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Role of final-state interactions in the inelastic structure function of quantum liquids

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We critically review the current status of models for the final state interaction in the inelastic structure function of quantum liquids close to asymptotic conditions. We find that the approximations underlying the Stringari model are too drastic and cause, as a rule, serious discrepancies. We show that the series of Sears, though formally correct, is not always rapidly convergent.

I. INTRODUCTION

Inclusive scattering is a subject, which in spite of its long history, continues to attract interest in various branches of physics. For instance, inclusive scattering of electrons from atomic nuclei[1,2] invokes intriguing questions regarding the role of subnucleonic degrees of freedom, possibly needed in the description of the data. In condensed-matter physics there is a body of data which has not yet been analyzed in a satisfactory manner, and some crucial questions still remain open.

The property of matter measured in inclusive scattering is the (inelastic) structure function $S(q,\omega)$, a function of center-of-mass momentum and energy transfer q and ω. Its definition

$$S(q,\omega)=\frac{d^2\sigma}{d\Omega\, d\omega}\Big/\frac{d\sigma}{d\Omega} \tag{1}$$

relates it to the inclusive cross section of a weakly interacting (scalar) probe and the elementary differential cross section. A calculation of $S(q,\omega)$ requires in general a realistic solution of the underlying many-body problem.[3] However, with increasing momentum transfer q asymptotic conditions may prevail, in which case [m is the mass of a particle, $E(\mathbf{p})=p^2/2m$; $\hbar=c=k_B=1$]

$$\begin{aligned}(q/m)S(q,\omega) &\underset{q\to\infty}{\longrightarrow} F_0(y)\\ &=(q/m)\int n(p)\delta(\omega+E(\mathbf{p})-E(\mathbf{p}+\mathbf{q}))d\mathbf{p}\\ &=2\pi\int_{|y|}^{\infty} n(p)p\,dp\ .\end{aligned} \tag{2}$$

The structure function is then simply related to the (lowest-order) scaling function $F_0(y)$ in terms of some scaling variable y. Here we chose the so-called West variable,[4] y_W (denoted as y, unless specifically needed),

$$y=-\frac{q}{2}+\frac{m\omega}{q}\ . \tag{3}$$

If valid, Eqs. (1) and (2) permit extraction of the single-particle momentum distribution $n(p)$ from data.

In the asymptotic limit only the kinematics of the struck particle matters; its final-state interaction (FSI) with the medium is negligible. However, in actual situations one frequently deals with moderate, but non-negligible FSI's. These are sometimes displayed by writing

$$S(q,\omega)=S^e(q,\omega)+S^o(q,\omega)\ . \tag{4}$$

Here S^e and S^o are parts, respectively, even and odd in $\omega-\omega^{\mathrm{QEP}}$, with ω^{QEP}, the position of the quasielastic peak (QEP), where S is maximal.[5,6] For the asymptotic form (2) and the choice of $y=y_W$, Eq. (3), $\omega^{\mathrm{QEP}}=q^2/2m$ is the free-particle recoil energy. A nonvanishing S^o is then taken as proof for the existence of FSI's.[7] A theoretical study of the role of FSI's is not only needed to reach an understanding of the data, but also to enable the isolation of $F_0(y)$, which contains the information on the momentum distribution $n(p)$. Several model and approximations have been proposed in the past.[8–11] The present paper is devoted to a critical study of two of them.[12,13]

In Sec. II we study the series of Sears and the model of Stringari, and confront these with data in Sec. III. In Sec. IV we remark on consequences of the use of alternative scaling variables. Conclusions and an outlook are summarized in Sec. V.

