Experimental study of the critical scattering from a two-dimensional random Ising antiferromagnet

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The magnetic critical fluctuations in the $d=2$ random-exchange antiferromagnet $Rb_2Co_{0.7}Mg_{0.3}F_4$ have been studied by neutron diffraction at temperatures both above and below the Néel temperature $T_N=40.8$ K. At temperatures above T_N, the line shape of the critical scattering is well described by the Lorentzian form, and the critical exponents ν and γ for the inverse correlation length $\kappa^+=\kappa_0^+ \mid t \mid^{\nu}$ and the staggered susceptibility $\chi^+=C_0^+ \mid t \mid^{-\gamma}$ are in good agreement with the values for the pure $d=2$ Ising model. The critical exponent β, which, along with the critical amplitude M_0, describes the temperature dependence of the $d=2$ long-range antiferromagnetic order just below T_N through $M=M_0 \mid t \mid^{\beta}$ is also in good agreement with the value of β for the pure $d=2$ Ising model. Furthermore, the combination of critical amplitudes $R_s=C_0^+(\kappa_0^+)^2/M_0^2$, which, from the hypothesis of two-scale-factor universality, is a universal quantity also agrees well with the values of R_s for the pure $d=2$ Ising model and the value of R_s found experimentally for the pure $d=2$ Ising antiferromagnet K_2CoF_4. The line shape of the critical scattering below T_N has been analyzed both by using a Lorentzian form and by using the form proposed by Tarko and Fisher for the pure $d=2$ Ising model below T_N. The analysis using the Lorentzian form leads to a physically unreasonable description of the critical scattering below T_N and hence to the conclusion that the Lorentzian form is an inappropriate description of the critical scattering below T_N in a $d=2$ random Ising system. The results of the analysis using the Tarko-Fisher form provide a better description of the critical scattering below T_N and the critical exponents for the inverse correlation length $\kappa^-=\kappa_0^- \mid t \mid^{\nu}$ and the staggered susceptibility $\chi^-=C_0^- \mid t \mid^{-\gamma}$ agree well with the values for the pure $d=2$ Ising model. However, the values of the universal critical amplitude ratios κ_0^-/κ_0^+ and C_0^+/C_0^-, which result from this latter analysis, differ significantly from the values for the universality class of the pure $d=2$ Ising model. This difference in the amplitude ratios may arise because the $d=2$ random Ising model is in a different universality class than the pure $d=2$ Ising model or because the Tarko-Fisher form for the line shape may not be appropriate for the $d=2$ random Ising model below T_N. An investigation of this latter possibility using a modified version of the Tarko-Fisher form which preserved the amplitude ratio of the inverse correlation lengths at the pure $d=2$ Ising model value suggests that this is a strong possibility.

INTRODUCTION

The critical properties of magnetic systems with random exchange constants are generally believed to be dependent upon the sign of the specific-heat exponent, α, of the analogous pure material. If α is positive, the behavior is expected[1] to be different from that of the pure material, while if α is negative, the behavior is expected to be the same as that of the pure material. In recent years, both experimental[2] and theoretical[3] work has concentrated on the random-exchange $d=3$ Ising model for which α is positive. Detailed experimental studies on homogeneously random crystals of $Fe_xZn_{1-x}F_4$ and $Mn_xZn_{1-x}F_4$ have shown that the critical exponents of these disordered systems are only slightly different from those of the pure material, but that the critical-amplitude ratios are substantially different from those of the pure material. The results of theoretical calculations for the $d=3$ random Ising model are in good accord with the values of the measured critical exponents but are in less satisfactory agreement with the experimental critical-amplitude ratios.

There has been less theoretical and experimental attention focused on systems for which the pure material has $\alpha=0$, such as the two-dimensional Ising model. It might be expected that since $\alpha=0$ is the marginal value for the change in critical behavior that the critical exponents of such a random system would be the same as those of the pure system but that there would be logarithmic corrections to the pure-system behavior. A theoretical calculation of the specific-heat capacity[4] and the real-space correlation function[5] at the critical temperature has been performed by Dotsenko and Dotsenko for the random $d=2$ Ising model. These authors found that the temperature dependence of the specific-heat capacity of the random system was given by $\ln(-\ln t)$ instead of the $-\ln t$ dependence of the pure system, where $t=(T-T_N)/T_N$. This agrees with the expectation of a logarithmic correction to the critical behavior of the pure system. The result of

Dotsenko and Dotsenko for the real-space correlation function at the critical temperature is that for large spatial separation R the correlation function is proportional to $\exp(-Bf^2+Df)$, where $f=\ln(\ln R)$ and B and D are constants. This is quite different from the behavior of the pure $d=2$ Ising model where the correlation function is proportional to $R^{-0.25}$ at the critical temperature, and predicts that for the random system the exponent η is 0 rather than the value of 0.25 for the pure system. Experimentally, the critical properties of the $d=2$ random Ising model have been studied previously both by neutron scattering measurements[6] above the transition temperature and by birefringence measurements[7] both above and below the transition temperature. The neutron scattering measurements were performed on samples of $Rb_2Co_xMg_{1-x}F_4$ with $0.75 \leq x \leq 1.0$. These critical scattering measurements were analyzed to obtain values of the critical exponents ν and γ which were found to be in reasonable accord with the values of the pure $d=2$ Ising model. The birefringence measurements were performed on a sample of $Rb_2Co_{0.85}Mg_{0.15}F_4$ and were analyzed to obtain the behavior of the specific-heat capacity. This behavior was found to be a symmetric logarithmic dependence on the reduced temperature, as in the pure $d=2$ Ising model.

