Finite-size effects in the electrical conduction of thin wires

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We have studied the electrical properties of small-diameter Au-Pd wires of various lengths. At temperatures below about 10 K, the effects of electron-electron interactions become significant, causing the resistance to increase as the temperature is decreased. We have found that for wires with lengths less than a few μm, the magnitude of this resistance increase becomes smaller as the wire is made shorter, while for longer wires the behavior is independent of the length. The observed length dependence is in qualitative, but not quantitative, agreement with the theory.

I. INTRODUCTION

One of the most interesting features of recent theories of the electrical properties of disordered systems is the fundamental importance of various microscopic and macroscopic length scales.[1–4] Quantities such as the resistance are predicted to be strong functions of these length scales. For example, localization effects cause the resistance of a thin (i.e., small diameter) wire to vary exponentially with its length at absolute zero.[1] At nonzero temperatures the inelastic diffusion length (also commonly referred to as the phase breaking length[5]) is, in the simplest case, the relevant length scale,[6] but the remnants of this exponential variation are still observable through the temperature dependence of the resistance.[1,3,7] Electron-electron interaction effects are characterized by a different length scale, the so-called thermal length[8,3] $L_T=(\hbar D/k_BT)^{1/2}$. Both the phase breaking length, L_ϕ, which is important in localization, and the thermal length are manifest through the temperature dependence of the resistance, and L_ϕ also plays a role in the magnetoresistance.[3,8] One length scale which can be directly controlled in an experiment is the sample size. In the one-dimensional case, i.e., for a thin wire, the important scale we have in mind here is just the length of the wire, L_w. Ordinarily this length is much longer than other lengths in the problem such as L_ϕ and L_T, and as a result, the shorter length scales dominate. However, if L_w is made comparable to or smaller than these scales, then we would expect that it would contribute to, and in the appropriate limit even determine, the relevant length scale. This situation permits a very direct probe of the length scales which are important in localization and interaction effects.

In order to test these ideas we have performed experiments with wires whose lengths are comparable to L_ϕ and L_T. We find that the behavior is strongly dependent on L_w, indicating that it can indeed affect the relevant length scale. The experiments reported in this paper were all performed in zero magnetic field, and previous work on similar wires[9] has shown that in this case interaction effects are dominant. We will see that our results for the length-dependent behavior are in qualitative, but not quantitative, accord with the theory of finite size effects with regard to electron-electron interactions.[10] Some of our results have been reported previously,[11] but at that time there was no quantitative theory with which to compare. Such a theory is now available, and this has allowed us to draw much more definitive conclusions than were possible in our earlier report.

II. THEORY

Magnetoresistance measurements[9] have shown that in the absence of a magnetic field, interactions are dominant for the Au-Pd wires we have studied. We will therefore only concern ourselves here with the predictions of interaction theory. We should note, however, that our results are not even qualitatively consistent with the predictions of finite-size effects due to localization.[12,13] For a long wire, the theory predicts that interactions make a contribution to the resistance of the form[8,10]

$$\frac{\Delta R}{R_0}=BL_T\ , \tag{1}$$

where R_0 is the ordinary Boltzmann resistance due to elastic (i.e., impurity) scattering, and B is a constant which will be discussed shortly. The prediction (1) is applicable only for a long wire, that is, $L_w \gg L_T$. In the opposite limit we expect L_w to be the relevant length scale, so that (1) would be replaced by

$$\frac{\Delta R}{R_0}=B'L_w\ , \tag{2}$$

where the constant B' may be different from B. The key question for the present work concerns the nature of the behavior as one goes from a very long wire, for which $L_w \gg L_T$ [i.e., Eq. (1)], to a very short wire, for which $L_w \ll L_T$ [Eq. (2)]. This question has been addressed by Al'tshuler *et al.*[10] They find in the limit $L_w \gg L_T$

$$\frac{\Delta R}{R_0}=\frac{3\zeta(\frac{3}{2})e^2\rho L_T}{(2\pi)^{3/2}\hbar A}\left[1-\frac{3\sqrt{(1+F/2)}-1}{2\sqrt{(1+F/2)}+1}\right]-\frac{e^2\rho L_T^2}{6\hbar A L_w}\left[5-\frac{3[F+4\sqrt{(1+F/2)}-4]}{2[F+4\sqrt{(1+F/2)}+4]}\right]+\frac{5\sqrt{2}\zeta(\frac{5}{2})\rho e^2 L_T^3}{\pi^{5/2}\hbar A L_w^2} . \tag{3}$$