II. APPROACH TO PERFECT SCALING

In the following we shall continue to use the West scaling parameter. Assuming interparticle forces $V(\mathbf{r}_i-\mathbf{r}_j)$ to be nonsingular, the structure function $S(q,\omega)$ may be expanded[14] (see, for instance, Ref. 12),

$$\frac{q}{m}S(q,\omega)=\sum_{n=0}^{\infty}\left[\frac{m}{q}\right]^n F_n(y)\ . \tag{5}$$

$F_n(y)$ in (5) are successive scaling functions, of which $F_0(y)$ [Eq. (2)] is asymptotically the dominant one. Below we focus mainly on $F_1(y)$, which governs the approach to perfect scaling, if q is large but not asymptotic. Although formal expressions for $S(q,\omega)$ may be given,[15] it appears that manageable forms are complicated, even for the leading correction term $F_1(y)$. We start with an expression for F_1 which can be derived from Rosenfelder's general formula for $S(q,\omega)$ (Ref. 10),

$$F_1(y)=\frac{i}{2\pi}\int_{-\infty}^{\infty}ds\, e^{-isy}\left\langle\sum_k e^{is\mathbf{p}_k\cdot\hat{\mathbf{q}}}\sum_{l\neq k}\left[\int_0^s d\sigma\, V(\mathbf{r}_k-\mathbf{r}_l-\sigma\hat{\mathbf{q}})-sV(\mathbf{r}_k-\mathbf{r}_l)\right]\right\rangle \tag{6a}$$

$$=\frac{i}{2\pi}\int_{-\infty}^{\infty}ds\, e^{-isy}\int\int d\mathbf{r}\,d\mathbf{r}'\rho^{(2)}(\mathbf{r}-s\hat{\mathbf{q}},\mathbf{r}';\mathbf{r},\mathbf{r}')\left[\int_0^s d\sigma\, V(\mathbf{r}-\mathbf{r}'-\sigma\hat{\mathbf{q}})-sV(\mathbf{r}-\mathbf{r}')\right]. \tag{6b}$$

$\rho^{(2)}(12,1'2')$ in Eq. (6b) is an element of the two-particle density matrix. Except for a study of inclusive electron scattering[10] on ^{12}C and exact evaluations of $F_1(y)$ for a particle bound in various potentials,[16] we do not know of any other application of Eqs. (6).

We now discuss a review by Sears,[12] who greatly expanded and summarized an approach initiated by Placzek[17] in his paper on neutron scattering on liquids and solids. Sears writes S as a series in derivatives of $F_0(y)$,

$$(q/m)S(q,\omega)=\left[1+\sum_{n=3}^{\infty}(-1)^n A_n(q)d_y^n\right]F_0(y). \tag{7}$$

Except for F_0 itself, no other term F_n stands out in (7), nor do we know of systematic expansions of $A_n(q)$ in powers of q^{-1}, eventually generating the series (5). In the expectation that (7) converges rapidly, a few A_n, or some parts of them, have been computed in the past. Thus (e.g., Refs. 17 and 12),

$$A_3(q)=\frac{m}{q}\tfrac{1}{36}\langle\Delta^2 V\rangle,$$
$$A_4(q)=\left[\frac{m}{q}\right]^2\tfrac{1}{72}\langle\nabla V\cdot\nabla V\rangle. \tag{8}$$

Unfortunately, not only A_3, but all $A_n(q)$ of odd order n contribute to $O(q^{-1})$, e.g.,[12]

$$A_5(q)=\frac{m}{q}\tfrac{1}{600}(a_{54}-\tfrac{5}{3}\langle\Delta^{(2)}V\rangle\langle p_z^2\rangle)+O(q^{-3}). \tag{9}$$

We are not aware of a previous determination of a_{54} in (9) and with a convergence test for $F_1(y)$ in Eq. (5) in mind we need that coefficient. It is then convenient to apply the moment method[12,18] not for the full $S(q,\omega)$, but for $F_1(y)$. Using the representation (6a) with $\hat{\mathbf{q}}=\hat{\mathbf{z}}$, one derives

$$M_n^{(1)}\equiv\int_{-\infty}^{\infty}y^n F_1(y)dy$$
$$=(i/2\pi)\int_{-\infty}^{\infty}dy\, y^n\int_{-\infty}^{\infty}e^{iys}\langle\cdots\rangle$$
$$=i^{n+1}\int_{-\infty}^{\infty}ds\, d_s^n[\delta(s)]\langle\cdots\rangle, \tag{10}$$

with identical $\langle\ \rangle$ brackets in Eqs. (6a) and (10). After partial integration, Eq. (10) becomes

$$M_n^{(1)}=\sum_{m=1}^{n-1}i\begin{bmatrix}n\\ m+1\end{bmatrix}\times\left\langle\sum_k(p_{z,k})^{n-m-1}\sum_{l\neq k}d_{z,l}^m V(|\mathbf{r}_k-\mathbf{r}_l|)\right\rangle. \tag{11}$$

