Temperature dependence of the conductivity and the susceptibility in metallic Si:P reanalyzed from the paramagnon point of view

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The previously observed temperature dependence of, in particular, the conductivity $\sigma(T)$ in metallic Si:P is reanalyzed according to a recent conjecture; such a conjecture proposed a scenario for reaching the metal-insulator transition at 0 K despite strong electron interactions, supposed to be of Hubbard-type contact. In the present paper we pursue the same scenario at finite T. When the disorder increases, the role of the interactions upon $\sigma(T)$ first increases but then weakens, while the localization contribution from the inelastic scattering process via paramagnon exchange becomes sizable. Moreover, according to a recent theoretical result, for low-T ranges reasonably easy to attain experimentally, the interaction and the localization terms in $\sigma(T)$ both vary with $\sqrt{T}$ but with opposite signs. The competition between these two contributions results more and more in favor of localization for increasing disorder. Moreover, the present analysis could also explain why perturbative results appear to account for experimental observations even in the strong-disorder regime near the metal-insulator transition. The resulting behavior of $\sigma(T)$ agrees qualitatively well with previous observations in metallic Si:P for varying donor densities. The most recently measured temperature dependence of the susceptibility $\chi(T)$ of Si:P is also discussed. This analysis is also compared with previous ones using screened Coulomb interactions.

I. INTRODUCTION

The theoretical problem of the understanding of electron localization in the presence of strong electron-electron interactions is not yet completely solved.[1] Simple perturbative theories[2–5] as well as more sophisticated ones using the renormalization-group procedure,[6] all agree on the general tendency that for increasing disorder the system develops strong spin fluctuations which drive the system away from the metal-insulator (M-I) transition: the electrical conductivity at 0 K, $\sigma(T=0)$, cannot vanish as it ought to, in order to account for experimental data.[1,7] However, the above results are all based on lowest-order expansion in the disorder parameter and, therefore, cannot make definitive statements concerning the M-I transition in the regime of strong disorder.

On the other hand, a number of changes occur experimentally close to the M-I transition. For instance, the archetypal Si:P system, which is believed[1] to contain strong correlations among the electrons, exhibits a temperature-dependent contribution in the electrical conductivity $\sigma(T)$, whose coefficient changes sign when the donor density n varies as one approaches the transition from the metallic side.

In an attempt to account for the occurrence of an M-I transition despite the presence of strong electron correlations, a conjecture was recently proposed[3] in the specific case of three-dimensional, disordered, nearly magnetic fermion systems, containing, already in absence of disorder, strong spin fluctuations, the paramagnons.

The purpose of the present paper is to examine further this conjecture and its consequences in connection with the above experimental observations: we will show that it may, in particular, allow one to qualitatively understand the variation observed in the behavior of the electrical conductivity of Si:P (Ref. 7) and the change of sign recalled above in the T-dependent coefficient of $\sigma(T)$.

We must first recall the main points of Ref. 3. The model interaction chosen was the Hubbard-type contact repulsion I among opposite spins, of order ε_F (the Fermi energy of the free fermions), so that, in the pure system, the Stoner criterion for magnetism to occur is almost fulfilled ($\bar{I}\sim 1$ but <1, with $\bar{I}\sim I/\varepsilon_F$). Such a system, free of impurities, is thus nearly magnetic at 0 K and exhibits strong spin fluctuations, the paramagnons.

When random and independent impurities are added to such a system, it has been shown[2–4] that, at least in the weakly disordered ("weakly localized") regime, $(\varepsilon_F\tau)\gg 1$, where τ is the elastic lifetime, the system shifts closer to the magnetic instability for increasing disorder among the fermions [i.e., when $(\varepsilon_F\tau)^{-1}$ increases]. The effective interaction $\bar{I}_{\rm eff}$ increases above the initial value $\bar{I}$ and closer to 1; the paramagnons become stronger, yielding the 0-K conductivity $\sigma(T=0)$, to increase above the Boltzmann value σ_0, thus preventing the M-I transition $[\sigma(T=0)=0]$ from occurring. But it was also shown in Refs. 2 and 3 that these paramagnons not only become stronger in presence of impurities, but altogether tend to switch from uniform paramagnons to local ones, i.e., tend to form local moments at random. Assuming that this tendency still holds when $(\varepsilon_F\tau)^{-1}$ increases further, the author of Ref. 3 examined the case where the crossover between uniform and local paramagnons occurs *before* the magnetic instability is reached, i.e., with $\bar{I}_{\rm eff}$ remaining smaller than 1.[8] It was then argued in Ref. 3 that such a crossover yields a $\bar{I}_{\rm eff}$ to stop increasing and, instead, starts to decrease continuously towards 0, as implied in an earlier result [9] according to which local paramagnon theories scale to free ones, so that the tendency to form local moments becomes less and less plausible and is ultimately completely suppressed. Thus $\bar{I}_{\rm eff}$, before vanishing, would eventually reach a sufficiently modest value ($\bar{I}_0$ in the following) below which, when $\bar{I}_{\rm eff}<\bar{I}_0$, $\sigma(T=0)$ can possibly vanish, as would be the case in absence of any interaction,[10,1] and thus the *M-I* transition can occur (this will be illustrated later on with Fig. 1). In the following we pursue the study of the same case as that in Ref. 3 (i.e., crossover, for increasing disorder, from uniform to local paramagnons, with $\bar{I}_{\rm eff}$ remaining smaller than 1). We examine the physical implications of such a model in order to facilitate comparison with experimental observations.[7]

II. THE CONTRIBUTION TO THE TEMPERATURE DEPENDENCE OF THE CONDUCTIVITY

To lowest order in the disorder $[(\varepsilon_F\tau)^{-1}\ll 1]$, $\sigma(T)$ reads[1]

$$\sigma(T)=\sigma_0(1+\delta\sigma_L/\sigma_0+\delta\sigma_{I,L}/\sigma_0)\ , \tag{1}$$

with, in a.u., the Drude value $\sigma_0=2k_F\varepsilon_F\tau/(3\pi^2)$ (k_F the Fermi momentum); $\delta\sigma_L$ is the localization contribution in the absence of interactions[1,10] and $\delta\sigma_{I,L}$ the electron-interaction quantum correction.[1]

At the lowest temperatures, $T\tau<1$, the T dependence of $\sigma(T)$ is usually[1] believed to be dominated by the one in $\delta\sigma_{I,L}$ $[\sim(T\tau)^{1/2}$, compared to that in $\delta\sigma_L\sim(T\tau)^{p/2}$ with $p>1]$. However, in our strongly enhanced case, the relevant T range is no longer unique ($T\tau<1$): $T\tau$ has to be compared not only to 1, but also to $1-\bar{I}$ and $(1-\bar{I})^{1/2}$. We examine both $\delta\sigma_L(T)$ and $\delta\sigma_{I,L}(T)$ in the following.

