PHYSICAL REVIEW B VOLUME 36, NUMBER 8 15 SEPTEMBER 1987-I

Effect of partial phase coherence on Aharonov-Bohm oscillations in metal loops

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(Received 11 May 1987; revised manuscript received 20 July 1987)

The averaging of Aharonov-Bohm oscillations caused by incomplete phase coherence has been studied in two Sb rings. In these devices, the phase-coherence length L_ϕ can be determined independently from an analysis of the weak localization contribution to the magnetoconductance (ΔG_C) or from the amplitude of the conductance fluctuations. We find that the Aharonov-Bohm oscillations decrease as $\exp(-L/L_\phi)$ where L is the sample length. The values of L_ϕ inferred from ΔG_C qualitatively predict the temperature dependence of the conductance fluctuations.

The recent observation of quantum-interference fluctuations in the resistance of metal samples[1-3] has revised conventional notions of diffusion in metals. At low temperatures, most of the scattering of carriers is merely elastic because inelastic scattering becomes weak or infrequent. In this case, the sample length L can be comparable to the distance L_ϕ the electron diffuses before losing memory of the wave-function phase. Classical approximations break down completely then: the interference of the wave functions of the carriers is observed directly in the resistance.[4] In the presence of a magnetic field, the Aharonov-Bohm mechanism[5] causes the resistance of a loop to oscillate periodically in field[6] and the resistance of a wire to fluctuate randomly.[7] In fact, the average amplitude of such oscillations and fluctuations in a device of length L_ϕ is a "universal" value $\langle \Delta G\rangle \sim e^2/h$ for any metallic sample.[8-13] Here, $\langle\,\rangle$ denotes ensemble average. The question naturally arises concerning the effect of partial phase coherence: what happens when $L_\phi \lesssim L$? This has been addressed for the case of aperiodic fluctuations (AF's) in a wire,[14] and it was found that the amplitude of the fluctuations falls off as $(L/L_\phi)^{-3/2}$. A similar decay of the amplitude of the periodic oscillations was observed for the case of ensemble averaging of completely-phase-coherent loops connected in series.[15] In both cases, the conductance fluctuations obeyed

$$\Delta G_{\rm AF} = G_1 \frac{e^2}{h}\left[\frac{\pi^2 \hbar D}{L_\phi^2 k_B T}\right]^{1/2}\left(\frac{L_\phi(T)}{L}\right)^{3/2}, \qquad (1)$$

where the factor in brackets accounts for energy averaging.[7,12,16] G_1 is a number of order 1 which accounts for the details of sample geometry. In this work we present evidence that the oscillations in a loop suffer a more severe averaging as the phase-coherence length shrinks to less than the sample size L.

The Aharonov-Bohm effect which causes the periodic oscillations of the resistance with the amplitude of the magnetic field (see Fig. 1) requires that the carriers involved retain phase coherence throughout the traversal of the sample. By the definition of L_ϕ, the fraction of carriers which arrive at the voltage probe retaining phase memory is $\exp(-L/L_\phi)$, where L is the distance between the voltage probes.[17] [The length L of a sample (with $L > L_\phi$) is the distance across which the resistance is measured.] For $L \gtrsim L_\phi$, we expect that the amplitude of the oscillations will be proportional to the number of carriers retaining phase memory, i.e., that the amplitude of the oscillations will also be proportional to the exponential. We further expect that energy averaging also reduces the amplitude of the h/e oscillations. The scale for energy averaging is $\Delta E = \pi^2 \hbar D/L^2$, where D is the diffusion constant of the carriers.[8,18] Assuming that the universal amplitude will result as $L_\phi \rightarrow \infty$, we expect

$$\Delta G_{h/e} = G_2 \frac{e^2}{h}\left[\frac{\pi^2 \hbar D}{L^2 k_B T}\right]^{1/2} \exp[-L/L_\phi(T)] \; . \qquad (2)$$

This equation differs from Eq. (1) because here we are accounting for conductance fluctuations ΔG which result from the carriers which travel the distance L without losing phase coherence, whereas Eq. (1) predicts the amplitude of the "ensemble average" of the fluctuations in L/L_ϕ phase-coherent segments. The energy-averaging factor contains L rather than L_ϕ because the electrons which contribute to the h/e period in the magnetoresistance are those which complete the entire path length L without losing phase memory. We emphasize that Eq. (2) is an estimate based on the above argument; it is not a quantitative

