PHYSICAL REVIEW B VOLUME 36, NUMBER 9 15 SEPTEMBER 1987-II

Dipole radiation in a multilayer geometry

C. E. Reed and J. Giergiel
Department of Physics, School of Physical Sciences, University of California, Irvine, California 92717

J. C. Hemminger
Department of Chemistry, School of Physical Sciences, University of California, Irvine, California 92717

S. Ushioda
Research Institute of Electrical Communication, Tohoku University, Sendai 980, Japan
(Received 27 April 1987)

There are several kinds of experiments that can be done with multilayer stacks of dielectric media which require an understanding of light emission by sources within the stack for their analysis. These experiments may involve, for example, light-emitting tunnel junctions, Raman scattering in Kretschmann and other multilayered geometries, and Rayleigh scattering by small amounts of surface or interface roughness, either alone or in combination with other processes. A set of electromagnetic Green's functions for a multilayer stack of isotropic dielectric media [D. L. Mills and A. A. Maradudin, Phys. Rev. B **12**, 2943 (1975)] gives the electric fields produced everywhere by a point source of current oscillating at a frequency f. These Green's functions can thus be used to solve this type of problem. In this paper we show how these Green's functions can be written in terms of 2×2 transfer matrices of the type commonly used to find the fields in a dielectric stack due to an incident plane wave. With this simplification we can easily evaluate the Green's functions for a stack with an arbitrary number of layers. We further show that, when the electric fields generated by a point source within the stack are evaluated far away, they can be written directly in terms of the electric fields that would be generated at the location of the current source by plane waves incident from the direction of the observation point. We show that this follows from the Lorentz reciprocity theorem. Thus, in this case the formalism of Green's functions is not needed.

I. INTRODUCTION

Understanding the propagation and generation of light in multilayered structures is necessary in order to correctly interpret experiments with light emission from metal-oxide-metal tunnel junctions, or with light scattering in multilayered structures. In the case of light scattering, one must first calculate the electric fields due to the incident beam at the locations of the scatterers in the multilayered structures. Then the oscillating electric dipoles of the scatterers radiate light into the multilayer structure. Thus one needs to know how to calculate the fields or the power radiated by oscillating dipoles in the structures. In light-emitting tunnel junctions, the radiation source is current fluctuations due to the tunneling currents, and the surrounding medium is multilayered by the nature of the device. Thus analyses of these experiments require calculation of local (macroscopic) fields in a multilayered stack due to oscillating electric dipoles located in the stack.

A theoretical framework to deal with these problems has been developed by Maradudin and Mills (MM).[1,2] They and others have addressed both these problems for various geometries. One example of such a geometry is an experiment with a light-emitting tunnel junction on a prism coupler,[3] a structure consisting of five layers; the prism, an aluminum film, a very thin layer of aluminum oxide, a gold film, and air. Another example is a recent experiment on Raman scattering in Kretschmann geometry[4] using a sample with four layers; a prism, a silver film, an MgF_2 film, and a liquid Raman-scattering sample. In analyzing both these experiments, some of the present authors made attenuated total reflectivity measurements, and compared them to calculations of reflectivity done using the method of 2×2 transfer matrices, in order to characterize the structures. In the second experiment the method of 2×2 transfer matrices was also used to calculate the fields due to a plane wave incident from the semi-infinite prism. However, to calculate the fields radiated by the dipoles, it was necessary to use the results of the present paper without presenting a rigorous proof.

In this paper we show how to calculate the electromagnetic Green's functions for a general n-layered structure using the 2×2 transfer matrix method originally due to Abeles.[5] One can use these Green's functions based on 2×2 matrices to calculate the radiation fields emitted by an oscillating electric dipole located in a stack. We also show that the radiation fields due to such an oscillating dipole take on a simple form when observed in the top or bottom layer far away from the rest of the stack, and that they can be written in terms of the local electromagnetic fields created in the stack by an incident wave without any use of the apparatus of

Green's functions. This reciprocity is a consequence of the more general Lorentz reciprocity theorem, as shown in the Appendix.

This calculation is based upon a set of Green's functions derived by MM.[1,2] The Green's functions give the electric fields caused by a point source of current oscillating at frequency f and having arbitrary location and direction. The sinks and sources of charge necessary for such a point source of current to obey the continuity equation are assumed to exist.

We do not repeat the derivation[2] of the Green's functions of Mills and Maradudin. Since an oscillating electric dipole of frequency f can be thought of as a point source of current together with the necessary sources and sinks of electric charge, the Green's functions of MM solve the present problem completely. However, we rewrite the Green's functions presented by MM in terms of 2×2 transfer matrices, a form which is well adapted to computer calculation.

The method of stationary phase is then used to find the limiting form of the electric field as $|z|\to\infty$, in much the same way that Lax and Mills[6] did. We then write out the explicit results for the electric field amplitude of the s- and p-polarized radiation emitted into the (θ,φ) direction by an oscillating electric dipole $\mathbf{p}_0 e^{-i\omega t}$ located at $\mathbf{r}=(0,0,z)$ in the dielectric stack. We compare this result with the electric field $\mathbf{E}(\mathbf{r})$ at $\mathbf{r}=(0,0,z)$ produced by s- and p-polarized radiation incident from the (θ,φ) direction as calculated using 2×2 transfer matrices. There is an exact reciprocity between these two results, in the sense that

$$\mathbf{E}(\mathbf{r})=\underline{A}\begin{bmatrix}E_s^{\rm in}\\E_p^{\rm in}\end{bmatrix}e^{i\mathbf{k}_\parallel\cdot\mathbf{x}_\parallel},$$

and

$$\begin{bmatrix}E_s^{\rm out}\\E_p^{\rm out}\end{bmatrix}=\frac{\omega^2}{c^2}\underline{A}^T\mathbf{p}_0,$$

where the matrix $\underline{A}^T$ is the transpose of $\underline{A}$, $E_s^{\rm out}$ and $E_p^{\rm out}$ are the amplitudes of the s- and p-polarized light emitted into the (θ,φ) direction, respectively, as seen from far away, and $E_s^{\rm in}$ and $E_p^{\rm in}$ are the amplitudes of the s- and p-polarized light of a plane wave incident from the (θ,φ) direction. This is not surprising; in fact, it follows from the Lorentz reciprocity theorem. This relation serves as a useful check on the algebra. The application of this method to Raman scattering in Kretschmann geometry in ultra-high vacuum will be treated in a forthcoming paper.[7]

Sipe[8] gave an expression in terms of transfer matrices for the power radiated by an arbitrarily oriented point dipole located in the bottom layer of a stack of layered media. He arrived at his result by a direct and intuitive approach which is justified by the Lorentz reciprocity theorem. He stated that it is easy to generalize his formulas to the case of a dipole located within any layer but did not do so.

Finally, we note that the Green's functions of MM can be applied to several kinds of problems involving surface roughness.[1,2,8–10]

II. A RADIATING ELECTRIC DIPOLE IN A MULTILAYER

In this section we indicate how the radiation emitted by an arbitrarily oriented electric dipole oscillating at frequency $f=\omega/2\pi$ and located within a stack of dielectrics (see Fig. 1) can be calculated. This calculation is purely classical. The dielectric layer j is assumed to be described by a local frequency-dependent isotropic dielectric constant $\tilde{\epsilon}(\omega)$, which may be complex.

The dielectric layers are assumed to be stacked along the z direction, as shown in Fig. 1, with layer 1 extending to $z=\infty$ and layer n to $z=-\infty$. The interface between layer j and layer $j+1$ is the plane $z=z_j+1$. We also take $z_1\equiv z_2$ for convenience. We are interested in the amount of radiation emitted by the dipole into the (θ,φ) where $\varphi=\arctan(y/x)$ and $\theta=\arctan[z/(x^2+y^2)^{1/2}]$.

We first consider an oscillating electric dipole having moment $\mathbf{p}(t)=\mathbf{p}_0e^{-i\omega t}$ located at $\mathbf{r}=\mathbf{r}_0$ in any isotropic medium. Consider the dipole as consisting of two charges of opposite sign of size $\rho(\mathbf{r},t)=|\mathbf{p}(t)|/|\Delta\mathbf{r}|$, where $\Delta\mathbf{r}$ is their relative displacement, and take the limit $|\Delta\mathbf{r}|\to0$. Then the charge distribution is

$$\begin{aligned}\rho(\mathbf{r},t)&=\lim_{|\Delta\mathbf{r}|\to0}\left\{\frac{|\mathbf{p}_0|}{|\Delta\mathbf{r}|}e^{-i\omega t}[\delta(\mathbf{r}-(\mathbf{r}_0+\Delta\mathbf{r}))-\delta(\mathbf{r}-\mathbf{r}_0)]\right\}\\&=|\mathbf{p}_0|e^{-i\omega t}\nabla\delta(\mathbf{r}-\mathbf{r}_0)\cdot\frac{-\Delta\mathbf{r}}{|\Delta\mathbf{r}|}\ .\end{aligned}\tag{1}$$

Therefore $\rho(\mathbf{r},t)=-[\nabla\delta(\mathbf{r}-\mathbf{r}_0)]\cdot\mathbf{p}_0e^{-i\omega t}$.