Notice in particular the presence of derivative operators in (11). This implies that A_n for $n\geq 5$ contains quantum fluctuations, or differently formulated, requires for its calculation knowledge of the nondiagonal two-body density matrix of the system [cf. Eq. (6b)]. Reference 12 summarizes the algorithm linking $M_n^{(1)}$ to A_n and, in particular ($\Delta^{(4)}V_r\equiv[d_r^4-(4/r)d_r^3]V(r)$),

$$A_5(q)=\frac{1}{600}\frac{m}{q}[\langle\Delta^{(4)}V\rangle+5\langle(\partial_z^2 V)p_z^2\rangle-\tfrac{5}{3}\langle\Delta V\rangle\langle p_z^2\rangle]+O(q^{-3}) \tag{12a}$$

$$\approx\frac{1}{600}\frac{m}{q}\langle\Delta^{(4)}V\rangle+O(q^{-3}). \tag{12b}$$

Equation (12b) holds if quantum fluctuations are small, which, when neglected, cause a cancellation of the last two terms in the square brackets of (12a). As a result,

$$F_1(y)=(\tfrac{1}{36}\langle\Delta^{(2)}V\rangle+\tfrac{1}{600}\langle\Delta^{(4)}V\rangle d_y^2+\cdots)d_y^3F_0(y). \tag{13}$$

The terms left out in (13) contain higher-order derivative terms, quantum fluctuation corrections, or combinations of the two. Assuming a Gaussian momentum distribution ($\langle K\rangle$ is the average kinetic energy),[19]

$$n(p)=n(0)e^{-p^2/p_0^2},\quad \langle K\rangle=\left[\frac{3}{4m}\right]p_0^2, \tag{14}$$

which leads by means of Eq. (13) to ($z=y/p_0$)

$$F_1(y)=\frac{2\pi n(0)}{18p_0}\langle\Delta^{(2)}V\rangle z\left[(3-2z^2)-\frac{3}{25p_0^2}\frac{\langle\Delta^{(4)}V\rangle}{\langle\Delta^{(2)}V\rangle}(15-20z^2+4z^4)+\cdots\right]e^{-z^2}. \tag{15}$$

Although the various terms in (15) have different z dependence, it will be useful for the following to define $\langle\Delta^{(2)}(V)\rangle_{\rm eff}$ such that

$$F_1(y)\cong\frac{2\pi n(0)}{18p_0}\langle\Delta^{(2)}V\rangle_{\rm eff}z(3-2z^2)e^{-z^2}. \tag{16}$$

A measure for convergence of the series (15) will be the ratio

$$r_1(y)=\rho_1\frac{15-20z^4+4z^4}{3-2z^2},$$
$$\rho_1=\frac{3}{25p_0^2}\left|\frac{\langle\Delta^{(4)}V\rangle}{\langle\Delta^{(2)}V\rangle}\right|, \tag{17}$$

which varies with y, or may be taken for some average $\langle y^{2n}\rangle=\int F(y)y^{2n}dy$. If $|r_1|<1$,

$$r_2(y,q)=\rho_2(q)z(3-2z^2)\ ,\quad \rho_2(q)=\frac{m}{9q}\frac{|\langle\Delta^{(2)}V\rangle|}{p_0^3} \tag{18}$$

will be a measure for the importance of FSI's. Again one may consider $r_2(\langle y\rangle,q)$ with $\langle y\rangle$ as above. Its value comes close to an estimate for the same given by Sears.[12]

Finally, if the series (5) can be terminated after two terms, one may define a new, V-dependent scaling variable, such that $(q/m)S(q,\omega)\sim F_0(\tilde{y})$. $\tilde{y}$ and the associated shift $\delta\omega^{\rm QEP}$ are[13,16]

$$\tilde{y}=y+(m/q)F_1(y)/F_0'(y)\ ,\quad \delta\omega^{\rm QEP}=F_1(y)/F_0'(y)\approx\langle\Delta^{(2)}V\rangle_{\rm eff}/8m\langle K\rangle\ , \tag{19}$$

with the last relation resulting from Eqs. (2), (14), and (16).

