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Behavior of anisotropic superconductors under uniaxial stress

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We investigate theoretically the behavior of anisotropic superconductors under uniaxial stress using the Landau expansion for the free energy. In the cases where the solution of the linearized gap equation belongs to a degenerate representation, there can be a linear coupling to the applied stress leading to a splitting of the transition from a normal to a superconducting state into two (or in some cases even three) second-order transitions. A complete analysis is presented of the resulting phase diagrams. The experimental observation of such splitting would give much information about the symmetry of the superconducting order parameter of the system.

The occurrence of a superconducting phase in the heavy-fermion metals UBe_{13}, $CeCu_2Si_2$, and UPt_3 has raised questions about their symmetry. Some experimental data point to unconventional, i.e., anisotropic Cooper pairing because of certain analogies to superfluid ^{3}He.[1–3] Volovik and Gor'kov,[4] Ueda and Rice,[5] and Blount[6] have investigated the possible anisotropic pairing states using a Ginzburg-Landau theory. A clear identification of the phase has not until now been achieved, partly because the normal phase of heavy-fermion systems is not still sufficiently well understood.

This paper proposes a method for tackling this problem. A qualitative effect will be investigated which depends only on the symmetry properties of the superconducting state and therefore avoids microscopic considerations. This objective was already pursued in the paper of Joynt and Rice,[7] where the spontaneous crystal symmetry lowering at the phase transition was discussed as a possibility to help identify the phase. In this paper we consider the consequences of applying a uniaxial stress to the crystal and we investigate the qualitative change of the properties of the superconducting phase. We examine both the cubic system (UBe_{13}), which can be converted to a tetragonal and rhombohedral structure by a uniaxial stress, and the hexagonal system (UPt_3), converted to an orthorhombic structure.

Section I introduces the unperturbed symmetry (cubic and hexagonal) and reproduces essentially the standard theory.[4–6] In Sec. II we show the consequences of a lattice distortion for the Ginzburg-Landau (GL) expansion. For that purpose Ozaki's group-theoretical method is used.[8] The new effects will be discussed in Sec. III. We will show that it is possible to observe additional phase transitions in the superconducting phase, if the superconductivity is anisotropic. These possibilities will be displayed in some phase diagrams with the dependence on certain parameters of the GL theory. In Sec. IV it will be shown that in most cases there are second-order transitions and in some simple examples the size of the discontinuity in specific heat will be calculated.

A preliminary version of this work has already appeared.[9]

I. p-WAVE PAIRING IN CUBIC AND HEXAGONAL SYMMETRY

We start by reviewing the theory for p-wave pairing assuming strong spin-orbit coupling using the approach of Ueda and Rice.[5] (The case of d-wave pairing is completely analogous, as we discuss below.) The gap function $\hat{\Delta}(\mathbf{k})$ is a 2×2 matrix, which may be written in vector notation as

$$\hat{\Delta}(\mathbf{k})=i(\mathbf{d}(\mathbf{k})\cdot\boldsymbol{\sigma})\sigma_y \ , \tag{1}$$

where $\boldsymbol{\sigma}$ denotes the three Pauli spin matrices. In a system with spin-orbit coupling there are only the following symmetries: the proper rotation transformation $\hat{R}$, which is the point group of the lattice structure, the time reversal K, the inversion P, and the U(1) gauge transformation. $\mathbf{d}(\mathbf{k})$ possesses, as a three-dimensional (3D) vector, the following simple transformation properties:

$$\begin{aligned} \hat{R}d_l(\mathbf{k}) &= \sum_m [\hat{D}^{(1)}(R)]_{ml} d_m(R\mathbf{k}) \ , \\ K\mathbf{d}(\mathbf{k}) &= -\mathbf{d}^*(-\mathbf{k}) \ , \\ P\mathbf{d}(\mathbf{k}) &= -\mathbf{d}(\mathbf{k}) \ . \end{aligned} \tag{2}$$

$\hat{D}^{(1)}(R)$ is the three-dimensional representation of the point-group element R. A U(1) gauge transformation corresponds to the multiplication with a phase factor $e^{i\phi}$. The antisymmetry of the fermion wave function requires $\mathbf{d}(\mathbf{k})=-\mathbf{d}(-\mathbf{k})$. At the transition point T_c it is allowable to write the gap equation as a homogeneous, linearized eigenvalue integral equation,

$$\omega\mathbf{d}(\mathbf{k})=-\frac{N(0)}{2}\langle V(\mathbf{k},\mathbf{k}')\mathbf{d}(\mathbf{k}')\rangle_{\mathbf{k}'} \ . \tag{3}$$

$V(\mathbf{k},\mathbf{k}')$ denotes the attractive pair interaction, $N(0)$ the density of states, and $\langle \cdots \rangle_{\mathbf{k}'}$ the average over the Fermi surface. The eigenvalue ω is linked with the transition temperature by the equation

$$T_c=1.14\varepsilon_c e^{-1/\omega} \ , \tag{4}$$

where ε_c is the cutoff energy. Because of the consideration of the spin-orbit coupling, $\mathbf{d}(\mathbf{k})$ belongs to the product of the three-dimensional representations, which have

(k_x, k_y, k_z) and $\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$, respectively, as their basis,

$$\mathbf{d}(\mathbf{k}) = f(\mathbf{k}) \sum_{m,l} d_{lm} \hat{l} k_m \ , \tag{5}$$

$\hat{l}$ denotes a unit vector in the l direction, d_{lm} are the Clebsch-Gordan coefficients, and $f(\mathbf{k})$ is an invariant function under all the operations of the point group. For the cubic symmetry group O, $\mathbf{d}$ belongs to $\Gamma_4 \otimes \Gamma_4$, and for the hexagonal group D_6 to $(\Gamma_2' \oplus \Gamma_5') \otimes (\Gamma_2' \oplus \Gamma_5')$. We write down two possible choices for the basis functions for $\mathbf{d}$ in cubic symmetry and also one choice in hexagonal symmetry (Table I). We distinguish bases I and II for the cubic structure, because I will be convenient for the tetragonal symmetry change; on the other hand, II will be better for the rhombohedral one. Because for all three cases the system of these basis functions is complete, we can write in every p-wave state the vector $\mathbf{d}$ as a linear combination,

TABLE I. The basis functions for the different irreducible representations. (a) Cubic basis functions I; (b) cubic basis functions II; (c) hexagonal basis functions.

	(a) $\Gamma_4 \otimes \Gamma_4 = \Gamma_1 \oplus \Gamma_3 \oplus \Gamma_4 \oplus \Gamma_5$
Γ_1:	$\mathbf{d}(\Gamma_1) = \frac{1}{\sqrt{3}}(\hat{\mathbf{x}}k_x + \hat{\mathbf{y}}k_y + \hat{\mathbf{z}}k_z)$
Γ_3:	$\mathbf{d}(\Gamma_3, u) = \frac{1}{\sqrt{6}}(2\hat{\mathbf{z}}k_z - \hat{\mathbf{x}}k_x - \hat{\mathbf{y}}k_y)$
	$\mathbf{d}(\Gamma_3, v) = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}}k_x - \hat{\mathbf{y}}k_y)$
Γ_4:	$\mathbf{d}(\Gamma_4, x) = \frac{1}{\sqrt{2}}(\hat{\mathbf{y}}k_z - \hat{\mathbf{z}}k_y)$
	$\mathbf{d}(\Gamma_4, y) = \frac{1}{\sqrt{2}}(\hat{\mathbf{z}}k_x - \hat{\mathbf{x}}k_z)$
	$\mathbf{d}(\Gamma_4, z) = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}}k_y - \hat{\mathbf{y}}k_x)$
Γ_5:	$\mathbf{d}(\Gamma_5, \xi) = \frac{1}{\sqrt{2}}(\hat{\mathbf{y}}k_z + \hat{\mathbf{z}}k_y)$
	$\mathbf{d}(\Gamma_5, \eta) = \frac{1}{\sqrt{2}}(\hat{\mathbf{z}}k_x + \hat{\mathbf{x}}k_z)$
	$\mathbf{d}(\Gamma_5, \zeta) = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}}k_y + \hat{\mathbf{y}}k_x)$
	(b)
Γ_1:	$\mathbf{d}(\Gamma_1) = \frac{1}{\sqrt{3}}(\hat{\mathbf{x}}k_x + \hat{\mathbf{y}}k_y + \hat{\mathbf{z}}k_z)$
Γ_3:	$\mathbf{d}(\Gamma_3, \alpha) = \frac{1}{\sqrt{3}}(\hat{\mathbf{z}}k_z + \omega\hat{\mathbf{x}}k_x + \omega^2\hat{\mathbf{y}}k_y)$
	$\mathbf{d}(\Gamma_3, \beta) = \frac{1}{\sqrt{3}}(\hat{\mathbf{z}}k_z + \omega^2\hat{\mathbf{x}}k_x + \omega\hat{\mathbf{y}}k_y)$
Γ_4:	$\mathbf{d}(\Gamma_4, 1) = \frac{1}{\sqrt{6}}[\hat{\mathbf{x}}(k_y - k_z) + \hat{\mathbf{y}}(k_z - k_x) + \hat{\mathbf{z}}(k_x - k_y)]$
	$\mathbf{d}(\Gamma_4, 2) = \frac{1}{\sqrt{6}}[\hat{\mathbf{x}}(\omega k_y - \omega^2 k_z) + \hat{\mathbf{y}}(k_z - \omega k_x) + \hat{\mathbf{z}}(\omega^2 k_x - k_y)]$
	$\mathbf{d}(\Gamma_4, 3) = \frac{1}{\sqrt{6}}[\hat{\mathbf{x}}(\omega^2 k_y - \omega k_z) + \hat{\mathbf{y}}(k_z - \omega^2 k_x) + \hat{\mathbf{z}}(\omega k_x - k_y)]$
Γ_5:	$\mathbf{d}(\Gamma_5, 1) = \frac{1}{\sqrt{6}}[\hat{\mathbf{x}}(k_y + k_z) + \hat{\mathbf{y}}(k_z + k_x) + \hat{\mathbf{z}}(k_x + k_y)]$
	$\mathbf{d}(\Gamma_5, 2) = \frac{1}{\sqrt{6}}[\hat{\mathbf{x}}(\omega k_y + \omega^2 k_z) + \hat{\mathbf{y}}(k_z + \omega k_x) + \hat{\mathbf{z}}(\omega^2 k_x + k_y)]$
	$\mathbf{d}(\Gamma_5, 3) = \frac{1}{\sqrt{6}}[\hat{\mathbf{x}}(\omega^2 k_y + \omega k_z) + \hat{\mathbf{y}}(k_z + \omega^2 k_x) + \hat{\mathbf{z}}(\omega k_x + k_y)]$
	(c) $(\Gamma_2' \oplus \Gamma_5') \otimes (\Gamma_2' \oplus \Gamma_5') = \Gamma_1' \oplus \Gamma_1' \oplus \Gamma_2' \oplus \Gamma_5' \oplus \Gamma_5' \oplus \Gamma_6'$
Γ_1':	$\mathbf{d}(\Gamma_1', a) = \hat{\mathbf{z}}k_z$
Γ_1':	$\mathbf{d}(\Gamma_1', b) = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}}k_x + \hat{\mathbf{y}}k_y)$
Γ_2':	$\mathbf{d}(\Gamma_2') = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}}k_y - \hat{\mathbf{y}}k_x)$
Γ_3':	$\mathbf{d}(\Gamma_5', u) = \hat{\mathbf{x}}k_z$
	$\mathbf{d}(\Gamma_5', v) = \hat{\mathbf{y}}k_z$
Γ_5':	$\mathbf{d}(\Gamma_5' 1) = \hat{\mathbf{z}}k_x$
	$\mathbf{d}(\Gamma_5', 2) = \hat{\mathbf{z}}k_y$
Γ_6':	$\mathbf{d}(\Gamma_6', 1) = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}}k_x - \hat{\mathbf{y}}k_y)$
	$\mathbf{d}(\Gamma_6', 2) = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}}k_y + \hat{\mathbf{y}}k_x)$

