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Stability of the McCumber curve for long Josephson tunnel junctions

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The stability of the McCumber solution of the perturbed sine-Gordon equation that describes the dynamics of a long Josephson junction may conveniently be studied within the context of a Fourier-Galerkin approximation. In the absence of an externally applied magnetic field, this procedure predicts analytically how the number, locations, and widths of the unstable regions of the McCumber curve depend on the junction parameters. These instabilities are of physical interest because they evolve into the fluxon oscillations associated with zero-field steps. In the presence of a small applied magnetic field, the same procedure provides a technique for studying Fiske steps.

I. INTRODUCTION

The determination of instability regions associated with the McCumber curve in the current-voltage (I-V) characteristics of long, narrow (hysteretic) Josephson tunnel junctions has attracted research interest because the existence of such instability regions is directly connected with the experimental observation of zero-field steps (ZFS's) in the I-V characteristics of such junctions. This connection was first pointed out in 1973 by Fulton and Dynes,[1] on the basis of observations on a mechanical analog of the long Josephson junction. Later, the problem was studied analytically by Burkov and Lifsic.[2] More recently, Pagano *et al.*[3] have considered the problem in some detail, reporting analytical, numerical, and experimental results. Briefly, the picture that emerges from these studies is as follows: To observe ZFS's experimentally, one raises the bias current applied to the junction from zero up to the critical value, whereupon the junction switches from the zero-voltage state to the gap state. The bias current is then reduced to some nonzero value; during this phase the McCumber curve in the I-V plane is traced out. Raising the current again then allows tracing out the ZFS's.

This situation may be understood theoretically by performing a stability analysis of the particular solution, corresponding to the McCumber curve, of the perturbed sine-Gordon equation that describes the dynamics of the junction. The simplest case is that in which there is no external magnetic field applied to the junction. In this case it is particularly convenient to perform a multimode, i.e., Fourier-Galerkin, decomposition of the model equation since the McCumber solution corresponds to excitation of only the zero-order mode. The stability of this solution is governed by the higher-order mode equations, which, in the linear approximation, reduce to a set of uncoupled, damped Lamé equations, for which exact, analytic solutions have been found. Such Lamé equations exhibit parametrically excited unstable solutions in some regions of their parameter space; these instabilities evolve with time into the fluxon oscillations associated with the ZFS's.

The application of an external magnetic field provides a mixing mechanism between the various mode equations, thus rendering the analysis less tractable. For a sufficiently small field, however, we can once again linearize the higher-order mode equations. In this approximation, the essential effect of the field is simply to add an inhomogeneous driving term to the odd-order mode equations. This term is responsible for the appearance of (odd-order) Fiske steps (FS's) in the I-V characteristic of the junction.

II. MATHEMATICAL MODEL

The mathematical model of the overlap-geometry Josephson junction is, in normalized form, the perturbed sine-Gordon equation[4]

$$\phi_{xx}-\phi_{tt}-\sin\phi=\alpha\phi_t-\beta\phi_{xxt}-\gamma \ , \tag{1a}$$

$$\phi_x(0,t)=\phi_x(L,t)=\eta \ . \tag{1b}$$

Here, $\phi(x,t)$ is the usual Josephson phase variable, x is distance along the junction normalized to the Josephson penetration length, and t is time normalized to the inverse of the Josephson plasma angular frequency. The model contains five parameters: α, β, γ, L, and η. The term in α represents shunt loss due to quasiparticle tunneling (assumed Ohmic), the term in β represents dissipation due to the surface resistance of the superconducting films, γ is the spatially uniform bias current normalized to the maximum zero-voltage Josephson current, L is the normalized junction length, and η is the normalized external magnetic field, applied in the plane of the junction and perpendicular to its long dimension. In recent years this model has been shown to describe a wide range of experimentally observed Josephson phenomena, often to a surprising level of detail.