In view of the predictions of Dotsenko and Dotsenko, and because of the surprisingly large changes in the critical-amplitude ratios found in the $d=3$ random Ising systems, we have performed a detailed study using neutron scattering techniques of the critical behavior both above and below the Néel temperature T_N in the $d=2$ random Ising antiferromagnet $Rb_2Co_{0.7}Mg_{0.3}F_4$. These measurements were performed and the results analyzed in a similar manner to previous studies of the critical fluctuations below T_N in K_2CoF_4 (Ref. 8) and the hypothesis of two-scale-factor universality in Rb_2CoF_4 (Ref. 9).

The rest of this paper is set out as follows. In the next section we describe the experimental details and the way in which the results were analyzed. The third section contains the experimental results, and in the fourth and final section, the conclusions from these results are given.

EXPERIMENTAL TECHNIQUES AND DATA ANALYSIS

$Rb_2Co_{0.7}Mg_{0.3}F_4$ is an ideal example of a nearly two-dimensional random Ising system. The crystal field on the Co ions produces a Kramers-doublet ground state within which the magnetic properties can be described by an effective spin, $s=\frac{1}{2}$. The magnetic interactions have been studied[10] and are predominately between nearest-neighbor Co ions in planes perpendicular to the crystallographic c axis, and these planes order antiferromagnetically below the Néel temperature $T_N=40.8$ K. The interaction between the planes is much weaker and leads to some degree of three-dimensional order depending upon the cooling rate through T_N and the waiting time after cooling.[11]

The crystal used in the measurements reported here has been previously used for neutron scattering studies and its growth is described in that paper.[6] It was of very good crystallographic quality with a mosaic spread of less than 0.05°, there was no evidence for any chemical ordering of the Co and Mg ions, and finally, the concentration uniformity as measured from the spread in T_N was less than 0.3% [full width at half maximum (FWHM)].

The crystal was mounted in a variable-temperature displex refrigerator with the magnetic [100] axis and the crystallographic c axis in the scattering plane. The temperature was controlled to within a stability of ± 0.02 K. The neutron scattering measurements were performed using the HB3 spectrometer at the High Flux Isotope Reactor of Oak Ridge National Laboratory in a two-axis mode. A pyrolytic-graphite monochromator was used to produce an incident neutron beam with an energy of 14.8 meV, and a pyrolytic-graphite filter was used to suppress neutrons scattered from higher-order planes in the monochromator. The in-plane collimation used was 20′ between reactor and monochromator, monochromator and sample, and sample to detector. The resolution function was measured at the (1,0,0) magnetic lattice point and found to have a FWHM along [ζ00] of 0.0162 reciprocal-lattice units ($2\pi/a$) and a FWHM vertically along [0ζ0] of 0.089 reciprocal-lattice units. The magnetic unit-cell lattice constants were measured to be $a=5.7915$ Å and $c=13.71$ Å.

The critical scattering was measured by varying the wave-vector transfer along the direction [ζ 0 -0.4] in reciprocal space, through the point $(1,0,-0.4)$. This scan was chosen because the outgoing wave vector is then aligned along the c-axis so that any energy dependence of the scattering does not alter the value of ζ. The results of scans taken above T_N are shown in Fig. 1, and similar scans taken below T_N are shown in Fig. 2.

As studied by Ikeda,[11] the extent of the $d=3$ ordering is dependent upon the cooling rate and upon the waiting time after cooling. Consequently, we have therefore determined the order parameter by integrating scans along [10ζ] with ζ varying from -0.5 to $+0.5$, and then correcting these scans for the background and critical scattering as discussed below. Above T_N the intensity in these scans along [10ζ] was independent of ζ, but below T_N, the intensity peaked at $\zeta=0$ with a width and intensity which depended on the prior history of the sample.