A. The localization contribution $\delta\sigma_L$

The T dependence of $\delta\sigma_L$ (Ref. 1) contained in the inelastic time $\tau_{\rm in}$ is

$$\delta\sigma_L=-[k_F/(2\pi^3\varepsilon_F\tau)][1-(3\tau/\tau_{\rm in})^{1/2}]\ . \tag{2}$$

$\tau/\tau_{\rm in}$ is proportional to $(T\tau)^p$, where p is equal[1] to $\frac{3}{2}$, 2, or 3 depending on whether electron-electron interactions in the dirty or clean limit, or electron-phonon interactions, respectively, determine the inelastic scattering. The lowest value of p, $p=\frac{3}{2}$, and thus the strongest T dependence in $\delta\sigma_L$, has been found in the case where screened Coulomb interactions are responsible for the inelastic scattering.[11] So far, the only calculation which considered Hubbard-type contact interactions instead of Coulomb ones (to be the source of $\tau_{\rm in}$) was that in Ref. 12, which was restricted to two dimensions. We have recently[13] extended the calculation of Ref. 12 to the case of three dimensions. In particular, when $\bar{I}$ is close to 1, we found the following contribution:

$$\left[\frac{\tau}{\tau_{\rm in}}\right]^{(1)}\sim\begin{cases}\dfrac{9\sqrt{3}}{2\sqrt{2}}\dfrac{1}{(\varepsilon_F\tau)^2}\left[\dfrac{T\tau}{1-\bar{I}}\right]^{3/2}, & T\tau<1-\bar{I},\bar{I}\sim 1 \quad (3a)\\[2ex] \dfrac{9\sqrt{3}}{4}\dfrac{1}{(\varepsilon_F\tau)^2}\dfrac{T\tau}{1-\bar{I}}, & 1-\bar{I}<T\tau<1,\bar{I}\sim 1\ . \quad (3b)\end{cases}$$

(The complete formula, not restricted to $\bar{I}\sim 1$, was given in Ref. 13.) When the system is close to the magnetic instability $\bar{I}\sim 1$, the range $T\tau<1-\bar{I}$ may be difficult to study experimentally, so that the range $1-\bar{I}<T\tau<1$ would instead dominate the observations.

Moreover, for screened Coulomb interactions it has been emphasized[14] that some contributions due to diagrams dropped in previous papers (as well as in Ref. 12) are important and yield to an extra contribution $(\tau/\tau_{\rm in})^{(2)}$ as follows:

$$\left[\frac{\tau}{\tau_{\rm in}}\right]_{\rm Ref.\,13}=\left[\frac{\tau}{\tau_{\rm in}}\right]^{(1)}+\left[\frac{\tau}{\tau_{\rm in}}\right]^{(2)}, \tag{4}$$

with

$$\left[\frac{\tau}{\tau_{\rm in}}\right]^{(1)}_{\rm Ref.\,13}\propto\frac{1}{(\varepsilon_F\tau)^2}(T\tau)^{3/2}\ , \tag{5a}$$

$$\left[\frac{\tau}{\tau_{\rm in}}\right]^{(2)}_{\rm Ref.\,13}\propto\frac{1}{(\varepsilon_F\tau)^2}(T\tau)\ . \tag{5b}$$

In Ref. 13 we first extended to three dimensions the calculations of Ref. 12, thus dropping the same diagrams, resulting in the contribution $(\tau/\tau_{\rm in})^{(1)}$ [formulas (3) above]. Then we pointed out that the reason for neglecting these diagrams, which was explained in Ref. 15 in the case of screened Coulomb interactions, is no longer relevant in the case of the Hubbard contact one.[16] Therefore it was expected in Ref. 13 that an extension of the treatment of Ref. 14, with $T\tau/(1-\bar{I})$ replacing $(T\tau)$, would yield

$$\frac{\tau}{\tau_{\rm in}}=\left[\frac{\tau}{\tau_{\rm in}}\right]^{(1)}+\left[\frac{\tau}{\tau_{\rm in}}\right]^{(2)}, \tag{6}$$

with $(\tau/\tau_{\rm in})^{(1)}$ given by formulas (3) and

$$\left[\frac{\tau}{\tau_{\rm in}}\right]^{(2)}\propto\frac{1}{(\varepsilon_F\tau)^2}\frac{T\tau}{1-\bar{I}}\ . \tag{7}$$

In that case one expects

$$\frac{\tau}{\tau_{\rm in}}\propto\frac{1}{(\varepsilon_F\tau)^2}\frac{T\tau}{1-\bar{I}} \tag{8}$$

both in the $T\tau<1-\bar{I}$ range where $T\tau/(1-\bar{I})$ of $(\tau/\tau_{\rm in})^{(2)}$ would prevail over the $[T\tau/(1-\bar{I})]^{3/2}$ of $(\tau/\tau_{\rm in})^{(1)}$, and in the $1-\bar{I}<T\tau<1$ range, where both terms are proportional to $T\tau/(1-\bar{I})$. The consequence for the T dependence of $\delta\sigma_L$ is that

$$\delta\sigma_L=-[k_F/(2\pi^3\varepsilon_F\tau)]\{1-a(\varepsilon_F\tau)^{-1}[T\tau/(1-\bar{I})]^{p/2}\}\sim-[k_F/(2\pi^3\varepsilon_F\tau)]\{1-a(\varepsilon_F\tau)^{-1}[T\tau/(1-\bar{I})]^{1/2}\},\quad \bar{I}\sim 1 \tag{9}$$

where a is a constant. Such a T dependence is important when compared to a similar one in $\delta\sigma_{I,L}$, recalled below.

B. The interaction contribution $\delta\sigma_{I,L}$

$\delta\sigma_{I,L}(T)$ has been extensively studied in the literature,[1] mostly for the screened Coulomb interaction, and, for the Hubbard-type contact interaction, in Refs. 17 and 18. In the following, as in Ref. 3, we use the notations of Ref. 18. (We found it helpful, to benefit nonspecialists, to show in the Appendix the connections between the various notations found in the literature.)

$\delta\sigma_{I,L}(T)$ for a nearly magnetic disordered system, in three dimensions, is

$$\delta\sigma_{I,L} = -\frac{\sqrt{3}}{8\pi^2}\frac{k_F}{\varepsilon_F\tau}g(1-1.83\sqrt{T\tau}) . \tag{10}$$

g is a complicated coupling constant, depending on the interaction $\bar{I}$ and the disorder $(\varepsilon_F\tau)^{-1}$. In Ref. 3, following Refs. 17 and 5, g was separated into the "singlet" part g_s and the "triplet" one g_t, due to the particle-hole diffusion processes, and g_c, due to the particle-particle diffusion processes:

$$g = g_s + g_t + g_c . \tag{11}$$

According to the results of Ref. 18, $g_s > o$ while $g_t < o$. Moreover, $g_s + g_c$ varies little as a function of $\bar{I}$ in the range $o < \bar{I} < 1$. In contrast, g_t varies from o to infinity in the same range. Due to mathematical difficulties, only an estimate of g_c was given in Ref. 18; as it plays a minor role in g in most of the $\bar{I}$ range, we neglect it in the following. Close to $\bar{I}=1$, one has

$$\left.\begin{array}{l} g_s \sim 1.3 \\ g_t \sim -(1-\bar{I})^{-1/2} \end{array}\right\} \text{ for } \bar{I}\to 1 . \tag{12}$$

We rewrite (10) as

$$\delta\sigma_{I,L} \propto -[g_s - |g_t|](\varepsilon_k\tau)^{-1}(1-\sqrt{T\tau}) , \tag{13}$$

where $g_s - |g_t|$ is a function of $\bar{I}$.