A number of approaches have been employed in the literature to solve Eqs. (1). One of these, which has been found convenient in particular for the study of periodic limit cycle behavior, is the Fourier-Galerkin approximation, i.e., projection onto a truncated series of Fourier spatial modes whose amplitudes are unknown functions of time. To illustrate this approach we consider first the case of homogeneous boundary conditions, i.e., $\eta=0$ in Eq. (1b). We take as a solution ansatz the form

$$\phi(x,t)=\sum_{j=0}^{N}\phi_j(t)\cos(j\pi x/L)\ , \tag{2}$$

where N is some finite number. The choice of this form stems from considering, at any instant of time, a reflection of the function $\phi(x,t)$ in the interval $x=0$ to L onto the interval $x=0$ to $-L$ in such a way as to construct a periodic, continuous, smooth, even function with spatial periodicity $2L$. In the limit $N\to\infty$ the representation of Eq. (2) is exact; the practical usefulness of this approach depends upon being able to obtain a "reasonable" description of the system behavior using a relatively small value of N.

Inserting Eq. (2) into Eqs. (1), and using the orthogonality properties of the trigonometric functions, we obtain the following set of ordinary differential equations for the mode amplitudes $\phi_j(t)$:

$$\ddot{\phi}_0+\alpha\dot{\phi}_0=\gamma-(1/L)\int_{x=0}^{L}\sin\phi\,dx\ , \tag{3a}$$

$$\ddot{\phi}_m+(\alpha+\beta\omega_m^2)\dot{\phi}_m+\omega_m^2\phi_m =-(2/L)\int_{x=0}^{L}\sin\phi\cos(m\pi x/L)dx\ , \quad m=1,2,\ldots,N \tag{3b}$$

in which $\omega_m\equiv m\pi/L$, and ϕ is given by Eq. (2), and overdots denote derivatives with respect to t.

III. McCUMBER STABILITY ANALYSIS

A McCumber solution of Eqs. (1) is one without spatial structure; in terms of the elastically coupled pendulum-chain analog of the sine-Gordon system it has all of the pendula rotating in synchronism "over the top." In terms of Eqs. (3) it is represented by a configuration having $\phi_0\neq 0$ and $\phi_m(t)\equiv 0$, $m=1,2,\ldots,N$. In this situation Eq. (3a) becomes

$$\ddot{\phi}_0+\alpha\dot{\phi}_0=\gamma-\sin\phi_0\ , \tag{4}$$

and all of Eqs. (3b) become identically zero. In the absence of loss and bias ($\alpha=\gamma=0$), the rotating solution of Eq. (4) is exactly

$$\phi_0(t)=2\,\mathrm{am}(t/k;k)\ , \tag{5}$$

where am is the Jacobian elliptic amplitude function[5] of modulus k, with $0\le k\le 1$. For nonzero α and γ we assume that Eq. (5) solves Eq. (4) in the power-balance approximation, i.e., we equate the average power furnished by the bias supply, $P_{\mathrm{in}}=\gamma\langle\dot{\phi}_0\rangle$, to the average power dissipated, $P_{\mathrm{out}}=\alpha\langle\dot{\phi}_0^2\rangle$, where angular brackets denote a time-averaged value. Carrying out this operation yields the following expressions for the McCumber branch of the I-V characteristic of the junction:

$$\gamma=4\alpha E(k)/\pi k\ , \tag{6a}$$

$$V\equiv\langle\dot{\phi}_0\rangle=\pi/kK(k)\ , \tag{6b}$$

where $K(k)$ and $E(k)$ are, respectively, the complete elliptic integrals of first and second kinds.[5]

To study the stability of this solution we suppose now that the $\phi_m(t)$, $m=1,2,\ldots,N$, are all small but nonzero. Defining

$$\epsilon\equiv\sum_{j=1}^{N}\phi_j(t)\cos(j\pi x/L)\ , \tag{7}$$

we expand $\sin\phi$ to linear terms as

$$\sin\phi=\sin(\phi_0+\epsilon)=\sin\phi_0+\epsilon\cos\phi_0\ . \tag{8}$$

Inserting Eq. (8) into Eqs. (3), and utilizing once again the orthogonality of the cosines, we obtain

$$\ddot{\phi}_0+\alpha\dot{\phi}_0=\gamma-\sin\phi_0\ , \tag{9a}$$

$$\ddot{\phi}_m+(\alpha+\beta\omega_m^2)\dot{\phi}_m+\omega_m^2\phi_m=-\phi_m\cos\phi_0\ , \quad m=1,2,\ldots,N\ . \tag{9b}$$