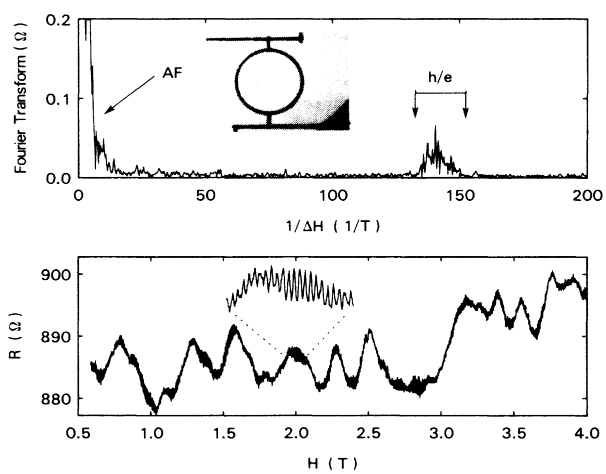


FIG. 1. The magnetoresistance $R(H)$ at $T = 0.06$ K in sample 1 (the inset illustrates the h/e oscillations which pervade the entire field range) and the Fourier transform of $R(H)$ (inserted is a photograph of sample 1).

prediction from the perturbation theory. This exponential decay was not observed in the experiments on noble-metal loops[15,16] because, in these samples, L_ϕ was temperature independent below $T \sim 1$ K.

It is the aim of this work to test Eqs. (1) and (2) quantitatively by independently determining the phase coherence length and the amplitudes of the various fluctuations and oscillations. Toward this purpose we studied two Sb loops made by two different lithographic techniques: sample 1, a round loop having a resistance $R = 876$ Ω ($D = 0.0028$ m^2/s), diameter 0.85 μm, length $L = 1.64$ μm, $w = 0.038$ μm, and $t = 0.08$ μm made by the contamination resist method,[19] and sample 2, a square loop (1.05 μm on a side) with $R = 86$ Ω ($D = 0.0095$ m^2/s), $L = 2.51$ μm, $w = 0.16$ μm, and $t = 0.08$ μm made by the same high-resolution scanning transmission microscope but using a negative resist[20] (w is the width and t is the thickness of the wires forming the device). We chose Sb as sample material because L_ϕ is large and because Sb exhibits weak-localization behavior. Also Sb is a semimetal with a low carrier concentration, and therefore we can achieve large resistances in small samples. However, we sacrifice the simple band structure of the noble metals used in other experiments[3] for the more complicated bands in Sb.[21]

The phase coherence length can be determined from weak localization independently from any assumption about the fluctuation amplitudes, except that there is sufficient ensemble averaging to make ΔG_C large enough to observe. It is extracted from the magnetoresistance by fitting the weak localization contribution (ΔG_C). Near $H = 0$, the coherent backscattering between time-reversed pairs of particles (weak localization) increases the resistance of the sample. The destruction of the weak localization by magnetic fields has been widely used to obtain the phase coherence length.[22] Specifically, for a one-dimensional sample (the width w of the wires forming the loop being small compared with L_ϕ) in a magnetic field H, the dominant correction to the conductance is[23]

$$\Delta G_C = -\frac{e^2}{2\pi L \hbar}(3L_1 - L_0) \ . \tag{3}$$

The lengths L_0 and L_1 are related to L_ϕ by

$$L_0^{-2} = L_\phi^{-2} + L_H^{-2} = L_{\text{in}}^{-1} + 2L_S^{-2} + L_H^{-2} \ ,$$

$$L_H^2 = 3(\hbar/eHw)^2 \ ,$$

and

$$L_1^{-2} = L_{\text{in}}^{-2} + \tfrac{4}{3}(L_{\text{SO}}^{-2}) + \tfrac{2}{3}(L_S^{-2}) + L_H^{-2} \ ,$$

where the subscripts denote inelastic (in), spin-orbit (SO), and spin-flip (S) scattering lengths. In short samples where $L \sim L_\phi$, it is important to account for the boundaries of the device.[24] Here we employ the correction derived by Santhanam[25] [his Eq. (7)] which should be applicable to four-terminal measurements, for $i = 1, 0$,

$$\frac{L_i}{L} \rightarrow \frac{L_i}{L}\left[\frac{5 + 4\tanh(L/L_\phi) - 3(L_\phi/L)\tanh(L/L_\phi)}{4 + 5\tanh(L/L_\phi)}\right] \ . \tag{4}$$