C. The total conductivity $\sigma(\tau)$

From now on we will replace I by $\bar{I}_{\text{eff}}$ everywhere, taking into account the renormalization of $\bar{I}$ by disorder and disordered paramagnon effects according to the scenario of Ref. 3. Combining formulas (9) and (13), (1) reads as follows:

$$\sigma(T) = \sigma_0\left\{1 - \frac{3}{4\pi}\frac{1}{(\varepsilon_F\tau)^2}\left[1-\left[\frac{3\tau}{\tau_{\text{in}}}\right]^{1/2}\right] - \frac{3\sqrt{3}}{16}\frac{g}{(\varepsilon_F\tau)^2}(1-1.83\sqrt{T\tau})\right\} . \tag{14}$$

Next, we discard all purely numerical constants that are not important in the qualitative scenario that we study here,

$$\sigma(T) \simeq \sigma(o) + A(T/T_F)^{1/2} , \tag{15}$$

with

$$\begin{aligned} \sigma(0) &\simeq \sigma_0\left[1 - \frac{1}{(\varepsilon_F\tau)^2}(1+g_s-|g_t|)\right] , \\ A &\simeq (\varepsilon_F\tau)^{-1/2}[g_s - (1-(\varepsilon_F\tau)^{-1})(1-\bar{I}_{\text{eff}})^{-1/2}] , \end{aligned} \tag{16}$$

with

$$\begin{aligned} T\tau &= (\varepsilon_F\tau)(T/T_F) , \\ \sigma_0 &\propto \varepsilon_F\tau , \end{aligned} \tag{17}$$

T_F being the Fermi temperature of the free electrons. In the next section we examing how the expected variation of $\bar{I}_{\text{eff}}$ with $(\varepsilon_F\tau)^{-1}$ affects the T dependence of $\sigma(\tau)$.

III. ANALYSIS OF THE THEORETICALLY EXPECTED VARIATION OF σ WITH T ACCORDING TO REF. 3

We illustrate on Fig. 1 the ideas of Ref. 3 by drawing a qualitative variation of $\bar{I}_{\text{eff}}$ when the disorder $(\varepsilon_F\tau)^{-1}$ increases from the pure case $(\tau^{-1}=0)$. The figure is schematically separated into five different regions (a)–(e). To each region will correspond a certain behavior for $\sigma(T)$ (Fig. 2).

A. Region (a)

One starts with a pure homogeneous system $[\bar{I}_{\text{eff}}(\tau^{-1}=0)\equiv\bar{I}]$, paramagnetic $(\bar{I}<1)$, but close to becoming an itinerant magnet ($\bar{I}$ close to 1) which contains strong uniform paramagnons. When a weak disorder is introduced $[0<(\varepsilon_F\tau)^{-1}\ll 1]$ and increases, so do $\bar{I}_{\text{eff}}$ $(\bar{I}<\bar{I}_{\text{eff}}<1)$ and $|g_t|$; the system becomes inhomogeneous and the paramagnons get stronger. As $(1-\bar{I}_{\text{eff}})^{-1}\gg 1$, $1+g_s$ and g_s are negligible compared to $|g_t|$ in (16), and, due to the smallness of $(\varepsilon_F\tau)^{-1}\ll 1$, $\sigma(T)$ is dominated by $\delta\sigma_{I,L}$:

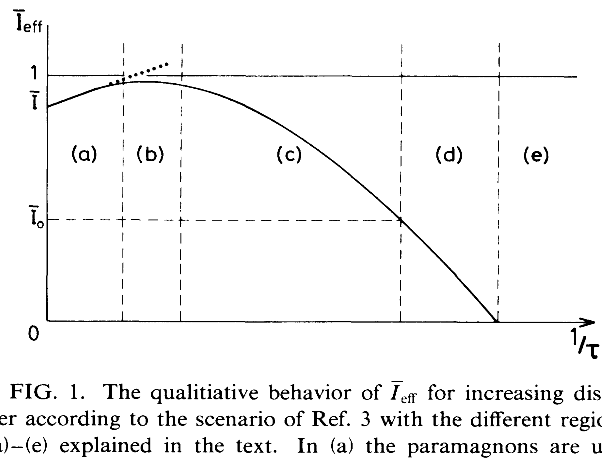

FIG. 1. The qualitiative behavior of $\bar{I}_{\text{eff}}$ for increasing disorder according to the scenario of Ref. 3 with the different regions (a)–(e) explained in the text. In (a) the paramagnons are uniform; they cross over in region (b) to local ones in regions (c), etc. $\bar{I}_{\text{eff}}<1$ corresponds to a paramagnetic regime and $\bar{I}_{\text{eff}}>1$ to a magnetically ordered one. The dotted line would describe the situation where $\bar{I}_{\text{eff}}$ would reach 1 (and the susceptibility would diverge) before the paramagnons have crossed over to local ones. $\bar{I}_0$ is the value of the effective interaction below which the localization transition can occur for higher τ^{-1} values.

$$\sigma_{(a)}(T) \simeq \sigma_{(a)}(0) + A_{(a)}(T/T_F)^{1/2} , \tag{18a}$$

$$\sigma_{(a)}(0) \simeq \sigma_0[1+(\varepsilon_F\tau)^{-2}|g_t|] \simeq (\varepsilon_F\tau)[1+(\varepsilon_F\tau)^{-2}(1-\bar{I}_{\rm eff})^{-1/2}] , \tag{18b}$$

$$A_{(a)} \simeq -(\varepsilon_F\tau)^{-1/2}|g_t| = -(\varepsilon_F\tau)^{-1/2}(1-\bar{I}_{\rm eff})^{-1/2} . \tag{18c}$$

Although $\sigma_{(a)}$ (0) remains larger than σ_0, $\sigma_{(a)}$ (0) decreases for increasing $(\varepsilon_F\tau)^{-1}$ [within the validity of perturbation theory, $(\varepsilon_F\tau)^{-2}|g_t| < 1$]. The main results in this region are

$$\begin{aligned} &\sigma_{(a)}(0) > \sigma_0 , \\ &\sigma_{(a)}(0) \text{ decreases when} (\varepsilon_F\tau)^{-1} \text{ increases} , \\ &A_{(a)} < 0 , \\ &|A_{(a)}| \text{ increases when} (\varepsilon_F\tau)^{-1} \text{ increases} . \end{aligned} \tag{19}$$

The results (19) are illustrated by the two "(a)" lines on Fig. 2, where $\sigma(\tau)$ is plotted versus $\sqrt{T}$ [$(\varepsilon_F\tau)^{-1}$ increases upon going from the top lines to the bottom ones].

The most sophisticated theories so far[6] stop here, faced with the puzzling problem that $\sigma_{(a)}$ (0) does not decrease below σ_0 and that $(1-\bar{I}_{\rm eff})^{-1} \to \infty$. The following remark was added in Ref. 3: while $\bar{I}_{\rm eff}(\omega=0,\ q=0)$ certainly increases toward 1, the q dependence of the static enhancement $[1-\bar{I}_{\rm eff}(\omega=0,\ q\neq 0)]^{-1}$ tends to disappear; the paramagnon correlation length shortens compared to the pure case [see formula (3) in Ref. 3(a)] and the uniform enhancement becomes a local one. The result was[3] that the paramagnons altogether get stronger and tend to become local paramagnons. Consequently, a tendency to form local moments develops.

