RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 36, NUMBER 9 15 SEPTEMBER 1987-II

Quantum theory of the cyclotron-resonance line shape in the presence of hole-phonon interactions in p-type multi-quantum-well structures

M. Singh
Department of Physics and Centre for Chemical Physics, The University of Western Ontario, London, Canada N6A 3K7
(Received 10 December 1986; revised manuscript received 8 June 1987)

A theory of the cyclotron-resonance line shape in the presence of hole-phonon interactions in multi-quantum-well structures (MQWS's) has been developed. The cyclotron-resonance linewidth (CRLW) has been calculated using the effective-mass and the elastic-scattering approximations to keep the calculation algebraically simple. We studied the effect of internal strains and found that the CRLW increases with the inclusion of strains. The contributions of the transverse-acoustic (TA) and longitudinal-acoustic phonons on the CRLW have also been studied. It is found that the TA phonons also contribute to the CRLW. This is the opposite situation from the usually studied case of the electron-phonon interaction where the above contribution is zero. The variation of the CRLW with magnetic field, temperature, and the MQWS period has also been investigated. It is found that the CRLW increases with increasing magnetic field and temperature but decreases with increasing MQWS period.

Recently there has been considerable interest in the magnetotransport and magnetooptics of two-dimensional systems such as p-type multi-quantum-well structures (MQWS's).[1-3] The experiments show two types of carriers, light holes ($J=\frac{3}{2}$, $m=\pm\frac{1}{2}$) and heavy holes ($J=\frac{3}{2}$, $m=\pm\frac{3}{2}$) present in the system. Cycloton-resonance absorption experiments on these materials have been used to try to find the effective masses of the light and heavy holes and the transition energies as a function of magnetic field for the cyclotron transition. Explicit results for the cyclotron-resonance linewidth (CRLW) are not reported in the above references. On the other hand, there are experimental results for the cyclotron-resonance linewidth in a two-dimensional (2D) electron gas.

I have calculated the CRLW in the presence of the electron–acoustic-phonon interaction for MQWS's, and the theoretical results are in qualitative agreement with experiments.[4] There are many works[5] on the electron-phonon interaction in 2D systems both in the presence and in the absence of a magnetic field. But there are very few theoretical calculations[3] which study the effect of hole-phonon interactions on the transport and optical properties of the above system.

The aim of the present paper is to develop a theory for the cyclotron-resonance line shape (CRLS) in p-type MQWS's in the presence of the hole–acoustic-phonon interaction. The effect of internal strains resulting from the lattice mismatch between two superlattice layers, dislocations, and disorder has been included. The contributions of transverse-acoustic (TA) and longitudinal-acoustic (LA) phonons to the CRLW are also investigated. We used the effective-mass and elastic-scattering approximations and neglected the layering effects in the calculation of the CRLW to keep the calculation simple. The simplification due to these approximations will occur at the expense of a quantitative understanding. However, all the salient features of the CRLW will be present in the present theory.

The present theory predicts the increase of the CRLW in the presence of internal strains. The CRLW increases with increasing magnetic field and temperature but decreases with increasing MQWS period. It is also found that the contribution of TA phonons is important in the calculation of the CRLW. Their contribution to the CRLW depends on the choice of the deformation-potential constants, and also upon other parameters such as temperature, magnetic field, and MQWS parameters. This is the opposite situation to the usually studied case of electron-phonon interactions where the above contribution is zero. The present theory can be applied to any p-type MQWS systems, but only GaAs/AlAs is treated here as an example for the sake of brevity.

Recently Wallace and I[6] calculated our expression for the cyclotron-resonance-active components of the conductivity tensor (σ_{+-}) for right circular polarization in a three-dimensional system. The present theory can easily be extended to p-type MQWS's in the presence of hole-phonon interactions and is written as

$$\mathrm{Re}(\sigma_{+-})=\frac{1}{4\pi^2 l^2\omega}\sum_{n,m,p}\langle n,m,p\,|\,j_+\,|\,n+1,m,p\rangle\langle n+1,m,p\,|\,j_-\,|\,n,m,p\rangle \times\Gamma^n_{n,p}[f(E^m_{n+1,p})-f(E^m_{n,p})][(E^m_{n+1,p}-E^m_{n,p}-\omega+\Delta^m_{n,p})^2+|\Gamma^m_{n,p}|^2]^{-1}\,, \tag{1}$$

where $j_\pm=(j_x\pm ij_y)/\sqrt{2}$. Here $j_\alpha=\partial H/\partial k_\alpha$ is the αth component of the current operator j. H is the hole Hamiltonian in the presence of a magnetic field and k_α is the αth component of momentum in the presence of magnetic field. $E^m_{n,p}$ is the Landau energy of the $|n,m,p\rangle$ state where n,m,p represents the Landau-level, valence-band ($m=\frac{3}{2},\frac{1}{2},-\frac{1}{2},-\frac{3}{2}$), and subband quantum numbers, respectively. ω is the laser frequency, and l is the Landau

