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Characteristic times for resonant tunneling in one dimension

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It is shown that the properties of the propagator for transmission through an arbitrary one-dimensional potential lead in a natural way to two characteristic times τ^0 and τ^L for decay, respectively, through the end points of the system, in terms of which the lifetime τ may be written as $2\tau^0\tau^L/(\tau^0+\tau^L)$. We obtain the traversal times t_{0L} and t_{L0} for scattering, respectively, from the left and right ends of the system. In terms of these characteristic times, it is found that, in general, for asymmetric or random potentials, the rate is given by $t_{0L}/t_{L0}=(1-R^{1/2})/(1+R^{1/2})$, where R stands for the reflection coefficient evaluated at resonance energy. A comparison with experiment shows that t_{0L} may be even an order of magnitude different from t_{L0}.

In recent years there has been an increasing interest in traversal times for tunneling.[1,2] This is of particular importance in the context of the remarkable developments in the fabrication of solid-state devices at nanometric scales as exemplified by multilayer heterostructures.[3] Over the years there has been a number of controversial definitions of the concept of traversal time on a one-dimensional barrier.[1,2] All those approaches refer to a situation where resonant effects are absent. In the presence of resonant processes, as in the multilayer heterostructures mentioned above, which originate from a complicated constructive interference phenomenon occurring in a time scale given by the reciprocal of the resonance width $\tau\sim\hbar/\Gamma$, a different approach is required. Clearly, the lifetime τ does not provide information on whether the electron decays through the left or right end points of the system, and therefore it cannot be identified with a traversal time as considered by some authors.[3-7]

The purpose of this work is to show, for a general arbitrary one-dimensional potential, how resonant processes give rise to two characteristic times τ_n^0 and τ_n^L, associated, respectively, with electronic decay through the end points of the system. The result is of particular interest for the case, normally found in physical cases, of scattering by asymmetric or random potentials. It is important to emphasize that we are concerned here with single-potential arrangements and not with ensemble averages. It is shown below that the traversal times for scattering at resonance energy ε_n, respectively, from the left and right end points of a one-dimensional system may be expressed in terms of the above characteristic times. We show, when compared with practical cases that these times may be quite different.

Let us, therefore, consider an electron of energy E and mass m incident from the left, $x<0$, on an arbitrary one-dimensional potential of finite length, i.e., $V(x)=0$; $x\leq 0$ and $x\geq L$. The solutions of the corresponding Schrödinger equation, outside the interaction range, may be written as[3] $\psi_l(k,x)=\exp(ikx)+r(k)\exp(-ikx)$, $x\leq 0$, and $\psi_l(k,x)=t(k)\exp(ikx)$, $x\geq L$, where the subindex l denotes that the electron approaches the system from the left, $k=(2mE)^{1/2}/\hbar$, $r(k)$, and $t(k)$ stand, respectively, for the reflection and transmission amplitudes.

It is well known that resonant processes may be described in terms of the complex poles of the outgoing Green's propagator $G^+(x,x';k)$ and its corresponding residues.[8,9] Making use of the equation for $G^+(x,x';k)$ and its boundary conditions at the end points of the system, enables one in a straightforward way, to obtain the relationship[10]

$$\psi_l(k,x)=2ikG^+(0,x;k);\ 0\leq x\leq L\ , \tag{1}$$

which is valid along the internal region of the interaction. From Eq. (1) and the value of $\Psi_l(k,x)$ at $x=L$, one obtains the expression for the transmission amplitude[10]

$$t_l(k)=2ikG^+(0,L;k)\exp(-ikL)\ . \tag{2}$$

Near an isolated pole $k_n=a_n-ib_n$, it is possible to write the propagator[8-10] as

$$G^+(x,x'k)\simeq u_n(x)u_n(x')/2k_n(k-k_n)\ , \tag{3}$$

where the functions $u_n(x)$ obey the Schrödinger equation with complex eigenvalues $E_n=\hbar^2k_n^2/2m=\varepsilon_n-i\Gamma_n/2$, and $\varepsilon_n=\hbar^2(a_n^2-b_n^2)/2m$ represents the position of the resonance, and $\Gamma_n=\hbar^2(4a_nb_n)/2m$ the corresponding width. The functions $u_n(x)$ satisfy purely outgoing boundary conditions at the end points of the system,[9] i.e., $u_n'(0)=-ik_nu_n(0)$, and $u_n'(L)=ik_nu_n(L)$. Here the prime denotes derivative with respect to x. Using Eq. (3) in Eqs.(1) and (2) allows one to write $|\Psi_l(k,x)|^2$ and $|t_l(k)|^2$ near resonance, respectively, as

$$|\Psi_l(k,x)|^2\simeq k^2b_n^0|u_n(x)|^2/a_n^2[(k-a_n)^2+b_n^2]\ , \tag{4}$$

and

$$|t_l(k)|^2 \simeq k^2 b_n^0 b_n^L / a_n^2 [(k-a_n)^2 + b_n^2] \ , \quad (5)$$

where $b_n^0 = |u_n(0)|^2$ and $b_n^L = |u_n(L)|^2$ denote, respectively, the resonant eigenfunction $u_n(x)$ evaluated at each end point of the system. The quantities b_n^0 and b_n^L are not independent. It can be easily shown[10] by using Green's theorem between the equations for $u_n(x)$ and $u_n^*(x)$ and the corresponding boundary conditions, that the pole width b_n may be written for an isolated resonance term as $b_n = (b_n^0 + b_n^L)/2$ or equivalently,

$$\Gamma_n = (\Gamma_n^0 + \Gamma_n^L)/2 \ . \quad (6)$$

Equations (4) and (5) are convenient expressions to discuss resonant processes in one-dimensional systems. In particular the quantities Γ_n^0 and Γ_n^L may be seen as partial decay widths through the corresponding end points of the system. One may define the characteristic times

$$\tau_n^0 \sim \hbar/\Gamma_n^0; \ \tau_n^L \sim \hbar/\Gamma_n^L \ . \quad (7)$$