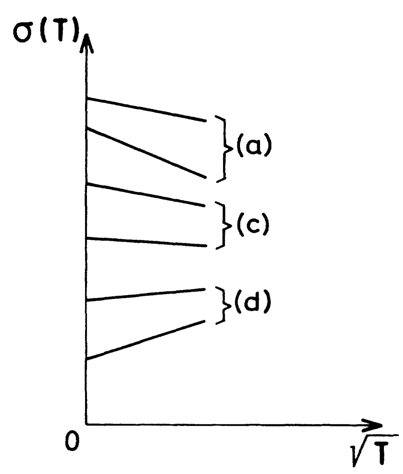

FIG. 2. The qualitative behavior expected for various degrees of disorder from the scenario of Ref. 3 and T-dependent analysis of the present paper, for $\sigma(T)$ vs $\sqrt{T}$; from the top to the bottom of the figure, at fixed T, the disorder increases; the curves (a), (b), etc. give the T dependence of $\sigma(T)$ corresponding to the various regions (a), (b), etc. of Fig. 1.

B. Region (b)

Such a region exists only within the hypothesis[3] that the uniform paramagnons in the left-hand part of region (a) cross over completely to local ones when $(\varepsilon_F\tau)^{-1}$ further increases [in regions (c)–(e)], and that they do so before the magnetic instability is reached ($\bar{I}_{\rm eff}$ remains, <1). If a more elaborate calculation could prove that, contrary to the assumption considered in Ref. 3, the magnetic instability is reached before the above crossover is achieved, then $\bar{I}_{\rm eff}$ would evolve as shown by the dotted line of Fig. 1. This case was considered in Ref. 4, where it was suggested that after $\bar{I}_{\rm eff}$ has reached the value 1 and the susceptibility has thus become infinite, a "spin-glass-type" order could set in. One must remember, though, that in real spin glasses the magnetic susceptibility remains finite at the transition. $\bar{I}_{\rm eff}$ crossing the borderline $\bar{I}_{\rm eff}=1$ should instead be visualized, as was done in Ref. 6(c), as the formation of a random spin-density wave. In the present paper, instead, local moments never form, although they are close to doing so.

C. Region (c)

From this region on, the paramagnons are local when $(\varepsilon_F\tau)^{-1}$ increases. Moreover, interactions among these local paramagnons rescale the effective electron-electron interaction $\bar{I}_{\rm eff}$ to smaller values according to the arguments developed in Ref. 9, so that $\bar{I}_{\rm eff}$ stops increasing and starts to decrease instead. Now we argue that the behavior of $\sigma(\tau)$ can still be described by the perturbative formula (14). Indeed, the perturbation results obtained for $(\varepsilon_F\tau)^{-1} << 1$ were supposed to hold as long as, essentially, $g/(\varepsilon_F\tau)^2 < 1$ or, replacing g by its most important contribution, $(1-\bar{I}_{\rm eff})^{-1/2} < (\varepsilon_F\tau)^2$; when the disorder increases $\varepsilon_F\tau$ decreases; but since in region (c) $\bar{I}_{\rm eff}$ is expected to also decrease,[3] so does $(1-\bar{I}_{\rm eff})^{-1/2}$, and the condition $g < (\varepsilon_F\tau)^2$ can still be fulfilled. This may explain why, experimentally,[6] the observed variation of $\sigma(T)$ can be accounted for by the same T dependence as that given by perturbation theory, even in the neighborhood of the metal-insulator transition (where $\varepsilon_F\tau \sim 1$). So we continue to consider (14) and (15) with $\bar{I}_{\rm eff}$ decreasing when $(\varepsilon_F\tau)^{-1}$ increases. As a consequence $|g_t|$ still predominates over g_s and $1+g_s$, but less and less so, so that A is still negative but $|A|$ weakens; in other words, $|A|$ would have reached a maximum value in the crossover region (b), reflecting the behavior of $\bar{I}_{\rm eff}$ versus $(\varepsilon_F\tau)^{-1}$. On the other hand, $(\varepsilon_F\tau)^{-1}$ is no longer very small compared to 1, so that $1-(\varepsilon_F\tau)^{-1}$ in (16) is no longer reducible to 1. The localization contribution to $\sigma(T)$ starts to play a role, although still minor. At first, $\bar{I}_{\rm eff}$ is still sufficiently close to 1 for $|g_t|$ to still be approximated by $(1-\bar{I}_{\rm eff})^{-1/2}$ and we write

$$\sigma_{(c)}(T) \simeq \sigma_{(c)}(0) + A_{(c)}(T/T_F)^{1/2} , \tag{20a}$$

$$\sigma_{(c)}(0) \simeq \sigma_0[1+(\varepsilon_F\tau)^{-2}(|g_t|-1-g_s)] \simeq (\varepsilon_F\tau)\{1+(\varepsilon_F\tau)^{-2}[(1-\bar{I}_{\rm eff})^{-1/2}-1-g_s]\} , \tag{20b}$$

$$A_{(c)} \simeq -(\varepsilon_F\tau)^{-1/2}\{(1-\bar{I}_{\rm eff})^{-1/2}[1-(\varepsilon_F\tau)^{-1}]-g_s\} . \tag{20c}$$

The main points in this region are

$$\begin{aligned}&\sigma_{(c)}(0) > \sigma_0 ,\\ &\sigma_{(c)}(0) \text{ decreases when } (\varepsilon_F\tau)^{-1} \text{ increases} ,\\ &A_{(c)} < 0 ,\\ &|A_{(c)}| \text{ decreases when} (\varepsilon_F\tau)^{-1} \text{ increases} .\end{aligned} \tag{21}$$

This is illustrated by the two "c" curves of Fig. 2.

D. Region (d)

This region is characterized as follows: In the renormalization process of $\bar{I}_{\rm eff}$ towards vanishing values and the corresponding decrease of $|g_t|$, $\bar{I}_{\rm eff}$ reaches a value $\bar{I}_0$ at the border between regions (c) and (d) where $\varepsilon_F\tau$ has decreased enough to approach 1 and where g_s+g_t has lowered enough so that $1+g_s-|g_t|$ is now positive in (16). There $\sigma(0)$ starts to be smaller than σ_0 and the metal-insulator transition may be reached at 0 K. More precisely, restoring all terms from (14) in $\sigma(0)$, with g_s and g_t extracted from Ref. 18 and recalled in Ref. 3,

$$\sigma(0) \simeq \sigma_0 \left[1 - \frac{1}{(\varepsilon_f\tau)^2}\frac{3}{4\pi}[1+1.36(g_c+g_s+g_t)] \right] , \tag{22}$$

$$g_s+g_t = \frac{4}{3\bar{I}_{\rm eff}} \left[2(1+\bar{I}_{\rm eff}) + \frac{1}{(1+\bar{I}_{\rm eff})^{1/2}} - \frac{3}{(1-\bar{I}_{\rm eff})^{1/2}} \right] . \tag{23}$$

$\sigma(0)$ will be smaller than σ_0 for $1+1.36g>0$. One writes

$$1+1.36g = 1.36g_c + f(\bar{I}_{\rm eff}) , \tag{24a}$$

$$\begin{aligned} f(\bar{I}_{\rm eff}) &= 1+1.36(g_s+g_t)\\ &= 4.63 + \frac{5.44}{\bar{I}_{\rm eff}} \left[\frac{2}{3} + \frac{1}{3(1+\bar{I}_{\rm eff})^{1/2}} - \frac{1}{(1-\bar{I}_{\rm eff})^{1/2}} \right] .\end{aligned} \tag{24b}$$