 © 1987 The American Physical Society

length. $\Gamma^m_{n,p}$ is the cyclotron-resonance linewidth (CRLW) due to the hole-phonon interaction and $\Delta^m_{n,p}$ is the energy shift of the Landau state $|n,m,p\rangle$. The CRLS or power absorption, $P(\omega)$, of holes in MQWS's under the influence of circularly polarized light of frequency ω and electric field strength E_0 is obtained with the help of the above equation as $P(\omega)=E_0^2\,\mathrm{Re}[\sigma_{+-}(\omega)]/2$.

The most important function to calculate in the CRLS is the CRLW, $\Gamma^m_{n,p}$. We shall therefore confine our attention to the calculation of the CRLW in the rest of this paper. The matrix elements of current operators j_α appearing in Eq. (2) are calculable on the same line as in Ref. 6.

The hole–acoustic-phonon interaction Hamiltonian in the presence of internal strains is taken as[3]

$$H_{h\text{-ph}}=D_a(\epsilon_{xx}+\text{c.p.})+3D_b(J_x^2\epsilon_{xx}+\text{c.p.})+\sqrt{3}D_d(\{J_x,J_y\}\epsilon_{xy}+\text{c.p.})\ , \qquad (2)$$

where $\{J_\alpha,J_\beta\}=\frac{1}{2}(J_\alpha J_\beta+J_\alpha J_\beta)$ and $\epsilon_{\alpha\beta}$ is the conventional strain tensor. D_α are the deformation potentials, c.p. refers to cyclic permutation, and J_α is the αth component of the total angular momentum $J=\frac{3}{2}$. This Hamiltonian includes the coupling of light- and heavy-hole bands in hole-phonon interaction. We assume that the heavy holes ($J=\frac{3}{2}$, $m=\pm\frac{3}{2}$) and light holes ($J=\frac{3}{2}, m=\pm\frac{1}{2}$) are free to move in the x-y plane and are confined in the z direction by a periodic square quantum well of height W in MQWS's. The MQWS period is $d=a+b$. Here a and b are the width of the quantum well and separation between the wells. We consider the magnetic field along the z direction.

To calculate the CRLW due to the hole–acoustic-phonon interaction we follow the same method as in Ref. 4. To keep the calculation algebraically simple, we used the effective-mass approximation to calculate the Landau energies of holes, and neglected the layering effect on acoustic-mode phonons. The simplification due to this approximation will occur at the expense of the quantitative understanding. However, all salient features of the CRLW will be present in the following description as in the case of LLW calculations.[3] The expression of the CRLW in the above approximation is

$$\Gamma^m_{n,p}=\sum_{n',p',m'}\sum_{\mathbf{Q},t}\sum_{\pm}|V_{Qt}|^2|C^{mm'}_{Qt}|^2|F_{pp'}(q_z)|^2N_Q^{\pm}\Gamma^{m'}_{n',p'}\{K(nn')[(E^{m'}_{n',p'}-E^m_{n+1,p}+\omega+\omega_Q+\Delta^{m'}_{n',p'})^2+|\Gamma^{m'}_{n',p'}|^2]^{-1}+K(n,n')[(E^{m'}_{n',p'}-E^m_{n,p}+\omega+\omega_Q+\Delta^{m'}_{n',p'})^2+|\Gamma^{m'}_{n',p'}|^2]^{-1}\}\ , \qquad (3)$$

where $N_Q^{\pm}=N_Q+\frac{1}{2}\pm\frac{1}{2}$ is the phonon distribution function. **Q** and t are the phonon wave vector and polarization branch, respectively. $t=1$ corresponds to the longitudinal-acoustic phonon mode and $t=2$ and $t=3$ correspond to the transverse-acoustic phonon modes. $K(n,n')$ are well-known functions and are defined in Ref. 5. $C^{mm'}_{Qt}$ are called coupling constants and $F_{pp'}(q_z)$ is the matrix element between p and p' in z direction and their values are given in Ref. 3. $V_{QT}=(3\hbar QD_d^2/2\rho v_t)^{1/2}$. Here ρ is the density and v_t is the phonon velocity in the tth branch and ω_Q is the phonon frequency.