TABLE II. Invariant terms of fourth order. These terms are composed of all linear independent, invariant terms of fourth order in λ. The coefficients η_i of the individual invariant terms are not fixed in a strong-coupling theory. The terms of the two cubic bases I and II are linked by the same transformation as the corresponding basis functions. $(i,j,k)=(x,y,z)$, (ξ,η,ζ) and $(l,m)=(u,v)$, (α,β), (ξ,η).

Γ	$F(\lambda^4)$
	Cubic I
Γ_1	$\eta_1\lvert\lambda\rvert^4$
Γ_3	$\eta_1(\lvert\lambda_u\rvert^2+\lvert\lambda_v\rvert^2)+\eta_2(\lambda_u^*\lambda_v-\lambda_u\lambda_v^*)^2$
Γ_4,Γ_5	$\eta_1(\lvert\boldsymbol{\lambda}\rvert^2)^2+\eta_2\lvert\boldsymbol{\lambda}^2\rvert^2+\eta_3(\lvert\lambda_i\rvert^2\lvert\lambda_j\rvert^2+\lvert\lambda_i\rvert^2\lvert\lambda_k\rvert^2+\lvert\lambda_j\rvert^2\lvert\lambda_k\rvert^2)$
	Cubic II
Γ_1	$\eta_1\lvert\lambda\rvert^4$
Γ_3	$\eta_1(\lvert\lambda_\alpha\rvert^2+\lvert\lambda_\beta\rvert^2)^2+\eta_2(\lvert\lambda_\alpha\rvert^2-\lvert\lambda_\beta\rvert^2)^2$
Γ_4,Γ_5	$\eta_1(\lvert\boldsymbol{\lambda}\rvert^2)^2+\eta_2\lvert\lambda_1^2+2\lambda_2\lambda_3\rvert^2+\frac{\eta_3}{3}(\lvert\lambda_1^2-\lambda_2\lambda_3\rvert^2+\lvert\lambda_2^2-\lambda_1\lambda_3\rvert^2+\lvert\lambda_3^2-\lambda_1\lambda_2\rvert^2)$
	Hexagonal
Γ_1',Γ_2'	$\eta_1\lvert\lambda\rvert^4$
Γ_5',Γ_6'	$\eta_1(\lvert\lambda_l\rvert^2+\lvert\lambda_m\rvert^2)^2+\eta_2(\lambda_l^*\lambda_m-\lambda_l\lambda_m^*)^2$

$$\mathbf{d}(\mathbf{k})=\sum_{\Gamma,m}\lambda(\Gamma,m)\mathbf{d}(\Gamma,m;\mathbf{k})\ , \tag{6}$$

where Γ means the irreducible representation and m the basis function. Further, following Monien *et al.*,[10] the pair interaction in Eq. (3) may be expanded in the spectral form

$$V(\mathbf{k},\mathbf{k}')=-\sum_{\Gamma}V(\Gamma)\sum_{m}\mathbf{d}(\Gamma,m;\mathbf{k})\mathbf{d}^{\dagger}(\Gamma,m;\mathbf{k}')\ , \tag{7}$$

and we assume that one $V(\Gamma)$ is essentially larger than all the others $[V(\Gamma)>0]$. It follows that only one representation Γ is predominant and, as pointed out by Monien *et al.*,[10] small admixtures of other representations can appear for certain states. This latter property does not affect our results. So we restrict the GL expansion to the predominant representation Γ. We regard the λ's in Eq. (6) as order parameters and we can therefore write the GL expansion,

$$F=F_s-F_N=\tfrac{2}{3}N(0)\left[A(T)\sum_{m}\lvert\lambda(\Gamma,m)\rvert^2+f(\lambda^4)\right]\ , \tag{8}$$

where $A(T)=\ln(T/T_c)$ and $f(\lambda^4)$ denotes the terms of fourth order in the order parameters. These are listed in Table II for all possible irreducible representations in these two symmetries. The coefficients η_i of the invariant terms have to be regarded as undetermined, independent parameters. The isotropy in the second-order term is due to the degeneracy of the basis functions of the same representation, i.e., their transition temperature is equal. This is also obvious from Eq. (7). The fourth-order terms describe the anisotropic features of the phase transition.

Detailed investigations about the possible stable superconductivity states for different choices of the η_i parameters can be found in the papers of Volovik and Gor'kov,[4] Ueda and Rice,[5] Blount,[6] and Monien *et al.*[10]

II. CHANGE OF SYMMETRY BY UNIAXIAL STRESS

Uniaxial stress applied on a crystal destroys the original symmetry and leads to a new lower symmetry. In this way, a cubic system, which is stressed uniaxially in the (0,0,1) and (1,1,1) directions, respectively, is changed to a tetragonal and rhombohedral structure, respectively. A hexagonal system with stress in the (1,0,0) direction becomes orthorhombic. Here will investigate these three examples of a change of structure.

We can include the lattice distortion in Eq. (3) by averaging over a corresponding deformed Fermi surface (e.g., an ellipsoid). The results of this perturbation are changes in the transition temperatures and new eigenfunctions $\mathbf{d}'$, which now belong to the new symmetry. We avoid here an explicit calculation with Eq. (3) and instead show the outcome qualitatively using group-theoretical methods. For example, a tetragonal change of the symmetry splits the 2D cubic representation Γ_3 into two different 1D representations of the tetragonal symmetry with their corresponding basis function. (In Appendix A the irreducible representations are catalogued for all needed symmetries.) Similarly, Γ_4 and Γ_5 each change over into one 1D and one 2D irreducible representation. The new eigenfunctions $\mathbf{d}'(\mathbf{k})$ are basis functions of the new representations and are combinations of the original functions. Such a combination can only take place between original basis functions, which change into the same new representation [e.g., between $\mathbf{d}(\Gamma_1)$ and $\mathbf{d}(\Gamma_3,u)$ for a tetragonal change of symmetry]. We would like to point out that the mixing between different basis functions of the cubic representations, which takes place through the existence of certain fourth-order terms as discussed by Monien *et al.*,[10] is implicitly induced in our new basis functions, since this mixing can also only take place between basis functions belonging to the same new representation. The representations and their new basis function are listed in Table III for all three considered cases of uniaxial stress. It must be remarked that in the rhombohedral case the eigenfunctions of $\hat{\Gamma}_2$ and $\hat{\Gamma}_3$ remain degenerate because of time-reversal invariance. In all the other cases a splitting of the high-dimensional representation is followed by a splitting of the eigenvalues in Eq. (3) and therefore of the transition temperatures too.

Regarding group-theoretical formulation of the new GL free energy, we present the following. Ozaki[8] developed a very simple and straightforward method for the treatment of a change of symmetry in the GL expansion. The free energy for a certain representation Γ has the following form in the presence of external stress and a resulting strain ε,

$$F(\varepsilon,\lambda)=F(0,\lambda)+\sum_{\gamma}C(\gamma)\sum_{m}\mathcal{V}(\gamma,\lambda)_m\,\varepsilon(\gamma,m)+\sum_{\gamma}B(\gamma)\sum_{m}\varepsilon(\gamma,m)^2+\sum_{\gamma,m}p(\gamma,m)\varepsilon(\lambda,m)\ . \tag{9}$$

$F(0,\lambda)$ is the original free energy in Eq. (8). $\varepsilon(\gamma,m)$ is the strain parameter with the symmetry of the corresponding basis function m in the representation γ, where γ denotes an irreducible component of the tensor product $\Gamma\otimes\Gamma$ of the relevant representation Γ. $p(\gamma,m)$ is the stress parameter with the same properties as $\varepsilon(\gamma,m)$ (Appendix B). $\mathcal{V}(\gamma,\lambda)_m$ is an irreducible bilinear form of λ^* and λ of the relevant representation Γ in the product $\Gamma\otimes\Gamma$. But only real $V(\gamma,\lambda)_m$ may be included in order to preserve the time-reversal invariance of $F(\varepsilon,\lambda)$. $C(\gamma)$ and $B(\gamma)$ are real numbers. The third term describes Hooke's law of an elastic lattice. $F(0,\lambda)$ has the complete original symmetry (e.g., cubic or hexagonal), whereas $F(\varepsilon\neq 0,\lambda)$ has a lower symmetry (e.g., tetragonal, rhombohedral, or orthorhombic) in the presence of an anisotropic strain. In Appendix B the strain parameter $\varepsilon(\gamma,m)$ are tabulated for the different distortions.

In the case of a 1D representation the strain simply shifts T_c. We analyze the more interesting representations with anisotropic character, i.e., $\dim\Gamma>1$ ($\Gamma_3,\Gamma_4,\Gamma_5,\Gamma_5',\Gamma_6'$). The correction of F is restricted to the terms second order in λ according to the ansatz (9) and in many cases it will destroy the above-mentioned isotropy. We let the terms of fourth order be unchanged.