The line shape of the critical scattering in the pure $d=2$ Ising model has been discussed in detail theoretically[12] and has been studied experimentally in a sample of K_2CoF_4.[8] Above the transition temperature, theory[12] predicts and experiment[8] confirmed that a Lorentzian form gives the best description of the line shape. The Lorentzian form for the wave-vector-dependent staggered susceptibility is given by

$$\chi_L^+(q,T)=\frac{\chi^+(0,T)}{1+y^2}\ , \tag{1}$$

where $y=q/\kappa^+$, and the inverse correlation length κ and staggered susceptibility $\chi^+(0,T)$ are given by

$$\kappa^{\pm}=\kappa_0^{\pm}\,|\,t\,|^{\,\nu} \tag{2}$$

and

$$\chi^{\pm}(0,T)=C_0^{\pm}\,|\,t\,|^{-\gamma}\ , \tag{3}$$

respectively, with $t=(T-T_N)/T_N$. Theoretically,[12] the Lorentzian form given in Eq. (1) is not the correct form for the wave-vector-dependent susceptibility below T_N in the pure $d=2$ Ising model. However, it was found that experimentally[8] the Lorentzian form gave a reasonable description of observed scattering, but the resulting critical-amplitude ratios were very different from those predicted by theory. A better description of the experimentally observed line shape was found by fitting to the form suggested by Tarko and Fisher[12] and given by

$$\chi_{\mathrm{TF}}(q,T)=\frac{\chi^{-}(0,T)(1+\phi^2y^2)^{\eta/2}}{[1-\lambda+\lambda(1+y^2)^{1/2}]^2}\ , \tag{4}$$

where $\eta=0.25$ and λ and ϕ^2 have the values of 0.45 and 0.16, respectively, as derived by Tracy and McCoy.[12] The resulting critical-amplitude ratios from these latter fits were in good accord with the theoretically predicted values.

Above T_N, the critical scattering from $Rb_2Co_{0.7}Mg_{0.3}F_4$ was fitted to the two-dimensional Lorentzian form given in Eq. (1), convolved with the experimental resolution function plus a flat background, in order to obtain the parameters κ^+ and $\chi^+(0,T)$. Below T_N, similar fits were performed using the Lorentzian form and also using the Tarko-Fisher form. These fits also included a Gaussian to account for the long-range-order component of the scattering below T_N.

EXPERIMENTAL RESULTS

The critical scattering was measured at sixteen temperatures above T_N, and the results analyzed to obtain the susceptibility $\chi^+(0,T)$ and the inverse correlation length, κ^+, by fitting the experimental data to the Lorentzian form, Eq. (1). In each case a good fit was obtained and the goodness-of-fit parameter χ^2 varied between 1.40 and 0.64 and averaged 1.1. Some typical fits to scans above the Néel temperature are shown in Fig. 1. The results for κ are shown in Fig. 3 and for $\chi^+(0,T)$ in Fig. 4. These results were then fitted to the power laws in Eqs. (2) and (3) and the results of these fits are listed in Tables I and II, and illustrated in Figs. 3 and 4. The results for T_N from both fits are in excellent agreement with the T_N found from measurements of the onset of the three-dimensional correlations. The value obtained for γ is in excellent accord with the result for the pure $d=2$ Ising model, 1.75, but the value for ν is slightly larger than 1.0. Figure 3 shows that this value for ν arises largely because

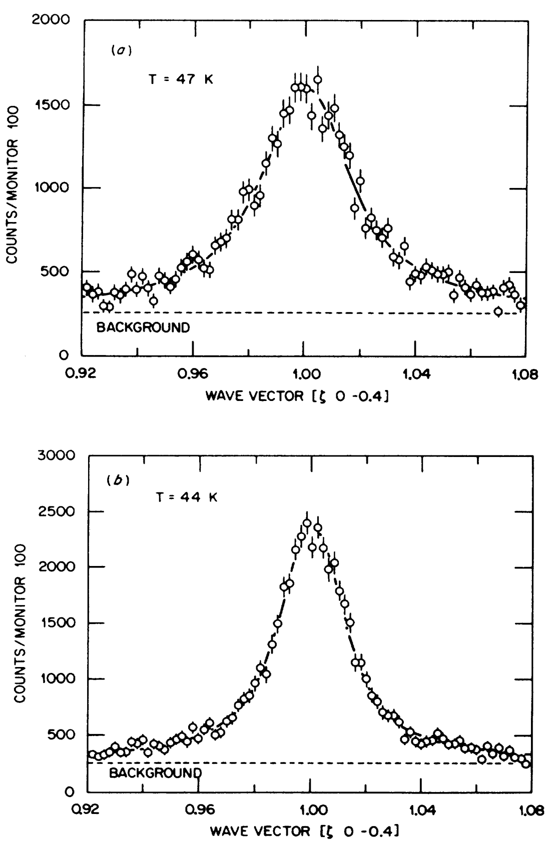

FIG. 1. Critical scattering measured at temperatures of (a) 47 K and (b) 44 K is shown, along with the fits to a Lorentzian line shape convolved with the resolution function (solid line) and the background level (dashed line).