Typical results of fitting the weak localization term near $H = 0$ are displayed in Fig. 2(a), where we have taken $L_{\text{SO}} \ll L_{\text{in}}$ since Sb is a heavy element. (This is consistent with measurements on large two-dimensional films.) The difference between a wire, for which the theory has been done,[25] and a loop has been accounted for in a classical approximation for series and parallel networks. The analysis of the data near $H = 0$ is, in principle, straightforward; however, since the amplitude of the conductance fluctuations is comparable to ΔG_C there is considerable danger of systematic errors in the fitting procedure which determines L_ϕ. From the results of such fits throughout the range of temperatures it was determined (under the standard assumption that $L_{\text{in}} \propto AT^{-p}$) that for sample 1, $L_S = 0.98$ μm and $L_{\text{in}} = (0.41\ \mu\text{m}) \times T^{-3/4}$, and for sample 2, $L_S = 2.84$ μm and $L_{\text{in}} = (1.53\ \mu\text{m}) \times T^{-3/4}$. (The fitted power $p = 0.75 \pm 0.1$ is somewhat different from the other published results in metal wires.[26]) We observe that L_ϕ is longer in sample 2 than in sample 1 by more than a factor of 2 at all temperatures. Although the absolute magnitude of the inelastic diffusion length is different for the two samples, the temperature dependence is approximately the same. The quality of these fits can be judged from Fig. 2(b) where the values of L_ϕ obtained from the magnetoresistance are plotted along with the above parametric equations (solid lines) for L_ϕ. To illustrate the importance of the boundary conditions we also display the results obtained upon ignoring the boundary corrections to Eq. (3) (dotted lines). Clearly the two equations for ΔG_C yield different results. For sample 1, where L/L_ϕ is larger, the difference is $\sim 15\%$, and for sample 2, where $L/L_\phi \sim 1.2$, the correction is as much as 30%.

In principle, we can use these values to predict the amplitude of the aperiodic fluctuations and the h/e oscillations as a function of temperature. Except for the prefactors G_1 and G_2, Eqs. (1) and (2) contain no free parame-

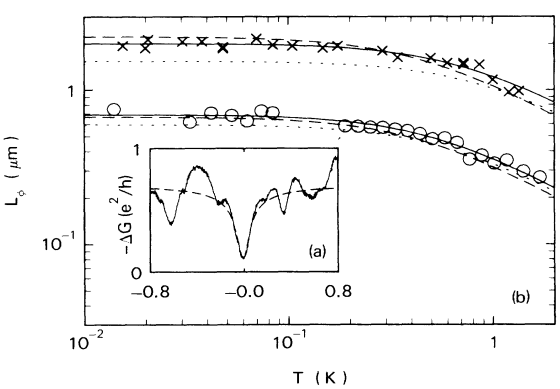


FIG. 2. (a) A representative fit of the weak localization term Eq. (3) to the magnetoresistance near zero magnetic field. The solid line is the experimental data, and the dashed curve is the fit which yields $L_\phi = 0.58$ μm. (b) The values of L_ϕ obtained from fitting Eq. (4) to the magnetoresistance in sample 1 (O) and sample 2 (×). The solid lines are the parametric fits to the weak localization data including the boundary corrections, and the dotted lines are the fits without boundary corrections. The dashed lines are the estimates of L_ϕ obtained from the amplitude of the h/e oscillations.

ters, since the diffusion coefficient can be calculated from the band parameters of Sb ($E_F \sim 0.1$ eV and $N_e = N_h = 5.36 \times 10^{25}/\mathrm{m}^3$) and the measured sample resistivity. We have measured the temperature dependence of the root-mean-square (rms) amplitudes ΔG_{AF} and $\Delta G_{h/e}$ for the aperiodic fluctuations and h/e oscillations respectively. We note that to determine these amplitudes it is necessary to take into account the noise in the measurement. The noise in each measurement has been calculated from the "dead" regions in the Fourier spectrum (see Fig. 1), and for each point in Fig. 3, the mean square noise has been subtracted from $(\Delta G)^2$. $\Delta G_{h/e}$ was measured in the range $-1 < H < 1$ T for both samples, and ΔG_{AF} was measured over wider field ranges. (Measurements from different ranges of magnetic field yielded the same results to within the experimental errors.) The first point to notice is that the oscillations and the random fluctuations do *not* follow the same function of temperature. As expected the oscillations are reduced more rapidly than the random components as temperature increases and L_ϕ decreases.