In Appendix B we show that the tetragonal deformation requires $\varepsilon(\Gamma_1)\neq 0$ and $\varepsilon(\Gamma_3,u)\neq 0$, whereas all other $\varepsilon(\Gamma,m)$ are zero. So we obtain the following second-order terms and "transition temperatures" in the different cubic representations: For Γ_3,

TABLE III. Splitting of the irreducible representation by uniaxial stress and their new basis functions. (a) Cubic system with tetragonal distortion, (b) cubic system with rhombohedral distortion, and (c) hexagonal system with orthorhombic distortion. *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, and *j* are some complex functions of the strain ε, different in every listed case.

(a) $\Gamma_{\text{cubic}}\rightarrow\Gamma_{\text{tetragonal}}$		$\mathbf{d}'(\Gamma,m)$	Basis functions
$\Gamma_1\rightarrow$	$\tilde{\Gamma}_1$	$\mathbf{d}'(\Gamma_1)$	$a(\hat{\mathbf{x}}k_x+\hat{\mathbf{y}}k_y)+b\hat{\mathbf{z}}k_z$
$\Gamma_3\rightarrow$	$\tilde{\Gamma}_1$	$\mathbf{d}'(\Gamma_3,u)$	
	$\tilde{\Gamma}_3$	$\mathbf{d}'(\Gamma_3,v)$	$\hat{\mathbf{x}}k_x-\hat{\mathbf{y}}k_y$
$\Gamma_4\rightarrow$	$\tilde{\Gamma}_2$	$\mathbf{d}'(\Gamma_4,z)$	$\hat{\mathbf{x}}k_y-\hat{\mathbf{y}}k_x$
	$\tilde{\Gamma}_5$	$\mathbf{d}'(\Gamma_4,x),\mathbf{d}'(\Gamma_4,y)$	$c\hat{\mathbf{y}}_z-d\hat{\mathbf{z}}k_y$
$\Gamma_5\rightarrow$	$\tilde{\Gamma}_5$	$\mathbf{d}'(\Gamma_5,\xi),\mathbf{d}'(\Gamma_5,\eta)$	$d\hat{\mathbf{z}}k_x-c\hat{\mathbf{x}}k_z$
	$\tilde{\Gamma}_4$	$\mathbf{d}'(\Gamma_5,\zeta)$	$\hat{\mathbf{x}}k_y+\hat{\mathbf{y}}k_x$

(b) $\Gamma_{\text{cubic}}\rightarrow\Gamma_{\text{rhombohedral}}$		$\mathbf{d}'(\Gamma,m)$	Basis functions
$\Gamma_1\rightarrow$	$\hat{\Gamma}_1$	$\mathbf{d}'(\Gamma_1)$	$\hat{\mathbf{x}}(ak_x+bk_y+ck_z)+\hat{\mathbf{y}}(ak_y+bk_z+ck_y)+\hat{\mathbf{z}}(ak_z+bk_y+ck_z)$
$\Gamma_3\rightarrow$	$\hat{\Gamma}_3$	$\mathbf{d}'(\Gamma_3,\alpha)$	$\hat{\mathbf{x}}\omega^2(dk_x+ek_y+fk_z)+\hat{\mathbf{y}}\omega(dk_y+ek_z+fk_y)+\hat{\mathbf{z}}(dk_z+ek_x+fk_y)$
	$\hat{\Gamma}_2$	$\mathbf{d}'(\Gamma_3,\beta)$	$\hat{\mathbf{x}}\omega(dk_x+e^*k_y+f^*k_z)+\hat{\mathbf{y}}\omega^2(dk_y+e^*k_z+f^*k_y)+\hat{\mathbf{z}}(dk_z+e^*k_x+f^*k_y)$
$\Gamma_4\rightarrow$	$\hat{\Gamma}_1$	$\mathbf{d}'(\Gamma_4,1)$	$\hat{\mathbf{x}}(k_y-k_z)+\hat{\mathbf{y}}(k_z-k_x)+\hat{\mathbf{z}}(k_x-k_y)$
	$\hat{\Gamma}_2$	$\mathbf{d}'(\Gamma_4,2)$	
	$\hat{\Gamma}_3$	$\mathbf{d}'(\Gamma_4,3)$	see above
$\Gamma_5\rightarrow$	$\hat{\Gamma}_1$	$\mathbf{d}(\Gamma_5,1)$	
	$\hat{\Gamma}_2$	$\mathbf{d}'(\Gamma_5,2)$	
	$\hat{\Gamma}_3$	$\mathbf{d}'(\Gamma_5,3)$	

(c) $\Gamma_{\text{hexagonal}}\rightarrow\Gamma_{\text{orthorhombic}}$		$\mathbf{d}'(\Gamma,m)$	Basis functions
$\Gamma_1'\rightarrow$	Γ_1''	$\mathbf{d}'(\Gamma_1',a)$	$a\hat{\mathbf{x}}k_x+b\hat{\mathbf{y}}k_y+c\hat{\mathbf{z}}k_z$
$\Gamma_1'\rightarrow$	Γ_1''	$\mathbf{d}'(\Gamma_1',b)$	
$\Gamma_2'\rightarrow$	Γ_2''	$\mathbf{d}'(\Gamma_2')$	$d\hat{\mathbf{x}}k_y+e\hat{\mathbf{y}}k_x$
$\Gamma_5'\rightarrow$	Γ_3''	$\mathbf{d}'(\Gamma_5',u)$	$f\hat{\mathbf{x}}k_z+g\hat{\mathbf{z}}k_x$
	Γ_4''	$\mathbf{d}'(\Gamma_5',v)$	$h\hat{\mathbf{y}}k_z+j\hat{\mathbf{z}}k_y$
$\Gamma_5'\rightarrow$	Γ_3''	$\mathbf{d}'(\Gamma_5',\alpha)$	
	Γ_4''	$\mathbf{d}'(\Gamma_5',\beta)$	see above
$\Gamma_6'\rightarrow$	Γ_1''	$\mathbf{d}'(\Gamma_6',\xi)$	
	Γ_2''	$\mathbf{d}'(\Gamma_6',\eta)$	

$$[A(T)+C(\Gamma_1)\varepsilon(\Gamma_1)+c(\Gamma_3)\varepsilon(\Gamma_3,u)]\,|\lambda_u|^2 +[A(T)+C(\Gamma_1)\varepsilon(\Gamma_1)-C(\Gamma_3)\varepsilon(\Gamma_3,u)]\,|\lambda_v|^2=A_u(T)\,|\lambda_u|^2+A_v(T)\,|\lambda_v|^2\;,$$

with

$$A_i(T)=\ln(T/T_i)$$

and

$$T_u\approx T_c(1-C(\Gamma_1)\varepsilon(\Gamma_1)-C(\Gamma_3)\varepsilon(\Gamma_3,u))\;,$$

$$T_v\approx T_c(1-C(\Gamma_1)\varepsilon(\Gamma_1)+C(\Gamma_3)\varepsilon(\Gamma_3,u))\;;$$

for Γ_4,

$$[A(T)+C(\Gamma_1)\varepsilon(\Gamma_1)+2C(\Gamma_3)\varepsilon(\Gamma_3,u)]\,|\lambda_z|^2 +[A(T)+C(\Gamma_1)\varepsilon(\Gamma_1)-C(\Gamma_3)\varepsilon(\Gamma_3,u)](\,|\lambda_x|^2+|\lambda_y|^2)=A_z(T)\,|\lambda_z|^2+A_x(T)(\,|\lambda_x|^2+|\lambda_y|^2)\;,$$

with

$$T_z\approx T_c(1-C(\Gamma_1)\varepsilon(\Gamma_1)-2C(\Gamma_3)\varepsilon(\Gamma_3,u))\;,$$

$$T_x=T_y\approx T_c(1-C(\Gamma_1)\varepsilon(\Gamma_1)+c(\Gamma_3)\varepsilon(\Gamma_3,u))\;;$$

The case for Γ_5 is analogous to Γ_4.

A rhombohedral deformation requires $\varepsilon(\Gamma_1)\neq 0$ and $\varepsilon(\Gamma_5,\xi)=\varepsilon(\Gamma_5,\eta)=\varepsilon(\Gamma_5,\xi)\neq 0$, whereas all other $\varepsilon(\Gamma,m)$ are zero. Here we use basis II (Table I) in order to obtain a diagonal bilinear form for the second-order term in λ. For Γ_3 the second-order part remains isotropic because here $F(\varepsilon,\lambda)$ contains no expressions with $\varepsilon(\Gamma_5,m)$. This confirms our earlier statement about the degeneracy of $\mathbf{d}(\Gamma_3,\alpha)$ and $\mathbf{d}(\Gamma_3,\beta)$. For Γ_4,

$$[A(T)+C(\Gamma_1)\varepsilon(\Gamma_1)+2C(\Gamma_5)\varepsilon(\Gamma_5,\xi)]\,|\lambda_1|^2+[A(T)+C(\Gamma_1)\varepsilon(\Gamma_1) -C(\Gamma_5)\varepsilon(\Gamma_5,\xi)](\,|\lambda_2|^2+|\lambda_3|^2)=A_1(T)\,|\lambda_1|^2+A_2(T)(\,|\lambda_2|^2+|\lambda_3|^2)\;,$$

with

$$T_1\approx T_c(1-C(\Gamma_1)\varepsilon(\Gamma_1)-2C(\Gamma_5)\varepsilon(\Gamma_5,\xi))\;,$$

$$T_2=T_3\approx T_c(1-C(\Gamma_1)\varepsilon(\Gamma_1)+C(\Gamma_5)(T_5,\xi))\;;$$

the case for Γ_5 is analogous to Γ_4.

Also, the degeneracy of $\mathbf{d}'(\Gamma_4,2)$ and $\mathbf{d}'(\Gamma_4,3)$ and $\mathbf{d}'(\Gamma_5,2)$ and $\mathbf{d}'(\Gamma_5,3)$, respectively, becomes obvious.