Simple algebra shows that

$$f(\bar{I}_{\rm eff}) > 0 \quad \text{for} \bar{I}_{\rm eff} \lesssim 0.4 . \tag{25}$$

As already mentioned at the end of Sec. II, g_c is not known from Ref. 18 at 0 K but plays a minor role: at finite T given by formula (2.29) of Ref. 18, g_c is negative but $|g_c|$ amounts to only a fraction of $\bar{I}_{\rm eff}$. Therefore, in the absence of a better knowledge of g_c, we can consider, as a very rough estimate, $\bar{I}_0 \sim 0.4$. On the other hand, at finite temperature an in a T range where $(\tau/\tau_{\rm in})^{(1)}$ is linear in T, an underestimate of $\tau/\tau_{\rm in}$ is given by $(\tau/\tau_{\rm in})^{(1)}$ since both $(\tau/\tau_{\rm in})^{(1)}$ and $(\tau/\tau_{\rm in})^{(2)}$ are positive. Then, neglecting g_c in g compared to (g_s+g_t) one gets approximately, for A in (15) [when $1-\bar{I} < T\tau < 1$, i.e., when $(1-\bar{I})/(\varepsilon_F\tau) < T/T_F < 1/(\varepsilon_F\tau)$],

$$A \simeq \frac{k_F}{(\varepsilon_F\tau)^{1/2}} \frac{\sqrt{3}}{2\pi^2} \frac{1}{(1-\bar{I}_{\rm eff})^{1/2}} \left\{ \frac{3}{2\pi(\varepsilon_F\tau)} \left[\frac{\sqrt{3}\,\bar{I}_{\rm eff}}{2-\bar{I}_{\rm eff}} \right]^{1/2} - \frac{1.83}{\bar{I}_{\rm eff}} \left[1 - \tfrac{2}{3}(1+\bar{I}_{\rm eff})(1-\bar{I}_{\rm eff})^{1/2} - \frac{1}{3}\left[\frac{1-\bar{I}_{\rm eff}}{1+\bar{I}_{\rm eff}}\right]^{1/2} \right] \right\} , \tag{26}$$

$$A > 0 \quad \text{for} \frac{1}{\varepsilon_F\tau} > \frac{1}{(\varepsilon_F\tau)_0} \text{ and } \bar{I}_{\rm eff} < \bar{I}_0 , \tag{27}$$

$$\bar{I}_0 \sim 0.4, \quad (\varepsilon_F\tau)_0 \sim 1.2 . \tag{28}$$

Again, these values are only orders of magnitude. To render these calculations more precise, one would need to calculate g_c and $(\tau/\tau_{\rm in}^{(2)})$. This is out of the scope of the present paper, which only aims to qualitatively understand the behavior of $\sigma(0)$ and $\sigma(T)$ in the various regimes for $\bar{I}_{\rm eff}$ versus $(\varepsilon_F\tau)^{-1}$. The main point in region (d) is that the inelastic scattering processes may now dominate over interaction.[19] In this region we have

$$\sigma_{(d)}(T) \simeq \sigma_{(d)}(0) + A_{(d)}(T/T_F)^{1/2} , \tag{29a}$$

$$\sigma_{(d)}(0) < \sigma_0 \quad \text{for} \bar{I}_{\rm eff} < \bar{I}_0 , \tag{29b}$$

$$\sigma_{(d)}(0) \text{ decreases when } (\varepsilon_F\tau)^{-1} \text{ increases} , \tag{29c}$$

$$A_{(d)} > 0 \quad \text{for} (\varepsilon_F\tau)^{-1} > (\varepsilon_F\tau)_0^{-1} , \tag{29d}$$

$$|A_{(d)}| \text{ increases when} (\varepsilon_F\tau)^{-1} \text{ increases} . \tag{29e}$$

This is illustrated by the two bottom curves "(d)" of Fig. 2. The key results here are the following: First, although the electron-electron interaction is still present, its renormalized value $\bar{I}_{\rm eff}$ has become weak enough for the localization transition to occur "as if" there were no interaction at all; second, the role of the inelastic processes becomes crucial and governs all behaviors close to the localization transition (in agreement with Ref. 19).

E. Region (e)

Finally, when $\bar{I}_{\rm eff}$ reaches zero, one recovers results analogous to those from the noninteracting case[10] and the T dependence of $\sigma(T)$ arises only from inelastic processes.

In the next section we reexamine the experimental work[7] from the point of view of the present analysis.

IV. COMPARISON WITH EXPERIMENTAL OBSERVATION IN Si:P

We now turn to the examination of the experimental situation of Ref. 7 for metallic Si:P [in particular, Figs. 1 and 2 of Ref. 7(a)], and compare it with our Fig. 2 [decreasing the P density $n = \bar{n} \times 10^{18}$ cm^{-3} in Ref. 7 corresponds to increasing the disorder $(\varepsilon_F\tau)^{-1}$ in our analysis; the metal-insulator transition occurs at

$\tilde{n}_c=3.74$]. The experimentalists found in the metallic region ($\tilde{n} \geq \tilde{n}_c$) a low-temperature dependence of $\sigma(T)$ which could be fitted almost as well by a $T^{1/2}$ or $T^{1/3}$ law. They, however, confirmed the $T^{1/2}$ power law later on.[7(b),20] They attributed that behavior to interaction effects [i.e., $\sigma(T) \sim \sigma(0)+\delta\sigma_{I,L}(T)$], especially since the inelastic process $\delta\sigma_L(T)$ was believed[11] to yield a much weaker temperature dependence in $T^{p/2}$ with $p>\frac{3}{2}$. They also remarked that the $T^{1/2}$ formula, obtained in perturbation theory to first order in the disorder, surprisingly is still obeyed in the region of strong disorder when $\varepsilon_F\tau \rightarrow 1$. It is generally believed[21] that metallic Si:P near the metal-insulator transition may be considered an enhanced fermion system with strong spin fluctuations; therefore our disordered paramagnon model could be reasonably applied. It is clear that our schematic Fig. 2 qualitatively agrees with the general trends of the observations [see, in particular, the inset of Fig. 1 in Ref. 7(a), as well as the change in the variation of the coefficient of $\sqrt{T}$ versus disorder in Fig. 2 of that same reference]. As already stressed earlier in this paper, our scenario of Ref. 3 allows one to understand why a formula obtained in perturbation theory $[(\varepsilon_F\tau)^{-1} \ll 1]$ still holds in the region of strong disorder, $(\varepsilon_F\tau)^{-1} \rightarrow 1$, due to the weakening of $\bar{I}_{\rm eff}$ in that region. Second, to explain the change of sign of the coefficient of $\sqrt{T}$, the authors of Ref. 7(a), using the screened Coulomb interactions, invoked the change in the Thomas-Fermi screening length as increasing when $\tilde{n}$ decreases towards $\tilde{n}_c$. But this was shown to be erroneous later on, as, actually, the Thomas-Fermi screening length does not change dramatically.[22] Instead, in our scenario we invoke a decrease in the strength of the interaction $\bar{I}_{\rm eff}$ which weakens the contribution of $\delta\sigma_{I,L}$; moreover, we also take into account the inelastic processes $\delta\sigma_L$ neglected in the previous analyses, since, following Ref. 13, they may introduce a similar contribution $\sqrt{T}$ more and more important when the disorder increases (or $\tilde{n}$ decreases towards $\tilde{n}_c$).

