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Linear temperature behavior of the resistivity in the new high-T_c superconductors

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We propose that the linear temperature behavior of the resistivity observed in the new high-T_c materials might be linked to their two-dimensional character for electron transport. For three-dimensional electron transport a linear temperature law for the resistivity is obtained for temperatures above the Debye temperature Θ_D. For two-dimensional transport, however, such a linear law holds above a characteristic temperature $T^* = 2\hbar s p_F$ which can be well below Θ_D (s denoting the sound velocity and p_F the Fermi momentum) provided that $p_F \ll \pi/a$, which is the case for these high-T_c materials.

The recent discovery of high-T_c superconducting materials by Bednorz and Müller[1] was first of all followed by worldwide initiatives in the search for even higher T_c: basically material research. By now, sample preparation and measurements have become refined enough to use these experimental results to try to put together a systematic theoretical picture of these new materials.

One of the unusual features of these materials is the linear temperature dependence of their resistivity observed in a large temperature regime starting just above T_c and extending, in general, to several hundreds degrees. These findings were reported, for both the first-generation high-T_c materials La-(Ba,Sr)-Cu-O (Refs. 2-7) having $T_c \approx 35$ K, as well as the second-generation materials Y-Ba-Cu-O (Refs. 8-13) having $T_c \approx 95$ K.

Structural analyses[3,5,14] and band-structure calculations[15,16] both indicate that electronically these materials may be considered to consist of well-separated stacked planes of Cu ions. It is therefore reasonable to examine the problem of two-dimensional electrical conductivity due to electron-phonon scattering.

Let us for this purpose write down the Kubo formula for the electrical conductivity tensor in the limit of an applied homogeneous and static field:

$$\sigma_{kl}^{(d)}(\mathbf{q}=0,\omega\rightarrow 0)=\frac{e^2}{m^2}\hbar^3\int\frac{d^dp}{(2\pi)^d}\frac{d}{dhp}\left[\frac{1}{2}\tanh\left(\frac{\beta h_p}{2}\right)\right]\frac{p_k p_l}{\Gamma^{(d)}(0)}\ , \tag{1}$$

where d denotes the dimensionality of the system, $p_{k(l)}$ indicates the kth (lth) component of the electron momentum p, and $h_p=(\hbar^2p^2/2m)-\mu$, where m is the electron mass and μ the Fermi energy. The expression for the relaxation lifetime $\hbar/\Gamma(h_p)$ is given in Ref. 17.

$$\Gamma^{(d)}(h_p)=\int\frac{d^dq}{(2\pi)^d}|v(\mathbf{p}-\mathbf{q})|^2\left(1-\frac{\mathbf{p}\cdot\mathbf{q}}{p^2}\right)\{[N(\omega_{\mathbf{p}-\mathbf{q}})+1-f(h_q)]2\pi\delta(h_q+\omega_{\mathbf{p}-\mathbf{q}}-h_p)$$
$$+[N(\omega_{\mathbf{p}-\mathbf{q}})+f(h_q)]2\pi\delta(h_q-\omega_{\mathbf{p}-\mathbf{q}}-h_p)\}\ , \tag{2}$$

where N and f denote the Bose and Fermi distribution functions, respectively. Without any loss of generality we consider here only one type of phonon, longitudinal ones, having frequency $\omega_{\mathbf{p}-\mathbf{q}}$. $v(\mathbf{p}-\mathbf{q})$ denotes the electron-phonon scattering matrix element.

Provided we are in a temperature regime $\beta^{-1}\equiv k_BT\ll\mu$, we can easily carry out the integral in Eq. (1) using the relation $d/dh_p[\frac{1}{2}\tanh(\beta h_p/2)]\simeq\delta(h_p)$. In particular, for the isotropic contribution to the conductivity we obtain