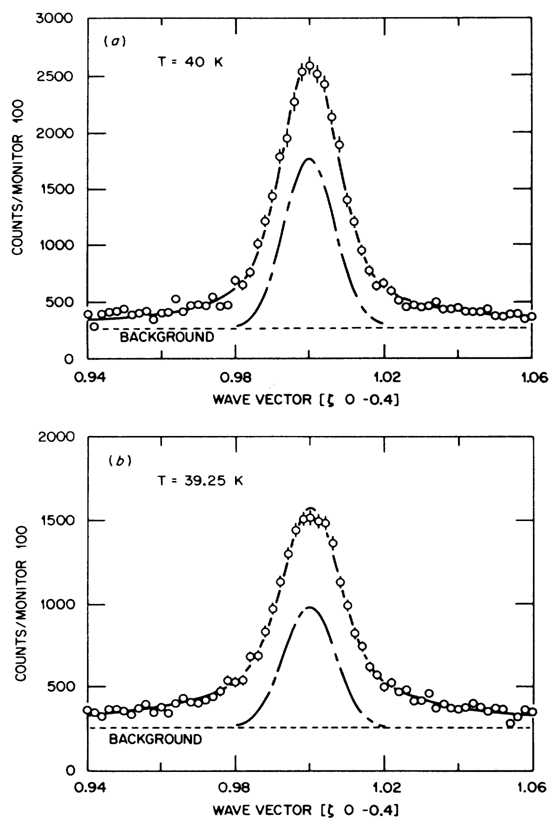

FIG. 2. Critical scattering measured at temperatures of (a) 40 K and (b) 39.25 K is shown along with the fits to the Tarko-Fisher line shape convolved with the resolution function and the magnetic $d=2$ long-range order (solid line). Also shown in (a) and (b) are the background level (dashed line) and the component of the scattering at these temperatures due to the $d=2$ long-range order (dotted-dashed line).

the points close to T_N have larger κ values than would be expected for a linear temperature dependence. At these temperatures, κ is ~ 0.1 of the FWHM of the resolution function, and so this discrepancy may arise because of the well known problem of extracting reliable values for κ when the value becomes small compared to the resolution width. We conclude that the value for ν is not significantly different from 1, the value for the pure $d=2$ Ising model as also found by Ikeda.[6] The value of the amplitude κ_0^+ is very much smaller, $\sim\frac{1}{5}$, than that observed in K_2CoF_4,[8] which was very close to the value for the pure $d=2$ Ising model. A part of this decrease arises from the smaller T_N in the expression for t, but it is surprising that κ^+ is a factor of 2 smaller for a fixed interval of $(T-T_N)$ (as opposed to t) in the disordered system than in the pure system.

Below T_N, measurements of the critical scattering were performed at nine temperatures, and the resulting scans used to obtain the parameters $\chi^-(0,T)$ and κ^- using the Tarko-Fisher form for the line shape, as described in the preceding section. In Fig. 2, some typical fits to scans below T_N are shown. The χ^2 goodness of fit parameter for these fits below T_N varied between 0.94 and 1.51, with an average of 1.20. The resulting values for κ^- and $\chi^-(0,T)$ are shown in Figs. 4 and 5. These values were fitted to the power laws given in Eqs. (2) and (3) with the results listed in Tables I and II. Unconstrained fits to these values below T_N gave values of T_N which were larger than those obtained from the fits to the data for $T>T_N$ and values for the exponents which were quite large but which had very considerable error bars. These large errors result from the uncertainties in the values of $\chi^-(0,T)$ and κ^- which reflect the difficulty of measuring the critical scattering below T_N. This difficulty is caused partly by the need to "subtract off" the component of the scattering from the long-range order and partly because the critical scattering of $d=2$ Ising systems is very much weaker below T_N than above T_N. Nevertheless, when the fits are constrained to have the expected critical exponents (of the pure $d=2$ Ising model) the values of T_N obtained are in excellent accord with those obtained from the fits for $T>T_N$. Although the χ^2 values of these constrained fits are larger than those of the unconstrained fits, they are still less than unity and we conclude that within the limits of error, the critical scattering below T_N is consistent with the exponents ν and γ of the pure $d=2$ Ising model.

We have also performed simultaneous fits of the results for the staggered susceptibility and inverse correlation length obtained from the Lorentzian fits above T_N and the Tarko-Fisher fits below T_N to the power-law equations (2) and (3). The results from these fits are listed in Tables I and II and the resulting exponents are in good accord with those of the pure $d=2$ Ising model. It is therefore surprising that the critical amplitude ratios, κ_0^-/κ_0^+ and C_0^+/C_0^-, are both much smaller than those obtained for K_2CoF_4 or given theoretically for the pure $d=2$ Ising model. These amplitude ratios are all given in Table IV.