We now find the current density corresponding to this dipole using the continuity equation $\nabla\cdot\mathbf{J}+\partial\rho/\partial t=0$. This yields

$$\sum_v\frac{\partial J_v}{\partial r_v}=-i\omega\sum_v\frac{\partial\delta(\mathbf{r}-\mathbf{r}_0)}{\partial r_v}p_{0v}e^{-i\omega t}\ .$$

There are an infinite number of possible solutions to this equation, we use the simplest;

$$\mathbf{J}=-i\omega\delta(\mathbf{r}-\mathbf{r}_0)\mathbf{p}_0e^{-i\omega t}\ .\tag{2}$$

Although we do not derive the Green's functions of MM, we now produce the equation which they solve. Start with Maxwell's equations in Gaussian units:

$$\nabla\cdot\mathbf{D}=4\pi\rho\ ,\tag{3a}$$

$$\nabla\cdot\mathbf{B}=0\ ,\tag{3b}$$

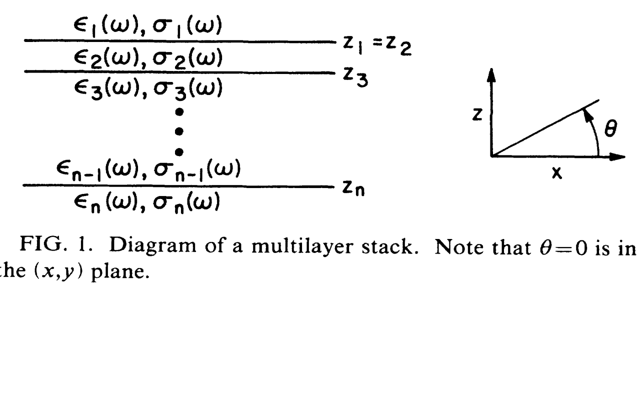

FIG. 1. Diagram of a multilayer stack. Note that $\theta=0$ is in the (x,y) plane.

$$\nabla\times\mathbf{E}(\mathbf{r},t)+\frac{1}{c}\frac{\partial\mathbf{B}(\mathbf{r},t)}{\partial t}=0\ , \tag{3c}$$

$$\nabla\times\mathbf{H}(\mathbf{r},t)-\frac{1}{c}\frac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t}=\frac{4\pi}{c}\mathbf{J}(\mathbf{r},t)\ . \tag{3d}$$

In our case all the fields and sources will have $e^{-i\omega t}$ time dependence. We assume that all media have magnetic permeability $\mu=1$, that $\mathbf{D}(\mathbf{r},\omega)=\epsilon(\mathbf{r})\mathbf{E}(\mathbf{r},\omega)$, and that some of the media may have finite conductivity $\sigma(\mathbf{r})$. The current $\mathbf{J}$ is given by $\mathbf{J}(\mathbf{r},\omega)=\sigma(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega)+\mathbf{J}_{\text{ext}}$. $\mathbf{J}_{\text{ext}}$ is the externally applied current, whose effect we are trying to determine. Finally, we assume that there are no charges except those required by the continuity equation $\nabla\cdot\mathbf{J}+\partial\rho/\partial t=0$. Thus $\mathbf{J}_{\text{ext}}$ requires a charge distribution ρ_{ext} and the nonzero conductivity requires a charge distribution ρ_s. That is, $\rho=\rho_{\text{ext}}+\rho_s$. Putting this together

$$\nabla\cdot[\epsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega)]=4\pi[\rho_{\text{ext}}(\mathbf{r},\omega)+\rho_s(\mathbf{r},\omega)]\ , \tag{4a}$$

$$\nabla\cdot\mathbf{B}(\mathbf{r},\omega)=0\ , \tag{4b}$$

$$\nabla\times\mathbf{E}(\mathbf{r},\omega)=\frac{i\omega}{c}\mathbf{B}(\mathbf{r},\omega)\ , \tag{4c}$$

$$\nabla\times\mathbf{B}(\mathbf{r},\omega)=-\frac{i\omega}{c}\tilde{\epsilon}(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega)+\frac{4\pi}{c}\mathbf{J}_{\text{ext}}(\mathbf{r},\omega)\ , \tag{4d}$$

where $\tilde{\epsilon}=\epsilon+4\pi i\sigma/\omega$.

Also, under our assumptions, Eq. (4a) can be rewritten as

$$\nabla\cdot[\tilde{\epsilon}(\mathbf{r}\omega)\mathbf{E}(\mathbf{r}\omega)]=4\pi\rho_{\text{ext}}(\mathbf{r}\omega)\ . \tag{4e}$$

Using Eq. (4c) to eliminate $\mathbf{B}$ from Eq. (4d), and writing the resulting equation in component form

$$\sum_{\mu}\left[-\frac{\partial^2}{\partial r_\mu\partial r_\lambda}+\nabla^2\delta_{\lambda\mu}+\frac{\omega^2}{c^2}\delta_{\lambda\mu}\tilde{\epsilon}(\mathbf{r},\omega)\right]E_\mu(\mathbf{r},\omega)=\frac{-4\pi i\omega}{c^2}J_{\lambda\text{ext}}(\mathbf{r},\omega) \tag{5}$$

is the end result. Thus Eqs. (5) and (4e) are the equations for $\mathbf{E}$. However, they are not independent. Any $\mathbf{E}$ that satisfies Eq. (5) will also satisfy Eq. (4e). The Green's functions of MM form a tensor $\underline{D}(\mathbf{r},\mathbf{r}',\omega)$. They are defined as the solutions of

$$\sum_{\mu}\left[\frac{\omega^2}{c^2}\delta_{\lambda\mu}\tilde{\epsilon}(\mathbf{r},\omega)-\frac{\partial^2}{\partial r_\lambda\partial r_\mu}+\nabla^2\delta_{\lambda\mu}\right]D_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)=4\pi\delta_{\lambda\nu}\delta(\mathbf{r}-\mathbf{r}')\ . \tag{6}$$

Multiplying both sides of Eq. (6) by $-i(\omega/c^2)\mathbf{J}_{\text{ext}}(\mathbf{r}',\omega)$, integrating the resulting equation over all $\mathbf{r}'$, and summing over the index ν, one finds that a solution of Eq. (5) is

$$E_\mu(\mathbf{r},\omega)=\frac{-i\omega}{c^2}\sum_\nu\int D_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)J_{\nu\text{ext}}(\mathbf{r}',\omega)d^3\mathbf{r}'\ . \tag{7}$$

In particular, if the external current results from an oscillating electric dipole located at $\mathbf{r}'$, Eq. (2) implies

$$E_\mu(\mathbf{r},\omega)=-\frac{\omega^2}{c^2}\sum_\nu D_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)p_{0\nu}\ . \tag{8}$$

This physical situation has translational symmetry in the x and y directions and therefore we can write[1,2]

$$D_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)=D_{\mu\nu}(\mathbf{x}_\parallel-\mathbf{x}_\parallel';z,z';\omega)=\int e^{i\mathbf{k}_\parallel\cdot(\mathbf{x}_\parallel-\mathbf{x}_\parallel')}d_{\mu\nu}(\mathbf{k}_\parallel,\omega\,|\,z,z')\frac{d^2k_\parallel}{(2\pi)^2}\ , \tag{9}$$

where $\mathbf{x}_\parallel\equiv(x,y,0)$ and $\mathbf{k}_\parallel\equiv(k_x,k_y,0)$. Here, the functions $d_{\mu\nu}$ are two-dimensional Fourier transforms of the $D_{\mu\nu}$. When $\mathbf{k}_\parallel=\hat{\mathbf{x}}k_\parallel$, the $d_{\mu\nu}(\mathbf{k}_\parallel,\omega\,|\,z,z')$ take on a comparatively simple form, which we write, following Refs. 1 and 2, as $g_{\mu\nu}(k_\parallel,\omega\,|\,z,z')$. The general $d_{\mu\nu}$ may be written in terms of the $g_{\mu\nu}$ by making the following rotation of coordinates:

$$d_{\mu\nu}(\mathbf{k}_\parallel,\omega\,|\,z,z')=\sum_{\mu'\nu'}g_{\mu'\nu'}(\mathbf{k}_\parallel,\omega\,|\,z,z')S_{\mu'\mu}(\mathbf{k}_\parallel)S_{\nu'\nu}(\mathbf{k}_\parallel)\ , \tag{10}$$

$$\underline{S}(\mathbf{k}_\parallel)=\frac{1}{k_\parallel}\begin{bmatrix}k_x & k_y & 0\\ -k_y & k_x & 0\\ 0 & 0 & k_\parallel\end{bmatrix}\ , \tag{11}$$

where $k_\parallel=|\mathbf{k}_\parallel|$. Of the $g_{\mu\nu}$,

$$g_{xy}=g_{yx}=g_{zy}=g_{yz}=0\ . \tag{12a}$$

As in Ref. 2,

$$g_{xx}(k_\parallel,\omega\,|\,z,z')=\frac{4\pi}{W_\parallel(k_\parallel\omega)}[E_x^>(k_\parallel,\omega\,|\,z)E_x^<(k_\parallel,\omega\,|\,z')\Theta(z-z')+E_x^<(k_\parallel,\omega\,|\,z)E_x^>(k_\parallel,\omega\,|\,z')\Theta(z'-z)]\ , \tag{12b}$$

$$g_{zx}(k_\parallel,\omega\,|\,z,z')=\frac{4\pi}{W_\parallel(k_\parallel,\omega)}[E_z^>(k_\parallel,\omega\,|\,z)E_x^<(k_\parallel,\omega\,|\,z')\Theta(z-z')+E_z^<(k_\parallel,\omega\,|\,z)E_x^>(k_\parallel,\omega\,|\,z')\Theta(z'-z)]\ , \tag{12c}$$

