Pinning by the local changes of the electron mean free path in anisotropic type-II superconductors

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(Received 25 September 1986; revised manuscript received 30 March 1987)

The bulk pinning force density F_p and the critical current j_c of a dirty type-II superconductor, with spatially varying electron mean free path l along a given direction, are calculated using the Ginzburg-Landau-Abrikosov theory. In high fields the critical force density F_c is proportional to $(1-B/B_{c2})^2(B/B_{c2})^m$, where $0<m<1$, but depends also on the size and form of inhomogeneity (grain boundary, dislocation wall, etc.) which causes the change of l. This may explain deviations from straight lines in the "Kramer plots" of $j_c^{1/2}B^{1/4}$ versus B/B_{c2} and the approximate scaling of F_c when the temperature is changed.

I. INTRODUCTION

Although "considerable progress has recently been made concerning the long-standing problem of the summation of forces exerted by individual pinning centers (pins) on the flux line lattice of a type-II superconductor in the mixed state,"[1] theory is still based on phenomenological theories dealing with elastic and plastic behavior of the vortex lattice.[2] These theories may give different explanations for experimental findings as, for example, in the case of saturation in high fields, when the bulk pinning force density F_p is independent of the pinning parameters. Its dependence on the reduced magnetic flux density B/B_{c2}, empirically known to obey the formula[2,3]

$$F_p \propto (1-B/B_{c2})^2(B/B_{c2})^{1/2} , \tag{1}$$

is explained as due to the plastic shearing deformation of the vortex lattice in Kramer's model,[3] while in the model of Matsushita *et al.*[2] the explanation is based on the occurrence of catastrophic flux flow caused by local plastic deformations. In the nonsaturation region of lower fields, the knowledge of the elementary vortex-pin interaction is necessary. However, it is not easy to make a simple model of pinning, even if the nature of the pins is known (dislocation clusters, particles of another phase, . . .) since various physical mechanisms may contribute.[4] Accordingly, for a microscopic approach, which may provide a better understanding of pinning phenomena, one needs simple model systems, with a well-determined spatial repartition of pins and a well-understood origin of pinning. In the last few years, several model systems[5] with regular distributions of inhomogeneity have been prepared and studied experimentally: thin films with periodically varying thickness, superconducting alloys with periodically modulated impurity concentration, lamellar superconductors including compounds, eutectics, and quite recently, superconducting superlattices.[6] These "regularly inhomogeneous" superconductors exhibit novel physical phenomena due to the spatial variation of various quantities of the conduction electrons, such as the density of states, the diffusion coefficient, the attractive interaction responsible for superconductivity, and so on.

One of the first microscopic models of pinning by regular inhomogeneity distribution was formulated theoretically by Ami and Maki.[7] The authors considered a dirty superconductor in the vicinity of H_{c2}, with periodic variation of the electron mean free path l and, consequently, of the diffusion coefficient $D=v_F l/3$ along a given direction. In the present work, we generalize this approach to treat various kinds of spatial variation $D(\mathbf{r})$. In Sec. II, we present our model of an inhomogeneous superconductor in the framework of the Ginzburg-Landau (GL) theory of dirty superconductors.[8] In Sec. III, we calculate the free energy of the vortex lattice as a function of the inhomogeneity characteristics. Section IV deals with some typical examples of inhomogeneity distribution, for which F_p and the critical current density (the maximum external current density which may flow without dissipation) are calculated as function of B and temperature T. The resulting formula

$$F_p \propto (1-B/B_{c2})^2(B/B_{c2})^m \tilde{D}(B/B_{c2}) , \tag{2}$$

where m is 0 or $\frac{1}{2}$, is similar to (1), except for the factor $\tilde{D}$, which reflects the shape of inhomogeneity. In Sec. V, possible applications of the model are discussed.

II. THE MODEL

When the presence of an inhomogeneity does not lead to a variation of the superconducting transition temperature T_c, the variation of the free energy is connected with gradients of the order parameter Δ and with the change of the local magnetic field $\mathbf{h}(\mathbf{r})$.[4,7] The spatial variation of $l=l(\mathbf{r})$ and, accordingly, of $D(r)=\frac{1}{2}V_F l(r)$ provides such an example. Physically, it may be realized with pinning on dislocations, grain boundaries, on the inhomogeneous impurity distribution within superconducting layers and so on.