We now discuss an approximation which appears to circumvent altogether the many-body aspects and complications inherent in $F_1(y)$.[13] Stringari suggests that a realistic $S(q,\omega)$ emerges if in Eq. (2) the energy of the struck particle, i.e., a *function* $E(p)$, is replaced by an assumed *constant* difference of single-particle binding and average potential energy $\varepsilon_0-\langle v\rangle$ (cf. also Ref. 9). Thus,

$$S^S(q,\omega)\rightarrow\int n(p)\delta(\omega+\varepsilon_0-\langle v\rangle-E(\mathbf{p}+\mathbf{q}))d\mathbf{p}\ . \tag{20}$$

Next, Stringari imposes on the so-constructed $S^S(q,\omega)$ the sum rule

$$\int_{-\infty}^{\infty}S(q,\omega)\omega\,d\omega=q^2/2m\ , \tag{21}$$

which leads to a determination of $\varepsilon_0-\langle v\rangle\Longrightarrow\langle K\rangle_{\rm eff}$. Stringari considers $\langle K\rangle_{\rm eff}$ as an adjustable parameter. For use below we shall continue to do so, although the sum rule (21) clearly demands $\langle K\rangle_{\rm eff}=\langle K\rangle$. Equation (20) then becomes

$$(q/m)S^S(q,\omega)\approx 2\pi\int_{|y^S|}^{\infty}n(p)p\,dp\ , \tag{22}$$

where

$$y^S=-q+[2m(\omega+\langle K\rangle_{\rm eff})]^{1/2}\ . \tag{23}$$

Equations (22) and (23) permit a series expansion (5), resulting in

$$F_1^S(y)=2\pi n(y)ym\langle K\rangle_{\rm eff}(1-2y^2/m\langle K\rangle_{\rm eff}) \tag{24a}$$

$$=2\pi n(y)(p_0^2/4)z(3-2z^2) \tag{24b}$$

$$=2\pi n(0)(p_0^2/4)z(3-2z)e^{-z^2}\ . \tag{25}$$

Equations (24a) and (24b) are Stringari's expressions for F_1, when $\langle K\rangle_{\rm eff}\neq\langle K\rangle$ and $\langle K\rangle_{\rm eff}=\langle K\rangle$, respectively, while (25) results if the distribution (14) is used in Eq. (24b). The associated expression for the shift in the QEP, Eq. (19), is seen to be simply

$$\delta^S\omega^{\rm QEP}=-\langle K\rangle_{\rm eff}\ . \tag{19$'$}$$

It is of interest to compare Eqs. (15), (16), and (25), which, we recall, hold for a Gaussian single-particle momentum distribution.

(1) For a Gaussian $n(p)$, Eq. (16) and Stringari's approximation produce exactly the same $y(z)$ variation of $F_1(y)$.

(2) Equations (24b) and (25) depend only on p_0 (or $\langle K\rangle$), whereas (16) contains, in addition, a dynamical quantity like $\langle\Delta^{(2)}V\rangle_{\rm eff}$. Equivalence demands

$$\langle\Delta^{(2)}V\rangle_{\rm eff}=8m(\langle K\rangle_{\rm eff})^2\ . \tag{26}$$

Such a relation does not appear supported by any theory.

(3) Both Eqs. (16) and (25) have ($z\neq0$) zeros and extrema z_M at

$$z_0=\pm(\tfrac{3}{2})^{1/2}\ ,\quad z_M=\pm\left[\frac{3\pm6^{1/2}}{2}\right]^{1/2}\ . \tag{27}$$

At the extrema closest to $z=0$, Stringari's model and Eq. (16) predict, for $\mu\equiv|F_1(y)|/F_0(0)$,

$$\mu^S=(m/q)(\langle K\rangle_{\rm eff}/3m)^{1/2}z_M(3-2z_M^2)e^{-z_M^2}\ , \tag{28a}$$

$$\mu=\tfrac{1}{9}(m/q)(\langle\Delta^{(2)}V\rangle_{\rm eff}/p_0^3)z_M(3-2z_M^2)e^{-z_M^2}\ . \tag{28b}$$

The two expressions become identical again if Eq. (26) holds. However, if $p_0=(4m\langle K\rangle/3)^{1/2}$ is kept fixed, μ in Eq. (28a) varies only like $\langle K\rangle_{\rm eff}^{1/2}$, and not much stronger like $(\langle K\rangle_{\rm eff}^2)^2$, as (26) suggests.