In summing over n', we use the high-magnetic-field approximation, keeping only the resonance terms which are characterized by matrix elements with the state of the same quantum numbers. Most of the experiments in MQWS's in which we are interested are done in extreme-quantum-subband limit $p=1$ where the interactions between subbands are neglected. Therefore, summation over p' has only one term $p'=p$. We denote $\Gamma^m_{n,p}$ by Γ^m_n in the rest of the paper. After using the elastic-scattering approximation[5] Eq. (3) reduces to

$$\Gamma^m_n=\sum_Q\sum_t\sum_{m'}(2N_Q+1)|v_{Qt}|^2|F_{11}(q_z)|^2|C^{mm'}_{Qt}|^2\Gamma^{m'}_n\{K(n,n)[(E^{m'}_n-E^m_{n+1}+\omega]^2+|\Gamma^{m'}_n|^2]^{-1}+K(n,n+1)[(E^m_n-E^{m'}_{n+1}+\omega)^2+|\Gamma^{m'}_n|^2]^{-1}\}\ . \qquad (4)$$

Equation (4) can be further simplified by replacing summation over **Q** by integration in the polar coordinates. For the cyclotron resonance, putting $\omega=\omega_c=E^m_{n+1}-E^m_n$ in Eq. (4) for the light and heavy hole, respectively, we get the follow-

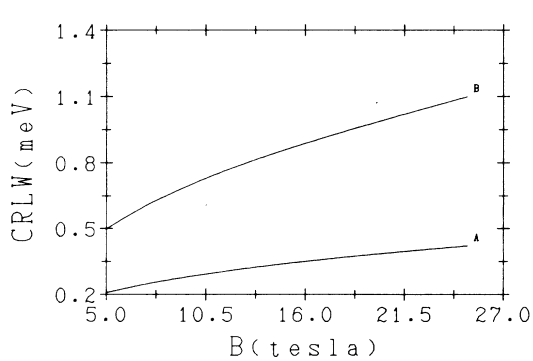

FIG. 1. CRLW vs magnetic field for constant temperature ($T=10$ K). Curves A and B correspond to heavy and light holes, respectively.

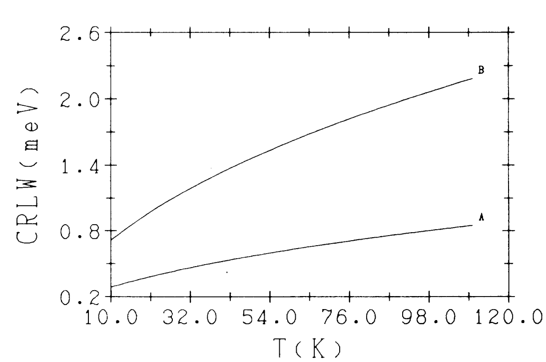

FIG. 2. CRLW vs temperature for constant magnetic field ($B=10$ T). Curves A and B denote the heavy and the light holes, respectively.

ing expressions for the CRLW:

$$|\Gamma_n^h|^2=\sum_t A_t\int \eta^3 d\eta \int \sin\theta d\theta |F_{11}(\eta,\theta)|^2(2N_Q+1)(K_n W_t^{hh}+W_t^{hl}B_n^h) , \quad (5)$$

$$|\Gamma_n^l|^2=\sum_t A_t\int \eta^3 d\eta \int \sin\theta d\theta |F_{11}(\eta,\theta)|^2(2N_Q+1)(K_n W_t^{ll}+W_t^{hl}B_n^l) , \quad (6)$$

$$B_n^h=\Gamma_n^h\Gamma_n^l\{K(n,n)[(E_n^l-E_n^h)^2+|\Gamma_n^h|^2]^{-1}+K(n,n+1)[(E_{n+1}^h-E_{n+1}^l)^2+|\Gamma_n^l|^2]^{-1}\} ,$$

$$B_n^l=\Gamma_n^h\Gamma_n^l\{K(n,n)[(E_n^l-E_n^h)^2+|\Gamma_n^l|^2]^{-1}+K(n,n+1)[(E_{n+1}^h-E_{n+1}^l)^2+|\Gamma_n^h|^2]^{-1}\} ,$$