The scans at temperatures below T_N have also been fitted to the Lorentzian form given in Eq. (1) with the results for $\chi^-(0,T)$ and κ^- shown in Figs. 5 and 6. They give a quite reasonable description of the observed scattering with χ^2 varying between 1.00 and 1.64 with an average of 1.26. The values of $\chi^-(0,T)$ and κ^- obtained from these fits were fitted to the power laws given in Eqs. (2) and (3) with the results listed in Tables I and II. The values of T_N given from these fits are considerably larger than the values of T_N determined directly from inspecting the raw data or from the other fits to the parameters. We conclude therefore the Lorentzian form is not an appropriate description of the critical below T_N in the $d=2$

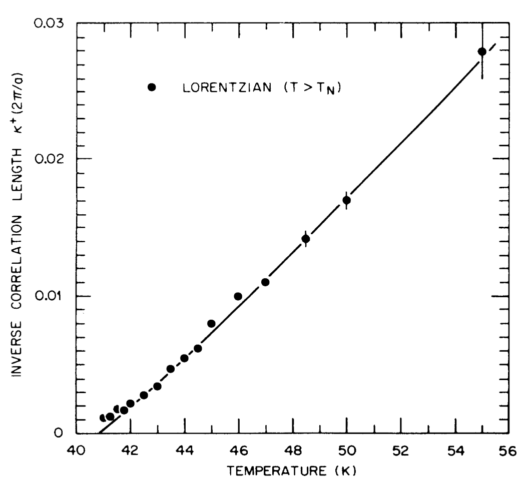

FIG. 3. Temperature dependence of the inverse correlation lengths κ^+ deduced from the Lorentzian fits for $T>T_N$ is shown, along with the fit result $\kappa^+=0.086t^{1.08}$ given by the solid line.

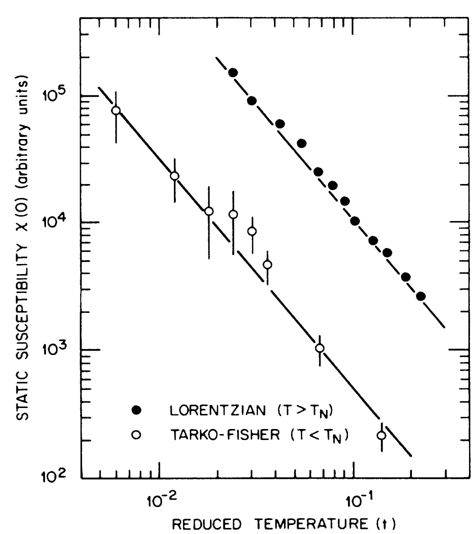

FIG. 4. Results for the static staggered susceptibilities $\chi^+(0,T)$ and $\chi^-(0,T)$ deduced from the fits to the Lorentzian line shape above T_N and the Tarko-Fisher line shape below T_N are shown as a function of the reduced temperature $t=|T-T_N|/T_N$, where $T_N=40.8$ K. The solid lines are the fit results listed in Table II with $\gamma=1.75$ and $T_N=40.8$ K fixed.

TABLE I. Fits to the inverse correlation length κ.

	κ_0^+	κ_0^-	T_N	ν	$(\chi^2)^{1/2}$
Lorentzian $(T>T_N)$	0.086±0.008		40.8±0.2	1.08±0.06	1.24
	0.076±0.002		40.9±0.1	1.0	
Tarko-Fisher $(T<T_N)$		0.34±0.36	41.4±0.6	1.58±0.52	0.52
		0.084±0.012	40.8±0.1	1.0	0.72
Lorentzian $(T>T_N)$	0.082±0.004	0.102±0.020	40.8±0.1	1.05±0.03	1.06
+ Tarko-Fisher $(T<T_N)$	0.076±0.002	0.078±0.008	40.9±0.1	1.0	
Lorentzian $(T<T_N)$		0.30 ±0.20	42.6±1.1	1.15±0.42	0.95
		0.24 ±0.02	42.2±0.3	1.0	0.91

random Ising model, just as it was not for the pure $d=2$ Ising model.

As described in the preceding section, the two-dimensional order parameter was obtained by integrating the scattering in scans along $[10\xi]$ with ξ varying from -0.5 to $+0.5$, and then subtracting the background and the critical scattering. The latter was achieved using the results of the fits to the critical scattering below T_N described above. The results, when the "Tarko-Fisher critical scattering" was subtracted, are shown in Fig. 7 together with a power-law fit of the data to the function

$$I_B(t)=M_0^2\,|t|^{2\beta} . \tag{5}$$

The parameters M_0^2 and β are listed in Table III for the fits to this data and to similar data obtained by subtracting the critical scattering deduced from the Lorentzian fits to the critical scattering below T_N. The results of both sets of fits are clearly consistent with the value $\beta=0.125$, the value for the pure $d=2$ Ising model.