The most recent[23] susceptibility (χ) measurement in Si:P down to 30 mK was made for $\tilde{n} \sim 4.08$. This would correspond [according to Fig. 2 of Ref. 7(a)] to a strongly negative coefficient of $\sqrt{T}$ in $\sigma(T)$, which, in turn, compared to our Fig. 2, would correspond to the border between regions (a) and (b) of our Fig. 1. In other words, this would be the point where $\bar{I}_{\rm eff}$ would be closest to 1. The experimental[23] log-log plot of χ versus T suggests a behavior $\chi(T) \sim T^{-1/2}$; extrapolation of the experimental points down to $T=0.01$ K would give an enhancement of $\chi(T=0.01)$ compared to $\chi_{\rm Pauli}$ (also indicated on the figure) of ~ 37.5. If χ would saturate at much lower T, one would thus expect $\chi(T=0)/\chi_{\rm Pauli}=(1-\bar{I}_{\rm eff})^{-1}>37.5$ and thus

$$0.97<\bar{I}_{\rm eff}<1 \ . \tag{30}$$

The experimental behavior may be understood as follows: from perturbation theories,[18,24] one may write[24]

$$\chi(T)=\frac{N_0+\delta\chi}{1-I(N_0+\delta\chi)} \tag{31}$$

when $N_0/(1-IN_0)$ would be the enhanced Stoner susceptibility in the absence of paramagnons and in the absence of disorder (N_0 is the density of states at the Fermi level),

$$\delta\chi=\delta\chi(T=0)+\delta\chi(T) \ , \tag{32}$$

$$I[N_0+\delta\chi(T=0)]=\bar{I}_{\rm eff} \ . \tag{33}$$

It was shown in Refs. 18 and 24 that

$$\delta\chi(T) \sim -BN_0\sqrt{T\tau} \ , \tag{34a}$$

where B depends on $\bar{I}$ (weakly) and of the amount of disorder. However, it was emphasized in Ref. 24 that such behavior holds as long as

$$T\tau<(1-\bar{I})^{1/2} \tag{34b}$$

(while, for $(1-\bar{I})^{1/2}<T\tau<1$, $\delta\chi$ rather behaves as $\sqrt{T\tau}[T\tau/(1-\bar{I})^{1/2}]$). On the other hand, it was also pointed out in Ref. 24(b) that $\delta\chi(T=0)$ strongly renormalizes the effective interaction, so that

$$\bar{I}<\bar{I}_{\rm eff}<1 \ . \tag{35}$$

Then, we write

$$\chi(T)=\frac{N_0+\delta\chi(T=0)-BN_0\sqrt{T\tau}}{1-\bar{I}_{\rm eff}+\bar{I}B\sqrt{T\tau}} \ . \tag{36}$$

Both $\delta\chi(T=0)$ and $BN_0\sqrt{T\tau}$ are small compared to N_0 (in perturbation theory); however, in the denominator of (36) both $1-\bar{I}_{\rm eff}$ and $\bar{I}B\sqrt{T\tau}$ are small quantities. Therefore one may reasonably approximate

$$\chi(T) \simeq \frac{N_0}{1-\bar{I}_{\rm eff}+\bar{I}B\sqrt{T\tau}} \ . \tag{37}$$

Now if (and this is apparently the case in Si:P) $\bar{I}_{\rm eff}$ is very close to 1, unless one is at extremely low temperatures, the measurements may instead be in the range

$$1-\bar{I}_{\rm eff} \ll \bar{I}B\sqrt{T\tau} \ , \tag{38}$$

in which case

$$\chi(T) \simeq \frac{N_0}{\bar{I}B\sqrt{T\tau}} \propto \frac{1}{\sqrt{T}} \ , \tag{39}$$

which would be in agreement with the experimental law. Note that the range of T in (38) is not incompatible with the T range where $\sigma(T)-\sigma(0) \sim \sqrt{T}$ both from the inelastic and the interaction processes, which was $T\tau>1-\bar{I}$. Combining all the conditions, and in terms of the unique effective value $\bar{I}_{\rm eff}$, our main point here is to show that when

$$\left[\frac{1-\bar{I}_{\rm eff}}{\bar{I}_{\rm eff}B}\right]^2<1-\bar{I}_{\rm eff}<T\tau<(1-\bar{I}_{\rm eff})^{1/2} \ , \tag{40}$$

one may have simultaneously

$$\sigma(T)-\sigma(0) \propto -\sqrt{T} \ , \qquad \chi(T) \propto 1/\sqrt{T} \ . \tag{41}$$

In order to observe the saturation of $\chi(T \rightarrow 0)$, i.e., in order for $1-\bar{I}_{\rm eff}$ in (37) to dominate over the T-dependent term, one should reach extremely low temperature

$T\tau<(1-\bar{I}_{\rm eff})^2/(\bar{I}_{\rm eff}B)^2$, in which case χ should saturate, but altogether the T dependence of $\sigma(T)$ would be more complicated since, for $T\tau<1-\bar{I}_{\rm eff}$, $(\tau/\tau_{\rm in})^{(1)}$ is no longer linear in T but rather behaves[13] as $(T\tau)^{3/2}$.

Note that the renormalization-group result[6] found a law

$$\chi(T)\propto\frac{1}{T^{4/3}}\ , \qquad (42)$$

which could not fit the measurements of Ref. 23. However, the authors of Ref. 6 emphasized that the $T^{-4/3}$ law should be taken too seriously since their renormalization-group procedure breaks down in the region where, in our language, $\bar{I}_{\rm eff}$ approaches 1.

Other low-T measurements of $\chi(T)$ in metallic Si:P were done in Ref. 25 down to 20 mK. Although the authors of Ref. 23 mentioned that a T dependence of χ well below T_F also found in Ref. 25, they did not comment on a detailed comparison between these two sets of measurements. Reference 25 exhibits for $\tilde{n}=4.5$ a variation of χ versus T which rather fits a Curie law between 30 and 100 mK,

$$\chi(T)\propto\frac{1}{T},\quad 30<T<100\ \text{mK}\quad(\text{Ref.}\,25)\ , \qquad (43)$$

uncompatible with Fig. 1 of Ref. 23, where in the same T range it was found that

$$\chi(T)\propto\frac{1}{\sqrt{T}},\quad 30<T<100\ \text{mK}\quad(\text{Ref.}\,23)\ . \qquad (44)$$

On the other hand, the authors of Ref. 25 found a sharp rise of $\chi(T)$ below 30 mK (but the data points stop at 20 mK. This may suggest a divergence of $\chi(T)$ where $T\to 0$ K as considered in Ref. 4 (corresponding to the dotted line in our Fig. 1).

It would thus be most useful if measurements of $\chi(T)$ on the same sample (with the same donor density) could become available[26] at the lowest possible temperature in order to check whether $\chi(T\to 0)$ saturates or diverges and also to get a definitive power-law dependence at finite temperatures. It would also be fruitful to measure $\chi(T)$ and $\sigma(T)$ for donor densities closer to the metal-insulator transition, where $\sigma(T)$ decreases with decreasing T.

