Theory of the absorption of electromagnetic radiation by hopping in p-type $Hg_{1-x}Mn_xTe$ in strong magnetic fields

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Absorption of polarized electromagnetic waves at 0 K in the wavelength region from 1.4 to 5 mm in p-type narrow-gap $Hg_{1-x}Mn_xTe$ has been investigated in a magnetic field within the quantum-limit regime. The model of photon-induced hopping transitions of electrons between the ground states of the acceptors was used. An integral formula for the absorption coefficient is derived. In addition to this, numerical results for electromagnetic waves with the electric vector perpendicular and parallel to the applied magnetic field are presented. The range of validity of our results is discussed. We obtain an absorption coefficient of the order of 10 cm^{-1} for a range of concentration of acceptors up to 1×10^{16} cm^{-3} and compensation ratios up to 0.2.

I. INTRODUCTION

In the last few years dc and ac impurity hopping conduction in narrow-gap p-type $Hg_{1-x}Mn_xTe$ has been studied both experimentally and theoretically.[1-4] In the presence of a magnetic field a strong decrease of the hopping resistivity was observed.[1-3] This effect was interpreted on the basis of an increase of the transverse radius of the acceptor wave function in the applied magnetic field. This occurs in p-type narrow-gap diluted magnetic semiconductors as a result of the very strong exchange-induced anisotropy of the valence band (decrease of the transverse mass). The anisotropy of the overlap integrals of neighboring acceptors makes the p-type hopping conductivity anisotropic, which was also observed experimentally.[1-3] For magnetic fields stronger than 5–6 T an increase of resistivity was observed.[2] This effect is due to an increase of the binding energy of the acceptor[5] [the energy difference between the ground-state impurity level and the uppermost valence Landau level $b_v(-1)$] and is always observed in nonmagnetic semiconductors. Gawron[4] has used a model of phonon-induced hopping of an electron from an ionized acceptor center to a neutral one in order to explain dc hopping conduction in p-type $Hg_{1-x}Mn_xTe$.

The purpose of the present work is to study the photon-induced transitions as a hopping mechanism and their effect on the absorption of electromagnetic radiation in narrow-gap p-type $Hg_{1-x}Mn_xTe$ at 0 K. This model of the absorption process was proposed by Tanaka and Fan[6] to explain some experimental results concerning the ac impurity conduction in p-type Si at wavelengths of the order of 1 cm. According to this model photons induce transitions between the ground states of singly ionized two-acceptor pairs in the presence of an electrostatic field of ionized donors (the compensation must be different from zero in order to have ionized acceptor centers).

The wavelength region of interest is from 1.4 to 5 mm where other absorption mechanisms such as lattice vibrations, free carriers, or excitations of neutral impurity centers are absent at 0 K. We shall neglect the electron-phonon interaction because the deformation-potential constants of the Γ_8 heavy-hole band in HgTe (and $Hg_{1-x}Mn_xTe$ with low x) are very small.[7]

The organization of our paper is as follows. First, we shall present the acceptor ground states of the two-center Hamiltonian, in the presence of an external magnetic field, in the quantum-limit regime with an additional electrostatic field provided by the ionized donor center. Then, we calculate the transition rate for photon-induced hopping. Finally, the formula for the absorption coefficient is reported and the range of validity of our results is discussed. Numerical results are presented for electromagnetic waves with the electric vector perpendicular and parallel to the applied magnetic field. Unfortunately, we cannot compare our results with experiment because to our knowledge, no experimental data have been reported.

II. TWO-CENTER ACCEPTOR STATES

For $Hg_{1-x}Mn_xTe$, the presence of an external magnetic field makes the effective mass of the valence bands anisotropic. The effective mass in the direction perpendicular to the magnetic field is much lower than that in the direction parallel to it. As a consequence, in the case of narrow-gap $Hg_{1-x}Mn_xTe$ (i.e., open-gap $Hg_{1-x}Mn_xTe$ with low x) for an external magnetic field sufficiently strong, the wave function of the shallow acceptor state may be constructed from the wave functions of the uppermost valence level $b_v(-1)$ as the Landau-level spacing is much larger than the binding energy of the acceptor ("quantum limit"). The quantum-limit regime in narrow-gap semimagnetic semiconductors is already achieved for slightly strong magnetic fields (of the order of a few tesla).[8] Experimental data suggest the onset of the quantum-limit regime at 5–8 T in $Hg_{1-x}Mn_xTe$ (Refs. 2 and 4) (Gawron used a somewhat smaller value of 4 T in his theoretical calculations of acceptor binding energy[9] and hopping magnetoresistivity[4]).