Turning to the hexagonal case, $\varepsilon(\Gamma_1')\neq 0$ and $\varepsilon(\Gamma_6',\xi)\neq 0$ describe the orthorhomic deformation. This case for Γ_5' and Γ_6' is completely analogous to the case of the cubic representation, Γ_3; for Γ_5' and Γ_6',

$$[A(T)+C(\Gamma_1')\varepsilon(\Gamma_1')+C(\Gamma_6')\varepsilon(\Gamma_6',\xi)]\,|\lambda_u|^2+[A(T)+C(\Gamma_1')\varepsilon(\Gamma_1') -C(\Gamma_6')\varepsilon(\Gamma_6',\xi)]\,|\lambda_v|^2=A_u(T)\,|\lambda_u|^2+A_v(T)\,|\lambda_v|^2\;,$$

with

$$T_u\approx T_c(1-C(\Gamma_1')\epsilon(\Gamma_1')-C(\Gamma_6')\epsilon(\Gamma_6',\xi))$$

$$T_v\approx T_c(1-C(\Gamma_1')\epsilon(\Gamma_1')+C(\Gamma_6')\epsilon(\Gamma_6',\xi))\;.$$

The occurrence of the anisotropy is an effect of the symmetry lowering.

III. DISCUSSION OF THE PHASE TRANSITIONS

The idea is now to discuss the GL free energy with this new symmetry in the second-order term and with its original fourth-order terms.

A. Tetragonal deformation of the cubic system

We begin with an explicit discussion of the case Γ_3 to illustrate the qualitatively new features. First we write for the order parameters $\lambda_j=|\lambda_j|e^{i\phi}$ $(j=u,v)$. The GL expansion takes the form $[\frac{2}{3}N(0)=1]$

$$F=A_u(T)\,|\lambda_u|^2+A_v(T)\,|\lambda_v|^2+\eta_1(\,|\lambda_u|^2+|\lambda_v|^2)^2 -4\eta_2\,|\lambda_u|^2\,|\lambda_v|^2\sin^2(\Delta\phi)\;. \tag{10}$$

It is obvious that $\eta_2>0$ leads to $\sin(\Delta\phi)=\pm 1$ and $\eta_2<0$ to $(\Delta\phi)=0$ $(\Delta\phi=\phi_u-\phi_v)$. If we assume that $T_u>T_v$, then F is minimized directly below T_u by the solution

$$|\lambda_u|^2=-\frac{A_u(T)}{2\eta_1},\quad |\lambda_v|=0\;. \tag{11}$$

The stability of this solution requires a positive-definite Jacobi matrix $(\partial^2F/\partial\,|\lambda_i|\,\partial\,|\lambda_j|\,)$,

$$B_{uu}=\frac{\partial^2 F}{\partial|\lambda_u|^2}=2A_u(T)+12\eta_1|\lambda_u|^2+4[\eta_1-2\eta_2\sin^2(\Delta\phi)]|\lambda_v|^2>0\ ,$$

$$B_{vv}=\frac{\partial^2 F}{\partial|\lambda_v|^2}=2A_v(T)+4[\eta_1-2\eta_2\sin^2(\Delta\phi)]|\lambda_u|^2+12\eta_1|\lambda_v|^2>0\ ,$$

$$B_{uv}=\frac{\partial^2 F}{\partial|\lambda_u|\partial|\lambda_v|}=8[\eta_1-2\eta_2\sin^2(\Delta\phi)]|\lambda_u|\,|\lambda_v|\ ,$$

and

$$B_{uu}B_{vv}-B_{uv}^2>0\ .$$

$B_{uu}>0$ is satisfied by the solution (11) in the range $0<T<T_u$. For B_{vv} we get, with $\eta_2<0$,

$$B_{vv}=2[A_v(T)-A_u(T)]>0 \text{ for } 0<T<T_u\ , \tag{12}$$

and $B_{uv}=0$. In this case the solution (11) remains stable below T_u. On the other hand, if $\eta_2>0$ $(\eta_1-\eta_2>0)$,

$$B_{vv}=2\left[A_v(T)-\frac{\eta_1-2\eta_2}{\eta_1}A_u(T)\right]. \tag{13}$$

There exists a temperature T_0 with $B_{vv}(T)\le 0$ for $T\le T_0$. Below T_0 another solution is stable:

$$|\lambda_{u,v}|^2=\frac{\pm\eta_1(A_v-A_u)-2\eta_2A_{v,u}}{8\eta_2(\eta_1-\eta_2)}\ , \tag{14}$$

where the + (−) sign belongs to u (v). There is a second-order transition for the λ's at T_0. T_0 is calculated by the equation $B_{vv}(T_0)=0$, leading to

$$T_0=T_u\left[\frac{T_v}{T_u}\right]^G<T_u\ , \tag{15}$$

with $G=\eta_1/2\eta_2>0$. In the next section we discuss the corresponding jumps in the specific heat. In Table IV the possible states are listed with the corresponding representations and degeneracies. The case $T_v>T_u$ is quite analogous.

The analysis of the 3D representations Γ_4 and Γ_5 is somewhat more complicated, because there are two anisotropic fourth-order terms;

$$\begin{aligned}F=&A_x(T)(|\lambda_x|^2+|\lambda_y|^2)+A_z(T)|\lambda_z|^2+\eta_1(|\lambda_x|^2+|\lambda_y|^2+|\lambda_z|^2)^2\\&+\eta_2[|\lambda_x|^4+|\lambda_y|^4+|\lambda_z|^4+2|\lambda_x|^2|\lambda_y|^2\cos(\phi_y-\phi_x)\\&+2|\lambda_x|^2|\lambda_z|^2\cos(\phi_z-\phi_x)+2|\lambda_y|^2|\lambda_z|^2\cos(\phi_y-\phi_z)]\\&+\eta_3(|\lambda_x|^2|\lambda_y|^2+|\lambda_x|^2|\lambda_z|^2+|\lambda_y|^2|\lambda_z|^2)\end{aligned} \tag{16}$$

is the GL free energy using $\lambda_j=|\lambda_j|\exp(i\phi_j/2)$. The η_3 term determines the anisotropy in the magnitude of the order parameter, whereas the η_2 term determines their phase factors. The degeneracy of $\mathbf{d}'(\Gamma_4,x)$ and $\mathbf{d}'(\Gamma_4,y)$ gives rise to two distinct cases, $T_x=T_y\gtrless T_z$.

Assuming $T_x>T_z$ and minimizing F, we can obtain first

$$|\lambda_x|^2=\frac{-A_x(T)}{2(\eta_1+\eta_2)},\quad \lambda_y=\lambda_z=0 \tag{17}$$

for $4\eta_2<\eta_3$, $\eta_3>0$. The Jacobi matrix B_{ij} is positive definite in $0<T<T_x$; thus solution (17) is stable. In the case $4\eta_2>\eta_3>0$ the solution has the following form and also remains stable until $T=0$,

$$|\lambda_x|^2=|\lambda_y|^2=\frac{-A_x(T)}{4\eta_1+\eta_3},\quad \lambda_z=0\ ,\qquad \phi_x-\phi_y=\pi\ . \tag{18}$$

A further transition appears in the range $\eta_3<0$. For $\eta_2<0$ the first solution (19) becomes unstable at T_0:

$$|\lambda_x|^2=|\lambda_y|^2=\frac{-A_x(T)}{4(\eta_1+\eta_2)+\eta_3},\quad \lambda_z=0\ ,\qquad \phi_x=\phi_y(\mathrm{mod}2\pi)=\phi_z(\mathrm{mod}2\pi)\ . \tag{19}$$

Below T_0, use of Cramer's rule leads to a stable solution,

TABLE IV. Phase table for the 2D representation Γ_3 with $T_u>T_v$. For $\eta_2>0$ two different phases exist, while for $\eta_2<0$ there is only one. In the former case the second transition takes place at the temperature $T_0=T_u(T_v/T_u)^G$ with $G=\eta_1/2\eta_2$. The first phase belongs to an irreducible representation of tetragonal symmetry.

η_2	T	λ	$\mathbf{d}(\mathbf{k})$	Degeneracy	Representation
$\eta_2<0$	$0<T<T_u$	$(\lambda_u,0)$	$a\hat{\mathbf{z}}k_z+b(\hat{\mathbf{x}}k_x+\hat{\mathbf{y}}k_y)$	1	$\tilde{\Gamma}_1$
$\eta_2>0$	$T_0<T<T_u$	$(\lambda_u,0)$	$a\hat{\mathbf{z}}k_z+b(\hat{\mathbf{x}}k_x+\hat{\mathbf{y}}k_y)$	1	$\tilde{\Gamma}_1$
	$0<T<T_0$	$(\lvert\lambda_i\rvert,i\lvert\lambda_v\rvert)$	$s(T)\hat{\mathbf{z}}k_z+r(T)\hat{\mathbf{x}}k_x+r^*(T)\hat{\mathbf{y}}k_y$	2	$\tilde{\Gamma}_1\oplus\tilde{\Gamma}_3$

$$|\lambda_x|^2=|\lambda_y|^2=\frac{2(\eta_1+\eta_2)(A_x-A_z)-\eta_3 A_z}{\eta_3(3(\eta_1+\eta_2)+\eta_3)},$$
$$|\lambda_z|^2=\frac{4(\eta_1+\eta_2)(A_z-A_x)+\eta_3(A_z-2A_x)}{2\eta_3(3(\eta_1+\eta_2)+\eta_3)}, \quad (20)$$
$$\phi_x=\phi_y(\mathrm{mod}2\pi)=\phi_z(\mathrm{mod}2\pi).$$

T_0 can be calculated using the fact $B_{zz}(T_0)=0$,

$$T_0=T_x\left[\frac{T_x}{T_z}\right]^G<T_x, \quad (21)$$

with $G=[4(\eta_1+\eta_2)+\eta_3]/\eta_3$.

The first solution for $\eta_2>0$ has the same form as Eq. (18). In the second superconducting phase we obtain order parameters with temperature-dependent phase factors,

$$|\lambda_x|^2=|\lambda_y|^2=\frac{2\eta_1(A_z-A_x)+2\eta_2(A_z\cos\phi-A_x)+\eta_3 A_z}{2[4\eta_1\eta_2(1-\cos\phi)^2-\eta_3(3\eta_1-\eta_2(1-4\cos\phi)+\eta_3)]},$$
$$|\lambda_z|^2=\frac{4\eta_1(A_x-A_z)+4\eta_2\cos\phi(A_x-A_z\cos\phi)+\eta_3(2A_x-A_z)}{2[4\eta_1\eta_2(1-\cos\phi)^2-\eta_3(3\eta_1-\eta_2(1-4\cos\phi)+\eta_3)]}, \quad (22)$$
$$\cos\phi=-\frac{|\lambda_z|^2}{|\lambda_x|^2+|\lambda_y|^2}=\frac{4\eta_1(A_z-A_x)+\eta_3(A_z-2A_x)}{4\eta_1(A_z-A_x)+2\eta_3 A_z}\geq -1,$$

where we use $\phi=\phi_x=-\phi_y$ by fixing the U(1) gauge as $\phi_z=0$. T_0 is obtained from Eq. (21) using $G=(4\eta_1+\eta_3)/\eta_3$. The spontaneous symmetry lowering at the first transition leads to an orthorhombic symmetry in every case and, at the second, to a monoclinic symmetry, if time reversal is dropped.