In the above analysis of Si:P we have neglected the mass-anisotropy and intervalley effects considered in Ref. 5. Although these effects are weaker in Si:P than in other systems,[1] they ought to be included in a more quantitative analysis together with a detailed account of $(\tau/\tau_{\rm in})^{(2)}$ and of g_c.

V. CONCLUDING REMARKS

In this paper we have reanalyzed the temperature dependence of the conductivity and partly of the susceptibility of metallic Si:P, which is one of the most widely studied examples of a metal-insulator transition. To do so we have developed our recent scheme[3] according to which, within the (disordered) paramagnon model, one may understand how the localization transition can be reached despite the presence of strong electron interactions. In our scenario, for increasing disorder, the effective interaction $\bar{I}_{\rm eff}$ first increases so much that the system becomes very close to the magnetic instability, in agreement with the renormalization-group theory,[6] but then $\bar{I}_{\rm eff}$ crosses over to decreasing values when the disorder further increases. Therefore in that last regime, first of all, perturbative results may still hold (due to the combined weakening of $\bar{I}_{\rm eff}$ and $\varepsilon_F\tau$), which would explain why perturbative results account for the observed behavior, even close to the metal-insulator transition; this would also justify *a posteriori* the additivity of the inelastic and interaction effects; secondly, in that same regime the interaction contribution to the conductivity weakens and the inelastic effects start to predominate with the same T dependence (according to Refs. 12 and 13) and with an opposite sign, so that, although the interactions are still present, they no longer prevent the localization transition to occur. In this regime, as explained in Ref. 3, the paramagnons have become local ones, as if one would have randomly distributed *almost* magnetic impurities in the system,[9] this picture is closely linked to the "pseudolocal spin fluctuations" of Ref. 6 or to the local spin fluctuations of the Brinkman-Rice-Gutzwiller form[21]. But, then, one has to take into account the interactions among those local spin fluctuations;[9] this yields a decrease in the strength of $\bar{I}_{\rm eff}$: the "almost magnetic impurities" become less and less close to becoming magnetic. At the earlier stage when $\bar{I}_{\rm eff}$ has increased to be very close to 1 and where such a strong-coupling regime induces the renormalization-group procedure to break down,[6] instead of $\bar{I}_{\rm eff}$ reaching 1, the model studied in Ref. 3 and here, is that the system crosses over towards a regime where $\bar{I}_{\rm eff}$ departs more and more from 1. If such a scenario actually happens, or if the system switches to a magnetically ordered phase, may be clarified by experiments performed at lower temperatures. On the other hand, in a metal where the disorder is due to random impurities, it has been shown[27] that *interactions between impurities* decreases the conductivity, i.e., short-range order among impurities act as a localizing agent for the electrons. More generally in cases like Si:P, fluctuations in the donor distribution or local concentrations of the P may play a role in the electron localization. From this last point of view of highly inhomogeneously distributed donors, one may have to face a more complicated behavior of $\bar{I}_{\rm eff}$. Instead of a unique one, either following the solid line of Fig. 1 or the dotted line, $\bar{I}_{\rm eff}$ may split into two parts, $(\bar{I}_{\rm eff})^{(1)}$ and $(\bar{I}_{\rm eff})^{(2)}$, corresponding to two sets of paramagnons existing simultaneously. Some remain uniform, retaining their q dependence with their $(\bar{I}_{\rm eff})^{(1)}$ reaching 1; thus, $(\bar{I}_{\rm eff})^{(1)}$ follows the dotted line on our Fig. 1 and local moments are formed. Other paramagnons would lose their q dependence before $(\bar{I}_{\rm eff})^{(2)}$ reaches 1, and would cross over to local paramagnons with the consequences studied in this paper; this $(\bar{I}_{\rm eff})^{(2)}$ would follow the solid line of our Fig. 1. This splitting of $\bar{I}_{\rm eff}$ would thus be linked to the degree of inhomogeneity in the composition of the system and could be tested by local measurements like NMR by varying the sample preparation for a given donor density. A model was recently proposed[28] for the nuclear-spin relaxation in Si:P involving both electrons near the Fermi energy and singlet pairs of localized spins deep below the Fermi energy. This is somewhat different from what we have here.

Finally, we emphasize that if our local paramagnon picture proves to be useful, much theoretical work is needed to render it quantitatively useful (for instance, concerning the possible modification in the ratio value of the low-T coefficient γ of the specific heat, to the susceptibility[29]). Indeed, the standard local paramagnon model[30] tells us that the exchange enhancement of the host is modified locally at the impurities sites, but ignores the fact that the impurities also introduce a finite mean free path for the electrons. Similarly, as emphasized by Anderson,[21] a disordered Brinkman-Rice-Gutzwiller theory does not yet exist. On the other hand, none of the localization theories that contain electron interactions[1–6] accounts for the fact that these interactions may be modified locally at the impurities sites. That is why it is impossible at present to provide quantitative results in the strongly disordered region.

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APPENDIX

Correspondence between various notations and expressions found in the literature for the interaction contribution $\delta\sigma_{I,L}$ to the conductivity is given below. This appendix is of no interest for theoreticians expert in the field. But it may appear helpful for experimentalists and also, of course, for readers not familiar with the problem.

In the following, only the contribution from particle-hole diffusion processes (called g_s+g_t in Ref. 3 and in the present paper) are considered.

In the review by Lee and Ramakrishnan[1] (LR) dealing with screened Coulomb interactions, we find [their formula (3.41c) in a.u.].

$$\delta\sigma_{I,L}(T)-\delta\sigma_{I,L}(0)=\frac{1}{4\pi^2}\frac{1.3}{\sqrt{2}}\left[\tfrac{4}{3}-\tfrac{3}{2}\tilde{F}_0\right]\left[\frac{T}{D}\right]^{1/2}, \quad \text{(A1)}$$

with

$$\tilde{F}_\sigma=-\frac{16}{d(d-2)}\frac{1+dF/4-(1+F/2)^{d/2}}{F/2}, \quad d\neq 2$$

so that

$$\tilde{F}_\sigma=-\frac{16}{3}\frac{1+\frac{3}{2}(F/2)-(1+F/2)^{3/2}}{F/2}, \quad d=3 \quad \text{(A2)}$$

(note that a fraction bar is missing just before the last F appearing in the formula for $\tilde{F}_\sigma$, in the printed version of LR, but was present in the prepublication version).

The expression $(\frac{4}{3}-\frac{3}{2}\tilde{F}_\sigma)$ in (A1) is identical to the expression $2(g_1-2g_3)$ (still for screened Coulomb interactions) given by Isawa and Fukuyama[17] (IF), when they used the T matrix approximation for the effective F, i.e., when their initial $F/2$ is replaced by $F/2(1+(F/2))$.