Using the values of L_ϕ obtained from fitting Eq. (4), we fitted Eq. (2) to the h/e data by optimizing the value of G_2 so that the fit coincided with the data near 0.5 K. This choice of normalization is arbitrary, and others yield different values of G_2 but do not affect the fit quality. Similarly the values of L_ϕ were inserted into Eq. (1) and fitted to the amplitude of the aperiodic fluctuations by adjusting G_1. The predicted dependences appear as solid lines in Fig. 3. The values of G_1 are 0.43 for sample 1 and 0.38 for sample 2, and G_2 is 0.12 and 0.25 for samples 1 and 2, respectively. This factor of 2 difference between the samples in G_2 is more than we expected, and it might indicate remnant errors in the values of L_ϕ (see below). The values for G_1 and G_2 are, however, consistent with theory[8] and with previous experiments.[13] Our results clearly demonstrate that an independent check of Eqs. (1) and (2) can be performed and that in this case it gives qualitatively correct results.

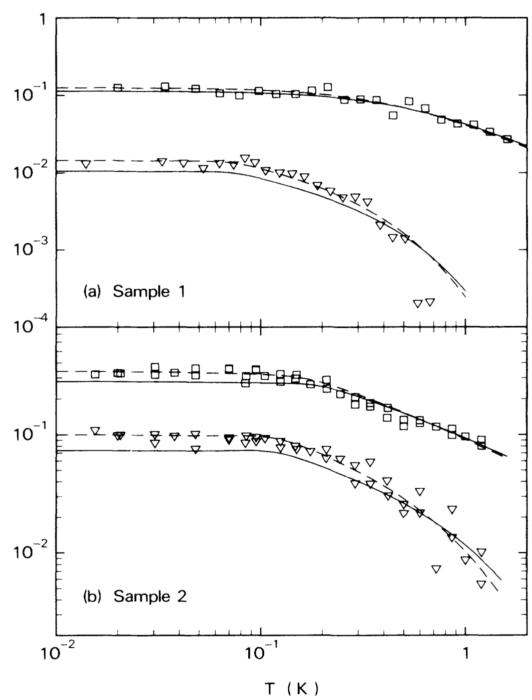


FIG. 3. The rms amplitude of the h/e oscillations (▽) and the aperiodic fluctuations (□) for samples 1 and 2 [(a) and (b), respectively]. The solid curves are the predictions from Eqs. (1) and (2) where L_ϕ is taken from fits to weak localization. The dashed lines are empirical fits to ΔG which are discussed in the text.

The difference between the measurements and the predictions (solid lines), however, is significant at low temperatures. In both samples the data have a somewhat stronger dependence on temperature than the predictions based on weak localization. For comparison, we fitted Eq. (2) to $\Delta G_{h/e}$ using L_ϕ as an adjustable parameter (again assuming a power law $L_{\mathrm{in}} \propto T^{-p}$ for the inelastic scattering). Using the L_ϕ inferred from the temperature dependence of $\Delta G_{h/e}$, we calculated the amplitude of the aperiodic fluctuations and the results are displayed as dashed lines in Fig. 3. We find excellent self-consistency within each sample. From these fits, G_1 is 0.55 and 0.39 in samples 1 and 2 respectively, and $G_2 = 0.30$ and $p = 0.75 \pm 0.1$ in both samples. The resulting L_ϕ are displayed as dashed lines in Fig. 2(b). In both samples, the conductance fluctuations (dashed lines) exhibit more temperature dependence than the L_ϕ inferred from ΔG_C would lead one to expect. This is clearly seen in Figs. 2 and 3 where the dashed curves are stronger functions of temperature than the solid curves. We conclude that the two different expressions, Eqs. (2) and (3), yield different values of the phase coherence length. Nevertheless, the qualitative agreement in Fig. 3 between ΔG and the predictions (solid lines) based on L_ϕ inferred from weak localization is respectable. There are several possible sources for the discrepancies seen at low temperatures. We propose the following possibilities.