The equation for ϕ_0, Eq. (9a), is exactly the same as in the unperturbed case, Eq. (4). Equations (9b) represent a set of N uncoupled, linear, parametrically excited oscillators in which the ϕ_0 term is the parametric driver. Since these equations are uncoupled, they may be solved independently, which greatly simplifies the analysis.

Inserting the expression of Eq. (5) for ϕ_0 into the generic member of Eqs. (9b), we obtain explicitly, for the nth mode, the equation

$$\ddot{\phi}_n+(\alpha+\beta\omega_n^2)\dot{\phi}_n+[\omega_n^2+1-2\,\mathrm{sn}^2(t/k;k)]\phi_n=0\ , \tag{10}$$

where sn is the Jacobian elliptic sine function of modulus k. Defining the new time variable, $\tau\equiv t/k$, we transform Eq. (10) into

$$\ddot{\phi}_n+k(\alpha+\beta\omega_n^2)\dot{\phi}_n+[k^2(\omega_n^2+1)-2k^2\mathrm{sn}^2(\tau;k)]\phi_n=0\ , \tag{11}$$

where overdots now denote derivatives with respect to τ. We may eliminate the first derivative term in Eq. (11) by means of the standard transformation

$$\phi_n(\tau)=y(\tau)\exp[-\tfrac{1}{2}k(\alpha+\beta\omega_n^2)\tau]\ , \tag{12}$$

under which Eq. (11) becomes

$$\ddot{y}+\{k^2[\omega_n^2+1-\tfrac{1}{4}(\alpha+\beta\omega_n^2)^2]-2k^2\mathrm{sn}^2(\tau;k)\}y=0\ . \tag{13}$$

Equation (13) is Lamé's equation. A detailed discussion of the exact analytic solution of this equation (in somewhat more generalized form) may be found in Whittaker and Watson[6] (who attribute the original solution to lecture notes of Hermite dating from 1872). Following their discussion, we find two linearly independent solutions of Eq. (13) to be

$$y_+(\tau)=[H(\tau-\tau_0)/\theta(\tau)]\exp[+Z(\tau_0)\tau]\ , \tag{14a}$$

$$y_-(\tau)=[H(\tau+\tau_0)/\theta(\tau)]\exp[-Z(\tau_0)\tau]\ , \tag{14b}$$

where H, θ, and Z are, respectively, the eta, theta, and zeta functions of Jacobi,[5] provided the constant τ_0

satisfies the equation

$$\mathrm{cn}^2(\tau_0;k)\mathrm{ds}^2(\tau_0;k)-\mathrm{ns}^2(\tau_0;k) = -k^2[\omega_n^2+1-\tfrac{1}{4}(\alpha+\beta\omega_n^2)^2] \ , \quad (15)$$

where cn, ds, and ns are Jacobian elliptic functions.[5] Using various identities amongst these functions,[5] we may simplify Eq. (15) to

$$\mathrm{sn}^2(\tau_0;k)=1/k^2-\omega_n^2+\tfrac{1}{4}(\alpha+\beta\omega_n^2)^2 \ . \quad (16)$$

The nature of the functions in Eqs. (14) depends on the value assumed by the right-hand side of Eq. (16). We may *a priori* distinguish four possible cases.

Case 1: $0\le 1/k^2-\omega_n^2+\frac{1}{4}(\alpha+\beta\omega_n^2)^2\le 1$. Using various results from Ref. 5, we may establish that τ_0 is real, $0\le\tau_0\le K(k)$, H/θ is real and periodic, and $Z(\tau_0)\ge 0$. Hence, from Eqs. (14) and (12), the stability boundaries of Eq. (11) are given by

$$\delta\equiv Z(\tau_0)-\tfrac{1}{2}k(\alpha+\beta\omega_n^2)=0 \ . \quad (17)$$

$\delta<0$ implies stability; $\delta>0$ implies instability.