$$g_{xz}(k_\parallel,\omega\,|\,z,z')=\frac{-4\pi}{W_\parallel(k_\parallel,\omega)}[E_x^>(k_\parallel,\omega\,|\,z)E_z^<(k_\parallel,\omega\,|\,z')\Theta(z-z')+E_x^<(k_\parallel,\omega\,|\,z)E_z^>(k_\parallel,\omega\,|\,z')\Theta(z'-z)]\ , \tag{12d}$$

$$g_{zz}(k_\parallel,\omega\,|\,z,z')=\frac{4\pi c^2\delta(z-z')}{\omega^2\epsilon_0(z',\omega)}-\frac{4\pi}{W_\parallel(k_\parallel,\omega)}[E_z^>(k_\parallel,\omega\,|\,z)E_z^<(k_\parallel,\omega\,|\,z')\Theta(z-z')+E_z^<(k_\parallel,\omega\,|\,z)E_z^>(k_\parallel,\omega\,|\,z')\Theta(z'-z)]\ , \tag{12e}$$

$$g_{yy}(k_\parallel,\omega\,|\,z,z')=\frac{4\pi}{W_\perp(k_\parallel,\omega)}[E_y^>(k_\parallel,\omega\,|\,z)E_y^<(k_\parallel,\omega\,|\,z')\Theta(z-z')+E_y^<(k_\parallel,\omega\,|\,z)E_y^>(k_\parallel,\omega\,|\,z')\Theta(z'-z)]\ , \tag{12f}$$

where $\Theta(z)$ is the Heaviside unit step function

$$\Theta(z)\equiv\begin{cases}1, & z\geq 0\\ 0, & z<0\ ,\end{cases}$$

$W_\perp(k_\parallel,\omega)$ and $W_\parallel(\mathbf{k}_\parallel,\omega)$ are Wronskians and do not depend on z. They are given by

$$W_\perp(k_\parallel,\omega)=W_{yy}(k_\parallel,\omega\,|\,z)\ , \tag{13}$$

$$W_\parallel(k_\parallel,\omega)=W_{xx}(k_\parallel,\omega\,|\,z)-W_{zz}(k_\parallel,\omega\,|\,z)\ , \tag{14}$$

where

$$W_{\mu\mu}(k_\parallel,\omega\,|\,z)=\frac{\partial E_\mu^>(k_\parallel,\omega\,|\,z)}{\partial z}E_\mu^<(k_\parallel,\omega\,|\,z)-\frac{\partial E_\mu^<(k_\parallel,\omega\,|\,z)}{\partial z}E_\mu^>(k_\parallel,\omega\,|\,z)\ . \tag{15}$$

In the following discussion we often suppress the argument $(k_\parallel,\omega)$. By $E_\mu^>(k_\parallel,\omega\,|\,z)$ we mean plane-wave solutions of Eq. (5) with $\mathbf{J}_{\text{ext}}=\mathbf{0}$ which obey boundary conditions (BC's) such that as z approaches positive infinity the plane-wave solution represent an outgoing plane wave [if $k_z=(\epsilon\omega^2/c^2-k_\parallel^2)^{1/2}$ is real] or approaches zero (if k_z is complex). Similarly by $E_\mu^<(k_\parallel\omega\,|\,z)$ we mean plane-wave solutions of Eq. (5) with $\mathbf{J}_{\text{ext}}=\mathbf{0}$ which obey the analogous BC's as z goes to negative infinity. We use the convention for taking the square root of k_z^2 such that

$$\text{Im}k_z\geq 0\ , \tag{16a}$$

and

$$\text{Im}k_z=0\Longrightarrow\text{Re}k_z\geq 0\ . \tag{16b}$$

In Sec. III we introduce the 2×2 transfer matrices which will be used later to find explicit forms for the $\mathbf{E}^>$, the $\mathbf{E}^<$, and thus $W_\parallel$ and $W_\perp$.

III. TRANSFER MATRICES

All the homogeneous solutions of Eq. (5) can be constructed by Fourier analyzing them into superpositions of plane-wave solutions. General plane-wave solutions can be constructed from the plane-wave solutions with $\mathbf{k}$ lying in the (x,z) plane by rotating coordinates. Consider such a plane-wave solution. The electric field in layer j is given by

$$\mathbf{E}^{(j)}(\mathbf{r})=[\mathbf{K}_+^{(j)}(\mathbf{k}_\parallel)e^{ik_z^{(j)}z}+\mathbf{K}_-^{(j)}(\mathbf{k}_\parallel)e^{-ik_z^{(j)}z}]e^{i(\mathbf{k}_\parallel\cdot\mathbf{x}_\parallel-\omega t)}\ . \tag{17}$$

We take $\mathbf{k}_\parallel=(k_x,0,0)$ since we assume $\mathbf{k}$ to be in the xz plane. We find the explicit values of $\mathbf{K}_\pm$ in Eq. (17) using the method of 2×2 "transfer matrices." This method, which we believe was first described by Abeles,[5] has been presented in numerous slightly different forms. For the sake of completeness we sketch its derivation and then present the form we are using, which was chosen to be compatible with the notation of MM. First we note that applying $\nabla\cdot\mathbf{E}=0$ to Eq. (17) and requiring the resulting equation to be true for all z in layer j yields

$$k_\parallel K_{+x}^{(j)}+k_z^{(j)}K_{+z}^{(j)}=0\ , \tag{18a}$$

$$k_\parallel K_{-x}^{(j)}-k_z^{(j)}K_{-z}^{(j)}=0\ . \tag{18b}$$

Equation (18) can be used to eliminate (for example) $K_{\pm z}$ in each layer j. At each boundary one requires that $E_\parallel$, $D_\perp$, and $H_\parallel$ be continuous. On writing these boundary conditions for the $K_{\pm\mu}$ one gets four linear equations for each interface. They are

$$K_{+x}^{(j)}e^{\alpha_j z_{j+1}}+K_{-x}^{(j)}e^{-\alpha_j z_{j+1}}=K_{+x}^{(j+1)}e^{\alpha_{j+1}z_{j+1}}+K_{-x}^{(j+1)}e^{-\alpha_{j+1}z_{j+1}}\ , \tag{19a}$$

$$K_{+y}^{(j)}e^{\alpha_j z_{j+1}}+K_{-y}^{(j)}e^{-\alpha_j z_{j+1}}=K_{+y}^{(j+1)}e^{\alpha_{j+1}z_{j+1}}+K_{-y}^{(j+1)}e^{-\alpha_{j+1}z_{j+1}}\ , \tag{19b}$$

$$\epsilon_j(K_{+z}^{(j)}e^{\alpha_j z_{j+1}}+K_{-z}^{(j)}e^{-\alpha_j z_{j+1}})=\epsilon_{j+1}(K_{+z}^{(j+1)}e^{\alpha_{j+1}z_{j+1}}+K_{-z}^{(j+1)}e^{-\alpha_{j+1}z_{j+1}})\ , \tag{19c}$$

$$k_z^j(K_{+y}^{(j)}e^{\alpha_j z_{j+1}}-K_{-y}^{(j)}e^{-\alpha_j z_{j+1}})=k_z^{j+1}(K_{+y}^{(j+1)}e^{\alpha_{j+1}z_{j+1}}-K_{-y}^{(j+1)}e^{-\alpha_{j+1}z_{j+1}})\ , \tag{19d}$$

where we have used $\alpha_j\equiv ik_z(j)$ to simplify the notation. Equations (19) were obtained from the continuity of E_x, E_y, D_z, and H_x, respectively. On inspecting Eqs. (18) and (19) one notes that the $\mathbf{K}_{\pm y}$ are decoupled from the $K_{\pm x}$ and $K_{\pm z}$. Thus we can express any solution for the $K_\pm$ as a linear combination of two kinds of solutions, one having $K_{\pm x}=K_{\pm z}=0$ and the other having $K_{\pm y}=0$. They are of course the well-known "s-polarized" or "transverse electric" (TE) and the "p-polarized" or "transverse magnetic" (TM) solutions, respectively.

Consider the TE solutions having $K_{\pm x}=K_{\pm z}=0$ in all layers. In any particular layer j, the electric field is completely determined by the two numbers $K_{+y}^{(i)}$ and $K_{-y}^{(j)}$. But by solving Eq. (19) one can find $K_{\pm y}^{(j+1)}$, and, if there is a layer $j-1$ one can replace j by $j-1$ in Eqs. (19b) and (19d) and solve the resulting equations for $K_{\pm,y}^{(j-1)}$. Thus the two numbers $K_{+y}^{(j)}$ and $K_{-y}^{(j)}$ determine the electric field in all the layers for the TE solutions.

Similarly, for the TM solutions, we can use Eq. (18) to eliminate $K_{\pm z}^{(j)}$ in favor of $K_{\pm x}^{(j)}$ in each layer j and also in the boundary conditions [Eqs. (19a) and (19c)]. Then again the two equations (19a) and (19c) suffice to determine $K_{\pm x}^{(j+1)}$ given $K_{\pm x}^{(j)}$, and, if there is a layer $j-1$, replacing j by $j-1$ lets one find $K_{\pm x}^{(j-1)}$. Thus, again, repeating this process lets one find $K_{\pm x}^{(j)}$ in all layers j.