$$\sigma^{(3)}(0,0)=\sum_{k=1,2,3}\sigma_{kk}^{(3)}(0,0)=\frac{e^2}{m}\frac{2}{3}\frac{\hbar}{(2\pi)^2}\frac{p_F^3}{\Gamma^{(3)}(0)}=\frac{e^2}{m}\frac{\hbar n}{\Gamma^{(3)}(0)}\ ,$$
$$\sigma^{(2)}(0,0)=\sum_{k=1,2}\sigma_{kk}^{(2)}(0,0)=\frac{e^2}{m}\frac{1}{2}\frac{\hbar}{(2\pi)}\frac{p_F^2}{\Gamma^{(2)}(0)}=\frac{e^2}{m}\frac{\hbar n}{\Gamma^{(2)}(0)}\ , \tag{3}$$

where we use $2(4\pi/3)p_F^3=n$ and $2p_F^2\pi(\pi/a)=n$ for $d=3$ and $d=2$, respectively; n denotes the number of electrons per unit volume and p_F the Fermi momentum.

In order to show that for $d=2$ the linear law for the resistivity can hold well below $k_B\Theta_D$, contrary to $d=3$, we shall now carry out explicitly the evaluation of the inverse transport lifetime Eq. (2) for $d=2$ and $d=3$. For that purpose let us suppose that the Fermi sea has a circular Fermi surface for $d=2$ and a spherical one for $d=3$. The volume integral in Eq. (2) becomes for $d=3$ and $d=2$, respectively,

$$\begin{aligned} d^3q &\equiv 4\pi\, d\cos\theta\, q^2 dq = 2\pi(E\,dE/\hbar^2 s^2)\,dq_{\parallel} \,, \\ d^2q &\equiv d\theta\, q\, dq = [E/(\hbar s)^2][(E/\hbar s)^2-(p-q_{\parallel})^2]^{-1/2}\,dq_{\parallel}dE \end{aligned} \tag{4}$$

after introducing the new variables $E=s\,|\mathbf{p}-\mathbf{q}|$ and $q_{\parallel}=q\cos\Theta$, where s denotes the sound velocity. We thus obtain for the inverse relaxation time

$$\begin{aligned} \Gamma^{(3)}(0) &= \frac{m}{(2\pi)(\hbar s)^4\hbar^2 p_F^3}\int_0^{k_B\Theta_D} dE\,E^3[N(E)+f(E)]\,|v(E)|^2 \,, \\ \Gamma^{(2)}(0) &= \frac{m}{(2\pi)(\hbar s)\hbar^2 p_F}\int_0^{k_B\Theta_D} dE[N(E)+f(E)]\,|v(E)|^2 \\ &\quad\times\left[\sum_{+-}\left(\frac{E^2}{2\hbar^2 p_F^2 s^2}\pm\frac{Em}{\hbar^2 p_F^2}\right)\left(1-\frac{m^2s^2}{\hbar^2p^2}\mp\frac{Em}{p_F^2\hbar}-\frac{E^2}{(\hbar s)^2(2p_F)^2}\right)^{-1/2}\right] . \end{aligned} \tag{5}$$

For $d=3$, $\Gamma^{(3)}(0)\propto k_BT$ for $T>\Theta_D$, which is the well-known result. For $d=2$, however, the square roots in expression $\Gamma^{(2)}(0)$ in Eq. (5) put upper limits on the E integration which are $2\hbar p_F s-2ms^2=[v_F-2s]2ms$ and $2\hbar p_F s+2ms^2=[v_F+2s]2ms$, where v_F is the Fermi velocity. For $v_F\gg 2s$—which seems to be justified in these new high-T_c materials—we obtain

$$\Gamma^{(2)}(0)=\frac{4\pi k_BT}{\pi\hbar^2}\int_0^1 dx\,\frac{x}{(1-x^2)^{1/2}}\,|v(x\hbar p_F 2s)|^2 \,, \tag{6}$$

which is valid for $k_BT>2\hbar p_F s$ provided $\hbar p_F s\ll\hbar s\pi/a=k_B\Theta_D$. Hence, we obtain a linear T law for the resistivity, which for $d=2$ is valid for $k_BT>2\hbar p_F s$, well below the Debye temperature.