Here F is a screening factor[3,8] whose value is estimated from previous work[9] to be ≈ 0.1, ρ is the impurity resistivity, A is the cross-sectional area of the wire, and ζ is the zeta function.[14] In the opposite limit, $L_w \ll L_T$, the prediction has the form

$$\frac{\Delta R}{R_0}=\frac{e^2\rho L_w}{\pi\hbar A}[0.36-\tfrac{3}{2}\chi((1+F/2)^{-1/2})] . \tag{4}$$

The function $\chi(y)$ can be expressed in terms of a complicated definite integral which cannot be performed in closed form, but in the limit $y \to 1$ it is given approximately by[10] $\chi(y)=(1-y)/3$. These predictions are only applicable when the ratio L_T/L_w is far from unity. Our experiments were conducted in the regime in which this ratio approaches unity, so in order to compare with our results it was necessary to carefully consider the limits of applicability of (3) and (4), and also determine if an interpolation is needed. This is discussed in the Appendix, where we show that (3) can in fact be used over the entire range of interest for our experiments.

While the theoretical predictions and the analysis required to derive them are somewhat complicated, the behavior they predict is quite simple. If we define $x=L_T/L_w$, (3) has the form

$$\frac{\Delta R}{R_0}=a_1 L_T(1-a_2x^{-1}-a_3x^{-2}) , \tag{5}$$

and is valid for small x, while in the opposite limit, (4) is simply equivalent in form to (2). Here a_1, a_2, and a_3 are just constants which depend on F, A, etc. One can see that the effect of interactions approaches a constant value, independent of L_w, for long wires (large x), while as L_w (or equivalently, x) is decreased, $\Delta R/R_0$ also decreases, as a simple power of x. From (2) it can also be seen that $\Delta R/R_0$ simply varies linearly with L_w in the limit $L_w \to 0$.

III. EXPERIMENTAL METHOD

Sample fabrication was essentially a two-step process. First, relatively long (≈ 100 μm) wires were made using substrate step techniques, which have been described in detail elsewhere.[15–17] They were composed of $Au_{40}Pd_{60}$ (referred to as Au-Pd below), which was deposited by dc sputtering in an Ar atmosphere with a pressure of 100 mtorr, and which had a low-temperature (i.e., elastic) resistivity of ≈ 375 $\mu\Omega$cm.[17,18] Next, fiber masking techniques[19,20] were used to make much shorter wires in the following manner. A glass fiber was drawn over a flame so as to reduce its diameter to approximately a few μm or less. The fiber was then laid over the top of a wire as shown in Fig. 1. A film of either Au or Ag, approximately 1000 Å thick was then deposited, and the fiber was removed. The portion of the wire under the fiber was thus not overcoated, and it connected the two (relatively) thick contact pads. The resistance of the contact pads was of order 0.1 Ω. This was small compared to the resistance of a typical wire, which was ~ 500 Ω. The resistance of the contact pads was also sufficiently independent of temperature that it did not affect measurements of the sample resistance.

After the measurements were completed, all of the samples were examined using scanning electron microscopy in order to measure their lengths, and to ensure that they were not damaged, etc., during the fabrication process. The lengths determined from the electron microscopy were in good agreement with the values inferred from the measured resistance before and after fiber masking, together with the previously determined cross-sectional areas. These samples were much more sensitive to "blowout" from electrostatic pickup than the long wires, and great care was required to avoid destroying samples in this way.[21] The length to width ratio of the short wires was in the range 4–50. By comparing the results for samples of different cross-sectional areas but the same length (and hence different length to width ratios), it was found that the behavior was independent of this ratio.