$A_t=(D_d^2/2\rho v_t \pi^2 l^2)$, $\eta=Ql$, $W^{hl}=W^{lh}$, $W_T^{ll}=W_T^{hh}$, $d_2=D_b/D_d$, $d_1=D_a/D_d$, $W_L^{hh}=W_1^{hh}=[d_1+\frac{3}{4}d_2(2\cos^2\theta+1)]$, $W_L^{ll}=W_1^{ll}=[d_1+\frac{3}{4}d_2(2\sin^2\theta+\frac{1}{3})]$, $W_L^{hl}=W_1^{hl}=3\sin^2\theta[\cos^2\theta+\sin^2\theta(D^2+1)/8]$, $W_T^{hh}=W_2^{hh}+W_3^{hh}=(q/4)d_2^2\sin^2\theta\cos^2\theta$, $W_T^{hl}=W_2^{hl}+W_2^{hl}=\frac{1}{16}[27\cos^2\theta-3+D^2(6\cos^2\theta+6)]$, $K_n=K(n,n)+K(n,n+1)$.

The present theory can be applied to any MQWS systems, but we will apply it to GaAs/AlAs here as an example, for the sake of brevity. The physical parameters used in the calculation are $D_a=8.9$ eV, $D_b=1.98$ eV, and $D_d=5.4$,[7] $\rho=5.3$ g/cm^3, $m_l/m=0.1$, $m_h/m=0.45$, $v_T=3.37\times10^5$ cm/s, and $v_L=4.77\times10^5$ cm/s. The calculations have been performed for the extreme-quantum-subband limit (i.e., $p=1$) and extreme-Landau-level limit (i.e., CR transition from $|0\rangle$ state to $|1\rangle$).

We used two coupled equations [(5) and (6)] to calculate the CRLW of heavy and light holes where we assumed $E_n^h-E_n^l=E_{n+1}^h-E_{n+1}^l$. The variation of the CRLW versus magnetic field at constant temperature $T=10$ K is given in Fig. 1. The curves *A* and *B* correspond to heavy and light holes, respectively. The results of CRLW for heavy (curve *A*) and light (curve *B*) holes versus temperature at a constant magnetic field $B=10$ T are presented in Fig. 2. The other parameters are $a=b=30$ Å and $W=0.05$ meV. It is clear from Figs. 1 and 2 that the theory predicts the increase of CRLW with the increase of magnetic field and temperature, and that the CRLW of the light hole is larger than that of the heavy hole.

We studied the effect of internal strains for the heavy and light holes, respectively. The results are shown only for the light holes in Fig. 3. The CRLW versus magnetic field are calculated with strain (curve *B*) and without strain (curve *A*). The parameters for these curves are $T=10$ K, $a=b=30$ Å, $W=0.05$ eV. The results show that in the presence of internal strains, the magnitude of the CRLW increased. We also calculated the CRLW versus the MQWS period *d*. The calculations also predict the decrease of CRLW with the increase of period.

We investigated the effect of LA phonons ($t=1$) and TA phonons ($t=2, t=3$) on CRLW. To make discussion clear, let us write Eqs. (5) and (6) into two parts such as $\Gamma_{CR}^m=\Gamma_{CR}^m(\mathrm{LA})+\Gamma_{CR}^m(\mathrm{TA})$ where $\Gamma_{CR}(\mathrm{LA})$ and $\Gamma_{CR}(\mathrm{TA})$ are a contribution to CRLW due to LA phonons and TA phonons, respectively. The CRLW versus magnetic field for the light holes only is presented here in Fig. 4. In this figure, curve *A* corresponds to $\Gamma_{CR}^l(\mathrm{LA})$ and curve *B* represents Γ_{CR}^l. The results are calculated by using the parameters as above. To our surprise the contribution of TA phonons to CRLW is almost equal to 10%–15% at low magnetic field and to 5% to 6% at high magnetic field, approximately. This is the opposite situation from the usually studied case of electron-phonon interaction[4] where the contribution of transverse phonons to Γ_{CR}^m is zero. It is because most of the theories considered the parabolic-band structure of electrons where the spectral intensity of TA phonons vanishes in the deformation-potential approximation. The value of $\Gamma_{CR}(\mathrm{TA})$ depends upon the choice of the deformation-potential constant. If one considers[8] the $D_b=1.7$ eV and $D_d=4.4$ eV, $\Gamma_{CR}(\mathrm{TA})$ is about 5% to 9% of the total CRLW value at low magnetic field and about 3% to 4% at high magnetic field. The value of the deformation-potential constant in the case of electrons lies between 7 and 16 eV and is very controversial in the case of quantum wells. Therefore, the value of $\Gamma_{CR}(\mathrm{TA})$ mentioned above may change according to the value of the deformation-potential constants. The contribution of TA phonons to CRLW depends on the values of deformation-potential constants and also other parameters such as magnetic field, temperature, and MQWS parameters.