The acceptor ground state in the quantum-limit regime

was observed by Mycielski and Mycielski[1] and investigated theoretically by Gawron and Mycielski.[8] The normalized wave function of the acceptor ground state has the form

$$\psi(\mathbf{r})=(2\pi\lambda)^{-1/2}\exp(-\rho^2/4\lambda^2)F(z/2^{1/2}\lambda)\Phi(\mathbf{r}) , \quad (1)$$

where the magnetic induction $\mathbf{B}$ is parallel to the z direction and the symmetric gauge with the vector potential $\mathbf{A}=\frac{1}{2}B(-y,x,0)$ was used. $\lambda=(\hbar c/eB)^{1/2}$ is the magnetic length and $\rho^2=x^2+y^2$. $\Phi(\mathbf{r})$ is the Bloch function corresponding to the band extremum (for total angular momentum equal to $-\frac{3}{2}$) and $F(z)$ is the normalized envelope function in the z direction which fulfills the following effective one-dimensional wave equation

$$\left[-\frac{1}{2}\frac{d^2}{d\xi^2}-\frac{\lambda}{a_\parallel}U(\xi)\right]F(\xi)=EF(\xi) , \quad (2)$$

where

$$U(\xi)=2^{1/2}\pi^{1/2}[1-\mathrm{erf}(|\xi|)]\exp(\xi^2) . \quad (3)$$

Here $\mathrm{erf}(x)$ is the error function and $\xi=z/2^{1/2}\lambda$. $a_\parallel=\hbar^2\epsilon/e^2m_\parallel$ is the effective Bohr radius calculated with the longitudinal effective mass (ϵ is the dielectric constant) and $E=2(E_{b_v(-1)}-E_A)/\hbar\omega_\parallel$, where $E_{b_v(-1)}$ is the energy of the uppermost Landau level $b_v(-1)$, E_A the acceptor ground-state energy, and $\omega_\parallel=eB/cm_\parallel$ the cyclotron frequency for the longitudinal effective mass. The envelope function in the z direction, $F(z)$, used in this work is a numerical solution of Eq. (2). A computer program developed by Gawron[4] was used for this. Recently, Wróbel *et al.*[5] have shown that the acceptor binding energy calculated numerically from (2) is in quite good agreement with the transport and optical experimental results. It should be stressed that the wave function given by (2) is appropriate for calculating the hopping transition rate since it has a correct asymptotical behavior.[4]

Now we have to obtain the two highest electron states (in the two-center acceptor problem) which are the eigenstates of the Hamiltonian

$$H=H_0+V+e^2/\epsilon r_a+e^2/\epsilon r_b . \quad (4)$$

H_0 is the valence-band electron Hamiltonian of the ideal crystal in the presence of an external magnetic field and includes the exchange interaction of the valence electron with the $3d^5$ electrons of the magnetic Mn^{2+} ions. V denotes the potential mainly due to the positively charged donor, the nearest one to the acceptors a and b. The position of the two acceptor centers are given by $\mathbf{R}_a$ and $\mathbf{R}_b$, and the vectors $\mathbf{r}_a$ and $\mathbf{r}_b$ are given by

$$\mathbf{r}_a=\mathbf{r}-\mathbf{R}_a, \quad \mathbf{r}_b=\mathbf{r}-\mathbf{R}_b .$$