Resistance measurements were made as a function of temperature in the absence of a magnetic field, using a cryostat of standard design which has been described elsewhere.[21,22] The temperature was measured with a calibrated germanium resistance thermometer, while the sample resistance was measured using a ratio-

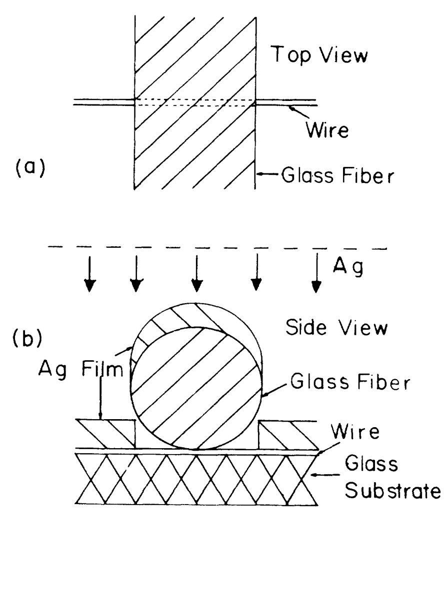

FIG. 1. Schematic description of the method used to make the short wires.

transformer bridge setup similar to that described by Gearhart *et al.*,[23] which compensated, in large part, for the effects of lead resistance. The residual effect of lead resistance was sufficiently small that the sample resistance could be measured with a resolution of 0.01 Ω.

IV. RESULTS

Typical results for the resistance as a function of temperature for several samples are shown in Fig. 2. Here we plot the normalized resistance change, $A\Delta R/CR_0$, where A is the cross-sectional area, and C is a constant ($=1.2\times10^{-13}$ cm^2) which is chosen, for convenience, so as to make the resistance rise between 1.5 K and 12 K unity for the longest wires. The results are plotted in this way so as to remove the dependence of the resistance rise on A, which is well known and understood.[8,16,18] Hence, in a plot of this type, the results for all wires should fall on a common curve, provided that the behavior is independent of their length. It can be seen from Fig. 2 that this is indeed the case for the two longest wires, but for the shorter wires the normalized resistance rise falls systematically below this curve. This is in qualitative accord with the behavior discussed in Sec. II.

In Fig. 3 we show the same data plotted as a function of $T^{-1/2}$, which is the temperature dependence predicted by the theory[8,3] for long wires (1). This temperature dependence has also been observed previously in long wires,[16] and our longer samples are seen to follow this form fairly well.[24] However, the shorter wires do not follow this dependence, but seem instead to approach a constant as $T\rightarrow0$. This is as expected, since in this limit the relevant length scale becomes L_w [see (2)], which is a constant, independent of temperature.

Figure 4 shows the normalized resistance rise between

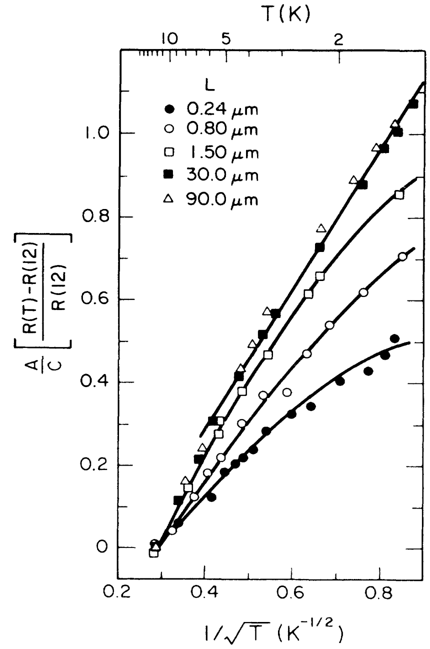

FIG. 3. Results for the normalized resistance rise as a function of $T^{-1/2}$.