That these conditions are satisfied follows from the various spectroscopic measurements[18] such as x-ray photoemission spectroscopy (XPS), ultraviolet photoemission spectroscopy (UPS), electron-energy-loss spectroscopy (EELS), and bremsstrahlung isochromat spectroscopy (BIS), all of which converge to the picture of a Fermi level which lies in a minimum of the density of states (having about 1.3 states/eV cell). This, together with the measurements of the dimension of the Fermi surface,[19] leads to very low carrier concentrations for these new high-T_c materials ($\sim 5\times10^{21}/\text{cm}^3$ for Y-Ba-Cu-O, and slightly higher for the Ba-La-Cu-O). Estimations of the Fermi velocity v_F and the effective mass of the carriers m_{eff} are[20] $v_F\approx10^7$ cm/sec and $m_{\text{eff}}\approx 5m_e$, where m_e denotes the free-electron mass. We take for the sound velocity a typical value of $s\approx10^6$ cm/sec which is in rough agreement with the values of the Debye temperatures for these materials; i.e., $\Theta_D\approx$ 400–500 K for Ba-La-Cu-O (Ref. 21) and $\Theta_D\approx$ 330–400 K for Y-Ba-Cu-O (Ref. 22). With these values we find that the conditions assumed in deriving expression (6) are justified, i.e., $v_F\gg s$ and $p_F\ll\pi/a$. The latter inequality follows from the relation $n=p_F^2\pi\times(4\pi/a)$ or $p_F=\sqrt{n}\,(a/\pi)^{3/2}(4\pi)^{-1/2}(\pi/a)$, which for the numerical values given above leads to $p_F\approx2.6\times10^{-2}(\pi/a)$. For the lattice constant a we have taken the value 3.8 Å.

Experimentally the linear-in-T behavior of the resistivity is sometimes observed down to temperatures of the order of $\Theta_D/2$ or even $\Theta_D/3$ although theoretically it is expected to hold only for $T>\Theta_D$. The fact that in those new high-T_c materials the linear-in-T law is observed down to $\Theta_D/4$ in Y-Ba-Cu-O and to $\Theta_D/10$ in Ba-La-Cu-O merits a serious questioning of the origin of this linear-in-T behavior. We have shown above how such a behavior can be explained within the standard electron-phonon dissipation mechanism, provided the electronic structure is bidimensional and the number of carriers is small. We should note that a different mechanism based on electron-electron scattering which also gives rise to a linear-in-T law for the resistivity has been proposed.[23]

In the present analyses we have totally neglected contributions coming from umklapp processes. Umklapp process only play a role provided that the Fermi radius p_F is bigger than half the Debye radius $\propto(\pi/a)$.[24] This does not seem to be the case for these high-T_c materials and we can restrict ourselves to normal processes only. For the sake of completeness let us briefly examine the situation for the low-temperature limit. For that purpose we have to make some assumptions about the electron-phonon scattering matrix element $v(E)$. Within the picture of the deformable potential approximation one can write $|v(E)|^2=\alpha E$ where α is a constant having dimensions of energy. Using this expression for $|v(E)|^2$ in Eqs. (5) we can easily see that for $k_BT\ll k_B\Theta_D$ for $d=3$ and for $k_BT\ll2\hbar p_F s$ for $d=2$ the integrations over E can be extended to infinity, yielding

$$\Gamma^{(3)}(0) = \frac{(k_B T)^5}{(2\pi)(\hbar s)^4} \frac{m}{p_F^3 \hbar^2} \alpha \int_0^\infty dx\, x^4 [N(x) + f(x)], \quad T \ll \Theta_D \;,$$

$$\Gamma^{(2)}(0) = \frac{(k_B T)^4}{(2\pi)(\hbar s)^3} \frac{m}{p_F^3 \hbar^3} \alpha \int_0^\infty dx\, x^3 [N(x) + f(x)], \quad T \ll 2\hbar p_{FS} \;. \tag{7}$$

In conclusion, we have shown that the linear temperature behavior of the resistivity well below the Debye temperature Θ_D observed in the newly discovered high-T_c superconductors is a consequence of the two-dimensional electronic character of these materials and the fact that $p_F \ll \pi/a$ is due to the small number of carriers in these materials. We have demonstrated that for $k_B T > 2\hbar p_{FS}$ (being much smaller than $k_B \Theta_D$) the resistivity $\rho \propto T/\sqrt{n}$, where n denotes the concentration of carriers, p_F the Fermi momentum, and s the sound velocity.

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