The two last terms in Eq. (4) give the interaction between the valence electron and the negative ions a and b. Following Miller and Abrahams,[10] we shall apply the variational procedure, choosing the two variational orthonormal functions (for the highest two-center acceptor states) as a linear combination of the ground-state wave functions $u(\mathbf{r}_a)$ and $u(\mathbf{r}_b)$ of acceptors a and b:

$$\psi_a(\mathbf{r})=c_a^+u(\mathbf{r}_a)+c_b^+u(\mathbf{r}_b) , \quad (5)$$

$$\psi_b(\mathbf{r})=c_a^-u(\mathbf{r}_a)+c_b^-u(\mathbf{r}_b) , \quad (6)$$

where

$$c_a^\pm=\varkappa^\pm c_b^\pm , \quad (7)$$

$$c_b^\pm=[1+|\varkappa^\pm|^2+2\,\mathrm{Re}(S\varkappa^{\pm *})]^{-1/2} \quad (8)$$

$$\varkappa^\pm=(\Delta/2W^*)(1\pm\{1+4\,\mathrm{Re}(S^*W)/\Delta+4|W|^2/\Delta^2-4[\mathrm{Im}(S^*W)]^2/\Delta^2\}^{1/2})+i\,\mathrm{Im}(S^*W)/W^* . \quad (9)$$

We are using the following abbreviations, similar to those used in the paper of Blinowski and Mycielski[11] (Paper I):

$$\begin{aligned}
&W=W_R-\tfrac{1}{2}S\Delta+Z ,\\
&W_R=L-SJ ,\\
&L=\langle u(\mathbf{r}_a)|e^2/\epsilon r_a|u(\mathbf{r}_b)\rangle ,\\
&J=\langle u(\mathbf{r}_a)|e^2/\epsilon r_b|u(\mathbf{r}_a)\rangle ,\\
&S=\langle u(\mathbf{r}_a)|u(\mathbf{r}_b)\rangle ,\\
&\Delta=\langle u(\mathbf{r}_a)|V|u(\mathbf{r}_a)\rangle-\langle u(\mathbf{r}_b)|V|u(\mathbf{r}_b)\rangle ,\\
&Z=\langle u(\mathbf{r}_a)|V|u(\mathbf{r}_b)\rangle-\tfrac{1}{2}S[\langle u(\mathbf{r}_a)|V|u(\mathbf{r}_a)\rangle+\langle u(\mathbf{r}_b)|V|u(\mathbf{r}_b)\rangle] .
\end{aligned} \quad (10)$$

W_R is the resonance energy; it is complex when the external magnetic fields is present because of the phase factors which appear in integrals L and S containing the wave functions from different acceptor centers. For the same reason, Z is complex too; J and Δ are real. If we select the acceptor a as the closest one to the donor center, we have $\Delta\leq 0$. The functions $\psi_a(\mathbf{r})$ and $\psi_b(\mathbf{r})$ have larger amplitudes near the acceptors a and b, respectively, and the higher energy corresponds to the state $\psi_b(\mathbf{r})$. The energy difference between the two states is

$$\Delta E=(1-|S|^2)^{-1}\{\Delta^2+4|W|^2+4\Delta\,\mathrm{Re}(S^*W)-4[\mathrm{Im}(S^*W)]^2\}^{1/2} . \quad (11)$$

It is easy to show that the expression ΔE becomes that reported in I when L and S are real. The next step is to calculate the photon-induced transition between the two states described above and then to derive the formula for the absorption coefficients.

III. ABSORPTION COEFFICIENT

We are interested in the case of small compensation, when the compensation ratios K fulfill the condition

$$K<0.2 . \quad (12)$$

In this case at 0 K only the acceptor nearest to the compensating ionized donor is ionized (in our case the acceptor a). Let us take the origin of the spatial coordinate system at the donor center and investigate its vicinity. The electron jumps from an ionized acceptor a situated at $\mathbf{R}_a$ to neutral acceptor b situated at $\mathbf{R}_b$. The potential V is

the Coulomb potential of the ionized donor, and we shall neglect the dipole potentials of other ionized-acceptor–donor pairs presented in the crystal (this assumption will be discussed later).