The case $T_z>T_x$ has a similar analysis. The first phase is always described by the nondegenerate state $\mathbf{d}'(\Gamma_4,z)$,

$$|\lambda_z|^2=\frac{-A_z(T)}{2(\eta_1+\eta_2)}, \quad \lambda_x=\lambda_y=0, \quad (23)$$

which has the full tetragonal symmetry. This state remains only stable until $T=0$ in the range $4\eta_2<\eta_3, \eta_3>0$. Otherwise we observe further transitions. The second state in $\eta_2<0, \eta_3<0$ has the form (20). For $4\eta_2>\eta_3>0$ we obtain

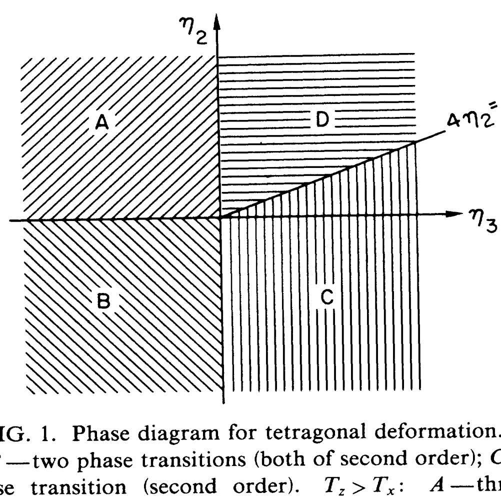

FIG. 1. Phase diagram for tetragonal deformation. $T_x>T_z$: A,B—two phase transitions (both of second order); C,D—one phase transition (second order). $T_z>T_x$: A—three phase transitions (all of second order); B,C—two phase transitions (both of second order); D—one phase transition (second order). The phase diagram shows the behavior with different values of the η_2 and η_3 parameters in the GL expansion.

$$|\lambda_x|^2=\frac{A_z(\eta_1-\eta_2)-A_x(\eta_1+\eta_2)}{4\eta_1\eta_2-\eta_3(\eta_1-\eta_2)}, \quad \lambda_y=0,$$
$$|\lambda_z|^2=\frac{A_x(2(\eta_1-\eta_2)+\eta_3)-A_z 2(\eta_1+\eta_2)}{2[4\eta_1\eta_2-\eta_3(\eta_1-\eta_2)]}, \quad (24)$$
$$\phi_z-\phi_x=\pi,$$

with an orthorhombic symmetry. In the range $\eta_2>0$, $\eta_3<0$ we find even three transitions. The third one is due to the complex nature of the order parameters, which begin to rotate in the complex plane after being fixed in the first two phases. This fact can be seen in solution (22), where in the second (orthorhombic) phase ϕ is constant ($\phi=\pi$), because the equation

$$\cos\phi=-|\lambda_z|^2/(|\lambda_x|^2+|\lambda_y|^2)<-1$$

has no solution. The third (monoclinic) phase has its onset when $|\lambda_z|^2/2|\lambda_x|^2=1$. In the phase diagram in Fig. 1 we show the ranges where further phase transitions take place as function of η_2 and η_3. Note that further transitions appear in the cases where the solution in the undeformed materials is not a single basis function. An important fact is that the phase transition from the normal to the superconducting state always leads to a gap function which belongs to only one irreducible tetragonal representation. In further transitions the states break out of a single irreducible representation and form combinations of more than one representation (e.g., $\tilde{\Gamma}_2\oplus\tilde{\Gamma}_5$). Further, all transitions in the tetragonal deformed crystal are second-order transitions. Table V gives information about the symmetry properties of the states in every phase.

TABLE V. Phase table for the 3D representation Γ_4 under tetragonal deformation. (a) $T_x > T_z$ and (b) $T_z > T_x$. The symbols A, B, C and D are defined in the phase diagram (Fig. 1). Note that in region A the phase factors $\phi(T)$ and $\phi'(T)$ depend on temperature in the lowest phase; this fact produces in (b) the threefold splitting of the phase transition. r, s, r', s', t, q, t', q', m, n, p, and u are functions of the temperature.

(a)

	T	$\mathbf{d}(\mathbf{k})$	Degeneracy	Representation	$T_0 = T_x(T_x/T_z)^G$
A	$T_0 < T < T_x$	$e^{i(\pi/4)}\mathbf{d}'(\Gamma_4,x)+e^{-i(\pi/4)}\mathbf{d}'(\Gamma_4,y)$	4	$\tilde{\Gamma}_5$	$G=\frac{4\eta_1+\eta_3}{\eta_3}$
	$0 < T < T_0$	$r(T)\mathbf{d}'(\Gamma_4,z)+s(T)[e^{i\varphi(T)}\mathbf{d}'(\Gamma_4,x)+e^{-i\varphi(T)}\mathbf{d}'(\Gamma_4,y)]$	8	$\tilde{\Gamma}_2\oplus\tilde{\Gamma}_5$	
B	$T_0 < T < T_x$	$\mathbf{d}'(\Gamma_x,x)+\mathbf{d}'(\Gamma_4,y)$	2	$\tilde{\Gamma}_5$	$G=\frac{4(\eta_1+\eta_2)}{\eta_3}$
	$0 < T < T_0$	$r'(T)\mathbf{d}'(\Gamma_4,z)+s'(T)[\mathbf{d}'(\Gamma_4,x)+\mathbf{d}'(\Gamma_4,y)]$	4	$\tilde{\Gamma}_2\oplus\tilde{\Gamma}_5$	
C	$0 < T < T_x$	$\mathbf{d}'(\Gamma_4,x)=c\hat{\mathbf{y}}k_z-d\hat{\mathbf{z}}k_y$	2	$\tilde{\Gamma}_5$	
D	$0 < T < T_x$	$\mathbf{d}'(\Gamma_4,x)+i\mathbf{d}'(\Gamma_4,y)$	4	$\tilde{\Gamma}_5$	

(b)

	T	$\mathbf{d}(\mathbf{k})$	Degeneracy	Representation	$T_0 = T_z(T_z/T_x)^G$
A	$T_0 < T < T_z$	$\mathbf{d}'(\Gamma_4,z)\sim\hat{\mathbf{x}}k_y-\hat{\mathbf{y}}k_x$	1		$G=\frac{2(\eta_1+\eta_2)}{\eta_3-4\eta_2}$
	$T_0' < T < T_0$	$t(T)\mathbf{d}'(\Gamma_4,z)+g(T)[e^{i(\varphi/2)}\mathbf{d}'(\Gamma_4,x)+e^{-i(\varphi/2)}\mathbf{d}'(\Gamma_4,y)]$	4		$T_1=T_x^{2(c-1)}/T_z^{2c-3}$
	$0 < T < T_0'$	$t'(T)\mathbf{d}'(\Gamma_4,z)+q'(T)[e^{iy'(T)}\mathbf{d}'(\Gamma_4,x)+e^{-iy'(T)}\mathbf{d}'(\Gamma_4,y)]$	8		$c=-4\eta_1/\eta_3$
B	$T_0 < T < T_z$	$\mathbf{d}'(\Gamma_4,z)\sim\hat{\mathbf{x}}k_g-\hat{\mathbf{y}}k_x$	1		$G=\frac{2(\eta_1+\eta_2)}{\eta_3}$
	$0 < T < T_0$	$m(T)\mathbf{d}'(\Gamma_4,z)+n(T)[\mathbf{d}'(\Gamma_4,x)+\mathbf{d}'(\Gamma_4,y)]$	4		
C	$0 < T < T_z$	$\mathbf{d}'(\Gamma_4,z)\sim\hat{\mathbf{x}}k_y-\hat{\mathbf{y}}k_x$	1		
D	$T_0 < T < T_z$	$\mathbf{d}'(\Gamma_4,z)\sim\hat{\mathbf{x}}k_y-\hat{\mathbf{y}}k_x$	1		$G=\frac{2(\eta_1+\eta_2)}{\eta_3-4\eta_2}$
	$0 < T < T_0$	$p(T)\mathbf{d}'(\Gamma_4,z)+iu(T)\mathbf{d}'(\Gamma_4,x)$	4		

B. Rhombohedral deformation of the cubic system

In a rhombohedral deformation only the two 3D representations must be considered, because, as mentioned above, Γ_3 splits into $\hat{\Gamma}_2$ and $\hat{\Gamma}_3$, but these new basis states (II) remain degenerate. Therefore in the case of Γ_3 no splitting appears in the phase transition under rhombohedral deformation.

On the other hand, Γ_4 and Γ_5 have anisotropic second-order terms,

$$F=A_1(T)|\lambda_1|^2+A_2(T)(|\lambda_2|^2+|\lambda_3|^2)+\eta_1(|\lambda_1|^2+|\lambda_2|^2+|\lambda_3|^2)^2$$
$$+\eta_2|\lambda_1^2+2\lambda_2\lambda_3|^2+\frac{\eta_3}{3}(|\lambda_1^2-\lambda_2\lambda_3|^2+|\lambda_2^2-\lambda_1\lambda_3|^2+|\lambda_3^2-\lambda_1\lambda_2|^2) . \quad (25)$$

In this case the fourth-order terms do not distinguish—in a simple way—between the anisotropy of the magnitude of the order parameters and their phase factors. They contain some linear components of single order parameters [e.g., in expressions like $(\lambda_1^*)^2, \lambda_2\lambda_3+\text{c.c.}$]. In some cases such terms suppress the solution which is favored by the anisotropic second-order terms, and cause all order parameters to be finite immediately below the normal transition.[10] Another important possibility is a first-order transition produced by such terms linked with "quasi-third-order" components like $\lambda_1^*\lambda_2^*\lambda_3^2+\text{c.c.}$, if λ_2 and λ_3 are approximately equal. But it must be remarked that such a first-order transition cannot take place at the onset of the superconducting phase.

Unfortunately, the minimization of the GL free energy F leads to analytically complicated equations when we assume $\lambda_j\neq 0$ for all j, and this makes numerical calculations necessary.