We do not know any results where the experimentalists have calculated CRLW for 2D hole systems in their papers. Therefore we cannot compare the present theory with the experiments. We encourage the experimentalist to calculate the CRLW in *p*-type modulation-doped MQWS where our theory can be applied. We compared the present theoretical results with the previous CRLW of two-dimensional electron gas.[4] We found that the CRLW due to hole-phonon interaction is higher in magnitude than that of electron-phonon interaction. In other words the hole-phonon interaction in 2D hole systems is stronger than electron-phonon interaction in a 2D electron system. This prediction of theory is consistent with the experimental predictions.[9]

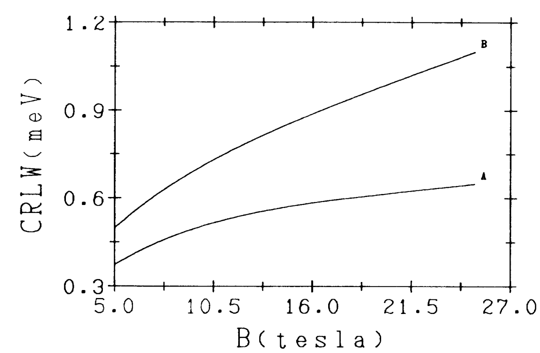

FIG. 3. CRLW vs magnetic field for the light hole. Curves *A* and *B* represent the longitudinal phonons and longitudinal and transverse phonon contribution to CRLW.

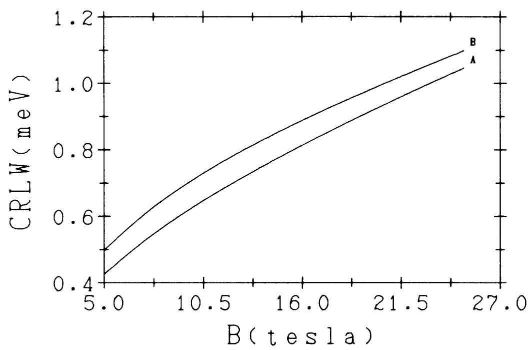

FIG. 4. CRLW vs magnetic field for light holes in the presence (curve B) and in the absence of strains (curve A). The temperature is constant $T=10$ K.

Finally, we would like to comment on the TA-phonon contribution and the layering effects on the hole-phonon interaction. We will mention some of the works from the literature where the experiments show that TA phonon contributes to carrier-phonon interaction and the layering effects can be neglected.

Mendez, Price, and Heiblum[10] measured the temperature dependence of the electron mobility in GaAs-AlAs hetrostructures between temperature 4°–40°. They explained their experiments and those of others by considering the theory of 2D electrons–3D phonons and neglecting the effect of layering on the acoustic phonons. The above approximation in the theory was supported by the results of magnetophonon experiments.[11] According to their results the contribution of TA phonons into electron-phonon interactions should be included. Hirakawa and Sakaki[12] explained their magnetotransport measurements of GaAs-AlAs system at low temperatures by using the 2D-electron–3D-acoustic-phonon coupling theory, and included the coupling of LA and TA phonons. Hensel, Dynes, and Tsui[13] reported the absorption of the ballastic phonons in 2D or Si metal-oxide-semiconductor field-effect transistor. They observed the coupling TA and LA phonons with the 2D electrons. There are many other experiments[14] where the coupling of electrons with TO phonons has also been reported in the literature along with LO phonons coupling. Xie, People, Bean, and Wecht[15] measured the power loss for 2D holes in strained $Ge_{0.2}Si_{0.2}$/Si system. They found excellent agreement with the experiment if they consider the scattering of 2D holes with the 3D acoustic mode phonons. They included the contribution of TA along with LA phonons in their analysis. They used the theory of electron theory to explain their hole experiments because there is no theory available except the present one. Recently there have been papers[16] where the effect of layering on the electron-optical phonon interaction is investigated. As far as I know there is no theoretical investigation of the above effect on electron-acoustic phonons. In principle, one should include this effect in hole-phonon interaction.