If one represents $K_{\pm y}^{(j)}$ as a two-element vector

$$\begin{bmatrix} K_{+y}^{(j)} \\ K_{-y}^{(j)} \end{bmatrix}$$

in each layer j, then since the two equations (19b) and (19d) are linear and determine a two-element vector as a function of a two-element vector, it follows that we can represent their effect by a 2×2 matrix, so that

$$\begin{bmatrix} K_{+y}^{(j+1)} \\ K_{-y}^{(j+1)} \end{bmatrix} = \underline{M}_y^{(j+1,j)} \begin{bmatrix} K_{+y}^{(j)} \\ K_{-y}^{(j)} \end{bmatrix} , \tag{20a}$$

and also

$$\begin{bmatrix} K_{+y}^{(j-1)} \\ K_{-y}^{(j-1)} \end{bmatrix} = \underline{M}_y^{(j-1,j)} \begin{bmatrix} K_{+y}^{(j)} \\ K_{-y}^{(j)} \end{bmatrix} , \tag{20b}$$

where

$$\underline{M}_{y,(j-1,j)} \equiv (\underline{M}_{y,j,j-1})^{-1} \tag{21}$$

is a 2×2 matrix.

We can do the same thing with the K_x coefficient:

$$\begin{bmatrix} K_{+x}^{(j\pm1)} \\ K_{-x}^{(j\pm1)} \end{bmatrix} = \underline{M}_x^{(j\pm1,j)} \begin{bmatrix} K_{+x}^{(j)} \\ K_{-x}^{(j)} \end{bmatrix} , \tag{22}$$

where again $\underline{M}_{x,(j-1,j)} = (\underline{M}_{x,j,j-1})^{-1}$.

And if one wishes, one can use Eq. (18) to eliminate the $K_{\pm x}^{(j)}$ so that instead of Eq. (22) one can use

$$\begin{bmatrix} K_{+z}^{(j\pm1)} \\ K_{-z}^{(j\pm1)} \end{bmatrix} = \underline{M}_z^{(j\pm1,j)} \begin{bmatrix} K_{+z}^{(j)} \\ K_{-z}^{(j)} \end{bmatrix} . \tag{23}$$

Repeated application of these equations lets us write

$$\begin{bmatrix} K_{+\mu}^{(j)} \\ K_{-\mu}^{(j)} \end{bmatrix} = \underline{M}_\mu^{(j,k)} \begin{bmatrix} K_{+\mu}^{(k)} \\ K_{-\mu}^{(k)} \end{bmatrix} , \tag{24}$$

where

$$\underline{M}_\mu^{(j,l)} \equiv \prod_{k=j-1}^{l} \underline{M}_\mu^{(k+1,k)}, \quad \text{if } j>l \tag{25a}$$

and

$$\underline{M}_\mu^{(j,l)} \equiv \prod_{k=j+1}^{l} \underline{M}_\mu^{(k-1,k)}, \quad \text{if } j<l \ . \tag{25b}$$

We will not explicitly derive the matrices $\underline{M}_\mu^{(j\pm1,j)}$ here, but will simply state them. They are

$$\underline{M}_\mu^{(j\pm1,j)} = \tfrac{1}{2} g'_\mu \begin{bmatrix} (1+Z_\mu) e^{(\alpha_j-\alpha_{j\pm1})z_>} & f_\mu (1-Z_\mu) e^{-(\alpha_j+\alpha_{j\pm1})z_>} \\ f_\mu (1-Z_\mu) e^{(\alpha_j+\alpha_{j\pm1})z_>} & (1+Z_\mu) e^{(\alpha_{j\pm1}-\alpha_j)z_>} \end{bmatrix} , \tag{26}$$

where the quantities Z_μ are optical impedances and are given by

$$Z_\mu \equiv \begin{cases} \alpha_j/\alpha_{j\pm1}, & \text{for } \mu=y \quad (27a) \\ \dfrac{\epsilon_j \alpha_{j\pm1}}{\epsilon_{j\pm1}\alpha_j}, & \text{for } \mu=x \text{ or } z \ . \quad (27b) \end{cases}$$

The other quantities are given by

$$z_> \equiv z_j, \quad j=\max\{j, j\pm1\} \tag{28}$$

$$g'_\mu \equiv \begin{cases} 1, & \mu=x \text{ or } y \quad (29a) \\ \alpha_j/\alpha_{j\pm1}, & \mu=z \quad (29b) \end{cases}$$

and

$$f_\mu \equiv \begin{cases} 1, & \mu=x \text{ or } y \quad (30a) \\ -1, & \mu=z \ . \quad (30b) \end{cases}$$

IV. TRANSFER MATRIX FORMULATION OF THE GREEN'S FUNCTIONS

In this section we write the basic quantities $\mathbf{E}^>$, $\mathbf{E}^<$, $W_\parallel$, and $W_\perp$, that appear in the Green's functions [Eqs. (9)–(15)] in terms of the transfer matrices derived in Sec. III. Start with $\mathbf{E}^>$ and $\mathbf{E}^<$. These can be written in the form analogous to that in Eq. (17)

$$E_\mu^>(z) = K_{+\mu}^{>(j)} e^{\alpha_j z} + K_{-\mu}^{>(j)} e^{-\alpha_j z} \quad \text{in layer } j \ , \tag{31}$$

and similarly for $K_{\pm,\mu}^<$. The equations in Sec. III hold also for both the $K_\mu^>$ and the $K_\mu^<$. In Eq. (31) the extra superscript $>$ on $K_{\pm\mu}^{(j)}$ signifies the boundary conditions these quantities have to satisfy. These conditions are

$$K_{-\mu}^{>(1)} = K_{+\mu}^{<(n)} = 0 \ , \tag{32a}$$

which follow from the desired outgoing boundary conditions and Eq. (16). We need to normalize the amplitude of $E_\mu^>(z)$ and $E_\mu^<(z)$. The particular normalization we use will not affect the functions $g_{\mu\nu}$, because if $E_\mu^>(z)$ (say) is multiplied by a constant C, so is the Wronskian $W_\perp$ (if $\mu=y$) or $W_\parallel$ (if $\mu=x$ or z). Then by examining Eq. (12) we see that the $g_{\mu\nu}$ and thus the Green's functions $D_{\mu\nu}$ will remain unchanged. Following MM we choose

$$1 = K_{+y}^{>(1)} = K_{+z}^{>(1)} = K_{-y}^{<(n)} = K_{-z}^{<(n)} \ . \tag{32b}$$

Then from Eqs. (18a) and (18b)

$$K_{+x}^{>(1)} = -\frac{k_z^{(1)}}{k_\parallel} \ , \tag{32c}$$

$$K_{-x}^{<(n)}=\frac{k_z^{(n)}}{k_\parallel} . \tag{32d}$$

Thus, substituting Eq. (32) into Eq. (24) and then Eq. (24) into Eq. (31) yields

$$E_\mu^{>(j)}(z)=[(\underline{M}_\mu^{(j,1)})_{1,1}e^{\alpha_j z}+(\underline{M}_\mu^{(j,1)})_{2,1}e^{-\alpha_j z}]K_{+\mu}^{>(1)} , \tag{33a}$$

$$E_\mu^{<(j)}(z)=[(\underline{M}_\mu^{(j,n)})_{1,2}e^{\alpha_j z}+(\underline{M}_\mu^{(j,n)})_{2,2}e^{-\alpha_j z}]K_{-\mu}^{<(n)} . \tag{33b}$$

We now evaluate the Wronskians. Substituting Eq. (31) into Eq. (15), we obtain

$$W_{\mu\mu}^{(j)}(z)=2\alpha_j(K_{+\mu}^{>(j)}K_{-\mu}^{<(j)}-K_{-\mu}^{>(j)}K_{+\mu}^{<(j)}) , \tag{34}$$

and after a little more algebra

$$W_{\mu\mu}^{(j)}=2\alpha_j K_{+\mu}^{>(1)}K_{-\mu}^{<(n)}\frac{(\underline{M}_\mu^{(1,n)})_{2,2}}{\|\underline{M}_\mu^{(1,j)}\|} .$$

From Eqs. (25)–(30) we have

$$|\underline{M}_x^{(j,l)}|=\frac{\epsilon_l\alpha_j}{\epsilon_j\alpha_l} , \tag{35a}$$

$$|\underline{M}_y^{(j,l)}|=\frac{\alpha_l}{\alpha_j} , \tag{35b}$$

$$|\underline{M}_z^{(j,l)}|=\frac{\epsilon_l\alpha_l}{\epsilon_j\alpha_j} . \tag{35c}$$

Using this together with Eqs. (32) we get

$$W_\perp\equiv W_{yy}^j=2\alpha_1(\underline{M}_y^{(1,n)})_{2,2} , \tag{36a}$$

and after some algebra

$$W_\parallel\equiv W_{xx}-W_{zz}=-2\frac{\epsilon_1\alpha_n\omega^2}{c^2k_\parallel^2}(\underline{M}_x^{(1,n)})_{2,2} . \tag{36b}$$

Note that $W_\perp(k_\parallel\omega)$ and $W_\parallel(k_\parallel\omega)$ are not functions of z. We have now defined all quantities needed to calculate the electric fields produced everywhere by an oscillating electric dipole in a layered stack of dielectrics.

V. FIELDS DUE TO A RADIATING DIPOLE

What we are interested in is the amount of light radiated into the (θ,ϕ) direction. By using the fact that the observation point is far away from the source the formulas can be simplified. We do this by using the method of stationary phase, as did Laks and Mills.[6]

Far away from the source the power flux is radial, which can be shown by the method of stationary phase. Therefore, the power flux per steradian is

$$\frac{dP}{d\Omega}=r^2|\mathbf{S}| , \tag{37}$$

where $\mathbf{S}$ is the Poynting vector.

