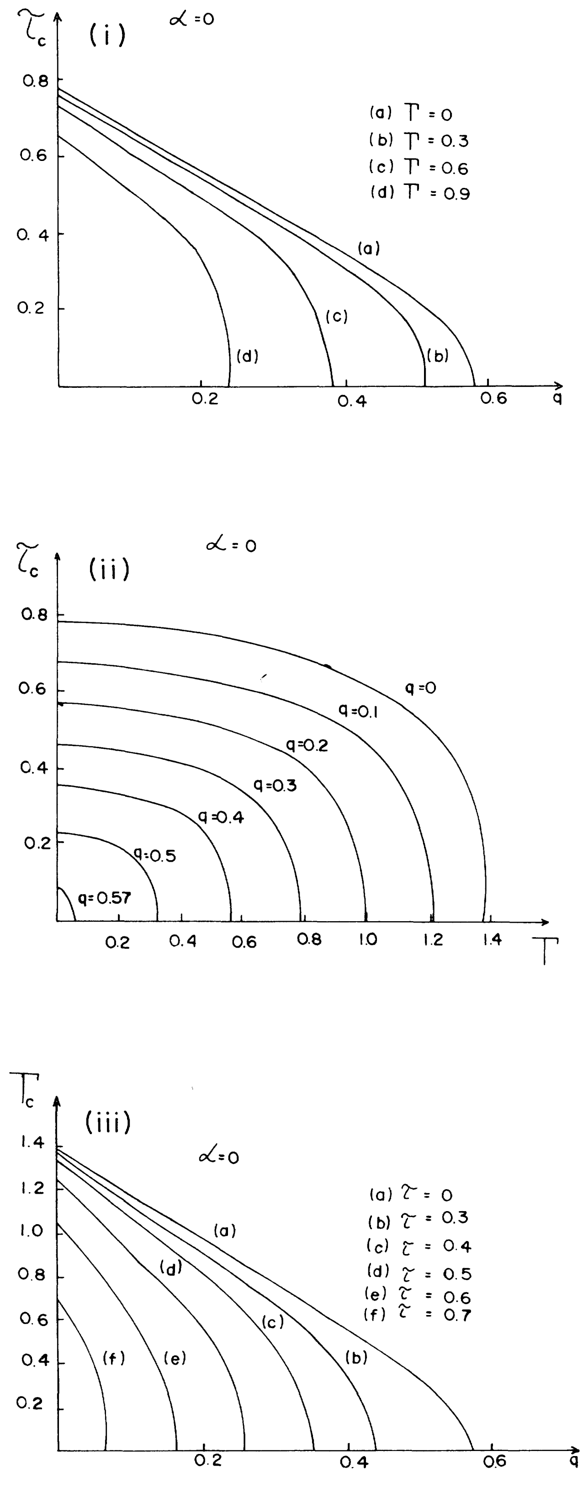

FIG. 3. (i) Concentration dependence of the critical temperature for several values of the transverse field for the diluted Ising model. (ii) Critical temperature as a function of the transverse field for several concentrations in the bond-diluted Ising model. (iii) Critical transverse field as a function of the concentration q at several temperatures for the bond-diluted Ising model.

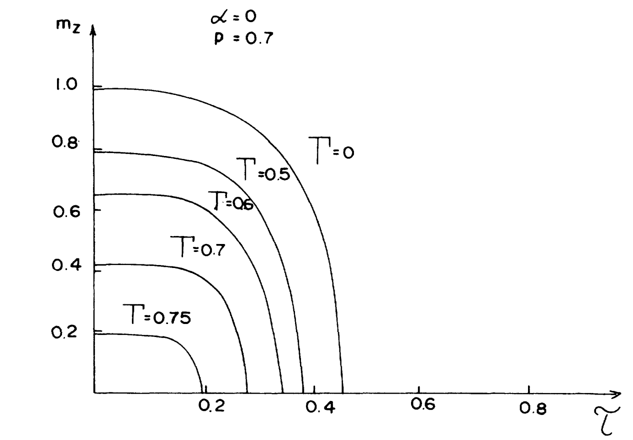

FIG. 4. Longitudinal magnetization curves as a function of the temperature (diluted Ising model; $\alpha = 0$) at $p = 0.7$ and for various values of the transverse field.