Assuming that T_c and the electronic density of states at the Fermi level $N(0)$ are not changed, we write the Ginzburg-Landau (GL) free energy, which applies[7,8] in the vicinity of T_c to a dirty superconductor with a given spatial variation $D(\mathbf{r})=D_c[1+D_1(\mathbf{r})]$,

$$\mathcal{F}_S-\mathcal{F}_N=\int dv\left[N(0)\left[-\frac{T_c-T}{T_c}\,|\Delta(\mathbf{r})|^2+\frac{7\zeta(3)}{16(\pi T_c)^2}\,|\Delta(\mathbf{r})|^4+\frac{\pi}{8T_c}D(\mathbf{r})\,|(\Delta-2ie\mathcal{A})\Delta(\mathbf{r})|^2\right]+\frac{h^2(\mathbf{r})}{8\pi}\right], \tag{3}$$

where $\mathcal{A}$ is the vector potential and $\zeta(3)$ is Riemann's zeta function. We notice that since in the dirty limit[7,8] the coherence length varies as $D^{1/2}$, and the magnetic penetration depth as $D^{-1/2}$ both these characteristic lengths vary locally with respect to their values ζ_0 and λ_0 in the absence of the perturbation. The free energy can be expressed in a more convenient dimensionless form if one introduces reduced units in the same way[7,9] as in the homogeneous case with $D=D_0$:

$$\mathcal{F}_{\rm SH}=\int dv\left[\tfrac{1}{2}-|\Delta|^2+\tfrac{1}{2}|\Delta|^4+h^2+D(\mathbf{r})\left[\frac{\nabla}{i\kappa_0}-\mathcal{A}\right]\Delta\,|^2\right], \tag{3'}$$

where κ_0 is the GL parameter of the homogeneous superconductor with $l=l_0$ and λ_0 is the unit of length. The magnetic field, the vector potential, and the current density are measured in units of $\sqrt{2}H_c$, $\sqrt{2}H_c\lambda_0$, and $\sqrt{2}H_c/4\pi\lambda_0$, respectively. The order parameter is scaled by

$$\Delta_\infty=\left[\frac{8(\pi T_c)^2(1-T/T_c)}{7\zeta(3)}\right]^{1/2}$$

and the diffusion coefficient by D_0. The spatial distribution of $\Delta(\mathbf{r})$ and $h(\mathbf{r})$ can be determined from the corresponding GL equations

$$\left[\left[\frac{\nabla}{i\kappa_0}-\mathcal{A}\right]D(\mathbf{r})\left[\frac{\nabla}{i\kappa_0}-\mathcal{A}\right]-1+|\Delta|^2\right]\Delta=0\ , \tag{4}$$

$$\mathbf{j}=\tfrac{1}{2}D(\mathbf{r})\left[\Delta^*\left[\frac{\nabla}{i\kappa_0}-\mathcal{A}\right]\Delta+\text{c.c.}\right], \tag{5}$$

together with relation $\mathbf{h}=\text{rot}\mathcal{A}$.

From these expressions one can deduce, as in the homogeneous case, two coupled equations dealing with $|\Delta|$ and $\mathbf{h}$. Putting

$$\Delta=fe^{i\chi},\quad \mathcal{A}'=\mathcal{A}-\nabla\chi/\kappa_0$$

and assuming that $\mathbf{h}=h\mathbf{e}_z$ is parallel to the applied field $\mathbf{H}$, directed along the z axis, one obtains

$$-\frac{1}{\kappa_0^2}(\nabla D\cdot\nabla)f+\frac{(\nabla h)^2}{Df^3}=f-f^3\ , \tag{6}$$

$$Df^2h=\nabla^2h-\frac{\nabla(Df^2)\cdot\nabla h}{Df^2}\ , \tag{7}$$

where $h=h(x,y)$. In the inhomogeneous case, $\mathbf{h}(\mathbf{r})$ will be parallel to $\mathbf{H}$ if the inhomogeneity distribution varies in the x-y plane only, i.e., when $D_1(\mathbf{r})=D_1(x,y)$. In the following, we restrict our attention to the uniaxial variation $D_1=D_1(y-y_0)$, where D_1 may be a periodic function, as it is the case in superconducting superlattices and in periodically modulated alloys, or may be centered at some $y=y_0$, decreasing to zero when $|y-y_0|\to\infty$. Even in the case of uniaxial modulation of D_1, the complete solution of Eqs. (6) and (7) appears to be a considerable task. However, close to the upper critical field H_{c2}, many properties of the mixed state may be obtained without constructing detailed solution of the GL equations, as was shown by Abrikosov[9,10] in the homogeneous case. We apply the same approach to the regularly inhomogeneous superconductors.

III. SOLUTION NEAR H_{c2}

In the presence of inhomogeneity, the structure of individual vortices and the global vortex configuration are changed.[11] The equilibrium structure, position, and orientation of the vortex lattice with respect to inhomogeneity are to be determined by minimizing the free energy density $F=\mathcal{F}/\int dV$. To evaluate F, we proceed by a generalization of the Abrikosov solution, based on the perturbative method. Close to H_{c2}, where $\Delta=0$, the order parameter increases but is still small and can be expanded in a series of orthonormalized functions, $\Delta=\Sigma c_n\phi_n$. A suitable choice of ϕ_n is the set of Eilenberger's functions,[7] where ϕ_0 is the solution of the linearized first GL equation for the homogeneous superconductor with $\kappa=\kappa_0$. The explicit form of this function, which represents a generalization of the Abrikosov solution at $H=H_{c2}$, is given in Sec. IV.