Case 2: $1/k^2-\omega_n^2+\frac{1}{4}(\alpha+\beta\omega_n^2)^2<0$. We may establish that τ_0 is pure imaginary, $Z(\tau_0)$ is pure imaginary, and H/θ is complex, but periodic. Hence, $\mathrm{Re}(\delta)=-\frac{1}{2}k(\alpha+\beta\omega_n^2)<0$, which implies stability.

Case 3: $1<1/k^2-\omega_n^2+\frac{1}{4}(\alpha+\beta\omega_n^2)^2\le 1/k^2$. We may establish that τ_0 is complex, $\mathrm{Re}(\tau_0)=K(k)$, $Z(\tau_0)$ is pure imaginary, and H/θ is complex, but periodic. Hence, as in case 2, $\mathrm{Re}(\delta)=-\frac{1}{2}k(\alpha+\beta\omega_n^2)<0$, which implies stability.

Case 4: $1/k^2<1/k^2-\omega_n^2+\frac{1}{4}(\alpha+\beta\omega_n^2)^2<\infty$, *i.e.*, $\alpha+\beta\omega_n^2>2\omega_n$. We may establish that τ_0 is complex, $\mathrm{Im}(\tau_0)=K(k')$, where k' is the complementary modulus, and H/θ is complex, but periodic; however, the nature of $Z(\tau_0)$ is not (at least to us) completely clear. However, we note that for "physically reasonable" parameters case 4 is unlikely: e.g., with $\alpha=0.05$ and $\beta=0.02$, case 4 is obtained only if $L/n<0.0314$ or if $L/n>126$, where L is the normalized junction length and n is the order of the ZFS.

Consequently, in practice, the only physically relevant situation is case 1. The computational procedure involves fixing the parameters α, β, L, and n, and iterating Eqs. (16) and (17) until a value of k is found which gives $\delta=0$. The stability boundaries in current and voltage are then found by inserting this value of k into Eqs. (6). The necessary calculations may be carried out readily using a programmable pocket calculator.

As a check on this theory we have compared our present results with those obtained by Pagano *et al.*[3] by means of a perturbation expansion of Eqs. (9b) in the high-voltage region of the McCumber curve. We have also compared our results with those obtained by a direct numerical implementation of Floquet theory.[7] Agreement was found in all cases.

IV. SOME PARTICULAR RESULTS

Suppose we wish to establish a stability boundary at $V=0$, i.e., we impose $\delta(V=0)=0$. From Eq. (6b), $V=0$ corresponds to $k=1$. For $k=1$, both $\mathrm{sn}(\tau_0;k)$ and $Z(\tau_0)$ reduce to $\tanh(\tau_0)$, so that, from Eqs. (16) and (17), the condition for $\delta(V=0)=0$ is

$$[1-\omega_n^2+\tfrac{1}{4}(\alpha+\beta\omega_n^2)^2]^{1/2}=\tfrac{1}{2}(\alpha+\beta\omega_n^2) \ , \quad (18)$$

i.e., $L/n=\pi$. For $L/n>\pi$, we have $\delta(V=0)>0$, which implies instability. From these facts one might be tempted to infer that for all $L/n\ge\pi$ the instability region in voltage extends from some maximum value, say V_m, down to $V=0$. Numerical calculations show this to be almost, but not quite, correct: For L/n *very* slightly greater than π, the instability region may consist of two disjoint pieces separated by a region of stability. For example, for $\alpha=0.05$, $\beta=0.02$, and $L/n=3.142$, the McCumber curve is unstable from $V=0$ to 0.287 and from $V=0.589$ to 2.39; but already for $L/n=3.143$ the intermediate stable region becomes unstable, and the instability region extends smoothly down to $V=0$.