If the fields are caused by an oscillating electric dipole of strength $\mathbf{p}=\mathbf{p}_0e^{-i\omega t}$ located at $\mathbf{r}'$, then $\mathbf{E}$ is given by Eq. (8) in which $D_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)$ is given by Eqs. (9)–(15). We now simplify the expression for $D_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)$.

From Eq. (32) it follows that in layers 1 and n $E_\mu^>(k_\parallel\omega\,|\,z)$ and $E_\mu^<(k_\parallel\omega\,|\,z)$, respectively, have the forms

$$E_\mu^>(k_\parallel\omega\,|\,z)=e^{ik_z^{(1)}z}K_{+\mu}^{>(1)}, \quad \text{if } z \text{ is in layer 1} \tag{38a}$$

$$E_\mu^<(k_\parallel\omega\,|\,z)=e^{-ik_z^{(n)}z}K_{-\mu}^{<(n)} \quad \text{if } z \text{ is in layer } n . \tag{38b}$$

Then from Eq. (12) it follows that the $g_{\mu\nu}(k_\parallel,\omega\,|\,z,z')$ have limiting forms

$$g_{\mu\nu}(k_\parallel,\omega\,|\,z,z')=e^{ik_z^{(1)}z}g'_{\mu\nu}(k_\parallel,\omega\,|\,z') \quad \text{if } z \text{ is in layer 1} . \tag{39a}$$

$$g_{\mu\nu}(k_\parallel,\omega\,|\,z,z')=e^{-ik_z^{(n)}z}g'_{\mu\nu}(k_\parallel,\omega\,|\,z') \quad \text{if } z \text{ is in layer } n . \tag{39b}$$

So the $g'_{\mu\nu}$ have no z dependence and are implicitly defined by Eq. (39). In the same way Eq. (10) implies that in layers 1 and n, $d_{\mu\nu}(\mathbf{k}_\parallel,\omega\,|\,z,z')$ has the form

$$d_{\mu\nu}(\mathbf{k}_\parallel,\omega\,|\,z,z')=\begin{cases} e^{ik_z^{(1)}z}d'_{\mu\nu}(\mathbf{k}_\parallel,\omega\,|\,z') & \text{if } z \text{ is in layer 1} \quad (40a)\\ e^{-ik_z^{(n)}z}d'_{\mu\nu}(\mathbf{k}_\parallel,\omega\,|\,z') & \text{if } z \text{ is in layer } n . \quad (40b)\end{cases}$$

Then defining

$$\mathbf{k}'=\begin{cases}(k_x,k_y,k_z^{(1)}) & \text{if } z \text{ is in layer 1}\\ (k_x,k_y,-k_z^{(n)}) & \text{if } z \text{ is in layer } n .\end{cases} \tag{41}$$

From Eq. (9) we obtain

$$D_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)=\int\frac{d^2k_\parallel}{(2\pi)^2}e^{i\mathbf{k}\cdot\mathbf{r}''}d'_{\mu\nu}(\mathbf{k}_\parallel,\omega\,|\,\mathbf{r}') ; \tag{42}$$

where

$$\mathbf{r}''\equiv(x-x',y-y',z) . \tag{43}$$

We now use the method of stationary phase similarly to Ref. 6 to get

$$D_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)\cong\frac{-ik\,|\sin\theta|}{2\pi r}e^{ikr}d'_{\mu\nu}(\mathbf{k},\theta,\phi,z') \tag{44}$$

for radiation into either the top or the bottom layer, where by $d'_{\mu\nu}(\mathbf{k},\theta,\phi,z')$, we mean $d'_{\mu\nu}(\mathbf{k}_\parallel,\omega,z')$ with

$$\mathbf{k}_\parallel=k(\cos\theta\cos\phi,\cos\theta\sin\phi,0) .$$

By substituting Eq. (44) into Eq. (8) we obtain

$$E_\lambda(\mathbf{r},\omega)=\frac{\omega^2}{c^2}\frac{ik}{2\pi r}|\sin\theta|e^{ikr}\sum_\nu d'_{\lambda\nu}(\mathbf{k},\theta,\phi,z')p_{0\nu} . \tag{45}$$

We use Eq. (7.13) of Ref. 11 for a plane wave in a medium of dielectric constant $\epsilon(\omega)$.

$$\mathbf{S}=\frac{c}{8\pi}(\epsilon/\mu)^{1/2}|\mathbf{E}_0|^2\hat{\mathbf{k}} ,$$

where $\mathbf{S}$ is the Poynting vector. Applying this to Eq. (45) and using Eq. (37) yields

$$\frac{dP}{d\Omega}=\frac{c}{32}\left[\frac{\omega k}{\pi c}\right]^3\sin^2\theta\sum_{\lambda,\nu,\nu'}p_{0\nu}p_{0\nu'}^*d'_{\lambda\nu}d'^*_{\lambda\nu} . \tag{46}$$

where we have suppressed the arguments of the $d'_{\mu\nu}$. We can go further than Eq. (46) and obtain separately the amounts of s- and p-polarized light radiated into the $(\theta\phi)$ direction rather than merely obtaining the total power, as in Eq. (46). To do so recall that the $d'_{\mu\nu}$ depend upon the $g'_{\mu\nu}$ in the same way as the $d_{\mu\nu}$ depend on the $g_{\mu\nu}$. Thus replacing $d_{\mu\nu}$ and $g_{\mu\nu}$ with $d'_{\mu\nu}$ and $g'_{\mu\nu}$ in Eq. (10) and carrying out the matrix multiplications explicitly yields

$$d'(\mathbf{k},\theta,\phi,z)=\begin{bmatrix} c^2g'_{xx}+s^2g'_{yy} & cs(g'_{xx}-g'_{yy}) & cg'_{xz} \\ cs(g'_{xx}-g'_{yy}) & c^2g'_{yy}+s^2g'_{xx} & sg'_{xz} \\ cg'_{zx} & cg'_{zx} & g'_{zz} \end{bmatrix} , \quad (47)$$

where we used c for $\cos\phi$ and s for $\sin\phi$.

We have suppressed the arguments of the $g'_{\mu\nu}$. Define unit vectors

$$\hat{\mathbf{s}}\equiv\frac{\hat{\mathbf{z}}\times\mathbf{k}}{|\hat{\mathbf{z}}\times\mathbf{k}|} , \quad (48a)$$

$$\hat{\mathbf{p}}\equiv\frac{-\mathbf{k}\times\hat{\mathbf{s}}}{|\mathbf{k}\times\hat{\mathbf{s}}|} , \quad (48b)$$

which correspond to the directions of the electric field for s- and p-polarized light with wave vector $\mathbf{k}=k\hat{\mathbf{k}}$, respectively. In terms of θ and ϕ we have

$$\hat{\mathbf{s}}=(-\sin\phi,\cos\phi,0) , \quad (49a)$$

$$\hat{\mathbf{p}}=(\sin\theta\cos\phi,\sin\theta\sin\phi,-\cos\theta) . \quad (49b)$$

Then the light emitted by a radiating dipole as seen in layer 1 or n is

$$\mathbf{E}(\mathbf{r},\omega)=[\mathbf{E}(\mathbf{r},\omega)\cdot\hat{\mathbf{s}}]\hat{\mathbf{s}}+[\mathbf{E}(\mathbf{r},\omega)\cdot\hat{\mathbf{p}}]\hat{\mathbf{p}}+[\mathbf{E}(\mathbf{r},\omega)\cdot\hat{\mathbf{k}}]\hat{\mathbf{k}} .$$

But far away from the dipole $\mathbf{E}(\mathbf{r},\omega)\cdot\hat{\mathbf{k}}=0$. We define quantities E_s^{out} and E_p^{out} such that, far away from the radiation source,

$$\mathbf{E}(\mathbf{r},\omega)=E_s^{\text{out}}\frac{e^{ikr}}{r}\hat{\mathbf{s}}+E_p^{\text{out}}\frac{e^{ikr}}{r}\hat{\mathbf{p}} . \quad (50)$$

By comparison to Eq. (45), we find that

$$E_{s\mu}^{\text{out}}=\frac{ik\omega^2}{2\pi c^2}|\sin\theta|\sum_\nu d'_{\mu\nu}p_{0\nu}(\hat{\mathbf{s}}\cdot\hat{\boldsymbol{\mu}}) , \quad (51a)$$

$$E_{p\mu}^{\text{out}}=\frac{ik\omega^2}{2\pi c^2}|\sin\theta|\sum_\nu d'_{\mu\nu}p_{0\nu}(\hat{\mathbf{p}}\cdot\hat{\boldsymbol{\mu}}) , \quad (51b)$$

where $\hat{\boldsymbol{\mu}}$ is the unit vector in the μ direction. Substituting in from Eq. (47) one finds, after some algebra, that

$$E_s^{\text{out}}=\frac{ik\omega^2}{2\pi c^2}|\sin\theta|g'_{yy}[-\sin\phi,\cos\phi,0]\cdot p_0 , \quad (52a)$$

$$E_p^{\text{out}}=\frac{ik\omega^2}{2\pi c^2}|\sin\theta|[\cos\phi(\sin\theta g'_{xx}-\cos\theta g'_{zx}),\sin\phi(\sin\theta g'_{xx}-\cos\theta g'_{zx}),\sin\theta g'_{xz}-\cos\theta g'_{zz}]\cdot p_0 . \quad (52b)$$