The author is thankful to Dr. S. Dassarma, Dr. Allen MacDonald, and Dr. S. R. Devreese for helpful discussions and to the National Science and Engineering Research Council, Canada, for the financial support in the form of a research grant.

[1]H. L. Störmer, A. M. Change, Z. Schlesinger, D. C. Tsui, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **51**, 126 (1983); A. Pinczuk, D. Heiman, R. Sooryakumar, A. C. Gossard, and W. Wiemann, Surf. Sci. **170**, 573 (1986); Z. Schlesinger, S. J. Allen, Y. Yafet, A. C. Gossard, and W. Wiegmann, Phys. Rev. B **32**, 5231 (1985); E. E. Mendez, W. I. Wang, L. L. Chang, and L. Esaki, *ibid.* **30**, 1087 (1984); G. Landwehr, in *Proceedings of the International Conference on the Application of High Magnetic Field in Semiconductor Physics, Würtzburg, West Germany,* edited by G. Landwehr (Springer Verlag, Heidelberg, 1987), and references therein; Y. Iwasa, N. Miura, S. Takeyama and T. Ando, *ibid.*, and references therein.

[2]D. A. Broido and L. J. Sham, Phys. Rev. B **31**, 888 (1985); V. Ekenberg and M. Altarelli, *ibid.* **30**, 3569 (1984); T. Ando, J. Phys. Soc. Jpn. **54**, 1528 (1985); E. Bangert and G. Landwehr, Surf. Sci. **170**, 593 (1986).

[3]M. Singh, Phys. Rev. B **36**, 1178 (1987).

[4]M. Singh, Phys. Rev. B **35**, 9301 (1987).

[5]M. Prasad and M. Singh, Phys. Rev. B **27**, 4803 (1984); M. Singh and M. P. Chaubey, *ibid.* **34**, 4026 (1986); M. P. Chaubey and M. Singh, *ibid.* **34**, 2385 (1986), and references therein.

[6]M. Singh and P. R. Wallace, J. Phys. C **16**, 3877 (1983).

[7]R. N. Bhargva and M. I. Nathan, Phys. Rev. **172**, 816 (1968).

[8]I. Blaslev, Solid State Commun. **5**, 315 (1967).

[9]J. Shah, A. Pinczuk, H. L. Störmer, A. L. Gossard, and W. Wiegmann, in *Proceedings of the 17th International Conference on the Physics of Semiconductors, San Francisco, CA, 1984* edited by D. J. Chadi and W. A. Harrison (Springer-Verlag, New York, 1985), p. 345.

[10]E. E. Mendez, P. J. Price, and M. Heiblum, Appl. Phys. Lett. **45**, 294 (1984), and references therein.

[11]D. C. Tsui, T. Englert, A.-Y. Cho, and A. C. Gossard, Phys. Rev. Lett. **44**, 341 (1980); T. Englert, D. C. Tsui, J. C. Portal, J. Beerens, and A. C. Gossard, Solid State Commun. **44**, 1301 (1982).

[12]K. Hirakawa and H. Sakaki, Appl. Phys. Lett. **49**, 889 (1986).

[13]J. C. Hensel, R. C. Dynes, and D. C. Tsui, Phys. Rev. B **28**, 1128 (1983).

[14]B. Vinter, Surf. Sci. **170**, 445 (1986); K. Kash, J. Shah, D. Block, A. C. Gossard, and W. Wiegmann, Physica **134B+C**, 189 (1985); L. C. Brunel, S. Haunt, R. J. Nicolas, M. A. Hopkins, M. A. Brummel, K. Karrai, J. C. Portal, M. Razeghi, K. Y. Chung, and A. Y. Cho, Surf. Sci. **170**, 582 (1986), and references therein; J. E. Zuker, A. Pinczuk, D. S. Chemla, A. C. Gossard, and W. Wiegmann, Phys. Rev. Lett. **53**, 1280 (1984).

[15]Y. H. Xie, R. People, J C Bean, and K. W. Wecht, Appl. Phys. Lett. **49**, 283 (1986).

[16]N. Sawaki and I. Akasaki, Physica B **134**, 494 (1984); S. Bakers, I. L. Mertz *et al.*, Phys. Rev. B **17**, 3181 (1978); N. Sawaki, Surf. Sci. **170**, 537 (1986); J. Phys. C **19**, 4965 (1986), and references therein.