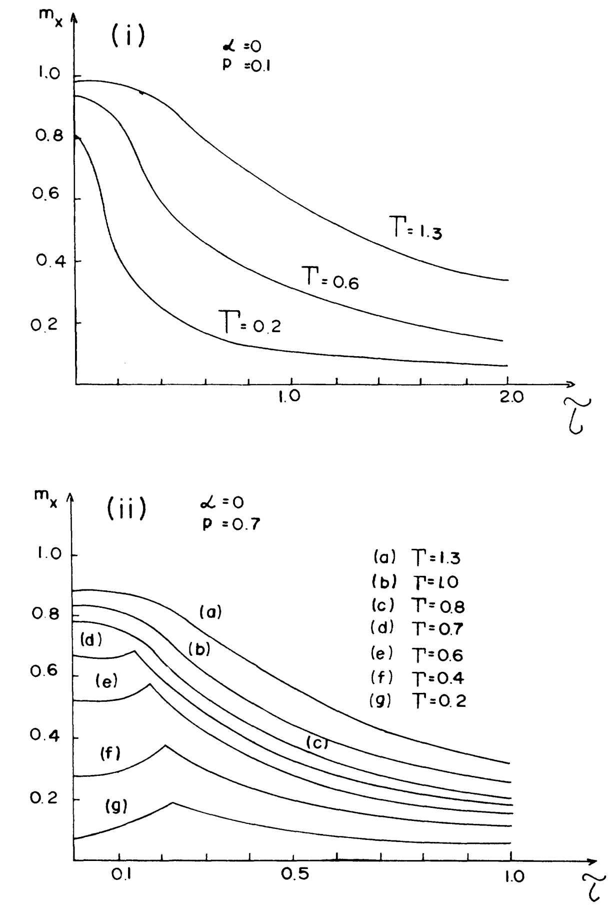

FIG. 5. Transverse magnetization curves for several values of the transverse field associated with the diluted Ising model, with (i) $p = 0.1$; (ii) $p = 0.7$.

longitudinal magnetization as a function of temperature T for several values of Γ at a fixed concentration $p = 0.7$. Now, we focus our attention in Eq. (13) in order to discuss the transverse magnetization m_x. We see that in the nonmagnetic regime, that is, $m_z = 0$, Eq. (13) reduces to

$$m^x = [\langle \cosh(J_{ij}D) \rangle_J]^4 g(x) \,|_{x=0} \; .$$

The behavior of the transverse magnetization m_x as a function of temperature τ for several values of Γ with fixed concentrations $p = 0.1$ and $p = 0.7$ is shown in Figs. 5(i) and 5(ii), respectively. These curves show clearly the qualitative difference in the behavior of m_x in the nonmagnetic regime from its behavior in the ferromagnetic regime. In the former case, there is only competition between the field and the temperature and therefore, as τ increases m_x decreases, the rate of this decrease being determined by the strength of the transversal field. On the other hand, in the ferromagnetic regime a new physical situation arises in the sense that the longitudinal ordering comes into play and, as τ increases from zero, this effect is diminished thus favoring the effect of the transversal field. Therefore, as τ increases up to τ_c from below, the transversal magnetization increases until it reaches its maximum value at $\tau = \tau_c$. This situation is shown in Fig. 5(i) where m_x displays a cusp at $\tau = \tau_c$. This sort of behavior is also found in the annealed model treated within the present framework[28] as well as in Monte Carlo simulation of the spin-∞ TIM.[19]

C. Bond-mixed TIM

Let us first consider the effect of the field on noncompeting systems, where $\alpha > 0$. We have seen in Sec. III A that in the absence of the field, such a system is ferromagnetic at $T = 0$ for all possible values of p. In the presence of the field however, as it acts to disrupt the longitudinal order, the critical temperature of the system is reduced. Therefore, for positive values of α at sufficiently high fields, the critical temperature can vanish beyond a field-dependent critical concentration p_c. When $p_c = 0$, we find from Eqs. (16)–(18) that the critical temperature vanishes when the ratio $(\Gamma/\alpha)_c \equiv 1.37$. Thus, for the particular case $\alpha = 1$, we recover the critical field, obtained in the last section.