By using $\mathbf{r}\cdot\mathbf{E}=0$, in Eqs. (45) and (47) one obtains

$$\cos\theta g'_{xz}+\sin\theta g'_{zz}=0 ,$$

$$\cos\theta g'_{xx}+\sin\theta g'_{zx}=0 .$$

These can be used to simplify Eq. (52b) by elimination of g'_{xz} and g'_{xx}. The result is

$$E_p^{\text{out}}=\frac{-\omega^2ik}{2\pi c^2}|\tan\theta|[g'_{zx}\cos\phi,g'_{zx}\sin\phi,g'_{zz}]\cdot\mathbf{p}_0 . \quad (52c)$$

We can write Eqs. (52a) and (52c) in terms of the more basic quantities $E_\mu^>(z)$, $E_\mu^<(z)$, $W_\parallel$, and $W_\perp$. Recall from Eqs. (12) that for g_{yy}, g_{zx}, and g_{zz} when $\mathbf{r}\neq\mathbf{r}'$

$$g_{\mu\nu}(z,z')=\pm\frac{4\pi}{W}[E_\mu^>(z)E_\nu^<(z')\theta(z-z')+E_\mu^<(z)E_\nu^>(z')\theta(z'-z)] ,$$

where $W=W_\perp$ for g_{yy} and $W=W_\parallel$ for g_{zx} and g_{zz}. The plus sign applies to g_{yy} and g_{zx} and the minus sign to g_{zz}. Recalling the implicit definition (Eq. 39) of the $g'_{\mu\nu}$ and the limiting forms (Eq. 38) of $E_\mu^>(z)$ and $E_\mu^<(z)$ in media 1 and n, respectively, it follows that for g'_{yy}, g'_{zx}, and g'_{zz} for $\mathbf{r}\neq\mathbf{r}'$,

$$g'_{\mu\nu}(z')=\begin{cases} f\dfrac{4\pi}{W}E_\nu^<(z'), & \text{in layer } 1 \quad (53a)\\[2ex] f\dfrac{4\pi}{W}E_\nu^>(z'), & \text{in layer } n , \quad (53b)\end{cases}$$

where $W=W_\perp$ for g'_{yy} and $W_\parallel$ for g'_{zx} and g'_{zz}.

$$f=\begin{cases} +1 & \text{for } g'_{yy} \text{ and } g'_{zx} \\ -1 & \text{for } g'_{zz} .\end{cases}$$

Using Eqs. (53) in Eqs. (52a) and (52c) we get

$$\begin{bmatrix} E_s^{\text{out}} \\ E_p^{\text{out}} \end{bmatrix}=-2ik_1\frac{\omega^2}{c^2}\begin{bmatrix} \sin\theta\sin\phi\dfrac{E_y^<(z')}{W_\perp} & -\sin\theta\cos\phi\dfrac{E_y^<(z')}{W_\perp} & 0 \\[2ex] \tan\theta\cos\phi\dfrac{E_x^<(z')}{W_\parallel} & \tan\theta\sin\phi\dfrac{E_x^<(z')}{W_\parallel} & -\tan\theta\dfrac{E_z^<(z')}{W_\parallel} \end{bmatrix}\mathbf{p}_0 \quad (54a)$$

for radiation into layer 1 where $k_1=(\omega/c)\sqrt{\epsilon_1}$. And

$$\begin{bmatrix} E_s^{\text{out}} \\ E_p^{\text{out}} \end{bmatrix} = 2ik_n \frac{\omega^2}{c^2} \begin{bmatrix} \sin\theta\sin\phi \dfrac{E_y^{>}(z')}{W_\perp} & -\sin\theta\cos\phi\dfrac{E_y^{>}(z')}{W_\perp} & 0 \\ \tan\theta\cos\phi\dfrac{E_x^{>}(z')}{W_\parallel} & \tan\theta\sin\phi\dfrac{E_x^{>}(z')}{W_\parallel} & -\tan\theta\dfrac{E_z^{>}(z')}{W_\parallel} \end{bmatrix} \mathbf{p}_0 \tag{54b}$$

for radiation into layer n where $k_n=(\omega/c)\sqrt{\epsilon_n}$.

Thus we have found the form of the s- and p-polarized radiation emitted by an oscillating electric dipole into the (θ,ϕ) direction, in terms of the Wronskians and the functions $E_\mu^{>}$, and $E_\mu^{<}$.

VI. PLANE WAVE INCIDENT ON THE STACK

We now consider the reciprocal problem (in a sense to be explained below) of calculating the electric fields produced at the point $\mathbf{r}'$ by incoming plane waves of s- and p-polarized light. Let the incident radiation have

$$\mathbf{E}_i(\mathbf{r},t)=(E_s^{\text{in}}\hat{\mathbf{s}}+E_p^{\text{in}}\hat{\mathbf{p}})e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} , \tag{55}$$

where $\hat{\mathbf{s}}$ and $\hat{\mathbf{p}}$ are defined by Eqs. (48). We consider the incident light to be coming from the (θ,ϕ) direction. Therefore

$$\mathbf{k}=-k(\cos\theta\cos\phi,\cos\theta\sin\phi,\sin\theta) .$$

E_s^{in} and E_p^{in} in Eq. (55) are the amplitudes of the s- and p-polarized light, respectively. Note, from Fig. 1, that when $\theta<0$ the light is incident from layer n and when $\theta>0$ it is incident from layer 1. For $\phi=\pi$, we have, from Eq. (49)

$$\hat{\mathbf{s}}=(0,-1,0) , \tag{56a}$$

$$\hat{\mathbf{p}}=(-\sin\theta,0,-\cos\theta) . \tag{56b}$$

The electric field at location $\mathbf{r}'$ in the stack is

$$\mathbf{E}(\mathbf{r}')=e^{i\mathbf{k}_\parallel\cdot\mathbf{x}'_\parallel}\mathbf{E}(z') .$$

Following Eq. (17) we express the electric field $\mathbf{E}(z')$ due to the incident light in the form

$$E_\mu(z')=K_{+\mu}^{(j)}e^{\alpha_j z'}+K_{-\mu}^{(j)}e^{-\alpha_j z'} ,$$

where z' is in layer j.

(a) *Let $\theta<0$.* Then for $\phi=\pi$, we have

$$E_\mu^{(j)}(z',\omega)=[(\underline{M}_\mu^{(j,1)})_{1,1}e^{\alpha_j z'} +(\underline{M}_\mu^{(j,1)})_{2,1}e^{-\alpha_j z'}]\frac{E_{i\mu}}{(\underline{M}_\mu^{(n,1)})_{1,1}} . \tag{57}$$

By comparison with Eq. (33a) we see that

$$E_\mu^{(j)}(z',\omega)=E_\mu^{>(j)}(z')\frac{E_{i\mu}}{K_{+\mu}^{>}(\underline{M}_\mu^{(n,1)})_{1,1}} . \tag{58}$$

From Eqs. (55) and (56)

$$\mathbf{E}_i=(-\sin\theta E_p^{\text{in}},-E_s^{\text{in}},-\cos\theta E_p^{\text{in}}) . \tag{59}$$

The values of $K_{+\mu}^{>(1)}$ are given by Eq. (32). So to put the results contained in Eq. (58) in a form similar to Eq. (54), we need to reexpress $(\underline{M}_\mu^{(n,1)})_{1,1}$.

Using Eqs. (35) and (36), we have

$$(\underline{M}_x^{(n,1)})_{1,1}=\frac{-\cos^2\theta}{2\alpha_1}W_\parallel , \tag{60a}$$

$$(\underline{M}_y^{(n,1)})_{1,1}=\frac{W_\perp}{2\alpha_n} . \tag{60b}$$

And now, making use of Eqs. (26)–(30) we obtain

$$(\underline{M}_z^{(n,1)})_{1,1}=\frac{-\cos^2\theta}{2\alpha_n}W_\parallel . \tag{60c}$$

Now using Eqs. (57), (60), and (32) for quantities E_μ^i, $(\underline{M}_\mu)_{1,1}$, and $K_{+\mu}^{>}$, respectively, we get (still for $\phi=\pi$)

$$\begin{bmatrix} E_x(z') \\ E_y(z') \\ E_z(z') \end{bmatrix} = -2ik_1 \begin{bmatrix} \dfrac{\sin\theta\sin\phi E_y^{<(j)}(z')}{W_\perp} & \dfrac{\tan\theta\cos\phi E_x^{<(j)}(z')}{W_\parallel} \\ \dfrac{-\sin\theta\cos\phi E_y^{<(j)}(z')}{W_\perp} & \dfrac{\tan\theta\sin\phi E_x^{<(j)}(z')}{W_\parallel} \\ 0 & \dfrac{-\tan\theta E_z^{<(j)}(z')}{W_\parallel} \end{bmatrix} \begin{bmatrix} E_s^{\text{in}} \\ E_p^{\text{in}} \end{bmatrix} . \tag{62b}$$

We obtain the general case when $\phi\neq\pi$ by multiplying by the appropriate rotation matrix.

$$\begin{bmatrix} E_x(z') \\ E_y(z') \\ E_z(z') \end{bmatrix} = 2ik_n \begin{bmatrix} \dfrac{\sin\theta\sin\phi E_y^{>(j)}(z')}{W_\perp} & \dfrac{\tan\theta\cos\phi E_x^{>(j)}(z')}{W_\parallel} \\ \dfrac{-\sin\theta\cos\phi E_y^{>(j)}(z')}{W_\perp} & \dfrac{\tan\theta\sin\phi E_x^{>(j)}(z')}{W_\parallel} \\ 0 & \dfrac{-\tan\theta E_z^{>(j)}(z')}{W_\parallel} \end{bmatrix} \begin{bmatrix} E_s^{\text{in}} \\ E_p^{\text{in}} \end{bmatrix} . \tag{62a}$$

This is the equation for the electric field everywhere produced by a plane-wave incident from the (θ,ϕ) direction with $\theta<0$ (i.e., from layer n).