The probability that the acceptor nearest to the donor center is in the element of volume dV_a and that there is a neutral acceptor in the element dV_b ($R_b > R_a$) is equal to

$$N^2 \exp(-4\pi N R_a^3/3) dV_a dV_b \ , \tag{13}$$

where N is the concentration of acceptors. If we multiply expression (13) by a transition rate W_{ab}, integrate over $\mathbf{R}_a$ and $\mathbf{R}_b$ (where $R_b > R_a$) and finally multiply the result by the concentration of donors KN, then the total number of jumps per unit time and volume will be obtained. The transition rate at 0 K from an arbitrary electron bound state a to the electron bound state b with the absorption of a photon can be written as

$$W_{ab} = (4\pi^2 e^2/c\hbar^2\epsilon^{1/2}) \mid \langle a \mid \mathbf{r} \mid b \rangle \hat{\mathbf{n}} \mid^2 I(\omega_0) \ , \tag{14}$$

where $\hat{\mathbf{n}}$ denotes the unit vector parallel to the electric vector of radiation and $I(\omega)$ the intensity of the radiation per unit interval of the angular frequency ω. The expression for the frequency ω_0 is

$$\omega_0 = \Delta E/\hbar \ ,$$

where ΔE is given by (11). For sufficiently low acceptor concentrations, it happens that for most acceptors pairs

$$|S| < 0.03 \ , \tag{15}$$

where

$$|S| = \exp[-R^2(1-x^2)/4\lambda^2] \times \int_{-\infty}^{\infty} dz\, F(z/2^{1/2}\lambda) F((z-Rx)/2^{1/2}\lambda) \ .$$

Thus, we can simplify expression (11) as follows:

$$\Delta E \simeq (\Delta^2 + 4 \mid W \mid^2)^{1/2} \ .$$

Using (5), (6), and (15), we have

$$\mid \langle a \mid \mathbf{r} \mid b \rangle \hat{\mathbf{n}} \mid^2 = (\mathbf{R}\cdot\hat{\mathbf{n}})^2 (\mid W_R \mid^2 + \mid Z \mid^2)(\Delta^2 + 4 \mid W \mid^2)^{-1} + (\mathbf{R}\cdot\hat{\mathbf{n}})[(\mathbf{R}\times\hat{\mathbf{n}})\cdot\hat{\mathbf{k}}]\Delta \, \mathrm{Im}(WS^*)(\Delta^2 + 4 \mid W \mid^2)^{-1} \ . \tag{16}$$

Here $\hat{\mathbf{k}}$ is the unit vector in the magnetic field direction and $\mathbf{R} = \mathbf{R}_b - \mathbf{R}_a$. Now, we will replace $I(\omega_0) = I[(\Delta^2 + 4 \mid W \mid^2)^{1/2}/\hbar]$ in expression (14) by $\delta(\omega - (\Delta^2 + 4 \mid W \mid^2)^{1/2}/\hbar)$. In this way, expression (14) becomes

$$W_{ab} = (4\pi^2 e^2/c\hbar^2\epsilon^{1/2}) \{ (\mid W_R \mid^2 + \mid Z \mid^2)(\Delta^2 + 4 \mid W \mid^2)^{-1} (\mathbf{R}\cdot\hat{\mathbf{n}})^2 + \Delta \, \mathrm{Im}(WS^*)(\Delta^2 + 4 \mid W \mid^2)^{-1} (\mathbf{R}\cdot\hat{\mathbf{n}})[(\mathbf{R}\times\hat{\mathbf{n}})\cdot\hat{\mathbf{k}}] \} \delta(\omega - (\Delta^2 + 4 \mid W \mid^2)^{1/2}/\hbar) \ . \tag{17}$$

The absorption coefficient α is the product of the total number of jumps per unit time and volume and energy $\hbar\omega$ of the photon

$$\alpha = \hbar\omega K N^3 \int dV_a \exp(-4\pi N R_a^3/3) \int_{R_b > R_a} dV_b \, W_{ab} \ , \tag{18}$$

where W_{ab} is given by (17). It is easy to verify that the contribution of the second term of the expression { } in formula (17) becomes zero when we try to evaluate the integrals in expression (18).

