RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 36, NUMBER 4 1 AUGUST 1987

Rapid Communications

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Positron mobility edge in dense gaseous helium

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(Received 3 November 1986)

We analyze the recent positron-lifetime experiments in dense helium in the presence of pulsed fields and show that all the data are quantitatively consistent with the idea of a field-dependent mobility edge.

A positron (e^+) in dense ($n \cong 4\times10^{21}$ cm^{-3}) low-temperature ($T \cong 4$ K) helium (He) gas is one of the most interesting and well-characterized prototypes of a purely disordered system.[1] The finite-mass He atoms act as a set of point scatters and provide a weak energy-loss mechanism. In addition, one can literally keep track of the e^+ as it trickles down in energy by observing the γ rays which are emitted on annihilation.

The lifetime spectra of e^+, with and without a pulsed electric field present, in such systems show a variety of striking phenomena which can be understood in terms of a mobility edge.[2] There are very few examples of systems which show unambiguous strong scattering or localization effects.[3] In an earlier paper,[4] we analyzed the annihilation spectrum in the absence of a field and suggested that pulsed-electric-field measurements could provide useful information about the behavior of e^+ near the mobility edge. In this Rapid Communication, we will analyze the pulsed-field data of I and show that it is consistent with the mobility-edge concept but that, strangely enough, this mobility edge occurs at (assuming our interpretation is correct)

$$k_c l \cong 40 \ , \tag{1}$$

where $k_c = \sqrt{2mE_c}$ ($\hbar = 1$) is the wave vector of the e^+ near the "mobility edge" E_c, and l is the mean free path. Such a striking deviation from the usual Ioffe-Regel[5] criterion $k_c l \cong 1$, which seems to be satisfied in other more complicated systems, suggests that some of our ideas regarding the universality[6] of the localization phenomenon may need revision.

In a typical e^+-lifetime experiment, e^+ with an energy of order 0.5 MeV, along with a prompt γ ray, are emitted from a radioactive source. The e^+ enters into a sample chamber containing helium gas and slows down, by virtue of electronic excitation and ionization of the helium, to an energy of about 10 eV in a time short compared to one nanosecond. Below this energy, the positrons lose energy slowly by quasielastic collisions with the helium atoms. Typical thermalization times are estimated to be about 100 nsec. During this rather long time interval, one monitors the positron decay rate which is to good approximation determined by the density of electrons at the positron.

For roughly the first 10 nsec of this period, the decay rate is fixed by the gas density $n \cong 10^{21}$ cm^{-3}. At some well-defined time (τ_R), there is a rapid increase (factor of 4) in the annihilation rate. This increase is due to the nucleation of a liquid drop around the e^+.[7] Furthermore, a Monte Carlo procedure[8] gives a *very* accurate description of the shape of the lifetime spectra. In this method, the motion of an e^+ through the host gas is treated classically ($kl \gg 1$), but the cross sections for momentum transfer and annihilation are high-quality quantum-mechanical quantities. A single parameter E_R, below which a liquid cluster forms, is used to fit the data. This threshold is a weak function of temperature and isotopic mass. In II we suggested that the nucleation threshold was not a mobility edge E_c but was, in fact, an energy E_R where $E_R < E_c$, so that an e^+ can remain trapped for a time equal to a crudely estimated nucleation time. Recent[8] Monte Carlo calculations show that in the presence of a static field the agreement between theory and experiments is significantly improved if one simply includes an additional sharp energy E_c below which the e^+ is not heated by the dc field (zero mobility). The pulsed-field data in I are much richer, and while no quantitative comparisons via Monte Carlo calculation have been made we can analyze such experiments within our existing two-threshold picture.

Let us assume that all of the observed experimental behavior in I is connected with a mobility edge E_c. We

know from the experiments the time at which the e^+ nucleates a drop. The Monte Carlo calculations in III (Ref. 8) give us a good estimate of the e^+ kinetic energy E_R at this time. It is roughly 10 meV. This means that the e^+ has a wave vector $k \cong 2\pi/\lambda \cong 5\times10^6$ cm^{-1}. The cross section for scattering from a single He atom is isotropic and roughly energy independent at these energies, with a cross section[8] $\sigma \cong 4\pi a_0^2 \lesssim 10^{-16}$ cm^2 ($a_0 \cong -0.3$ Å). Thus the *single-scattering* mean free path $l \cong 1/n\sigma \cong 800$ Å and $kl \cong 40$. At these densities the inter-He-atom spacing $d \cong 10$ Å (Ref. 9) and $kd \cong 0.5$. There are ten to a hundred scatters in an e^+ wavelength so that we would expect the *effective* mean free path (fl), which includes multiple scattering, to be larger since $kd \lesssim 1$ suggests an averaging of fluctuations of the He-atom density.