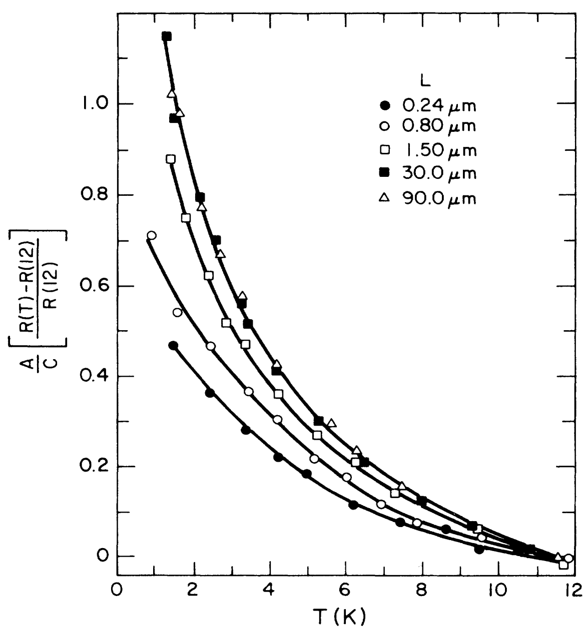

FIG. 2. Results for the normalized resistance rise as a function of temperature for wires of various lengths as indicated in the figure. The diameters of the samples were 590 Å, 500 Å, 430 Å, 370 Å, and 600 Å, where we have listed the value for the longest wire first, etc. The choice of the normalization factor is discussed in the text.

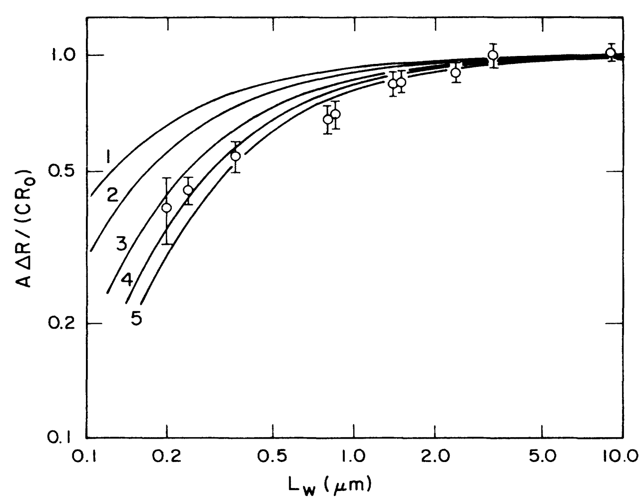

FIG. 4. Normalized resistance rise from 12 K to 1.5 K as a function of the length of the wire, L_w. The solid curves are predictions of the theory for $F=0.1$, and for different values of D as discussed in the text. The curves labeled 1–5 correspond to $D=2.2$, 4, 8, 12, and 16 cm^2/s respectively. The symbols are the experimental results.

12 K and 1.5 K as a function of the length of the wire. Note that because of the normalization we have used, the magnitude of the rise for a long wire is unity in Fig. 4. One can see that for wires shorter than a few μm the resistance rise falls systematically below the value found in the limit $L_w \rightarrow \infty$. The solid curves in Fig. 4 were obtained from (3) using different values of the diffusion constant, D, as we will now discuss.