First, we investigate the case $T_1 > T_2 = T_3$. In this case we have only one transition in the range B [Fig. 2(a)],

$$|\lambda_1|^2=\frac{-A_1(T)}{2(\eta_1+\eta_2+\eta_3/3)},\quad \lambda_2=\lambda_3=0 . \quad (26)$$

Indeed, this solution describes the first superconducting phase in the whole phase diagram, when $T_1 > T_2 = T_3$. One further second-order transition is found in A and D.

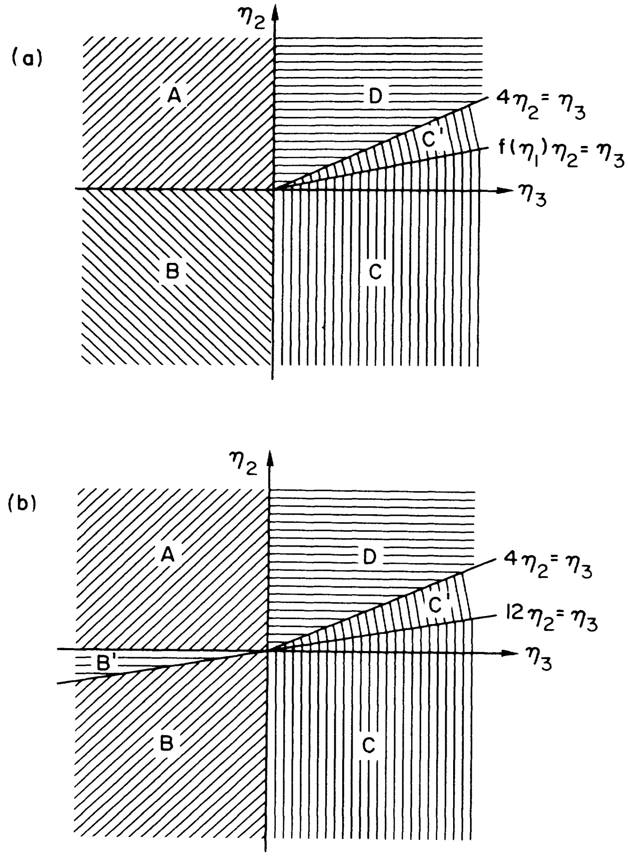

FIG. 2. Phase diagrams for rhombohedral deformation. (a) $T_1 > T_2$: C'—three phase transitions (the first two of second order, the third of first order); A,D—two phase transitions (both of second order); C—two phase transitions (the first of second order, the second of first order); B—one phase transition (second order). (b) $T_2 > T_1$: D—two phase transitions (both of second order); B',C'—two phase transitions (the first of second order, the second of first order); A,B,C—one phase

It is possible to calculate the transition point T_0 analytically using again the criterion for the stability of the solution (26) leading to

$$T_0 = T_1 \left[\frac{T_2}{T_1} \right]^G , \tag{27}$$

with $G = (\eta_1 + \eta_2 + \eta_3/3)/2\eta_2$. As $T_0 - T$ increases, the order parameters $\boldsymbol{\lambda}$ tends towards the value $\sim(1,1,-\frac{1}{2})$ in area A and towards $\sim(1,a,-b)$ in area D [$a = (\sqrt{3}-1)/2$, $b = (\sqrt{3}+1)/2$], which are both sixfold degenerate. Note in a single domain of either of these solutions the lattice symmetry is monoclinic. In region C there is a variable boundary depending on η_1 determined by the condition $f(\eta_1)\eta_2 = \eta_3$, where the limiting values of $f(\eta_1)$ obtained by numerical calculation are $f(\eta_1 = 0) = 5$ and $f(\eta_1 \to \infty) = 5\frac{1}{3}$. In this range we find an additional first-order transition. The small area C' has even three transitions. The first two are continuous, while the third is a first-order transition with the remarkable property that it lowers the degree of the degeneracy from sixfold to threefold (both monoclinic). The second transition has its onset at T_0 from Eq. (27). In both C and C' the lowest phase behaves as $\boldsymbol{\lambda} \to \sim(1,1,1)$ for $T \to 0$.

Turning to the case $T_2 = T_3 > T_1$, the degeneracy of the $\hat{\Gamma}_2$ and $\hat{\Gamma}_3$ states leads to a boundary ($12\eta_2 = \eta_3$) in the phase diagram [Fig. 2(b)], which separates two types of solutions. For one type ($12\eta_2 > \eta_3$) the first superconducting phase belongs to $\hat{\Gamma}_2$ and $\hat{\Gamma}_3$; for the other type ($12\eta_2 < \eta_3$) it belongs to $\hat{\Gamma}_2 \oplus \hat{\Gamma}_3$ ($\oplus \hat{\Gamma}_1$).

In areas B and C ($12\eta_2 < \eta_3$) we observe only one (second-order) transition caused by the above-mentioned linear terms (i.e., linear in λ_1). As $T \to 0$, then $\boldsymbol{\lambda} \to \sim(-\frac{1}{2},1,1)$ in region B and $\boldsymbol{\lambda} \to \sim(1,1,1)$ in region C. In the other half-plane of the phase diagram we obtain, as a first solution,

$$|\lambda_2|^2 = \frac{-A_2(T)}{2(\eta_1 + \eta_3/3)}, \quad \lambda_1 = \lambda_3 = 0 , \tag{28}$$

with rhombohedral symmetry or the time-reversal degenerate solution with $\lambda_3 \neq 0$, $\lambda_1 = \lambda_2 = 0$. The single additional transition in B' and C' is a first-order transition, whereas in D it is a second-order one with a transition temperature

$$T_0 = T_2 \left[\frac{T_1}{T_2} \right]^G , \tag{29}$$

with

$$G = (12\eta_2 - \eta_3)(\eta_1 + \eta_3/3)/2\eta_2\eta_3 .$$

The range A has only the single superconducting phase (28). The low-temperature limits in regions A and D are $\boldsymbol{\lambda} \to \sim(0,1,0)$ and $\boldsymbol{\lambda} \to \sim(1,a,-b)$, respectively.

The first-order transition points can be calculated only numerically by comparing different local minima of F. In Table VI the different states for Γ_4 are listed. Note that Γ_5 is quite similar.

C. Orthorhombic deformation in the hexagonal system

The internal degeneracy of the 2D representations Γ_5' and Γ_6' of the hexagonal symmetry is lifted under orthorhombic deformation. Since the fourth-order terms for this representations and the cubic Γ_3 are quite the same, their new GL expansion has also the form (10). So the discussion is similar to that in Sec. III. A. In Table VII we catalog the different phase transitions for Γ_5'. Note that Γ_6' behaves analogously.

IV. DISCUSSION OF THE PHASE TRANSITIONS

The entropy is related to the free energy by

$$S(T) = -\frac{\partial F}{\partial T} = -\frac{2}{3}N(0)\frac{1}{T}\sum_i |\lambda_i|^2 . \tag{30}$$

TABLE VI. Phase table for the 3D representation Γ_4 under rhombohedral deformation. (a) $T_1 > T_2$ and (b) $T_2 > T_1$. A, B, C, C', and D give the ranges in the phase diagram (Fig. 2). The stable states are combined from the perturbed basis states II. T_0 is always a second-order transition, whereas $\tilde{T}_0$ denotes a first-order transition. The need to resort to numerical analysis for the first-order transition prevents from obtaining an analytical term for $\tilde{T}_0$. a, b, c, d, a', b', e, f, and g are functions of the temperature.

(a)

	T	$\mathbf{d}'(\mathbf{k})$	Degeneracy	Representation	$T_0=T_1(T_2/T_1)^G$
A	$T_0<T<T_1$	$\mathbf{d}'(\Gamma_4,1)$	1	$\hat{\Gamma}_1$	$G=\frac{\eta_1+\eta_2+\eta_3/3}{2\eta_2}$
	$0<T<T_0$	$a(T)\mathbf{d}'(\Gamma_4,1)+b(T)\mathbf{d}'(\Gamma_4,2)-c(T)\mathbf{d}'(\Gamma_4,3)$	6	$\hat{\Gamma}_1\oplus\hat{\Gamma}_2\oplus\hat{\Gamma}_3$	
B	$0<T<T_1$	$\mathbf{d}'(\Gamma_4,1)$	1	$\hat{\Gamma}_1$	
C	$\tilde{T}_0<T_1$	$\mathbf{d}'(\Gamma_4,1)$	1	$\hat{\Gamma}_1$	
	$0<T<\tilde{T}_0$	$a'(T)\mathbf{d}'(\Gamma_4,1)+b'(T)\mathbf{d}'(\Gamma_4,2)+b'^*(T)\mathbf{d}'(\Gamma_4,3)$	3	$\hat{\Gamma}_1\oplus\hat{\Gamma}_2\oplus\hat{\Gamma}_3$	
C'	$T_0<T<T_1$	$\mathbf{d}'(\Gamma_4,1)$	1	$\hat{\Gamma}_1$	$G=\frac{\eta_1+\eta_2+\eta_3/3}{2\eta_2}$
	$\tilde{T}_0<T<T_0$	$d(T)\mathbf{d}'(\Gamma_4,1)+e(T)\mathbf{d}'(\Gamma_4,2)-f(T)\mathbf{d}'(\Gamma_4,3)$	6	$\hat{\Gamma}_1\oplus\hat{\Gamma}_2\oplus\hat{\Gamma}_3$	
	$0<T<\tilde{T}_0$	$a'(T)\mathbf{d}'(\Gamma_4,1)+b'(T)\mathbf{d}'(\Gamma_4,2)+b'^*(T)\mathbf{d}'(\Gamma_4,3)$	3	$\hat{\Gamma}_1\oplus\hat{\Gamma}_2\oplus\hat{\Gamma}_3$	
D	$T_0<T<T_1$	$\mathbf{d}'(\Gamma_4,1)$	1	$\hat{\Gamma}_1$	
	$0<T<T_0$	$d(T)\mathbf{d}'(\Gamma_4,1)+e(T)\mathbf{d}'(\Gamma_4,2)-f(T)\mathbf{d}'(\Gamma_4,3)$	6	$\hat{\Gamma}_1\oplus\hat{\Gamma}_2\oplus\hat{\Gamma}_3$	

(b)