In closing this section we remark that without a well-founded derivation, the underlying assumptions leading to Eqs. (24a) and (24b) seem to be too simplistic. In particular, the invoked sum rule is hardly a constraint on dynamics: the asymptotic limit (2), lacking *any* FSI's, also satisfies (21).

III. COMPARISON WITH DATA

We now reach a confrontation with data for which we chose ^{4}He at $T=1.2$ K, $q=10$ Å^{-1} (Ref. 5) and Ne at $T=26.9$ K, $9.5\geq q$ (Å^{-1}) ≥5.5 (Ref. 6). The former are in a quantum regime close to $T\sim0$, while the latter data are for a quantum liquid close to its classical limit. Table I contains experimental information on, and input

TABLE I. Kinematical data on the systems investigated. The last two columns give parameters for the Lennard-Jones potential (Refs. 23 and 6).

	T (K)	p_0 (Å^{-1})	$\langle K\rangle$ (K)	q (Å^{-1})	m/q (10^5)	σ (Å)	ε (K)
^{4}He	1.2	1.285	15.0	10	1.88	2.556	10.22
Ne	26.9	5.17	48.2	7.5	1.25	2.75	35.6

for, these liquids. The second and third columns give p_0,[19] the parameter in a Gaussian fit (14) to the momentum distribution $n(p)$,[20,21] and the corresponding $\langle K \rangle$. [Compare also an accurate theoretical result $\langle K \rangle^{\rm theor} = 14.77$ K, derived without the use of (14).[22]]

In Table II we assembled calculated results. Columns 2–4 contains values for $\langle V \rangle$, $\langle \Delta^{(2)} V \rangle$, and $\langle \Delta^4 V \rangle$. For ^{4}He we used the Aziz He-He potential considered to give the best agreement with data,[24] as well as a pair distribution function $g(r)$ for the same V, given by Kalos *et al.*[25] For later purposes we also report results for a Lennard-Jones potential,[23] employing in a not entirely consistent manner the same $g(r)$. For Ne we used only a Lennard-Jones potential[6] and results derivable from Verlet's calculation of $g(r)$ for the conditions: $\rho\sigma^3 = 0.75$ and $T = 30.6$ K (ρ is the number density),[26] as close as possible to the actual ones. Next we assembled information on $\langle K \rangle_{\rm eff}$. In column 5 we entered the equivalent $\langle K \rangle_{\rm eff}$, which follows from the correspondence, Eq. (26), if $F_1(y)$ were represented by Eq. (16). Columns 6 and 7 contain the same information, extracted from the shift $\delta\omega^{\rm QEP}$, Eq. (19), and the ratio μ, Eqs. (28). To these sources, one may for Ne add a third one.[21] With $T_{\rm eff} = \frac{2}{3} \langle K \rangle = 32.1$ K and thus close to $T = 26.9$ K, one may utilize a relation, valid for a quantum fluid close to its classical limit,[27,28]

$$\frac{T_{\rm eff}}{T} = 1 + \frac{1}{12} \left[\frac{\Theta}{T} \right]^2 - \frac{1}{240} \left[\frac{\Theta}{T} \right]^4 + \cdots ,$$
$$\Theta^2 = \langle \Delta^{(2)} V \rangle / 3m . \tag{29}$$

Using Eq. (26), the value entered in column 8 results. Columns 9 and 10 give the ratio of observed and predicted positions of $|z_0|$ $(z \neq 0)$ and extrema of

$$(q/m) S^o(q,\omega) = (m/q) F_1(y) .$$

Finally, the last two columns are the convergence parameters ρ_1, ρ_2, Eqs. (17),(18), for the series (13) and (7).