As well as evaluating the critical exponents and critical-amplitude ratios, we have also used the fitting results for the critical amplitudes with the critical exponents held fixed at their nominal values to obtain an experimental value for the quantity R_s given by

$$R_s=\left[\frac{1}{V_R}\right]\frac{C_0^+(\kappa_0^+)^2}{M_0^2} , \tag{6}$$

where V_R is the volume of the spectrometer resolution function within the a^*-b^* plane.[9] The value of R_s should, according to the hypothesis of two scale factor universality,[13] be a universal number for systems in the universality class of the $d=2$ Ising model. As shown in Table IV, the resulting value of R_s for $Rb_2Co_{0.7}Mg_{0.3}F_4$ is in very reasonable accord with the value obtained for K_2CoF_4 (Ref. 8) and is not significantly different from the theoretical value[13] for the pure $d=2$ Ising model. This is, in some ways, a quite surprising result when it is considered that $(\kappa_0^+)^2$ is 28 times larger in K_2CoF_4 than in $Rb_2Co_{0.7}Mg_{0.3}F_4$.

CONCLUSIONS

The measurements reported in this paper have studied the magnetic critical fluctuations in the $d=2$ random-exchange antiferromagnet $Rb_2Co_{0.7}Mg_{0.3}F_4$. The results for the critical scattering above T_N are well described by a Lorentzian line shape for the critical scattering. The inverse correlation length and staggered susceptibility deduced from fits to this line shape have critical exponents $\nu=1.08\pm0.06$ and $\gamma=1.75\pm0.07$, respectively, which are consistent with the values of $\nu=1.0$ and $\gamma=1.75$ for the pure $d=2$ Ising model. These values suggest that for the disordered $d=2$ Ising model $\eta\neq0$, in disagreement with the theoretical results of Dotsenko and Dotsenko,[5] but in agreement with the expectation that when $\alpha=0$, disorder in the exchange constants does not produce changes in the

TABLE II. Fits to the staggered susceptibility.

	C_0^+	C_0^-	T_N	γ	$(\chi^2)^{1/2}$
Lorentzian $(T>T_N)$	190±22		40.85±0.10	1.75±0.07	1.77
Tarko-Fisher $(T<T_N)$		1.36±1.16	41.2 ±0.4	2.6±0.6	0.55
		8.91±1.45	40.7 ±0.1	1.75	0.92
Lorentzian $(T>T_N)$	170±11	7.95±1.74	40.75±0.04	1.81±0.03	1.49
± Tarko-Fisher $(T<T_N)$	192±5	9.98±1.36	40.82±0.04	1.75	1.49
Lorentzian $(T<T_N)$		2.80±2.20	42.3 ±0.6	2.3±0.5	0.83
		7.33±0.86	41.6 ±0.1	1.75	0.92

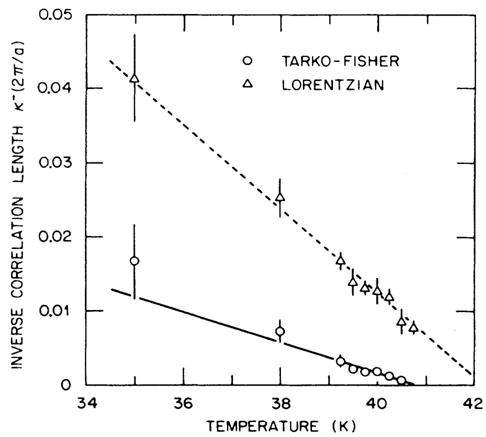

FIG. 5. Inverse correlation lengths κ^{-} deduced from the Tarko-Fisher and Lorentzian fits below T_N are shown. The solid line and the dashed line are the best fits of the Tarko-Fisher and Lorentzian results listed in Table I with ν fixed at 1.0.

critical exponents.

The temperature dependence of the long-range antiferromagnetic order below T_N also has a critical exponent, $\beta=0.13\pm0.02$, which is in good accord with the value of $\beta=0.125$ of the pure $d=2$ Ising model. Furthermore, when the critical amplitude for the long-range order is combined with the critical amplitudes for the inverse correlation length and the staggered susceptibility above T_N to evaluate the quantity R_s given in Eq. (6), a value consistent with that obtained for the pure $d=2$ Ising model (Ref. 13) and for the pure $d=2$ Ising antiferromagnet K_2CoF_4 (Ref. 8) is obtained. This value for R_s, listed

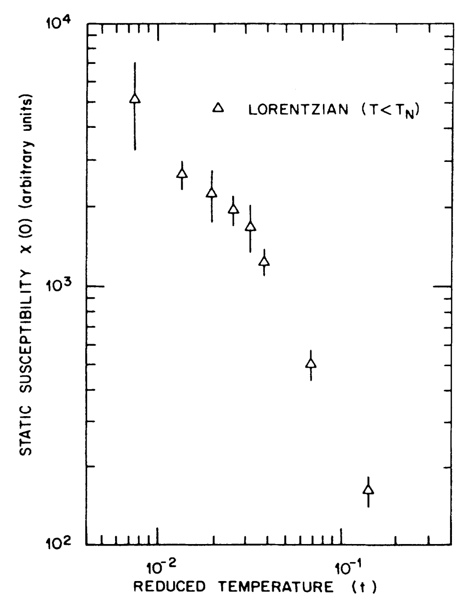

FIG. 6. Results for the static staggered susceptibility $\chi^{-}(0,T)$ as deduced from the Lorentzian fits below T_N are shown as a function of the reduced temperature, where $T_N=40.8$ K.