Localization of the e^+ is characterized by a localization length[6]

$$L_0(E) = fl\left(\frac{E_c}{E_c - E}\right)^{\nu} . \tag{2}$$

We have introduced the number f which is larger than one to roughly take into account the above-mentioned averaging. The quantity ν is expected to be about unity.[6] *The localization length cannot be smaller than the mean free path* and in this case is in the neighborhood of 10^4 Å. In II we argued that only a *localized* charge could nucleate a drop. A free e^+ moves too rapidly. The nucleation process is complicated, but it will occur when one He atom, attracted by a stationary bit of positron charge, moves some fraction δ to an inter-He-atom spacing d. In this case the e^+ is spread out [in some type of complicated quasistationary wave function $\psi(r)$] over a very large volume L_0^3. The charge density ψ^2 will surely have inhomogeneities on a length scale given by k^{-1} (or perhaps a bit smaller, depending on the details of the potential energy). In this case the nucleation time is

$$\tau_{N,\text{eff}} \cong \tau_N (kL_0)^{3/2} , \tag{3}$$

where τ_N is the time for nucleation assuming that a whole e^+ is localized at a distance k^{-1} from a He atom. τ_N was estimated in II, i.e., $\tau_N \cong 2\times10^{-13}$ sec for $d = 10$ Å, $\delta = 0.1$. The square root in Eq. (3) occurs because the time to move a given distance goes like the square root of the force. Thus for no electric field we suggest that

$$\frac{E_c - E_R}{E_c} \cong \sqrt{(m/M)(kT/E_c)}\sqrt{(\tau_{N,\text{eff}}/\tau_{\text{elastic}})} , \tag{4}$$

which, using Eqs. (2) and (4), implies that

$$\frac{E_c - E_R}{E_c} \cong (fkl)^{3/(4+3\nu)}\left(\frac{m}{M}\frac{kT}{E_c}\frac{\tau_N v}{l}\right)^{2/(4+3\nu)} , \tag{5}$$

where $v \cong \sqrt{2E_c/m}$. Equations (4) and (5) are statements of the fact (see II) that an e^+ at energy E at some instant random walks above E_c, due to Doppler scattering from the thermally moving He atoms. When this time is equal to the nucleation time we have arrived at our new threshold E_R. Thus, while E_c should be sharp, E_R should be smeared.[10]

Equation (5) is meant to be a very rough, but physically correct, expression, since it has several dimensionless quantities which are very small ($m/M \cong 10^{-4}$) and large [$(fkl) \cong 3\times10^2$ for $f \cong 6$]. It is interesting to note that for $\nu = 1$ and $\tau_N = 2\times10^{-13}$ sec [see Eq. (2) in II] $(E_c - E_R)/E_c \cong 0.6$ as compared with an experimentally determined (see I) ratio of 0.7.[11] One should certainly think of the agreement as a "fit" to the experimentally determined value since there is a great deal of uncertainty in the parameters inserted into Eq. (5) and all of the details of the nucleation process. The point is that the parameters used are reasonable.

Equation (5) also gives a good qualitative description of the dependence of the time (τ_R) at which a liquid drop nucleates as a function of temperature density and isotopic mass. Assuming that E_c depends only on density (disorder), which is good as long as the wavelength is still determined by a free-particle E-vs-k relationship, then $n\tau_R/M$, as far as its density and temperature dependence are concerned, should be proportional to $(E_c - E_R)/E_c$. This correlates correctly with the observed trend, i.e., it increases as T increases and as M decreases. The expected density dependence of E_c, i.e., higher E_c as n increases implies shorter times at higher densities, above and beyond the trivial linear term. This is also in agreement with the data.[1]

Having f to fit $(E_c - E_R)/E_c$ we will see that the measured electric field dependence of the delay time is also described correctly. For electric fields $F \neq 0$ (we take the charge on the e^+ equal to unity), the experimental results in I, shown in Fig. 1, indicate a significant decrease in the separation between E_c and E_R for fields F in the neighborhood of 100 V/cm. The experimental results are shown as a measured time difference

$$\Delta\tau \equiv (E_c^* - E_R^*)/\dot{E} , \tag{6}$$

where the starred quantities are in the presence of a field and $\dot{E}$ is a rather constant energy-loss rate due to collisions with the thermally moving helium atoms.