We now discuss the results in somewhat greater detail.

Stringari focussed on the ^{4}He data for fixed $q = 10$ Å^{-1}, which are shown in Fig. 1. For these (cf. Table I), $q\sigma \gg 1$, and also from the magnitude of ρ_2 (Table II) one expects FSI's to be small. Consequently $S(q,\omega)$ ought to be well represented by the first two terms in Eq. (5). The parts $S^{e,o}$ are separated on the basis that the QEP peak corresponds to $\omega^{\rm QEP} = q^2/2m$, as is the case for the West representation (see, however, Sec. IV). Stringari correctly emphasizes that the shape of $(q/m) S^o \approx (m/q) F_1$ follows exclusively from a Gaussian $n(p)$ (see Fig. 1). The quantities which provide the real test are thus the shift $\delta\omega$ [Eq. (19)] and the ratio μ [Eq. (28a)]. From columns 5 and 6 in Table II one finds, respectively, $\langle K \rangle_{\rm eff} \sim 13$–$16$ K, and a range $\langle K \rangle_{\rm eff} \sim 16$–$36$ K, compatible with the scatter of data. The value $\langle K \rangle_{\rm eff} \sim 14$ K $\sim \langle K \rangle^{\rm expt}$ leading to the dashed line in Fig. 1 is indeed compatible with all data and seems to support the model.

Next we discuss the series for $F_1(y)$. The large value of ρ_1 shows that the series (13) is not likely to converge rapidly, in spite of a relatively small ρ_2.

Taking only the first term in (15), the equivalent $\langle K \rangle_{\rm eff} = 23.1$ K from Eq. (26) is larger than Stringari's value ~ 14 K and, consequently, the prediction (dashed-dotted line in Fig. 1) overshoots the data for S^o. Notice that the latter $\langle K \rangle_{\rm eff}$ value lies within the range entered in column 6: the remark following Eq. (28) explains the apparent discrepancy. Of course, with a large ρ_2 there is no way to reach a conclusion, except that $\langle \Delta^4 V \rangle$ causes $\langle \Delta^{(2)} V \rangle_{\rm eff} < \langle \Delta^{(2)} V \rangle$ [cf. Eq. (16)].

The Ne data have not been analyzed by Stringari and we shall do so for those with fixed $q = 7.5$ Å^{-1}. All three sources, (23), (28), and (29), give $\langle K \rangle_{\rm eff} \sim 25$–$27$ K, about half the value $\langle K \rangle = 48.2$ K (Table I), and thus in clear disagreement with Stringari's prediction (Fig. 2). The relatively small value for ρ_1 (due to large p_0 or $\langle K \rangle$) this time enables a calculation of correction terms in the series (13) for $F_1(y)$. These again lead to a reduction $\langle \Delta^{(2)} V \rangle_{\rm eff} < \langle \Delta^2 V \rangle$ in line with the extracted $\langle K \rangle_{\rm eff}$ values. Predictions using Eq. (15) are given as long-dashed curves in Fig. 2 and are seen to lead to too large a reduction for S^o (see, however, Sec. V below).

IV. ALTERNATIVE SCALING VARIABLES

In the developments above we used the West scaling variable[4] (3), which is only a natural choice, if a Fermi

TABLE II. For the Aziz (Ref. 24) and Lennard-Jones potentials (Refs. 6 and 23) the columns 1–4 contain calculated values for $\langle V \rangle, \langle \Delta^{(2)} V \rangle$, and $\langle \Delta^{(4)} V \rangle$, as well as $\langle K \rangle^{\rm eff}$ from Eq. (26). In the next three columns are entered extracted $\langle K \rangle_{\rm eff}^{\rm expt}$ from experimental values of $\mu = |F_1(y)| / F_0(0)$ [Eqs. (28)], the shift in the QEP peak [Eq. (19)], and $T_{\rm eff}/T$ [Eq. (29)]. The last columns give positions of zeros and extrema of F_1 [Eq. (27)] and the convergence measures ρ_1, ρ_2 [Eqs. (17) and (18)].