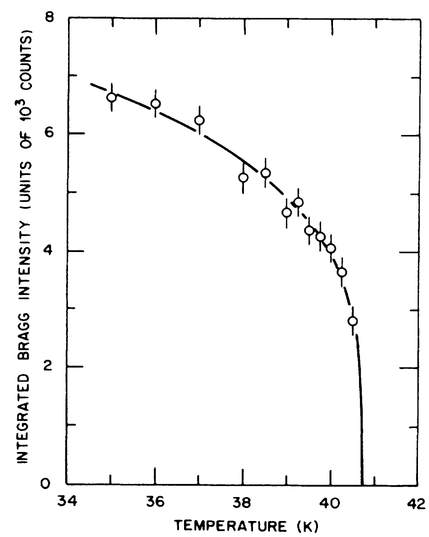

FIG. 7. Integrated intensity from scans along (10ζ) with $-0.5\leq\zeta\leq0.5$, with the background and Tarko-Fisher critical scattering subtracted is shown as a function of temperature below the Néel temperature. The solid line is the fit result given in Table III with $2\beta=0.26$ and $T_N=40.7$ K.

in Table IV, suggests the concept of two-scale-factor universality applies equally well to the disordered $d=2$ Ising system as to the pure system.

The results of the analysis of the critical scattering below T_N provide a less satisfactory picture. If the critical scattering below T_N is analyzed using a Lorentzian line shape, then physically unreasonable values for the inverse correlation length and staggered susceptibility are found. This is similar to the situation found for the pure $d=2$ Ising antiferromagnet K_2CoF_4. We conclude that the critical fluctuations below T_N are not appropriately described by the Lorentzian form. We have also analyzed the critical scattering data below T_N using the form proposed by Tarko and Fisher[12] for the pure $d=2$ Ising model which was found to provide a consistent description of the critical scattering below T_N in K_2CoF_4.[8] These fits yielded values for the inverse correlation length and staggered susceptibility whose temperature dependence was consistent with the critical exponents of the pure $d=2$ Ising model. However, as shown in Table IV,

TABLE III. Fits to the long-range-order parameter.

M_0^2	T_N	2β	$(\chi^2)^{1/2}$
	Tarko-Fisher subtraction		
11 185±950	40.7±0.1	0.26±0.04	0.71
10 922±1951	40.7±0.1	0.25	0.68
	Lorentzian subtraction		
11 834±1308	41.3±0.1	0.29±0.05	1.05
10 759±2024	41.1±0.1	0.25	1.05

TABLE IV. Critical-amplitude ratios.

	$Rb_2Co_{0.7}Mg_{0.3}F_4$	K_2CoF_4	Theory (pure $d=2$ Ising model)
κ_0^+	0.076 ± 0.006	0.40 ± 0.02	0.397
κ_0^-/κ_0^+	1.02 ± 0.20	1.85 ± 0.22	2.0
C_0^+/C_0^-	19.1 ± 5.0	32.6 ± 3.7	37.33
R_s	0.062 ± 0.010	0.0565 ± 0.0075	0.0507

the critical amplitude ratios κ_0^-/κ_0^+ and C_0^+/C_0^- which result from these latter fits differ substantially and significantly from the values predicted theoretically for the pure $d=2$ Ising model and those found experimentally in K_2CoF_4.

The difference between the critical-amplitude ratios for C_0^+/C_0^- and κ_0^-/κ_0^+ determined from the measurements and analysis described previously and those of the pure $d=2$ Ising model could perhaps be explained by any one of three different reasons. Firstly, it could be that the $d=2$ random Ising model is in a different universality class to that of the pure $d=2$ Ising model. However, because of the values of the critical exponents β, γ, and ν, the critical-amplitude combination R_s and the critical-amplitude ratio for the specific-heat capacity,[7] this would seem to be an unlikely explanation. Secondly, it is possible that the effect of an inhomogeneous magnetization below T_N may play a role in changing the critical-amplitude ratio C_0^+/C_0^- as discussed by Pelcovits and Aharony[14] for the $d=3$ random Ising model. Unfortunately, without a specific line shape for the structure factor due to this effect, in $d=2$ dimensions, it is not possible to include it in our analysis. Thirdly, it could be possible that the Tarko-Fisher line shape is not the correct line shape for the critical scattering below T_N in the $d=2$ random Ising model. In particular, the parameters λ and ϕ^2 may not be universal and could be different for the $d=2$ random Ising model. As emphasized in the work on the pure system K_2CoF_4,[8] critical-amplitude ratios deduced when the functional form of the critical scattering is incorrect can be very wrong.