The microscopic field can be expressed[9] as

$$h=\kappa_0+h_2+h_4+\cdots\ , \tag{8}$$

where the corrections $h_2,h_4,\ldots$ are of the order of $f^2,f^4,\ldots$, respectively, since Eqs. (6) and (7) contain only even powers of $f=|\Delta|$. Inserting the expression for h in Eqs. (6) and (7) the conditions to all orders in f are obtained as

$$-\frac{1}{\kappa_0^2}(\nabla D\cdot\nabla)f+\frac{1}{Df^3}(\nabla h_2)^2-f=0\ , \tag{9}$$

$$\kappa_0Df^2-\nabla^2h_2+\frac{\nabla(Df^2)\cdot\nabla h_2}{Df^2}=0\ , \tag{10}$$

$$\frac{2\nabla h_2\cdot\nabla h_4}{Df^3}+f^3=0\ , \tag{11}$$

$$Df^2h_2-\nabla^2h_4+\frac{\nabla(Df^2)\cdot\nabla h_4}{Df^2}=0\ . \tag{12}$$

In analogy with the homogeneous case, where the contribution to the microscopic field due to the supercurrents is proportional to f^2, we take

$$h_2=-\frac{1}{2\kappa_0}Df^2+\delta+\delta_0\ , \tag{13}$$

where $\delta=\delta(\mathbf{r})$ and δ_0 is a constant. This yields two equations for δ and f, instead of (9) and (10), which in the general case can be solved only numerically. The situation is much simpler in the case of small perturbation D_1, which we consider in the following. To first order in D_1, the order parameter can be approximated[7] by $\Delta = C(\phi_0 + \epsilon\phi_1)$, where $\phi_1 = -(2/\kappa_0 B)^{1/2}\partial\phi_0/\partial y$, B is the average flux density, and C is a normalization constant. (For the determination of ϵ, see the Appendix). As for the field variation, we do not calculate it explicitly at the moment, but proceed with the evaluation of the free energy density F. With the help of Eqs. (3′) and (4), F can be written in the same compact form as in the homogeneous case

$$F = \tfrac{1}{2} - \tfrac{1}{2}\langle f^4\rangle + \langle h^2\rangle \ , \tag{14}$$

where the angular brackets denote the space average. Taking $h = \kappa_0 + h_2$, where h_2 is given by Eq. (13), and $B = \langle h\rangle$, we get

$$F = \tfrac{1}{2} + B^2 - \frac{|C|^4}{4\kappa_0^2}\beta_A(2\kappa_0^2 - 1 - \alpha' + 2\gamma') - (\kappa_0 - B + \delta_0)^2 \ , \tag{15}$$

where

$$|C|^2\alpha = 2\langle D_1 f^2\rangle \ ,$$

$$\beta_A |C|^4\alpha' = 2\langle D_1 f^4\rangle \ ,$$

and

$$\beta_A \left|\frac{C}{\kappa_0}\right|^4 \gamma' = 2\langle \delta f^2\rangle \ .$$

As in the homogeneous case, the factor $\beta_A = \langle f^4\rangle/(\langle f^2\rangle)^2$ depends on the lattice geometry, whereas the coefficients α, α', γ', and γ, of the order of D_1, characterize the influence of inhomogeneity. The coefficient γ, defined via $(|C|^2/2\kappa_0)\gamma = 2\langle\delta\rangle$, enters into the expression for the normalization constant C. The latter can be suitably obtained in terms of δ_0 from the higher-order equations (11) and (12)

$$\frac{|C|^2}{2\kappa_0} = \frac{\delta_0(1+\alpha/2)}{\beta_A(1+\alpha' - 2\kappa_0^2 - \gamma')} \ . \tag{16}$$

The condition $B = \langle h\rangle$, written explicitly, provides a second relation between $|C|^2$ and δ_0,

$$\frac{|C|^2}{2k_0} = \frac{\delta_0 + k_0 - B}{1 + (\alpha - \gamma)/2} \ . \tag{17}$$

From Eqs. (15)–(17) one obtains

$$F = \tfrac{1}{2} + B^2 - \frac{(\kappa_0 - B)^2(1+\alpha/2)^2}{\left[\left(1 + \frac{\alpha-\gamma}{2}\right)(1+\alpha/2) - \beta_A(1+\alpha' - 2\kappa_0^2 - \gamma')\right]}\left[1 - \frac{\frac{\gamma}{2}\left(1 + \frac{\alpha-\gamma}{2}\right) - \beta_A\gamma'}{\left[\left(1+\frac{\alpha-\gamma}{2}\right)(1+\alpha/2) - \beta_A(1+\alpha'-2\kappa_0^2-\gamma')\right]}\right]$$

$$\cong \tfrac{1}{2} + B^2 - \frac{(\kappa_0 - B)^2}{1+\beta_A(2\kappa_0^2 - 1)}\left[1 + \beta_A\frac{\alpha(2\kappa_0^2-1)+\alpha'}{1+\beta_A(2\kappa_0^2-1)}\right] \ , \tag{18}$$

where in the second line on the right-hand side the terms of the order of D_1^2 and higher order have been neglected.