The normalization employed in Fig. 4 removes the dependence on the cross-sectional area of the sample. The theoretical predictions for this case then involve only two adjustable parameters, the screening factor, F, and the diffusion constant, D. Both are known approximately from previous studies[16,9] of Au-Pd. By its definition,[8,3] F lies between 0 and 1, and previous experiments[9] suggest a value of $\approx 0.1 \pm 0.1$. We have found that varying F over the range 0–0.5 has essentially no effect on the results and conclusions which follow. We have therefore used $F=0.1$ in all of the calculations below. The value of the diffusion constant is, however, very important, since it directly determines the length scale L_T. Using simple free-electron theory, it was previously estimated[9] that for Au-Pd made in the manner we have employed, $D \approx 2.2$ cm^2/s. The uppermost theoretical curve (labeled 1) in Fig. 4 was obtained with this value of D, which leads to $L_T = (\hbar D / k_B T)^{1/2} \approx 410$ Å at 1 K. It is clear from Fig. 4 that this value of D does not lead to predictions consistent with our results. However, since this value of D was obtained from free-electron theory, it could certainly be in error by a significant amount, so it is worth considering other possible values. One can obtain an independent estimate of D by considering the temperature dependence of $\Delta R / R_0$ for the long wires. This is described by the first term in (3), and it can be seen that this term is simply proportional to L_T, and hence to $D^{1/2}$. A comparison of this prediction for $\Delta R / R_0$ with results for our long Au-Pd wires (i.e., the two longest wires considered in Fig. 2) yields $D \approx 3.5 \pm 0.5$ cm^2/s. We believe that for the purposes of this paper, namely to test the theory (3) and (4), this is really the "best" way to estimate D, since we are in a sense using the theory self-consistently to determine D. Note, however, that this method for estimating D uses only the first term in (3) [or equivalently, (6)], and is therefore *independent* of the length dependent predictions which we wish to test. Using the value $D=4$ cm^2/s gives the curve labeled 2 in Fig. 4. While this is in better agreement with the experiments than the curve labeled 1, the agreement is still not very good. The curves labeled 3–5 in Fig. 4 were obtained using $D=8$, 12 and 16 cm^2/s respectively, and it is seen that the value $D \approx 12 \pm 4$ provides an approximate best fit to the experiments.[25] The residual discrepancy between the curve for $D=12$ cm^2/s and the experimental results could be due to the effects of localization, which are known[9] to make a small contribution to $\Delta R / R_0$, although the differences between the experiment and theory for $D=12$ cm^2/s in Fig. 4 are probably inside the experimental uncertainties.

Based on the results in Fig. 4, we conclude that the theory is certainly in at least qualitative agreement with our results. The overall functional form predicted for the length dependence is quite consistent with the experiments. However, there is a disturbing discrepancy concerning the value of D, or equivalently, L_T. The value of D inferred from the magnitude of the resistance rise exhibited by the long wires is a factor of 3 smaller than the value required to account for the length dependence. This difference appears to be outside the uncertainties in parameters, such as D, which are needed to evaluate the theoretical predictions.

V. DISCUSSION

We have performed a detailed study of the length dependence of electron-electron interaction effects in thin Au-Pd wires. Our results are in qualitative agreement with the behavior expected for this case. However, we find that the length scale L_T needed to account for the observed length dependence is almost a factor of 2 larger than that required to explain the behavior of the long wires as a function of temperature. The reason for this discrepancy is not understood.

It is interesting to note that in previous work[9] it was pointed out[26] that the observed one-dimensional behavior of 400 Å Au-Pd wires with respect to electron-electron interaction effects at temperatures above 1 K was inconsistent with the value of $L_T \approx 410$ Å (at 1 K) obtained from the theory using $D=2.2$ cm^2/s. Our results suggest that L_T is much longer than this. Using the value of D obtained from the results for the length dependence (Fig. 4) we find $L_T \approx 900$ Å at 1 K. This value of L_T is in much better accord with the one-dimensional behavior found for these wires. However, it does not address the problem of why the value of D (and hence L_T) needed to account for the behavior of the long wires differs from the value found from the length dependence. We should also point out that in recent work on Bi wires and films,[27] it has been found that the value of L_T inferred from the temperature dependence of the resistance can be substantially different from that inferred from the effective dimensionality of the inelastic electron-electron scattering.[28] While the work on Bi involves measurements which are very different from those reported here, the problems with understanding the different values of L_T obtained in different ways may well be related.