In the next step we will neglect the terms which contain $|Z|$. It means that the jump across the donor region cannot be taken into account (this assumption will be discussed later). As a consequence, we can consider the donor's field in good approximation as homogeneous in the region of the two acceptors a and b. It enables us also to set

$$\Delta \simeq -(e^2/\epsilon)(1/R_a - 1/R_b) \ .$$

After the change of variables it is possible to evaluate three of the six integrals in (18), and finally we have

$$\alpha = (16\pi^4 \epsilon^{1/2} K N^3/c\hbar) \left[(\hat{n}_x^2 + \hat{n}_y^2) \int_0^1 dx (1-x^2) L(x) + 2\hat{n}_z^2 \int_0^1 dx \, x^2 L(x) \right] \ , \tag{19}$$

where $\hat{n}_x$, $\hat{n}_y$, $\hat{n}_z$ are the components of $\hat{\mathbf{n}}$. Besides,

$$L(x) = \int_{R(x,\omega)}^{\infty} dR \, R^3 \mid W_R \mid^2 (\hbar^2\omega^2 - 4 \mid W_R \mid^2)^{-1/2} l(R, (\hbar^2\omega^2 - 4 \mid W_R \mid^2)^{1/2}) \ , \tag{20}$$

where $x = \cos\theta$, θ being the angle between $\mathbf{R}$ and the z axis of our coordinate system. The function $l(R, \mathcal{E})$, in expression (20), has the form

$$l(R, \mathcal{E}) = \int_{R_{a1}}^{R_{a2}} dR_a R_a^4 (1 - \epsilon R_a \mathcal{E}/e^2)^{-3} \exp(-4\pi N R_a^3/3) \ , \tag{21}$$

where

$$R_{a1} = e^2/\epsilon\mathcal{E} + R/2 - (e^4/\epsilon^2\mathcal{E}^2 + R^2/4)^{1/2}$$

and

$$R_{a2} = (e^2 R/\epsilon\mathcal{E} + R^2/4)^{1/2} - R/2 \ .$$

The lower limit in the integral over R results from the condition

$$2 \mid W_R(R,x) \mid \leq \hbar\omega \ , \tag{22}$$

where $\mid W_R(R,x) \mid$ has the form

$$\mid W_R \mid = (e^2/\epsilon)\exp[-R^2(1-x^2)/4\lambda^2]\left[(\pi/2)^{1/2}\lambda^{-1}\int_{-\infty}^{\infty} dz\, F(z/2^{1/2}\lambda)F((z-Rx)/2^{1/2}\lambda)\exp(z^2/2\lambda^2)\right.$$
$$\times[1-\mathrm{erf}(\mid z \mid /2^{1/2}\lambda)]$$
$$\left.-R^{-1}\int_{-\infty}^{\infty} dz\, F(z/2^{1/2}\lambda)F((z-Rx)/2^{1/2}\lambda)\right] . \tag{23}$$

In expression (23) we have substituted the matrix element J by $e^2/\epsilon R$.

Now we will discuss the assumption which permits us to neglect the elements which contain Z in (17). This assumption was already discussed in I, where the following restriction was found:

$$N < 8/\pi R^3(\omega) \ . \tag{24}$$

It was shown that if condition (24) is fulfilled, the contribution of jumps across the donor region to integral (21) is insignificant. Consequently, their contribution to the absorption coefficient is also insignificant. This is not exactly our case. First of all, we have $R(x,\omega)$ instead of $R(\omega)$ because we do not average over all directions of $\mathbf{R}$ as it was done in I. However, the main difference between the problem treated in I and ours comes from the phase effect introduced by the presence of external magnetic field.[12] The phase effect makes Z complex and even if the field of the donor is homogeneous in the region of the two acceptors (a and b), Z reduces to

$$i(2e^2/\epsilon)S[\mathbf{R}\times(\mathbf{R}_a+\mathbf{R}_b)]\cdot\hat{\mathbf{k}}/\mid \mathbf{R}_a+\mathbf{R}_b \mid^3 \tag{25}$$

and not to zero as is the case treated in I. The term (25) does not contribute significantly to the absorption coefficient if the function $R_a^4\exp(-4\pi NR_a^3/3)$ in integral (21) has its maximum at $R_a \gtrsim R(x,\omega)$ [the angular average $\langle \mid Z \mid^2 \rangle$ is less than $10^{-1} \mid W_R \mid^2$ for $R_a \gtrsim R(x,\omega)$]. It imposes the following restriction on the concentration of acceptors:

$$N \lesssim 1/\pi[\max R(x,\omega)]^3 \ , \tag{26}$$

which is stronger than (24).