An obvious problem in the comparison between theory and experiment is that the weak localization theory assumes an ensemble average, and in the experiment this average is far from complete. Using the L_ϕ inferred from Eq. (3), in sample 1 we find the ratio L/L_ϕ is as low as 2 at the lowest temperatures and for sample 2 it is as low as 1.2. Since the AF becomes a significant perturbation on ΔG_C at these low ratios [see Fig. 2(a)], it might contribute to a systematic error in the values of L_ϕ. In addition, in this limit both the magnitude and functional form of ΔG_C are very sensitive to the assumptions employed for the boundary corrections. It is possible that Eq. (4) is not appropriate for our system. Another possible problem with the analysis is that L_S is, in fact, a field-dependent parameter. The spin-flip scattering is quenched when $H > kT/\mu$ (μ is the effective magnetic moment of the impurities) which is less than 0.1 T at the lowest temperatures. The fluctuation amplitudes ΔG_{AF} are measured in a regime where magnetic scattering is probably suppressed. In the case of strong scattering from paramagnetic impurities, the h/e oscillations are completely quenched at low fields and regain their full amplitude at higher fields.[27,13] Although there is no clear signature of strong magnetic scattering in the present data, we would find it surprising if the process were completely absent.

The remaining possibility is that in Eq. (2) some numerical constants are absent which multiply the sample or phase coherence length, and the values of L_ϕ inferred from fitting $\Delta G_{h/e}$ are wrong. The self-consistency of the dashed lines in Fig. 3 implies that this is unlikely—the values of L_ϕ which fit the h/e oscillations also fit the aperiodic fluctuations. In particular, setting L [in Eq. (2)] to be half the perimeter of the loop, we obtain worse agreement with weak localization predictions and unreasonably small values of G_2. An error in the measured value of L can also affect the quality of the fit because the "cutoff" for the energy averaging factor in Eq. (2) shifts with L. From the h/e frequency and the photographs of the devices, we conclude that the error δL for sample 1 is less than 2% and for sample 2 it is about 10%. Inserting $L \pm \delta L$ into Eqs. (1) and (2), we find no significant difference in the quality of the fits.

We have shown experimentally that the phase memory length L_ϕ is a critical parameter determining the amplitude of various conductance fluctuations which result from Aharonov-Bohm effects in a normal metal. In agreement with previous experiments[14,15] the aperiodic fluctuations average away as a power law in the ratio L/L_ϕ, where L is the separation between the voltage probes. In contrast, the periodic h/e oscillations are reduced exponentially as L_ϕ/L shrinks. The phase coherence lengths determined from fitting the weak localization term (with the proper boundary conditions) in the conductivity yield predictions for the amplitudes of the conductance fluctuations (both aperiodic and periodic components) which are qualitatively correct. We attribute the quantitative discrepancies partly to approximations in the theoretical formulas used in the analysis of the data, and partly to incomplete ensemble averaging within the devices.

We acknowledge useful discussions of weak-localization theory and the materials properties of Sb with P. Santhanam. We are also grateful to P. A. Lee, D. Di Vincenzo, and D. Stone for helpful conversations about the conductance-fluctuation theory.

[1]D. Yu. Sharvin and Yu. V. Sharvin, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 285 (1981) [JETP Lett. **34**, 272 (1981)].

[2]R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, in *Localization, Interaction, and Transport Phenomena in Impure Metals,* edited by G. Bergmann, Y. Bruynseraede, and B. Kramer (Springer-Verlag, Heidelberg, 1985); C. P. Umbach, S. Washburn, R. B. Laibowitz, and R. A. Webb, Phys. Rev. B **30**, 4048 (1984).

[3]R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, Phys. Rev. Lett. **54**, 2696 (1985); V. Chandrasekhar, M. J. Rooks, S. Wind, and D. E. Prober, *ibid.* **55**, 1610 (1985); S. Datta, *et al. ibid.* **55**, 2344 (1985).

[4]R. Landauer, Philos. Mag. **21**, 863 (1970).

[5]Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959); B. L. Al'tshuler, A. G. Aronov, and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 94 (1981) [JETP Lett. **33**, 101 (1981)].

[6]Y. Gefen, Y. Imry, and M. Ya. Azbel, Surf. Sci. **142**, 203 (1984); Phys. Rev. Lett. **52**, 129 (1984); M. Büttiker, Y. Imry, R. Landauer, and S. Pinhas, Phys. Rev. B **31**, 6207 (1985).

[7]A. D. Stone, Phys. Rev. Lett. **54**, 2692 (1985).

[8]P. A. Lee and A. D. Stone, Phys. Rev. Lett. **55**, 1622 (1985); B. L. Al'tshuler, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 530 (1985) [JETP Lett. **41**, 648 (1985)]; B. L. Al'tshuler and D. E. Khmel'nitskii, *ibid.* **42**, 291 (1985) [*ibid.* **42**, 359 (1985)].