(b) *Let* $\theta>0$. The derivation is exactly parallel to that in (a) and need not be repeated. The end result is

$$\begin{bmatrix} E_x(z') \\ E_y(z') \\ E_z(z') \end{bmatrix} = -2ik_1 \begin{bmatrix} \dfrac{\sin\theta\sin\phi E_y^{<(j)}(z')}{W_\perp} & \dfrac{\tan\theta\cos\phi E_x^{<(j)}(z')}{W_\parallel} \\ \dfrac{-\sin\theta\cos\phi E_y^{<(j)}(z')}{W_\perp} & \dfrac{\tan\theta\sin\phi E_x^{<(j)}(z')}{W_\parallel} \\ 0 & \dfrac{-\tan\theta E_z^{<(j)}(z')}{W_\parallel} \end{bmatrix} \begin{bmatrix} E_s^{\text{in}} \\ E_p^{\text{in}} \end{bmatrix} . \tag{62b}$$

Notice that Eqs. (62) state that

$$\mathbf{E}(\mathbf{r}') = \underline{A} \begin{bmatrix} E_s^{\text{in}} \\ E_p^{\text{in}} \end{bmatrix} e^{i\mathbf{k}_\parallel\cdot\mathbf{x}'_\parallel} , \tag{63}$$

where $\underline{A}$ is a 3×2 matrix. On the other hand, Eqs. (54) can be written

$$\begin{bmatrix} E_s^{\text{out}} \\ E_p^{\text{out}} \end{bmatrix} = \frac{\omega^2}{c^2}\underline{A}^T\mathbf{p}_0 , \tag{64}$$

where $\underline{A}^T$ is the transpose of $\underline{A}$. It is shown in the Appendix that this relationship between Eq. (63) and Eq. (64) is a direct consequence of the Lorentz reciprocity theorem. The theorem implies that in a set of isotropic and nonmagnetic media, not necessarily layered (as we assume in this paper),

$$D_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega) = D_{\nu\mu}(\mathbf{r}',\mathbf{r},\omega) . \tag{65}$$

Equations (64) and (63) taken together are simply a variation of the Lorentz reciprocity theorem, as shown in the Appendix.

We finally note that, given the relationship between Eq. (64) and (63), one can calculate the light radiated by a dipole by simply calculating the field induced by the incoming plane wave at a location of the dipole in a multilayer stack. This can be done by using Eqs. (54a) and (54b) or much more simply by using the 2×2 transfer matrices, as has been described in numerous papers, including Ref. 4.

An example of how the 2×2 transfer matrices formulation of the Green's functions can be applied to in other problems, can be found in our recent publication[10] dealing with roughness induced "mode conversion," and also in our forthcoming paper in which we will discuss enhancement of radiation from the dipole placed on an optical resonator (Kretschmann geometry).[7]

ACKNOWLEDGMENTS

This work was supported by the United States Department of Energy, under Contract No. DE-FG03-85ER45196. One of us (S.U.) would like to acknowledge support by the U.S. National Science Foundation and the Japan Society for Promotion of Science through the U.S.–Japan Cooperative Science Program. We would like to thank D. L. Mills for helpful discussions. J.C.H. would like to thank the Alfred P. Sloan Foundation for support.

APPENDIX: IMPLICATIONS OF THE LORENTZ RECIPROCITY THEOREM

We now show that the close relation between the field induced at a point z in a dielectric planar stack by a plane wave incident from the (θ,ϕ) direction [Eq. (62)] and the light emitted into the (θ,ϕ) direction by a radiating dipole (at a location z) [Eq. (54)] can be obtained directly from the Lorentz reciprocity theorem without assuming that the media are layered and using only general properties of the (possibly hard to calculate) electromagnetic Green's functions. The Lorentz reciprocity theorem is one of several related theorems for electromagnetism.[12–14] The theorem is usually stated in the following terms. Let $\mathbf{J}_1(\mathbf{r},\omega)$ be a current source (not necessarily a point source) lying inside V_0, and let $\mathbf{E}_1$ be the electric field resulting from $\mathbf{J}_1$ (with outgoing wave-boundary conditions outside V; $V_0\in V$). Similarly let $\mathbf{E}_2$ be the electric field resulting from a source current $\mathbf{J}_2(\mathbf{r},\omega)$ inside V_0. Then

$$\int_V [\mathbf{J}_1(\mathbf{r},\omega)\cdot\mathbf{E}_2(\mathbf{r},\omega) - \mathbf{J}_2(\mathbf{r},\omega)\cdot\mathbf{E}_1(\mathbf{r},\omega)]d^3r = 0 , \tag{A1}$$

provided that both the magnetic permeability $\boldsymbol{\mu}(\mathbf{r},\omega)$ and the effective dielectric constant $\underline{\tilde{\epsilon}}(\mathbf{r},\omega)=\underline{\epsilon}(\mathbf{r},\omega)+(4\pi i/\omega)\underline{\sigma}(\mathbf{r},\omega)$ are symmetric tensors. They can, however, vary in any way in space. For a proof of Eq. (A1) see Refs. 12–14. The theorem obviously applies to the problem considered in this paper since we assume all media to be isotropic ($\underline{\mu}=\mu\underline{I}=1\underline{I}$ and $\underline{\tilde{\epsilon}}=\tilde{\epsilon}\underline{I}$). The electric fields produced by a current source are given by Eq. (7) in terms of the Green's functions $\underline{D}(\mathbf{r},\mathbf{r}',\omega)$. It follows that the Green's functions have to possess a symmetry property that leads to the condition contained in Eq. (A1). If we take the currents $\mathbf{J}_1$ and $\mathbf{J}_2$ to be

$$\mathbf{J}_{1\mu}(\mathbf{r},\omega) = \hat{\mathbf{x}}_\alpha\delta(\mathbf{r}-\mathbf{r}') , \tag{A2a}$$

$$\mathbf{J}_{2\mu}(\mathbf{r},\omega) = \hat{\mathbf{x}}_\beta\delta(\mathbf{r}-\mathbf{r}'') . \tag{A2b}$$

Then using Eq. (7) yields

$$E_{1\mu}(\mathbf{r},\omega) = -\frac{i\omega}{c^2}D_{\mu\alpha}(\mathbf{r},\mathbf{r}',\omega) , \tag{A3a}$$

$$E_{2\mu}(\mathbf{r},\omega) = -\frac{i\omega}{c^2}D_{\mu\beta}(\mathbf{r},\mathbf{r}'',\omega) . \tag{A3b}$$

After substituting Eqs. (A2) and (A3) into (A1), and doing the sum over μ and the integration, one gets

$$D_{\alpha\beta}(\mathbf{r},\mathbf{r}'',\omega)-D_{\beta\alpha}(\mathbf{r}'',\mathbf{r}',\omega)=0 \; . \tag{A4}$$

Equation (A4) is simply another form of the Lorentz reciprocity theorem. In fact, it is also possible to reverse the proof and get Eq. (A1) from Eq. (A4). It can also be shown that, as a consequence of their definition, the Green's functions for layered media of MM [Eqs. (9)–(15)] necessarily obey Eq. (A4).

Assume the current source associated with the radiating dipole to be

$$\mathbf{J}(\mathbf{r},\omega)=-i\omega\delta(\mathbf{r}-\mathbf{r}')\mathbf{p}_0(\omega) \; .$$

Assume also that for $z>z_a$ the dielectric constant is real and does not depend on z; i.e., $\tilde{\epsilon}(\mathbf{r},\omega)=\tilde{\epsilon}_1(\omega)$ and $\mathrm{Im}[\tilde{\epsilon}_1(\omega)]=0$ for $z>z_a$. For $z\le z_a$, the dielectric constant may depend on z and may be complex. We define θ and ϕ so that

$$\mathbf{r}=r(\cos\theta\cos\phi,\cos\theta\sin\phi,\sin\theta) \; .$$

Assuming that $\theta>0$. Let $\mathbf{r}$ take the place of $\mathbf{k}$ in Eq. (48), so that $\hat{\mathbf{r}}$, $\hat{\mathbf{s}}$, and $\hat{\mathbf{p}}$ are three mutually orthogonal unit vectors, and s- or p-polarized light incident from or emitted into the (θ,ϕ) direction has its electric field entirely parallel to $\hat{\mathbf{s}}$ or $\hat{\mathbf{p}}$, respectively. Far from $\mathbf{r}'$ in the (θ,ϕ) direction the electric field of the outgoing light should be, from Eq. (44),

$$\mathbf{E}^{\rm out}(\mathbf{r})\cong[E_s^{\rm out}(\theta\phi)\hat{\mathbf{s}}+E_p^{\rm out}(\theta\phi)\hat{\mathbf{p}}]\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \; , \tag{A5}$$

where $k=(\omega/c)\sqrt{\tilde{\epsilon}_1}$.