	T	$\mathbf{d}(\mathbf{k})$	Degeneracy	Representation	$T_0=T_2(T_2/T_1)^G$
A	$0<T<T_2$	$\mathbf{d}'(\Gamma_4,2)$	2	$\hat{\Gamma}_2$ $(\hat{\Gamma}_3)$	
B	$0<T<T_2$	$a(T)\mathbf{d}'(\Gamma_4,1)-b(T)\mathbf{d}'(\Gamma_4,2)-b^*(T)\mathbf{d}'(\Gamma_4,3)$	3	$\hat{\Gamma}_1\oplus\hat{\Gamma}_2\oplus\hat{\Gamma}_3$	
B'	$\tilde{T}_0<T<T_2$	$\mathbf{d}'(\Gamma_4,2)$	2	$\hat{\Gamma}_2$ $(\hat{\Gamma}_3)$	
	$0<T<\tilde{T}_0$	$a(T)\mathbf{d}'(\Gamma_4,1)-b(T)\mathbf{d}'(\Gamma_4,2)-b^*(T)\mathbf{d}'(\Gamma_4,3)$	3	$\hat{\Gamma}_1\oplus\hat{\Gamma}_2\oplus\hat{\Gamma}_3$	
C	$0<T<T_2$	$c(T)\mathbf{d}'(\Gamma_4,1)+d(T)\mathbf{d}'(\Gamma_4,2)+d^*(T)\mathbf{d}'(\Gamma_4,3)$	3	$\hat{\Gamma}_1\oplus\hat{\Gamma}_2\oplus\hat{\Gamma}_3$	
C'	$\tilde{T}_0<T<T_2$	$\mathbf{d}'(\Gamma_4,2)$	2	$\hat{\Gamma}_2$ $(\hat{\Gamma}_3)$	
	$0<T<\tilde{T}_0$	$c(T)\mathbf{d}'(\Gamma_4,1)+d(T)\mathbf{d}'(\Gamma_4,2)+d^*(T)\mathbf{d}'(\Gamma_4,3)$	3	$\hat{\Gamma}_1\oplus\hat{\Gamma}_2\oplus\hat{\Gamma}_3$	
D	$T_0<T<T_2$	$\mathbf{d}'(\Gamma_4,2)$	2	$\hat{\Gamma}_2$ $(\hat{\Gamma}_3)$	$G=\frac{(\eta_3-12\eta_2)(\eta_1+\eta_3/3)}{2\eta_2\eta_3}$
	$0<T<T_0$	$e(T)\mathbf{d}'(\Gamma_4,1)+f(T)\mathbf{d}'(\Gamma_4,2)-g(T)\mathbf{d}'(\Gamma_4,3)$	6	$\hat{\Gamma}_1\oplus\hat{\Gamma}_2\oplus\hat{\Gamma}_3$	

When the λ_i are continuous at the transition point, we obtain a phase transition of second order. This is always true at the first transition from the normal to the superconducting state, but also for all further transitions which are purely tetragonal and orthorhombic deformations, as we discussed above. The specific heat then has a discontinuity at this point,

$$\Delta C_0=\left[\frac{\partial}{\partial T}(TS_{\rm I})-\frac{\partial}{\partial T}(TS_{\rm II})\right]_{T_0}, \tag{31}$$

with $S(T)=\begin{cases}S_{\rm I}(T) & \text{for } T<T_0,\\ S_{\rm II}(T) & \text{for } T>T_0\,.\end{cases}$

Further, we find that the sum of all discontinuities ΔC_i satisfies the following relation:

$$\Delta C=\lim_{\varepsilon\to 0}\sum_i \Delta C_i\ , \tag{32}$$

where ΔC is the jump in specific heat without deformation. We give here only the explicit results for the case of tetragonal deformation of Γ_3:

$$C_u=\frac{N(0)}{6\eta_1 T_u}=\frac{N(0)}{6\eta_1 T_c}[1+C(\Gamma_1)\varepsilon(\Gamma_1)+C(\Gamma_3)\varepsilon(\Gamma_3,u)]\ ,$$

$$\Delta C_0=\frac{N(0)}{6(\eta_1-\eta_2)T_0}-\Delta C_u(T_0)=\frac{N(0)}{6(\eta_1-\eta_2)T_c}\left[1+C(\Gamma_1)\varepsilon(\Gamma_1)-\left[1-\frac{\eta_1}{\eta_2}\right]C(\Gamma_3)\varepsilon(\Gamma_3,u)\right]-\Delta C_u(T_0)\ , \tag{33}$$

and

$$\Delta C=\frac{N(0)}{6(\eta_1-\eta_2)T_c}\ .$$

TABLE VII. Phase table for the 2D representation Γ_5' for $T_u > T_v$. The behavior is completely analogous to the case of Γ_3 in Table I, because the anisotropic terms of fourth order are the same.

η_2	T	$\mathbf{d}'(\mathbf{k})$	Degeneracy	Representation
$\eta_2 > 0$	$T_0 < T < T_u$	$\mathbf{d}'(\Gamma_5', u) = f\hat{\mathbf{x}}k_z + g\hat{\mathbf{z}}k_x$	1	Γ_3''
	$0 < T < T_0$	$r(T)\mathbf{d}'(\Gamma_5', u) + is(T)\mathbf{d}'(\Gamma_5, v)$	2	$\Gamma_3'' \oplus \Gamma_4''$
$\eta_2 < 0$	$0 < T < T_u$	$\mathbf{d}'(\Gamma_5', u) = f\hat{\mathbf{x}}k_z + g\hat{\mathbf{z}}k_x$	1	Γ_3''

The η_i parameters come in a simple form in Eqs. (20). Similar expressions can be easily calculated for the other representations.

Lastly, we note that in the case of extremely strong coupling of the order parameter to the lattice, or coupling to a very soft distortion, there is the possibility of a first-order transition. This would involve a discontinuous change in the λ_i, and latent heat. This phenomenon is well known in the context of magnetic systems, and details may be found in Ref. 11. The criterion for its occurrence in our case is (for the cubic system with $\eta_2 < 0$) $C^2(\gamma) > 4B(\gamma)\eta_1$, and $V(\gamma)$ is the largest of the V's.

V. CONCLUSION

The stress-produced symmetry lowering leads to anisotropic second-order terms in a Ginzburg-Landau theory for p- or d-wave superconductivity (as treated in Appendix B) in higher-dimensional irreducible representations. This fact leads to possible splittings of the phase transitions, which depend on the relation between the different fourth-order terms. A key for the possible behaviors of a cubic system is given in Table VIII. Possibly, one is able to identify the phase definitively and also to give an estimate for η_i with help of the jumps of the specific heat. But note that this measurement is not suitable to distinguish even versus odd parity (or, equivalently, p-wave versus d-wave) superconductivity since there is a close parallel between the group-theoretical classification of both systems. It is clear that such effects do not occur in s-wave superconductivity, since it possesses only a one-dimensional representation. Therefore we believe that this effect can be a good

TABLE VIII. Key showing the different behaviors of superconductivity in cubic materials under uniaxial stress. The first two columns give the number of transitions with their order for tetragonal and rhombohedral deformation. The other columns show the corresponding possible cases. Symbols: $=$, second and third phase transition of second order; $\simeq$, second phase transition of second order, third phase transition of first order; $-$, second phase transition of second order; $\sim$, second phase transition of first order.

Tetragonal	Rhombohedral	$T_x > T_z$	$T_x < T_z$	$T_1 > T_2$	$T_1 < T_2$	η_2, η_3	Γ
3 (=)	2 (−)		*	*		$\eta_2 > 0, \eta_3 < 0$	Γ_4, Γ_5
3 (=)	1		*		*	$\eta_2 > 0, \eta_3 < 0$	Γ_4, Γ_5
2 (−)	2 (−)	*		*		$\eta_2 > 0, \eta_3 < 0$	Γ_4, Γ_5
			*	*		$4\eta_2 > \eta_3 > 0$	Γ_4, Γ_5
			*		*	$4\eta_2 > \eta_3 > 0$	Γ_4, Γ_5
2 (−)	2 (∼)		*		*	$0 > 12\eta_2 > \eta_3$	Γ_4, Γ_5
2 (−)	1	*			*	$\eta_2 > 0, \eta_3 < 0$	Γ_4, Γ_5
		*		*		$\eta_2 < 0, \eta_3 < 0$	Γ_4, Γ_5
			*	*		$\eta_2 < 0, \eta_3 < 0$	Γ_4, Γ_5
		*			*	$0 > \eta_3 > 12\eta_2$	Γ_4, Γ_5
			*		*	$0 > \eta_3 > 12\eta_2$	Γ_4, Γ_5
			$T_u > T_v$,	$T_u < T_v$		$\eta_2 > 0$	Γ_3
1	3 $\simeq$	*		*		$4\eta_2 < \eta_3 < f(\eta_1)\eta_2$	Γ_4, Γ_5
			*	*		$4\eta_2 < \eta_3 < f(\eta_1)\eta_2$	Γ_4, Γ_5
1	2 (−)	*		*		$4\eta_2 > \eta_3 > 0$	Γ_4, Γ_5
		*			*	$4\eta_2 > \eta_3 > 0$	Γ_4, Γ_5
1	2 (−)	*		*		$f(\eta_1)\eta_2 < \eta_3,\ \eta_3 > 0$	Γ_4, Γ_5
			*	*		$f(\eta_1)\eta_2 < \eta_3,\ \eta_3 > 0$	Γ_4, Γ_5
		*			*	$12\eta_2 > \eta_3 > 4\eta_2$	Γ_4, Γ_5
			*		*	$12\eta_2 > \eta_3 > 4\eta_2$	Γ_4, Γ_3
1	1	*			*	$12\eta_2 < \eta_3,\ \eta_3 > 0$	Γ_4, Γ_5
			*		*	$12\eta_2 < \eta_3,\ \eta_3 > 0$	Γ_4, Γ_5
			$T_u > T_v$,	$T_u < T_v$		$\eta_2 < 0$	Γ_3
							Γ_1

method for proving anisotropic superconductivity. A rough estimate of the temperature splitting leads to $|\Delta T_c| \approx 0.05$ K per 1 kbar uniaxial stress in UBe_{13} ($\Delta T_c = T_i - T_j$, $i,j = u,v$ or $x,z,\ldots$).[9] If no splitting is observed, three reasons can be responsible: (1) The coupling $[C(\gamma)]$ to the strain is too small. (2) The relevant representation is one dimensional. (3) The η_i parameters lie in one of the small regions of the phase diagram where no splitting takes place for any uniaxial stress direction. Not much can be concluded in this case.