[9]A. D. Stone and Y. Imry, Phys. Rev. Lett. **56**, 189 (1986).

[10]A. D. Benoit, C. P. Umbach, R. B. Laibowitz, and R. A. Webb, Phys. Rev. Lett. **58**, 2343 (1987).

[11]Y. Isawa, H. Ebisawa, and S. Maekawa, J. Phys. Soc. Jpn. **55**, 2523 (1986).

[12]A review of the perturbation theory is given in P. A. Lee, A. D. Stone, and H. Fukuyama, Phys. Rev. B **35**, 1039 (1987).

[13]For reviews, see Y. Imry, in *Directions in Condensed Matter Physics,* edited by G. Grinstein and E. Mazenko (World Scientific, Singapore, 1986), p. 101; S. Washburn and R. A. Webb, Adv. Phys. **35**, 375 (1986).

[14]W. J. Skocpol, P. M. Mankiewich, R. E. Howard, L. D. Jackel, D. M. Tennant, and A. D. Stone, Phys. Rev. Lett. **56**, 2865 (1986); **58**, 2347 (1987).

[15]C. P. Umbach, C. van Haesendonck, R. B. Laibowitz, S. Washburn, and R. A. Webb, Phys. Rev. Lett. **56**, 386 (1986). The data for ΔG in this experiment were plotted after the effect of the series addition of loops was factored out. In this method of displaying the data, the power law is $\Delta G \propto N^{-1/2}$.

[16]S. Washburn, C. P. Umbach, R. B. Laibowitz, and R. A. Webb, Phys. Rev. B **32**, 4789 (1985). The factor in brackets is the inverse of the number of uncorrelated bands contributing to ΔG_C; since this number is never less than one, the factor in brackets can never exceed one.

[17]D. Di Vincenzo (private communication); see also Ref. 11.

[18]This has been given erroneously in past publications (Washburn and Webb in Ref. 13) as $\Delta E = \pi^2 \hbar D/L_\phi^2$.

[19]A. N. Broers, J. Van. Sci. Technol. **10**, 979 (1973).

[20]C. P. Umbach, A. N. Broers, C. G. Willson, R. Koch, and R. B. Laibowitz (unpublished).

[21]Yu. A. Pospelov, G. S. Grachev, and S. G. Novikov, Zh. Eksp. Teor. Fiz. **87**, 2104 (1984) [Sov. Phys. JETP **60**, 1215 (1984)], and references cited therein.

[22]D. J. Bishop, R. C. Dynes, and D. C. Tsui, Phys. Rev. B **26**, 773 (1982); G. Bergmann, Phys. Rep. **107**, 1 (1984).

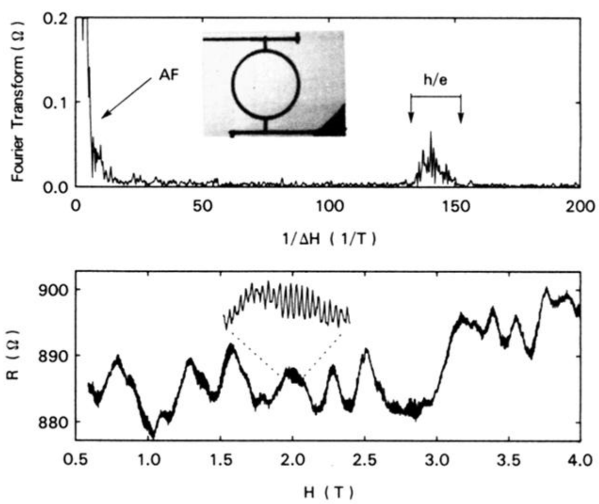
[23]J. C. Licini, D. J. Bishop, M. A. Kastner, and J. Melngailis, Phys. Rev. Lett. **55**, 2987 (1985).

[24]B. Doucot and R. Rammal, J. Phys. (Paris) **47**, 973 (1986); Phys. Rev. Lett. **55**, 1148 (1985).

[25]P. Santhanam, Phys. Rev. B **35**, 8737 (1987).

[26]P. Santhanam, S. Wind, and D. E. Prober, Phys. Rev. B **35**, 3188 (1987), and references cited therein.

[27]A. D. Benoit *et al.* (unpublished).



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