In order to investigate this last possibility, we have carried out fits of the critical scattering below T_N to the Tarko-Fisher form in which the parameters λ and ϕ were allowed to vary, while the values of κ^- were held fixed at the values $\kappa^-=2\kappa_0^+t$, where κ_0^+ and T_N were taken from the fits above T_N. The resulting values of λ and ϕ at each temperature were all reasonably close in value and were averaged to obtain "best fit" values of $\lambda=1.0\pm0.1$ and $\phi^2=0.40\pm0.16$. A value of $\lambda=1$ is a surprising result since it leads to a Lorentzian denominator for the modified Tarko-Fisher form. As a consistency test of the modified Tarko-Fisher form, the data taken below T_N was fitted again with λ and ϕ fixed at these new values and κ^- and $\chi^-(0,T)$ variable. The χ^2 goodness-of-fit parameters for these latter fits varied from 0.94 to 1.46 and had an average of 1.18. It is not possible from these values to say that this modified Tarko-Fisher form is any better than the Tarko-Fisher form. In Fig. 8, the values of κ^- which resulted from this fitting are shown, along with the best fit to the equation $\kappa^-=\kappa_0^-t$, where T_N was fixed at 40.8 K from the fits above T_N. The value of κ_0^- from this fit was 0.148 ± 0.008, which using the value for κ_0^+ of 0.076 ± 0.002 from Table I gives a ratio of κ_0^-/κ_0^+ of 1.95 ± 0.11. This is in reasonable accord with our initial condition of $\kappa_0^-/\kappa_0^+=2$ for the modified Tarko-Fisher function. In Fig. 9, the values of $\chi^-(0,T)$ from the fits to be modified Tarko-Fisher form are shown. The solid line in Fig. 9 is a fit to the form $\chi^-(0,T)=C_0^-t^{-1.75}$, where T_N was fixed at 40.8 K as before. The best-fit amplitude

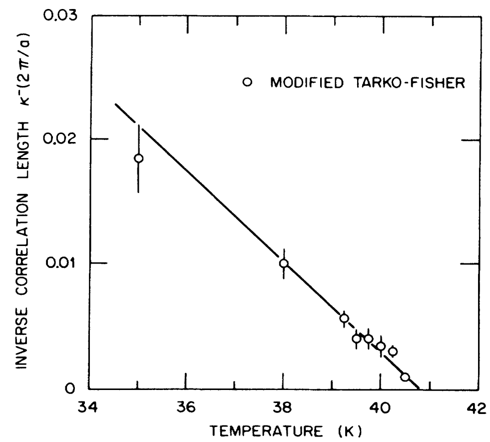

FIG. 8. Values of κ^- deduced from the fits to the modified Tarko-Fisher line shape are shown, along with the fit result with $T_N=40.8$ K and $\nu=1.0$ fixed.

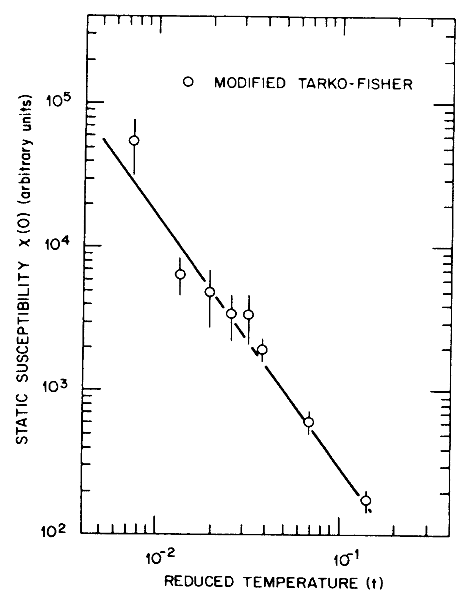

FIG. 9. Values of $\chi^-(0,T)$ deduced from the fits to the modified Tarko-Fisher line shape are shown. The solid line is the fit result with $T_N=40.8$ K and $\gamma=1.75$ fixed.

C_0^- is 5.33 ± 0.48, which when combined with the amplitude $C_0^+ = 190 \pm 22$ taken from Table II, gives an amplitude ratio $C_0^+/C_0^- = 35.65 \pm 5.23$. This ratio is much closer to the value for the pure $d=2$ Ising model of 37.33 than that found from the Tarko-Fisher fits.

Therefore in conclusion, the critical scattering below T_N in the $d=2$ random Ising antiferromagnetic $Rb_2Co_{0.7}Mg_{0.3}F_4$ is different to that of the pure $d=2$ Ising model. The origin of this difference may either be because the critical amplitude ratios κ_0^-/κ_0^+ and C_0^+/C_0^- are different to those of the pure $d=2$ Ising model or because the line shape of the critical scattering, which from the experimental point of view defines κ^- and $\chi^-(0,T)$, is different to that of the pure $d=2$ Ising model.

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