Let us now discuss the behavior of the competing model for which $\alpha < 0$ (with $J_1 < 0$ and $J_2 < 0$) in the presence of the transverse field. As one might expect, the critical temperature vanishes now at concentrations p higher than the pure model. Typical results are shown in Figs. 6(i) and 6(ii), where we plot τ_c as a function of q for several values of the transverse field with $\alpha = -\frac{1}{3}$ and -1, respectively. Note that there, as in the bond-diluted case, we also find that the slopes of these curves for small Γ are nearly the same.

Another interesting feature caused by the presence of the field which is easily deducible from Eqs. (16)–(18) is that the critical lines corresponding to $-\frac{1}{3} < \alpha < 0$, $-1 < \alpha < -\frac{1}{3}$, and $-3 < \alpha < -1$ do not fall on the same percolation concentration anymore. Thus now, for each α one a specific value $p_c(\alpha)$, as expected, because now the field plays the role of temperature. However for some values of Γ/α we may still find reentrant phases in the low temperature range.

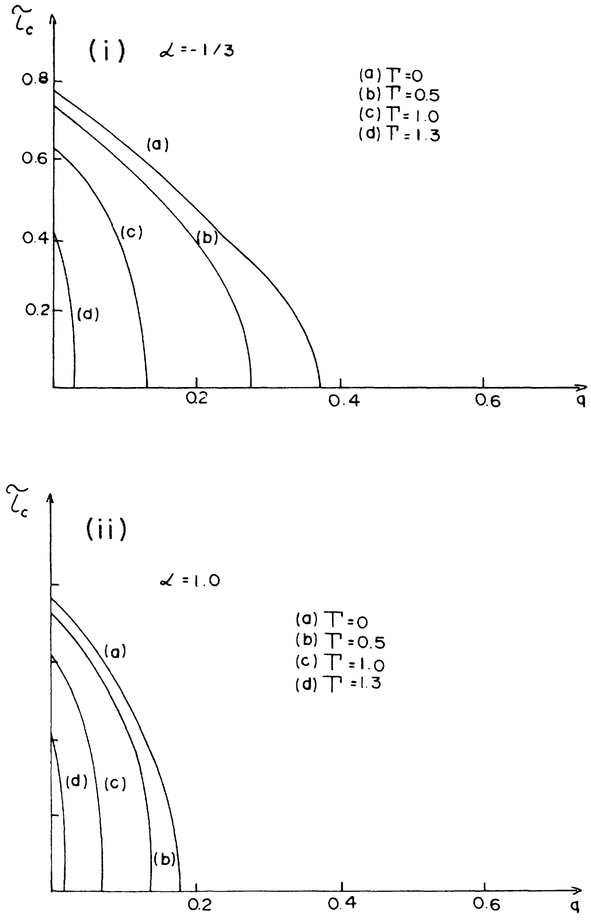

FIG. 6. Concentration dependence of the critical temperature associated with the quenched bond-mixed Ising model for several values of the transverse field with: (i) $\alpha=-\frac{1}{3}$; (ii) $\alpha=-1.0$.