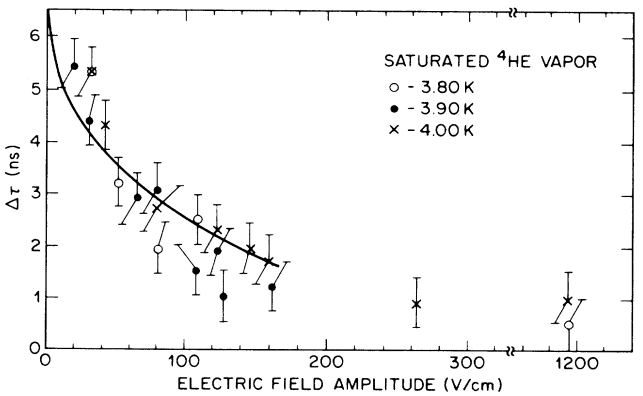

FIG. 1. The experimental data presented in I (points) along with a plot of Eq. (11) with $F_0 = 300$ V/cm.

We expect that

$$E_c^* = E_c - qFL_0(E_c^*) \ , \tag{7}$$

with $q \sim 1$. Equation (7) states the obvious. A particle bound by an energy $E_c - E_c^*$ will escape from its well when it can gain energy from the field roughly equal to its binding energy. The fact that we have used the zero-field form for L_0 [Eq. (2)] in Eq. (7) can only be accurate when F is small; nevertheless we will assume it is roughly correct for all F. Equation (2) suggests that

$$\frac{E_c - E_c^*}{E_c} \sim \left[\frac{flF}{E_c}\right]^{1/(\nu+1)} \equiv \xi \ . \tag{8}$$

Since Eq. (5) is a statement regarding nucleation time it is also valid in the electric field, i.e., the same equation with E_c and E_R replaced by E_c^* and E_R^*. This leads in a straightforward fashion to an expression for

$$\begin{aligned}\frac{E_R^* - E_R}{E_c} &\cong (1-\xi)^{1-\delta}[1(1-\xi)^{\delta}] - (\epsilon - 1)\xi \\ &\cong (1+\delta-\epsilon)\xi, \end{aligned} \tag{9}$$

for ξ and δ less than one. Here $\delta = 2/(4+3\nu)$ and $\epsilon = E_c/(E_c - E_R)$. Using the measured values (see I) of $E_c = 15 \pm 3$ meV and $E_R = 5.5 \pm 0.5$ meV we see that

$$(1+\delta-\epsilon) \cong 0.2 \left[\frac{+0.8}{-0.1}\right] \simeq 0 \ ,$$

i.e., E_R moves only a small amount relative to E_c, a prediction of these arguments. This implies if we take $\nu = 1$, that

$$\frac{(\Delta\tau)}{(\Delta\tau)_0} = 1 - \left[\frac{F}{F_0}\right]^{1/2} \ , \tag{10}$$

with $F_0 = E_c/fl$. For $E_c = 15$ meV and $fl = 5 \times 10^3$ Å, $F_0 = 300$ V/cm. The solid line in Fig. 1 is Eq. (11) with that F_0. One should really think of this comparison as a one-parameter fit to all the data. The "measured" $E_c - E_R/E_c$ and the data shown in Fig. 1. There is no *a priori* reason why f should be six, although it should be larger than one. Clearly, more accurate measurements are required to determine whether a fit with an exponent of one-half is, in fact, justified.

In summary, then, we feel that the e^+-lifetime spectra in pulsed electric fields give *convincing* evidence of an e^+ mobility edge at a $kl \cong 40$. The localization length is consistent with the known mean free paths and the density of He atoms. The measured values of the mobility edge and the nucleation threshold are in agreement with simple ideas relating to the nucleation process. In addition, the behavior of these edges in a field as deduced from the lifetime experiments seems to be quantitatively consistent with a very simple picture of the edge in the presence of a field.

We would like to thank K. Canter, E. Abrahams, and S. Coppersmith for several informative discussions.

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[9] We know of no other systems which have been studied where $|a_0| < d < \lambda < l$. Electrons in dense helium vapor (see Ref. 3) have an $a_0 \cong 1.2$ Å, i.e., $a_0 \cong d \cong \lambda$ $(T = 2\ \text{K}) \cong l$. For doped semiconductors (Ref. 6) near the metal-insulator transitions all lengths are also approximately equal.

[10] There is some indication that, in fact, a better fit to the data is obtained if E_R is smeared somewhat [K. Canter (private communication)].

[11] In II we had incorrectly taken the localization length to be of the order of d. This led to a very small $(E_c - E_R)/E_c$ (at that time not measured) and a somewhat negative view of the possibility of doing an interesting experiment. Our conclusion now is that the large localization length and our qualitative picture of the nucleation and mobility thresholds seems to be roughly correct.