Another consequence of this theory is that instability regions corresponding to the same value of α, β, and L, but different n, can overlap. This situation is illustrated in Fig. 1, which shows the instability regions for $\alpha=0.05$, $\beta=0.02$ and $n=1,2,3$, as a function of L. For different values of α and β the form of the instability regions remains quite similar to that shown, the major difference being that the "peak points," i.e., the points where the width of a region goes to zero, move to higher voltages for smaller α and/or β. The numbers in parentheses in Fig. 1 indicate which McCumber regions are unstable in the various zones of the *V-L* plane. The existence of overlapping instability regions *might* imply the existence of a switching mechanism between different ZFS's, but a verification of this hypothesis lies beyond the scope of a linear stability theory. In any case, the above two observations suggest a simple explanation for the frequently observed experimental fact that it is often difficult to bias on low-order ZFS's in longer junctions: If the low-order instability regions overlap, and if the lowest region or regions extend down to $V=0$, then it seems not unlikely that in descending along the

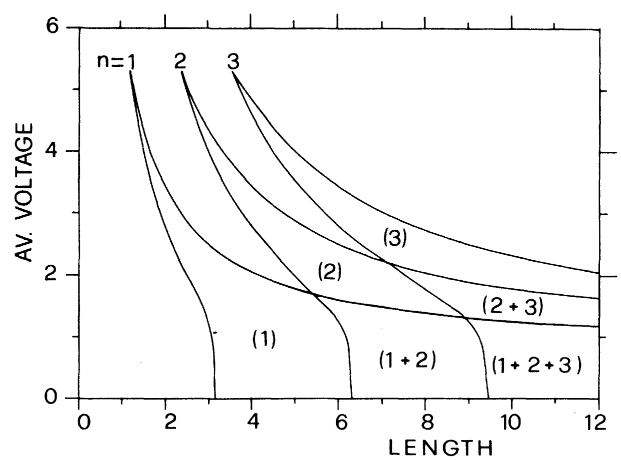

FIG. 1. Junction-length dependence of instability regions for $\alpha=0.05$ and $\beta=0.02$. Numbers in parenthesis indicate which McCumber regions are unstable.

McCumber curve one might switch directly to the zero-voltage state, skipping over the intervening ZFS's.

A linear stability analysis can provide estimates of the stability boundaries along the McCumber curve, but it cannot furnish the time evolution of an unstable solution. This question was addressed in Ref. 3, where it was shown by direct numerical integration of Eqs. (3) how such an unstable solution evolves into the fluxon oscillation associated with a ZFS. The existence of such a dynamic route is also suggested by the following analytic argument: Suppose in Eqs. (3) that only one spatial mode, say the nth, is excited, but that its amplitude is not restricted to be small. In this case Eqs. (3) can be written explicitly as

$$\ddot{\phi}_0+\alpha\dot{\phi}_0=\gamma-J_0(\phi_n)\sin\phi_0\ , \tag{19a}$$

$$\ddot{\phi}_n+(\alpha+\beta\omega_n^2)\dot{\phi}_n+\omega_n^2\phi_n=-2J_1(\phi_n)\cos\phi_0\ , \tag{19b}$$

where J_0 and J_1 are Bessel functions of the first kind. With the assumption that $\phi_0=\omega t$, with ω constant, Eq. (19b) becomes just the equation derived by Takanaka[8] to study the I-V profile of ZFS's. In fact, all of Takanaka's results are reproduced by applying the Krylov-Bogoliubov approximation procedure[9] to Eqs. (19). Similarly, a two-mode approximation to Eqs. (3), together with the Krylov-Bogoliubov procedure, gives rise to the results of Chang *et al.*,[10] and an N-mode approximation to those of Enpuku *et al.*[11]