Potential	$\langle V \rangle$ (K)	$\langle \Delta^{(2)} V \rangle$ (KÅ^{-2})	$\langle \Delta^{(4)} V \rangle$ (K Å^{-4})	$\langle K \rangle_{\rm eff}$ (K)	μ	$\langle K \rangle_{\rm eff}^{\rm expt}$ $\delta\omega^{\rm QEP}$	$T_{\rm eff}/T$	$z_0^{\rm expt}/z_0^{\rm theor}$	$z_M^{\rm expt}/z_M^{\rm theor}$	ρ_1	ρ_2
						He					
Aziz	-21.58	352.3	8447	23.1							
					16–36	13–16		0.97	1.12	1.74	0.152
LJ	-20.48	292.3	12 204	21.0							
						Ne					
LJ		3081	9697	30.5	25.3	26.5	27.0	0.93	0.84	0.141	0.138

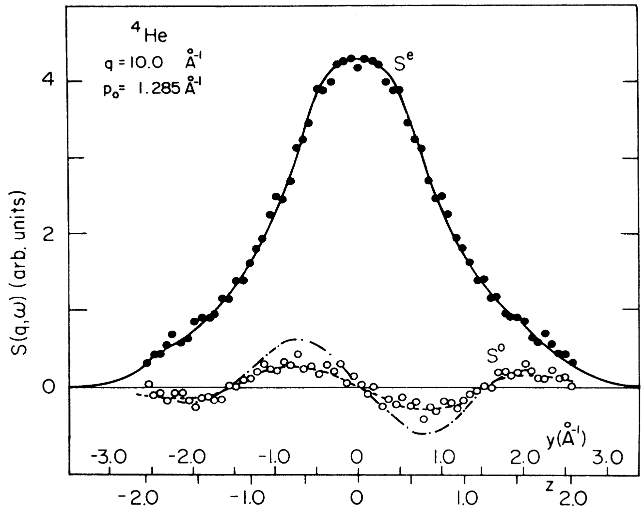

FIG. 1. Separated $S^e(q,\omega)$ and $S^o(q,\omega)$ for ^{4}He at $T=1.2$ K and $q=10$ Å^{-1}. Short-dashed curve is the Stringari fit (25) $\langle K\rangle_{\text{eff}}\sim 14$ K. Long-dashed—dotted curve corresponds to form (16).

gas is used as a lowest-order description for fermions in interaction.

Other scaling variables, $y_i=y_i(q,\omega)$, have been used in the past (see, for instance, Refs. 1 and 11). All satisfy

$$y_i-y_W=(m/q)\gamma_i(y_i)+O(q^{-2}) \ , \tag{30}$$

and the asymptotic behavior of S remains given by (2), irrespective of the choice of y_i.[29,30] Using (30), consider now the series (5),

$$\begin{aligned}(q/m)S(q,\omega)&=F_0(y_W)+(m/q)F_1^W(y_W)+O(q^{-2})\\&=F_0(y_i)+(m/q)F_1^{(i)}(y_i)+O(q^{-2}) \ ,\\F_1^i(y_i)&=F_1^W(y_i)+\gamma_i(y_i)F_0'(y_i) \ .\end{aligned} \tag{31}$$

The position of the quasielastic peak, ω_i^{QEP}, given by $y_i=0$, is clearly dependent on the choice of the scaling variable. This immediately reflects on the decomposition (4) of $S(q,\omega)$ as well as on all parameters discussed: the positions z_0 and z_M, the shift $\delta\omega^{\text{QEP}}$, and the ratio μ [Eqs. (27), (19), and (28)].

We illustrate our remarks for the choice $y_i=y_0$, a scaling variable emerging in an impulse approximation for S (ε_0 is an average separation energy),

$$y_0=-q+[2m(\omega+\varepsilon_0)]^{1/2} \ . \tag{32}$$

Its relativistic analog has frequently been used in the analysis of inclusive electron scattering on nuclei[1] which are always in their ground state. This is not the case for quantum liquids, but we shall assume that for the data ^{4}He at $T=1.2$ K the use of y_0 makes sense.