While the theory can account at least qualitatively for our one-dimensional length-dependent results (Fig. 4), we should also point out that we have performed similar experiments in two dimensions (i.e., with thin films).[29,21] In the two dimensional case our results are not in agreement with even the qualitative predictions of the theory. We found that in two dimensions the overall resistance rise does decrease as the length of the sample is reduced (in analogy with Fig. 2), but the form of the temperature dependence is *unchanged* even for the shortest samples, in marked contrast to the one-dimensional results (Fig. 3). Theoretical calculations[10] performed along the same lines as the one-dimensional theory, (3) and (4), cannot account even qualitatively for this behavior. We therefore conclude that the length dependent properties of one- and two-dimensional conductors are still only partially understood.

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APPENDIX

As noted in the discussion of the theoretical predictions (3) and (4), these expressions are applicable only in the limits $L_w \gg L_T$ and $L_w \ll L_T$, respectively. Since most of the experimental results were in the intermediate regime, it is necessary to consider the limits of applicability of (3) and (4). Figure 5 shows a plot of (3) and (4) for $F=0.1$ and $D=16$ cm^2/s. The results are essentially independent of the value of F, but as was noted in connection with Fig. 4 they are quite sensitive to D. From Fig. 5 we see that (3) and (4) match up fairly well, and that (4) is not badly behaved until L_w becomes smaller than about 0.15 μm. Our shortest sample was 0.20 μm long, so it appears safe for our purposes to simply use (3) to compare with our results. All of the theoretical curves in Fig. 4 were therefore obtained directly from (3). We note that the value of L_w below which (3) is clearly unreliable, which is ≈ 0.15 μm in Fig. 5, becomes smaller as D is made smaller. The value of D used for Fig. 5 is the largest one considered in this paper, and hence for the other values of D we have considered, (3) can be used down to even smaller values of L_w.

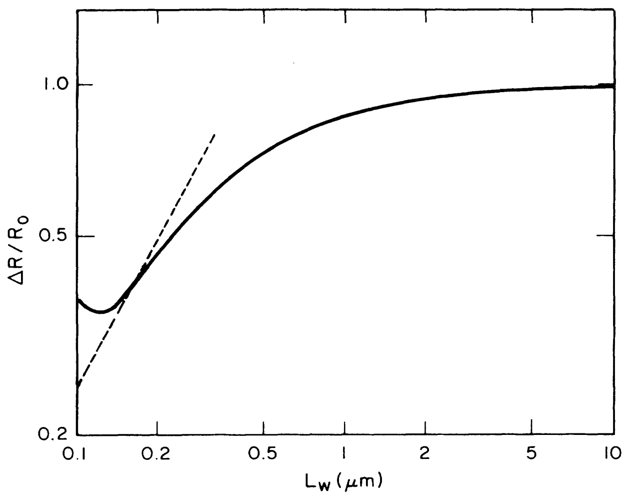

FIG. 5. Theoretical predictions (3) (solid curve) and (4) (dashed curve) for $\Delta R/R_0$ at 1.5 K as a function of L_w. The parameter values $F=0.1$, $D=16$ cm^2/s, $A=1.4\times10^{-11}$ cm^2, and $\rho=3.75\times10^{-4}$ Ω cm were used. Note that this plot differs slightly from Fig. 5 where we plotted the *difference* between $\Delta R/R_0$ at 1.5 K and at 12 K.

We have also considered the use of an interpolation scheme employing a Padé approximant. For the range of L_w of interest here, it gives results for Fig. 4 which are essentially identical to those obtained using (3) directly.

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[26]In Ref. 9 it was found that the use of $D=2.2$ cm^2/s gave reasonable (though not perfect) agreement with results for long Au-Pd wires. This is somewhat smaller than the value ($D=4$ cm^2/s) we find here. This difference is due to a slightly smaller impurity resistivity in the present case, and when this [and also the 10% difference between (3) and Ref. 8, which was used in the analysis of Ref. 9] is taken into account, the two experiments are in agreement to within the combined uncertainties.

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