V. MAGNETIC FIELD EFFECTS

In the presence of an external magnetic field, $\eta\neq0$ in Eq. (1b), the solution ansatz of Eq. (2) is no longer appropriate since it does not satisfy the boundary conditions. The expedient normally employed in this situation is to replace Eq. (2) by an ansatz of the form

$$\phi(x,t)=f(x)+\sum_{j=0}^{N}\phi_j(t)\cos(j\pi x/L)\ , \tag{20}$$

where $f(x)$ is some function that satisfies Eq. (1b). Several different such ansätze have been used by various authors: Enpuku *et al.*[11] use $f(x)=\eta x$. Watanabe and Ishii[12] use the procedure, due to Olsen and Samuelsen,[13] of choosing an $f(x)$ that corresponds to two static virtual fluxons placed outside the two ends of the junction. Kawamoto[14,15] uses, respectively, in the two papers cited, a static fluxon lattice array and a static fluxon-antifluxon array for $f(x)$.

The basic mathematical requirement on $f(x)$ is that it must satisfy Eq. (1b). In addition, a "good" choice for $f(x)$ presumably should be computationally simple and should lead to a relatively rapid convergence of the truncated Fourier series in Eq. (20). Since we are not aware of how to guarantee *a priori* this second condition, we shall, in what follows, use the Enpuku *et al.*[11] ansatz, $f(x)=\eta x$, because of its simplicity. This choice [with the substitution of Eq. (20) for ϕ] leaves the form of the dynamical equations, Eqs. (3), unchanged.

The main effect of the introduction of the magnetic field is that now there is, in general, always an excitation of the spatial modes that does not any longer allow a separation of the junction dynamics into independent oscillators, as in the discussion leading up to Eqs. (9). Consequently, in order to make some analytical progress, we now *assume* that $\eta L \ll 1$, and we *assume* that this implies that ϵ in Eq. (7) is small. Expanding $\sin\phi$ to linear terms in small quantities, we may write Eqs. (3) as

$$\ddot{\phi}_0+\alpha\dot{\phi}_0=\gamma-\sin(\phi_0+\eta L/2)\ , \tag{21a}$$

$$\ddot{\phi}_m+(\alpha+\beta\omega_m^2)\dot{\phi}_m+(\omega_m^2+\cos\phi_0)\phi_m=(4\eta L/m^2\pi^2)P_m\cos\phi_0\ , \tag{21b}$$

where $m=1,2,\ldots,N$, and $P_m=(0,1)$ for m (even, odd).

With the substitution $\phi_0'=\phi_0+\eta L/2$, Eq. (21a) becomes identical to Eq. (9a). Equations (21b) differ from Eqs. (9b) only by the presence of the inhomogeneous driving term $(4\eta L/m^2\pi^2)P_m\cos\phi_0$, which is present only for odd m. Since Eqs. (21b) are, by construction, once again linear, their total solution is just the sum of the homogeneous solution, i.e., that found in Sec. III above, plus a particular integral. The homogeneous solution shows exponential growth when the dominant frequency of $\cos\phi_0$ is approximately $2\omega_m$ (for any m). Outside of the instability regions the homogeneous solution tends asymptotically to zero. The particular integral, on the other hand, is essentially a resonance having a peak response when the dominant frequency of $\cos\phi_0$ is approximately ω_m (for odd m). Thus, in the context of the linear approximation, the appearance of ZFS's may be attributed to a parametrically excited resonance of the multimode equations, whereas the appearance of the odd-order FS's derives from a directly excited resonance of these equations. A complete analysis of magnetic field effects, and in particular an analysis of the even-order FS's, requires going beyond the linear approximation. This may be effected by perturbation theory, as in Ref. 3, or else by a direct numerical integration of Eqs. (3), in either case using the ansatz of Eq. (20) for ϕ.

VI. CONCLUSIONS

The linear stability analysis described above provides a simple explanation for a number of frequently observed experimental facts, e.g., the fact that it is often difficult to bias on low-order ZFS's in longer junctions. Moreover, it underscores the fact, first suggested by Chang *et al.*,[10] that both ZFS's and FS's might be described within the context of a single, unified model. It should be noted, however, that this analysis applies only to the mechanism of switching from the McCumber curve. There presumably exist also other mechanisms for biasing on steps (both ZFS's and FS's); the study of these will presumably require other tools.

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