Using $|\varepsilon|=6.96$ K (Ref. 22) one shows that $F_0(y_0)/F_0(y_W)$ grows for increasing $|y|$, and reaches ~ 2 for $y_0\sim 2$ Å^{-1}. Since $n(p)$ is extracted from $(q/m)S^e\sim F_0$, components of $n(p)$ with $p\gtrsim 1$ Å^{-1} will be affected. Without entering here questions as to a preferred y_i (see Ref. 11), one is at least warned against indiscriminate use of quantities derived from one selected scaling variable y.

V. CONCLUSIONS

A. Stringari model

As shown, the model appears to be in disagreement with the Ne data and the same has been observed in studies of F_1 for exactly solvable potential models.[16] The matter is serious, since the expressions (24) and (25) for F_1 are "universal," without any room for amendments. One then wonders whether the initial success for ^{4}He at $T=1.2$ K is not a numerical coincidence. It seems that, if at all possible, one ought to establish clear limits of applicability without which there is no basis for the use of the Stringari model in general.[31]

B. The Sears series

Contrary to the Stringari model the Sears series is formally correct. It is the evaluation in practice which meets with difficulties.

(i) A calculation of $F_1(y)$ requires derivatives of increasing order of $V(r)$ and $F_0(y)$. Even if one trusts the basic functions, derivatives of ever higher order clearly incur corresponding greater inaccuracies. A striking example is provided by the growing disparities in $\langle V\rangle$, $\langle\Delta^{(2)}V\rangle$, and $\langle\Delta^{(4)}V\rangle$ for the two He-He interactions considered, and displayed in Table II. We have observed similar sensitivities if for He a more accurate $n(p)$ than (14) is used.[20]

(ii) The uncertain size of the quantum corrections, present in (12), and simply ignored in Eq. (15), may well be responsible in part for the excessively large ρ_1 for ^{4}He.

All difficulties may be traced to the exact expressions (6) which clearly demand the evaluation of a nondiago-

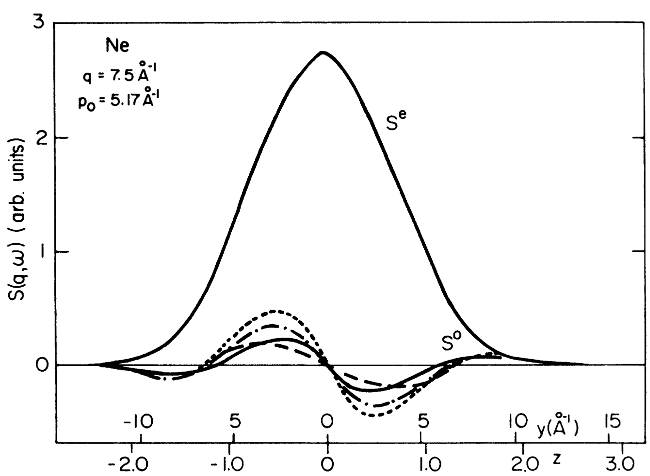

FIG. 2. Same for Ne ($T=26.9$ K, $q=7.5$ Å^{-1}). Solid lines correspond to smoothed data (Ref. 6). Long-dashed curve corresponds to Eq. (15).

nal two-particle density matrix. The latter embodies the required dynamics and quantum correlations, which the Stringari model avoids completely, and the low-order terms of the Sears series, in part. It may well be time to address Eq. (6) directly.

Note added in proof. We have recently become aware of work by H. A. Gersch and co-workers, who—15 years ago—derived Eq. (6b) above [Phys. Rev. A 5, 1547 (1972), Eq. (31)].

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[14]Equation (5) is an expansion in m/q or more precise: terms q^{-n} are "naturally" accompanied by powers m^n. For high-energy-electron scattering on atomic nuclei, m/q is $O(1)$. However, for n scattering on liquids it is of order 10^5, and the coefficients $F_n(y)$ must be correspondingly small for Eq. (5) to make sense. An expansion $(q/m)S(q,\omega) = \sum_n (q\sigma)^{-n} G_n(y)$ is of course identical to (5), yet emphasizes that for a typical range σ of the atom-atom interaction, frequently $g\sigma \gg 1$.

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