In terms of the above quantities one may write the lifetime $\tau \sim \hbar/\Gamma_n$ as

$$\tau = 2\tau_r = 2\tau_n^0 \tau_n^L / (\tau_n^0 + \tau_n^L) \ , \quad (8)$$

which features the lifetime in a rather curious way as twice a reduced time in terms of τ_n^0 and τ_n^L. The transmission coefficient $T(k) = |t_l(k)|^2$ evaluated at resonance may be written using (5) as

$$T(a_n) \simeq \Gamma_n^0 \Gamma_n^L / \Gamma_n^2 \ , \quad (9)$$

which indicates that in general the transmission peak at resonance depends on the values of the partial decay widths. Clearly its value may be less than unity. From Eqs. (6) and (9) one readily obtains an expression which gives Γ_n^0 and Γ_n^L in terms of the total width Γ_n and the corresponding value of the reflection coefficient $R = 1 - T$ evaluated at resonance energy, namely

$$\Gamma_n^0 \simeq \Gamma_n \{1 - [R(a_n)]^{1/2}\}; \ \Gamma_n^L \simeq \Gamma_n \{1 + [R(a_n)]^{1/2}\} \ , \quad (10)$$

which may be convenient for practical purposes.[11]

For scattering from the right, the transmission amplitude $t_r(k)$ depends on $G^+(L,0;k)$, which is manifestly symmetrical on b_n^0 and b_n^L as shown by Eq. (5). However, the corresponding probability density $|\Psi_r(k,x)|^2$ will be given in this case by

$$|\Psi_r(k,x)|^2 \simeq k^2 b_n^L |u_n(x)|^2 / a_n^2 [(k-a_n)^2 + b_n^2] \ , \quad (11)$$

which differs from Eq. (4) by the appearance of b_n^L instead of b_n^0. For a symmetric potential, it is well known that the transmission peak attains a value of unity at resonance energy, and therefore, using Eq. (5) it follows that $b_n^0 = b_n^L$. Consequently, in that case $|\Psi_l(k,x)|^2 = |\Psi_r(k,x)|^2$. However, in general, for asymmetric potentials $b_n^0 \neq b_n^L$, which tell us that the decay will occur with different probability and, therefore, with a different time scale through each end point of the system. Using Eqs. (4) and (11) enables one to obtain the traversal times for scattering from the left and the right, respectively, from the expressions

$$t_{0L}(a_n) \sim \frac{1}{a_n} \int_0^L |\Psi_l(a_n,x)|^2 dx = \frac{4\hbar}{\Gamma_n} \frac{\Gamma_n^0}{\Gamma_n} \ , \quad (12)$$

and

$$t_{L0}(a_n) \sim \frac{1}{a_n} \int_0^L |\Psi_r(a_n,x)|^2 dx = \frac{4\hbar}{\Gamma_n} \frac{\Gamma_n^L}{\Gamma_n} \ . \quad (13)$$

The above results mean that although the transmission coefficient for scattering from the left or from the right has the same value, the corresponding traversal times t_{0L} and t_{L0} are, in general, different. The integrals appearing in Eqs. (12) and (13) correspond to the definition of time delay. It follows, therefore, that the analysis of traversal times in terms of the phase of the transmission amplitude[2] is related to the present approach. A comparison will be considered elsewhere. From Eqs. (12) and (13) one obtains the ratio $t_{0L}/t_{L0} \sim \Gamma_n^0/\Gamma_n^L$ and using Eq. (10) leads to the result

$$t_{0L}/t_{L0} \sim \Gamma_n^0/\Gamma_n^L = \{1 - [R(a_n)]^{1/2}\}/\{1 + [R(a_n)]^{1/2}\} \ , \quad (14)$$

which allows one to estimate the rate of traversal times in terms of the reflection coefficient evaluated at resonance energy. For a symmetrical system one has $\Gamma_n^0 = \Gamma_n^L$ and, hence $t_{0L} = t_{L0}$. In practice, however, the potential profile is deformed by the applied voltage. It is well known that this affects the value of the transmission peak[4,7] and, as shown by Eq. (9), this is related to the characteristic times of the system, namely to the fact that $\Gamma_n^0 \neq \Gamma_n^L$. As a consequence, in this case the traversal time for scattering from the left is different from that from the right, i.e., $t_{0L} \neq t_{L0}$. For example, for the symmetrical double-barrier structure considered by Sollner, Goodue, Tannewald, and Parker,[12] with parameters $v = 0.23$ eV, $a = b = 50$ Å, and $m^* = 0.08 m_e$, we obtain a width $\Gamma = 0.481$ meV which corresponds to a lifetime $\tau \sim 1.368 \times 10^{-12}$ s, which is of the same order of magnitude as other estimates.[4-7] However, in the presence of an applied voltage of 0.1 eV as considered in Ref. 12, the transmission peak at resonance attains the value $T(\varepsilon_n) \sim 0.644$ and using Eq. (14) it is predicted that $t_{0L}/t_{L0} \sim 0.25$. Similarly, for an applied voltage of 0.24 eV, $T(\varepsilon_n) \sim 0.349$ and $t_{0L}/t_{L0} \sim 0.11$. The above results indicate that the traversal times for the electron through the system may be quite different when the resulting potential is asymmetrical.

The results of this work are relevant, in addition to asymmetric double-barrier structures, for the case of random one dimensional "superpotentials"[7,13,14] and, in general, for finite length one-dimensional problems.

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