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APPENDIX A: REPRESENTATIONS OF THE PROPER POINT GROUPS

For cubic O:

Γ	E	$8C_3$	$3C_2$	$6C_2'$	$6C_4$
Γ_1	1	1	1	1	1
Γ_2	1	1	1	-1	-1
Γ_3	2	-1	2	0	0
Γ_4	3	0	-1	1	-1
Γ_5	3	0	-1	-1	1

For hexagonal D_6:

Γ	E	C_2	$2C_3$	$2C_6$	$3C_2'$	$3C_2''$
Γ_1'	1	1	1	1	1	1
Γ_2'	1	1	1	1	-1	-1
Γ_3'	1	1	1	-1	1	-1
Γ_4'	1	-1	1	-1	-1	1
Γ_5'	2	-2	-1	1	0	0
Γ_6'	2	2	-1	-1	0	0

For tetragonal D_4:

Γ	E	$2C_4$	C_2	C_2'	$2C_2''$
$\tilde{\Gamma}_1$	1	1	1	1	1
$\tilde{\Gamma}_2$	1	1	1	-1	-1
$\tilde{\Gamma}_3$	1	-1	1	1	-1
$\tilde{\Gamma}_4$	1	-1	1	-1	1
$\tilde{\Gamma}_5$	2	0	-2	0	0

For rhombohedral C_3 ($\omega \geq e^{i(2\pi/3)}$):

Γ	E	C_3	C_3^2
$\hat{\Gamma}_1$	1	1	1
$\hat{\Gamma}_2$	1	ω	ω^2
$\hat{\Gamma}_3$	1	ω^2	ω

Γ	E	C_2	C_2'	C_2''
Γ_1''	1	1	1	1
Γ_2''	1	-1	1	-1
Γ_3''	1	1	-1	-1
Γ_4''	1	-1	-1	1

APPENDIX B: SYMMETRY-BREAKING TERMS

1. Pressure and strain parameters

The elastic free energy in the presence of a stress $p(\gamma,m)$ can be written as

$$F_H(p,\epsilon) = \sum_{\gamma} B(\gamma) \sum_{m} \varepsilon(\gamma,m)^2 + \sum_{\gamma,m} p(\gamma,m)\varepsilon(\gamma,m) .$$

Minimizing F_H, we get $\varepsilon(\gamma,m) = -p(\gamma,m)/2B(\gamma)$.

Because F_H is invariant under all cubic- and hexagonal-symmetry transformations, respectively, $p(\gamma,m)$ and $\varepsilon(\gamma,m)$ must possess the transformation properties of the corresponding basis functions of the irreducible representations γ. $p(\gamma,m)$ and $\varepsilon(\gamma,m)$ may be expressed by the pressure and strain tensor, respectively, in an analogous way. For cubic-symmetry transformations, we have

$$p(\Gamma_1) = \frac{1}{\sqrt{3}}(p_{xx} + p_{yy} + p_{zz}) ,$$

$$p(\Gamma_3,u) = \frac{1}{\sqrt{6}}(2p_{zz} - p_{xx} - p_{yy}) ,$$

$$p(\Gamma_3,v) = \frac{1}{\sqrt{2}}(p_{xx} - p_{yy}) ,$$

$$p(\Gamma_5,\xi) = \sqrt{2}p_{yz} ,$$

$$p(\Gamma_5,\eta) = \sqrt{2}p_{zx} ,$$

$$p(\Gamma_5,\zeta) = \sqrt{2}p_{xy} .$$

For hexagonal-symmetry transformations,

$$p(\Gamma_1') = a(p_{xx} + p_{yy}) + bp_{zz} ,$$

$$p(\Gamma_5',u) = \sqrt{2}p_{xz} ,$$

$$p(\Gamma_5',v) = \sqrt{2}p_{yz} ,$$

$$p(\Gamma_6',\xi) = \frac{1}{\sqrt{2}}(p_{xx} - p_{yy}) ,$$

$$p(\Gamma_6',\eta) = \sqrt{2}p_{xy} .$$

The remaining irreducible representations do not appear because of the symmetry of the tensor p_{ij}. The strain parameters $\varepsilon(\gamma,m)$ have an analogous relationship to the strain tensor ε_{ij}. The application of stress on a cubic system in a (0,0,1) direction ($p_{zz} \neq 0$) leads to $\varepsilon(\Gamma_1) \neq 0$ and $\varepsilon(\Gamma_3,u) \neq 0$, with all other components zero. For a (1,1,1) direction ($p_{xy} = p_{xz} = p_{yz} \neq 0$, $p_{xx} = p_{yy} = p_{zz} \neq 0$) only $\epsilon(\Gamma_1) \neq 0$ and $\varepsilon(\Gamma_5,\xi) = \varepsilon(\Gamma_5,\eta) = \varepsilon(\Gamma_5,\zeta) \neq 0$. The hexagonal system with stress in the (1,0,0) direction has only $\varepsilon(\Gamma_1') \neq 0$ and $\varepsilon(\Gamma_6,\xi) \neq 0$.

2. The bilinear forms $\mathcal{V}(\gamma,\lambda)_m$

The coefficients $\mathcal{V}(\gamma,\lambda)_m$ are bilinear in λ^* and λ and are basis functions of the irreducible representations in the product $\Gamma \otimes \Gamma$ of the cubic or hexagonal representation Γ under consideration. The invariance of F under time reversal requires V to be real. For $\Gamma_1 \otimes \Gamma_1 = \Gamma_1$,

$$\mathcal{V}(\Gamma_1)=|\lambda|^2 .$$

For $\Gamma_3\otimes\Gamma_3=\Gamma_1\oplus\Gamma_2\oplus\Gamma_3$,

$$\mathcal{V}(\Gamma_1)=|\lambda_u|^2+|\lambda_v|^2 ,$$
$$\mathcal{V}(\Gamma_3,u)=|\lambda_u|^2-|\lambda_v|^2 ,$$
$$\mathcal{V}(\Gamma_3,v)=\lambda_u^*\lambda_v+\lambda_u\lambda_v^* .$$

For $\Gamma_4\otimes\Gamma_4=\Gamma_1\oplus\Gamma_3\oplus\Gamma_4\oplus\Gamma_5$,

$$\mathcal{V}(\Gamma_1)=|\boldsymbol{\lambda}|^2 ,$$
$$\mathcal{V}(\Gamma_3,u)=2|\lambda_z|^2-|\lambda_x|^2-|\lambda_y|^2 ,$$
$$\mathcal{V}(\Gamma_3,v)=\sqrt{3}(|\lambda_x|^2-|\lambda_y|^2) ,$$
$$\mathcal{V}(\Gamma_5,\xi)=\lambda_y^*\lambda_z+\lambda_y\lambda_z^* ,$$
$$\mathcal{V}(\Gamma_5,\eta)=\lambda_z^*\lambda_x+\lambda_z\lambda_x^* ,$$
$$\mathcal{V}(\Gamma_5,\zeta)=\lambda_x^*\lambda_y+\lambda_x\lambda_y^* .$$

The case for $\Gamma_5\otimes\Gamma_5=\Gamma_1\oplus\Gamma_3\oplus\Gamma_4\oplus\Gamma_5$ is analogous to that for Γ_4. $\mathcal{V}(\Gamma_2)$ and $\mathcal{V}(\Gamma_4,m)$ are imaginary and therefore will not be taken into account. For $\Gamma_1'\otimes\Gamma_1'=\Gamma_1'$,

$$\mathcal{V}(\Gamma_1')=|\lambda|^2 .$$

For $\Gamma_2'\otimes\Gamma_2'=\Gamma_1'$,

$$\mathcal{V}(\Gamma_1')=|\lambda|^2 .$$

For $\Gamma_5'\otimes\Gamma_5'=\Gamma_1'\oplus\Gamma_2'\oplus\Gamma_6'$,

$$\mathcal{V}(\Gamma_1')=|\lambda_u|^2+|\lambda_v|^2 ,$$
$$\mathcal{V}(\Gamma_6',\xi)=|\lambda_u|^2-|\lambda_v|^2 ,$$
$$\mathcal{V}(\Gamma_6',\eta)=\lambda_u^*\lambda_v+\lambda_u\lambda_v^* .$$

The case for $\Gamma_6'\otimes\Gamma_6'=\Gamma_1'\oplus\Gamma_2'\oplus\Gamma_6'$ is analogous to that for Γ_4. Note that $\mathcal{V}(\Gamma_2')$ is imaginary.

3. *d*-wave superconductivity

d-wave pairing is spin-singlet pairing and such a state may essentially be described by a scalar $\psi(\mathbf{k})$[4]:

$$\hat{\Delta}(\mathbf{k})=i\hat{\sigma}_y\psi(\mathbf{k}) .$$

In the case of cubic symmetry the basis of the *d*-wave states contains both irreducible representations Γ_3 and Γ_5 ($\Gamma_3\oplus\Gamma_5$): For Γ_3,

$$\psi(\Gamma_3,u)=\frac{1}{\sqrt{6}}(2k_z^2-k_x^2-k_y^2) ,$$
$$\psi(\Gamma_3,v)=\frac{1}{\sqrt{2}}(k_x^2-k_y^2) ;$$

for Γ_5,

$$\psi(\Gamma_5,\xi)=\sqrt{2}k_yk_z ,$$
$$\psi(\Gamma_5,\eta)=\sqrt{2}k_zk_x ,$$
$$\psi(\Gamma_5,\zeta)=\sqrt{2}k_xk_y .$$

The hexagonal symmetry has the following distribution: $\Gamma_1'\oplus\Gamma_5'\oplus\Gamma_6'$; for Γ_1',

$$\psi(\Gamma_i')=\frac{1}{\sqrt{6}}(2k_z^2-k_x^2-k_y^2) ;$$

for Γ_5',

$$\psi(\Gamma_5',u)=\sqrt{2}k_xk_z ,$$
$$\psi(\Gamma_5',v)=\sqrt{2}k_yk_z ;$$

for Γ_6',

$$\psi(\Gamma_6',\xi)=\frac{1}{\sqrt{2}}(k_x^2-k_y^2)$$
$$\psi(\Gamma_6',\eta)=2k_xk_y .$$

Both systems of basis functions are complete. Every *d*-wave state $\psi(\mathbf{k})$ can be written as a linear combination:

$$\psi(\mathbf{k})=\sum_{\Gamma,m}\lambda(\Gamma,m)\psi(\Gamma,m) .$$

From this point on, all considerations go completely analogously to the *p*-wave case. The Ginzburg-Landau expansions for the different representations are exactly the same as before as is their behavior under the change of symmetry.

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