Then, with their formulas (2.14) and (2.19) one gets, with their (2.15b) and $\lambda=(2\pi\varepsilon_F\tau)^{-1}$, $\zeta(\frac{1}{2})=-1.46$,

$$\delta\sigma_{I,L}(T)=\sigma_0\left[\frac{\sigma_1'}{\sigma_0}+\frac{\sigma_3'}{\sigma_0}\right]$$
$$=-\frac{\sigma_0}{4\pi^2(\varepsilon_F T)^2}\frac{3\sqrt{3}\pi^2}{8}(2-1.46\sqrt{2\pi T\tau})$$
$$\times(g_1-2g_3), \quad \text{(A3)}$$

and inserting their formulas (4.2b) and (4.3b) one gets

$$\delta\sigma_{I,L}(T)-\delta\sigma_{I,L}(0)=\frac{1}{4\pi^2}\frac{1.3}{\sqrt{2}}2(g_1-2g_3)\left[\frac{T}{D}\right]^{1/2}, \quad \text{(A4)}$$

with

$$2(g_1-2g_3)=\frac{8}{3}\left[2+3\frac{1+F/2}{F/2}-3\frac{(1+F/2)^{3/2}}{F/2}\right]. \quad \text{(A5)}$$

Comparing with (A1) and (A2),

$$(\tfrac{4}{3}-\tfrac{3}{2}\tilde{F}_\sigma)_{\rm LR}\equiv 2(g_1-2g_3)_{\rm IF\ with\ T\ matrix}. \quad \text{(A6)}$$

On the other hand, formula (9) of Altshuler and Aronov[16] (AA) reads

$$\delta\sigma_{I,L}(T)-\delta\sigma_{I,L}(0)$$
$$=\frac{1}{8\pi^2}\left[\frac{T}{D}\right]^{1/2}[\tfrac{8}{3}+3\lambda_\sigma^{(j=1)}(F)]\times 0.915, \quad d=3 \quad \text{(A7)}$$

with their formula (10):

$$\lambda_\sigma^{(j=1)}=\frac{16}{d(d-2)}\frac{1-dF/4-(1-F/2)^{d/2}}{F/2}$$
$$=-\frac{16}{3}\frac{1-\frac{3}{2}F/2-(1-F/2)^{3/2}}{-F/2} \quad d=3. \quad \text{(A8)}$$

Then we have to make the following correspondence between the F of AA and that of LR:

$$(-F/2)\equiv F/2_{\rm LR}. \quad \text{(A9)}$$

This sign difference is just a different notation, as is clear from formula (7) of Ref. 16. Then,

$$(\lambda_\sigma)_{\rm AA}^{(j=1)}\equiv(\tilde{F}_\sigma)_{\rm LR}, \quad \text{(A10)}$$

and then (A7) may be rewritten

$$\delta\sigma_{I,L}(T)-\delta\sigma_{I,L}(0)$$
$$=\frac{1}{4\pi^2}\left[\frac{T}{D}\right]^{1/2}[\tfrac{4}{3}+\tfrac{3}{2}\lambda_\sigma^{j=1}(F)]\times 0.915. \quad \text{(A11)}$$

This formula would identify with the one by LR [formula (A1)] above, were it not for a different sign inside the square brackets of (A11). However, Ref. 16 contains a few misprints, so we assume that formulas (A7) and (A11) should be understood with a minus sign inside the square brackets instead of a plus sign, or, alternatively, there should be an overall minus sign missing in the expression of $\lambda_\sigma^{(j=1)}$.

As far as F is concerned, it is given by formula (3.36b) in the review by Altshuler and Aronov,[1] or formula (3.34) of Lee and Ramakrishnan[1] by the angular integral:

$$\frac{F}{2}=\frac{\int d\hat{\Omega}\, v(q=2k_F\sin(\theta/2))}{\int d\hat{\Omega}\, v(0)}\,, \tag{A12}$$

with

$$v(q)=\frac{4\pi}{\kappa_3^2}\frac{1}{1+q^2/\kappa_3^2}$$

when κ_3^{-1} is the three-dimensional Thomas-Fermi screening length; k_F is the Fermi momentum (actually, in Lee and Ramakrishnan[1] F appears on the left-hand side instead of $F/2$). The result as given in Altshuler and Aronov[1] is

$$\frac{F}{2}=\frac{1}{2}\int_0^{\pi}\sin\theta\, d\theta\frac{1}{1+(2k_f/\kappa_3)^2\sin^2(\theta/2)}$$
$$=\frac{\ln[1+(2k_F/\kappa_3)^2]}{(2k_F/\kappa_3)^2} \tag{A13}$$

[F has to be understood in the sense of Ref. 1, equal to minus that in Ref. 16. Note also that F in (A13) corresponds to what is called $F/2$ in formula (3.36b) of the review by Altshuler and Aronov[1]]. $F/2$ clearly varies between 1 and 0 when the screening length increases from 0 to ∞.

The experimentalists of Ref. 6a used an earlier expression $[1-\frac{3}{2}(F/2)]$ for the coefficient of $\sqrt{T}$ in $\delta\sigma_{I,L}$. However, Altshuler and Aronov[1,16] showed that 1 has to be replaced by $\frac{4}{3}$ as recalled in LR after their formula (3.41c). Moreover, as mentioned above, following Finkelstein,[5] Altshuler and Aronov[16,1] showed that it is rather $[\frac{4}{3}-3(\tilde{F}_\sigma/2)]$ which ought to appear [see also LR (Ref. 1), end of their Sec. III c] with $\tilde{F}_\sigma$ given above in (A2).

With all these ingredients in hand, one can proceed to the Hubbard model where, according to Ref. 17, in formula (A3) g_1 and g_3 should be given, respectively, by formulas (3.2) and (2.20b) of that reference, which yields (replacing now $F/2$ by $\bar{I}$)

$$(g_1-2g_3)_{\text{Hubbard}}=\frac{4}{3\bar{I}}\left[2(1+\bar{I})+\frac{1}{(1+\bar{I})^{1/2}}-\frac{3}{(1-\bar{I})^{1/2}}\right]. \tag{A14}$$

This is what was called, in Ref. 3, g_s+g_t:

$$(g_s+g_t)_{\text{Ref. 3}}=(g_1-2g_3)_{\text{Ref. 17, Hubbard model}}\,. \tag{A15}$$

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abstract of that paper (but not in the text) where the ranges of T should read $T\tau \ll (1-\bar{I})^{1/2}$ and $(1-\bar{I})^{1/2} < T\tau \ll 1$: $(1-\bar{I})^{1/2}$ should appear instead of $1-\bar{I}$; this is important in the discussion of $\chi(T)$ of Si:P in the present paper; (b) M. T. Béal-Monod, Ref. 2(c).

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[28]Z. Gan and P. A. Lee, Phys. Rev. B **33**, 3595 (1986); although this model is quite attractive, it yields $1/T_1 \sim T$, while the data fit $1/T \sim \sqrt{T}$. However, at the lowest available temperatures it might be difficult to distinguish between the two power laws as pointed out by G. Thomas (private communication).

[29]After the present paper was submitted for publication, there appeared for dimensions $d = 2+\varepsilon(\varepsilon \to 0)$ a detailed discussion of γ/χ in C. Castellani and C. di Castro, [Phys. Rev. B **34**, 5935 (1986)] along the lines of their preceding work [Refs. 6(d) and 6(e) when the renormalization-group techniques hold. However, it is difficult from that work to infer what would be γ/χ for $d = 3$. Note also that the T behavior of χ in that paper is different from the earlier result by the same authors [Ref. 6(e)] that we quote here as our formula (42).

[30]See, for instance, the review by D. L. Mills, M. T. Béal-Monod, and P. Lederer, in *Magnetism,* edited by H. Suhl (Academic, New York 1973), Vol. V, p. 89.