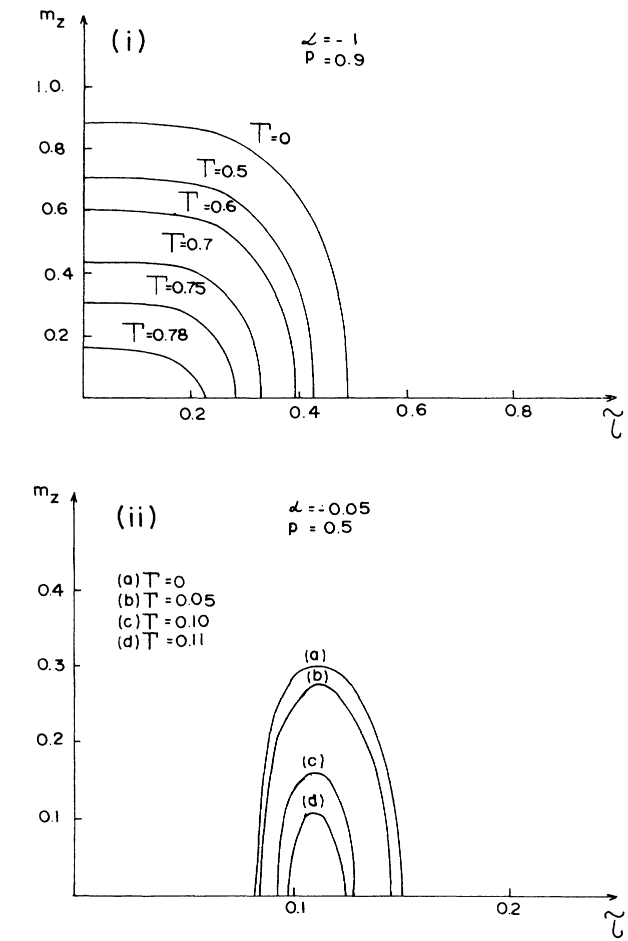

FIG. 7. Longitudinal magnetization curves associated with the quenched bond-mixed Ising model for several values of the transverse field, with: (i) $\alpha=-1.0$, $p=0.9$; (ii) $\alpha=-0.05$, $p=0.5$.

We proceed in order to illustrate the behavior of the longitudinal magnetization of the competitive model. Typical results are shown in Fig. 7(i) where we have plotted m_z as a function of τ, for $\alpha=-1$ and for a concentration of ferromagnetic bonds $p=0.9$ varying the transverse field from $\Gamma=0$ to $\Gamma=0.78$. In Fig. 7(ii) we have also plotted the longitudinal magnetization m_z as a function of τ, for $\alpha=-0.05$ and $p=0.5$ varying the transverse field from $\Gamma=0$ to $\Gamma=0.11$, to illustrate the persistence of a reentrant phase even in the presence of the field.

The behavior of the transverse magnetization m_x as a function of τ is depicted in Fig. (8), with $\alpha=-1$ and $p=0.9$, for several values of the transverse field. Note that from a qualitative point of view, the behavior of the transverse magnetization shown here is essentially the same behavior of m_x for the bond-diluted model, as discussed previously.

IV. CONCLUSIONS

We have applied an effective-field treatment to the quenched bond-mixed spin-$\frac{1}{2}$ transverse Ising model on a

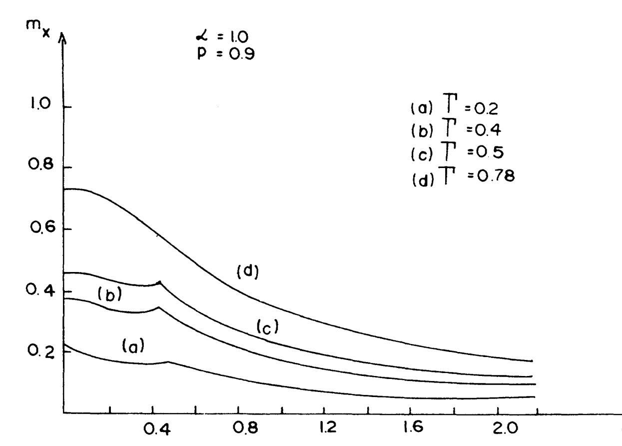

FIG. 8. Transverse magnetization curves associated with the quenched bond-mixed Ising model for several values of the transverse field with $\alpha=-1.0$ and $p=0.9$.

square lattice, considering both competing and noncompeting interaction. The effective field approach is based on a generalization of the Callen relation for the Ising model in the presence of a transverse field. Phase diagrams have been calculated as well as longitudinal and transverse magnetizations. The results described in the last section are quite remarkable considering that the approximation used within this simple effective-field approach neglects spin-spin correlations. As previous work on other models[38] have indicated, we find that the results obtained herein can be given qualitative, and to a certain extent, quantitative liability.

The discussion presented above still leaves open the question on the reasons for the peculiar results obtained here for the competing model in the low-temperature region. In particular, models with competing ferromagnetic and antiferromagnetic bonds pose a challenge to theoretical understanding. Considering that neglecting spin-spin correlations may lead to inaccurate results but it does not change the physics in an essential way; one is inclined to interpret these results as a consequence of the content of frustration existent in the system. These frustration effects being relevant only at low temperatures and for small values of the transverse field when ferromagnetic and antiferromagnetic interactions are allowed in the lattice.

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