It was shown in I that the main contribution to the integral (21) is given by $R_a \lesssim (\pi N)^{-1/3}$, where $(\pi N)^{-1/3}$ is the position of the maximum of the function $R_a^4\exp(-4\pi NR_a^3/3)$. Then, if we take into account conditions (12) and (26), the main contribution to the absorption coefficient is given by $R_a < 1/2(KN)^{1/3}$ and $R(x,\omega) < 1/2(KN)^{1/3}$. Thus, we can say that the hopping occurs predominantly between the acceptors within the volume which contains one donor and less frequently between acceptors from different volumes.

In this work we have also used the assumption that the dipole potentials of ionized donor-ionized acceptor pairs can be neglected. This assumption was analyzed in I and leads to the following restriction:

$$K < [\epsilon\hbar\omega/e^2\max R(x,\omega)]N^{-2/3} \ , \tag{27}$$

where $\max R(x,\omega)$, for a given ω, stands for $R(\omega)$ in I.

The validity of formula (19) for the absorption coefficient requires assumptions (12), (26), and (27). Besides, the pairs of R and x which contribute mostly to (19) must fulfill condition (15).

The last assumption made by us is $T=0$ K. This means that $kT \ll \hbar\omega$. In turn, $\hbar\omega$ must be much smaller than the ionization energy of an acceptor and the intensity of radiation must be small enough if we want to avoid the "saturation effect" (significant change of electron state occupation).

IV. NUMERICAL RESULTS

Our numerical calculations of the ratio α/K given by formula (19) were performed for $Hg_{1-x}Mn_xTe$ with the composition $x \simeq 0.14$. Following Gawron,[4,9] we have assumed the following material parameters: $\epsilon = 18$, $m_0/m_\parallel = 2.5$ (where m_0 is the free-electron mass). The calculations were performed for two values of the magnetic induction (4 and 6 T) and two concentrations of acceptors (0.5×10^{16} cm^{-3}, 1.0×10^{16} cm^{-3}).

In Fig. 1 we give the ratios $\alpha_\parallel/K$ and $\alpha_\perp/K$ (for $\hat{\mathbf{n}}\parallel\mathbf{B}$ and $\hat{\mathbf{n}}\perp\mathbf{B}$, respectively) as a function of the wavelength λ of radiation, for $B=4$ T and for the two concentrations mentioned above. In Fig. 2 we compare these ratios for the two values of magnetic induction (4 and 8 T) and for $N=1.0\times10^{16}$ cm^{-3}. We can see that the dichroic ratio $\alpha_\perp/\alpha_\parallel$ is larger for the case of smaller magnetic fields. Thus, for $\lambda=3$ mm this ratio has the values 1.68 and 1.48 for $B=4$ and 6 T, respectively.

The short-wavelength limitations in our numerical calculations were given by assumption (15) and the long-wavelength one by assumptions (26) and (27). In the case of the ratio $\alpha_\perp/K$, assumption (15) is fulfilled quite well only for $\lambda > 2$ mm. For a smaller value of λ our results are rather semiquantitative, and for $\lambda < 1.4$ mm the contribution of terms with $\mid S \mid$ larger than 0.2 starts to be significant. In the latter case our expression of the absorption coefficient is no longer meaningful.