Defining s and c as before ($s=\sin\phi$ and $c=\cos\phi$), we can write Eq. (A5) as

$$\mathbf{E}^{\rm out}=\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\begin{bmatrix}-s & c\sin\theta\\ c & s\sin\theta\\ 0 & -\cos\theta\end{bmatrix}\begin{bmatrix}E_s^{\rm out}(\theta\phi)\\ E_p^{\rm out}(\theta\phi)\end{bmatrix} \; . \tag{A6}$$

By the principle of superposition, $E_s^{\rm out}$ and $E_p^{\rm out}$ depend linearly on the source current. So we can write

$$\begin{bmatrix}E_s^{\rm out}(\theta\phi)\\ E_p^{\rm out}(\theta\phi)\end{bmatrix}=\begin{bmatrix}A_{11}(\theta\phi) & A_{12}(\theta\phi) & A_{13}(\theta\phi)\\ A_{21}(\theta\phi) & A_{22}(\theta\phi) & A_{23}(\theta\phi)\end{bmatrix}\mathbf{p}_0 =\underline{A}(\theta\phi)\mathbf{p}_0(\omega) \tag{A7}$$

Consider now an incoming plane wave from the $(\theta\phi)$ direction in medium 1

$$\mathbf{E}^{\rm in}(\mathbf{r})=(E_s^{\rm in}\hat{\mathbf{s}}+E_p^{\rm in}\hat{\mathbf{p}})e^{i\mathbf{k}\cdot\mathbf{r}} \; , \tag{A8}$$

where $\mathbf{k}=-k(\cos\theta\cos\varphi,\cos\theta\sin\varphi,\sin\theta)$. Then, as before, the electric fields induced at $\mathbf{r}'$ must depend linearly on $E_s^{\rm in}$ and $E_p^{\rm in}$ so that

$$\begin{bmatrix}E_x^{\rm in}(\mathbf{r}')\\ E_y^{\rm in}(\mathbf{r}')\\ E_z^{\rm in}(\mathbf{r}')\end{bmatrix}=\begin{bmatrix}B_{11}(\theta\phi) & B_{12}(\theta\phi)\\ B_{21}(\theta\phi) & B_{22}(\theta\phi)\\ B_{31}(\theta\phi) & B_{32}(\theta\phi)\end{bmatrix}\begin{bmatrix}E_s^{\rm in}\\ E_p^{\rm in}\end{bmatrix} \; . \tag{A9}$$

We will show that

$$\underline{A}(\theta\phi)=\frac{\omega^2}{c^2}\underline{B}^T(\theta,\phi) \; . \tag{A10}$$

That is, $\underline{A}$ is proportional to the transpose of $\underline{B}$. We first note that for the current distribution assumed above, Eq. (8) becomes

$$\mathbf{E}^{\rm out}(\mathbf{r})=\frac{-\omega^2}{c^2}\underline{D}(\mathbf{r},\mathbf{r}',\omega)\mathbf{p}_0 \; .$$

But from Eqs. (A6) and (A7) we have

$$\mathbf{E}^{\rm out}(\mathbf{r})=\frac{e^{ik|\mathbf{r}-\bar{\mathbf{r}}'|}}{|\mathbf{r}-\mathbf{r}'|}\begin{bmatrix}-s & c\sin\theta\\ c & s\sin\theta\\ 0 & -\cos\theta\end{bmatrix}\underline{A}(\theta\phi)\mathbf{p}_0 \; . \tag{A11}$$

If Eqs. (8) and (A11) are true for all values of $\mathbf{p}_0$ then we must have for large $|\mathbf{r}-\mathbf{r}'|$

$$\frac{-\omega^2}{c}\underline{D}(\mathbf{r},\mathbf{r}',\omega)=\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\begin{bmatrix}-s & c\sin\theta\\ c & s\sin\theta\\ 0 & -\cos\theta\end{bmatrix}\underline{A}(\theta\phi) \; . \tag{A12}$$

Now assume that we have an oscillating current source at $\mathbf{r}''$ where $\mathbf{r}''$ is located far away in the (θ,ϕ) direction. Define the current source to be

$$\mathbf{J}'(\mathbf{r})=-i\omega\delta(\mathbf{r}-\mathbf{r}'')\mathbf{p}' \; . \tag{A13}$$

Then it produces an electric field everywhere of

$$\mathbf{E}^{\rm in}(\mathbf{r}')=-\frac{\omega^2}{c^2}D(\mathbf{r}',\mathbf{r}'',\omega)\mathbf{p}' \; . \tag{A14}$$

But the electric fields $\mathbf{E}^{\rm in}(z)$ as seen at $\mathbf{r}'$ will seem to be made up only of s- and p-polarized plane waves incident from the (θ,ϕ) direction together with the other electric fields these plane waves give rise to. We want the formula for the electric field of the plane waves radiated by the dipole $\mathbf{p}'$ in a medium with dielectric constant $\tilde{\epsilon}_1(\omega)$, which is, for large r,

$$\mathbf{E}(\mathbf{r},\omega)=\frac{\omega^2}{c^2}\frac{e^{ikr}}{r}(\hat{\mathbf{r}}\times\hat{\mathbf{p}})\times\hat{\mathbf{r}} \tag{A15}$$

for the dipole located at the origin. If $\mathbf{r}'$ is located at (r,θ,ϕ), and r is very large, then for the incident electric field in medium 1 we use Eq. (A5), with $\mathbf{r}$ replaced by $-\mathbf{r}$, which does not change it.

So, using $\hat{\mathbf{s}}\cdot\hat{\mathbf{r}}=0$

$$E_s^{\rm in}=\hat{\mathbf{s}}\cdot\mathbf{E}(\mathbf{r})=\frac{\omega^2}{c^2}\frac{e^{ikr}}{r}\hat{\mathbf{s}}\cdot\mathbf{p}' \; ,$$

and similarly

$$E_p^{\rm in}=\hat{\mathbf{p}}\cdot\mathbf{E}(\mathbf{r})=\frac{\omega^2}{c^2}\frac{e^{ikr}}{r}\hat{\mathbf{p}}\cdot\mathbf{p}' \; .$$

Thus

$$\begin{bmatrix}E_s^{\rm in}\\ E_p^{\rm in}\end{bmatrix}=\frac{\omega^2}{c^2}\frac{e^{ikr}}{r}\begin{bmatrix}-s & c & 0\\ c\sin\theta & s\sin\theta & -\cos\theta\end{bmatrix}\mathbf{p}' \; . \tag{A16}$$

Therefore using Eq. (A9)

$$\mathbf{E}^{\text{in}}(\mathbf{r}',\omega)=\frac{\omega^2}{c^2}\frac{e^{ik|\mathbf{r}'-\mathbf{r}''|}}{|\mathbf{r}'-\mathbf{r}''|}\underline{B}(\theta\phi)\begin{bmatrix}-s & c & 0\\ c\sin\theta & s\sin\theta & -\cos\theta\end{bmatrix}\mathbf{p}' . \tag{A17}$$

For both Eq. (A14) and Eq. (A17) to be true for all $\mathbf{p}'$ we must have

$$\frac{-\omega^2}{c^2}\underline{D}(\mathbf{r}',\mathbf{r}'',\omega)=\frac{\omega^2}{c^2}\frac{e^{ik|\mathbf{r}'-\mathbf{r}''|}}{|\mathbf{r}'-\mathbf{r}''|}\underline{B}(\theta\phi)\begin{bmatrix}-s & c & 0\\ c\sin\theta & s\sin\theta & -\cos\theta\end{bmatrix} . \tag{A18}$$

Comparing Eq. (A18) and Eq. (A12) with $\mathbf{r}=\mathbf{r}''$, and using Eq. (A4) we have

$$\frac{\omega^2}{c^2}\underline{B}(\theta\phi)\begin{bmatrix}-s & c & 0\\ c\sin\theta & s\sin\theta & -\cos\theta\end{bmatrix}=\underline{A}^T\begin{bmatrix}-s & c & 0\\ c\sin\theta & s\cos\theta & -\cos\theta\end{bmatrix} .$$

Multiplying both sides of the above equation on the right by

$$\begin{bmatrix}-s & c\cos\theta\\ c & s\sin\theta\\ 0 & -\cos\theta\end{bmatrix} ,$$

we get

$$\frac{\omega^2}{c^2}\underline{B}=\underline{A}^T ,$$

thus justifying the claims made previously.

[1]A. A. Maradudin and D. L. Mills, Phys. Rev. B **11**, 1392 (1975).

[2]D. L. Mills and A. A. Maradudin, Phys. Rev. B **12**, 2943 (1975).

[3]S. Ushioda, J. E. Rutledge, and R. M. Pierce, Phys. Rev. Lett. **54**, 224 (1985); Phys. Rev. B **34**, 6804 (1986).

[4]K. Kurosawa, R. M. Pierce, J. C. Hemminger, and S. Ushioda, Phys. Rev. B **33**, 799 (1985).

[5]F. Abeles, Ann. Phys. **3**, 504 (1948); **5**, 706 (1950).

[6]B. Laks and D. L. Mills, Phys. Rev. B **20**, 4962 (1979).

[7]J. Giergiel, C. E. Reed, J. C. Hemminger, and S. Ushioda (unpublished).

[8]J. Sipe, Surf. Sci. **105**, 489 (1981).

[9]K. Arya, Phys. Rev. B **30**, 7242 (1984).

[10]J. Giergiel, C. E. Reed, J. C. Hemminger, and S. Ushioda, Phys. Rev. B (to be published).

[11]J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975).

[12]E. W. Cowan, *Basic Electromagnetism* (Academic, New York, 1968), pp. 82–84 and 383–387.

[13]L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Vol. 8 of *Course of Theoretical Physics*, translated from the Russian by J. B. Sykes and J. S. Bell (Pergamon, New York, 1960), pp. 288 and 289.

[14]*The Propagation of Electromagnetic Waves in a Plasma*, 2nd ed., by V. L. Ginsburg, translated from Russian by J. B. Sykes and R. J. Taylor (Pergamon, New York, 1970), pp. 404–407.