In the case of the ratio $\alpha_\parallel/K$, contrary to the case of $\alpha_\perp/K$, the main contribution to the result is given by

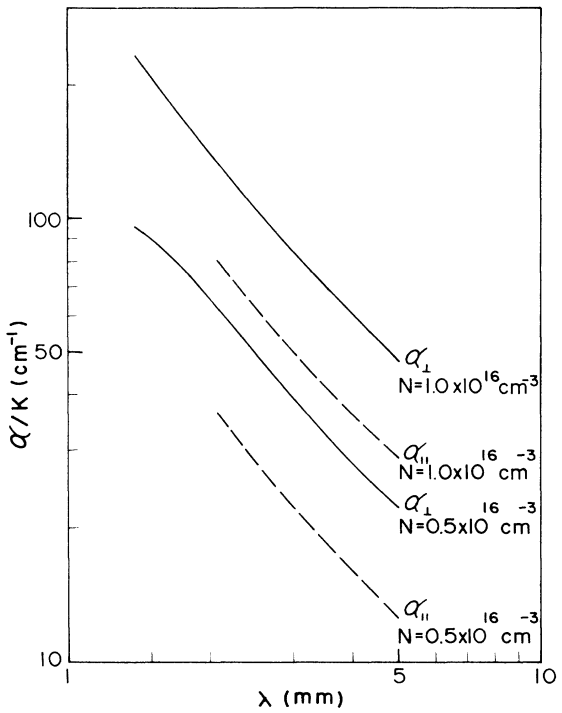

FIG. 1. The dependence of the absorption coefficient per unit compensation on the wavelength of radiation for $Hg_{1-x}Mn_xTe$ for $x \simeq 0.14$ at two concentrations of acceptors (0.5×10^{16} and 1.0×10^{16} cm^{-3}), and for magnetic induction $B=4$ T.

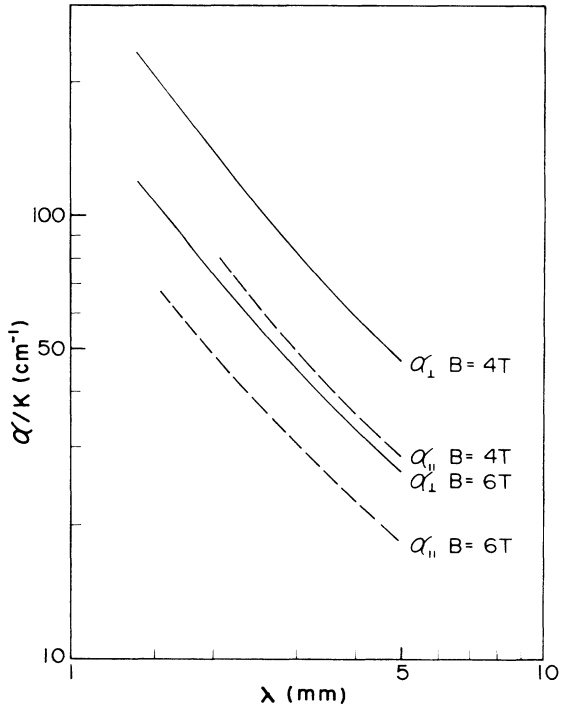

FIG. 2. The dependence of the absorption coefficient per unit compensation on the wavelength of radiation for $Hg_{1-x}Mn_xTe$ with $x \simeq 0.14$, for two values of magnetic induction (4 and 6 T), and for the concentration of acceptors $N=1.0\times10^{16}$ cm^{-3}.

terms with x near to one and for this value of x, the value of $|S|$, for a given R, is the largest one. Then we have to set a stronger short-wavelength limitation for the ratio $\alpha_{\parallel}/K$, and the results for $\lambda < 3$ mm are rather semiquantitative.

In our ranges of wavelength, assumption (27) is quite well fulfilled for both concentrations. For $Hg_{1-x}Mn_xTe$ with the composition $x \simeq 0.14$, the ionization energy of an acceptor[5] is more than 4 times higher than $\hbar\omega$ permitted by the assumptions discussed above. Thus, we could avoid the influence of the excited states of acceptors. The maximum value of the absorption coefficient in our ranges of wavelength, concentration, and compensation ratio is 47 cm^{-1}, for $N=1.0\times10^{16}$ cm^{-3}, $K=0.2$, and $\lambda=1.4$ mm.

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