Within the present approximation, the field correction δ does not appear in the free-energy variation, expressed via only two coefficients, $\alpha = 2\langle D_1|\phi_0|^2\rangle$ and $\alpha' = 2\langle D_1|\phi_0|^4\rangle/\beta_A$. Since the first of them is multiplied by the factor $(2\kappa_0^2 - 1)$, large when $\kappa_0 \gg 1$, a further simplification is possible in this case:

$$F \cong \tfrac{1}{2} + B^2 - \frac{(\kappa_0 - B)^2(1+\alpha)}{1+\beta_A(2\kappa_0^2 = 1)} \ . \tag{18′}$$

Accordingly, the minimum energy configuration is obtained when β_A is minimum, but at the same time α (>0) should attain its maximum value. Leaving the calculation of α (for chosen typical examples) for Sec. IV we conclude this section evaluating the inhomogeneity-induced change in the magnetization curve. The equilibrium relation between B and the applied field H, obtained from Eq. (20′), is given by

$$H = \frac{1}{2}\frac{\partial F}{\partial B} = B + \frac{(\kappa_0 - B)(1+\alpha)}{1+\beta_A(2\kappa_0^2-1)} \tag{19}$$

and, with the same accuracy, the magnetization curve

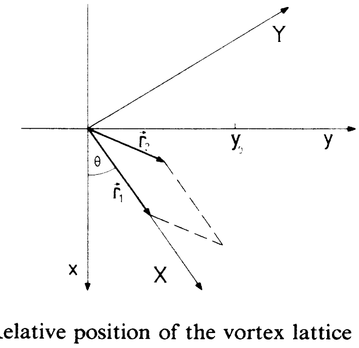

FIG. 1. Relative position of the vortex lattice (reference frame XY) and inhomogeneity, centered at y_0 in the xy reference frame.

shows the change in the slope near H_{c2},

$$H-B=\frac{(\kappa_0-H)(1+\alpha)}{\beta_A(2\kappa_0^2-1)} . \tag{20}$$

IV. PINNING AND CRITICAL CURRENT

The pinning energy density in a uniaxially modulated superconductor is a function of y_0, the center of inhomogeneity distribution in the reference frame of the vortex lattice (Fig. 1). By definition, the pinning energy density is the energy difference $\Delta F=F(y_0)-F_0$, where $F(y_0)$ is the energy density of the actual vortex configuration, optimally rotated with respect to y axis, and F_0 corresponds to the homogeneous superconductor with $\kappa=\kappa_0$ (i.e., $D=D_0$). To obtain $F(y_0)$ from Eq. (18′), where $\alpha=\alpha(y_0)$, we observe that the coefficients α and β_A depend on the vortex configuration via ϕ_0,

$$\phi_0(\mathbf{r})=(2\eta/d^2)^{1/4}\exp[-(\pi/\eta)Y^2]\sum_{p=-\infty}^{+\infty}\exp\left[-(2\pi/d)pY+\frac{2\pi}{d}pXi+\frac{\pi p^2}{d^2}(\zeta i-\eta)\right] .$$

This function describes, in the X-Y reference frame, a two-dimensional lattice with unit flux quantum per elementary cell, mapped on the basis vectors $\mathbf{r}_1=(d,0)$, $\mathbf{r}_2=(\zeta/d,\eta/d)$, where η is the unit-cell area (in reduced units) and $2\pi/\eta=\kappa_0 B$ (Fig. 1). To calculate $F(y_0)$ explicitly, we consider three types of variations of D_1, shown in Fig. 2:

(a) D_1 even: $D_1=ae^{-b|y-y_0|}$,

(b) D_1 odd: $D_1=ab(y-y_0)e^{-b^2(y-y_0)^2}$,

(c) D_1 harmonic: $D_1=a\cos[q_0(y-y_0)]$, $q_0=2\pi b$.

In cases (a) and (b) we assume that D_1 is defined within the interval of the length $L\gg b^{-1}$, much larger than the vortex spacing as well. In case (c), also considered in Ref. 7, the period b^{-1} is of the order of d, and the vortex lattice can be commensurate with the inhomogeneity distribution. In each case, we first calculate α and then minimize F in order to obtain the lattice structure (i.e., the equilibrium relations between d, ζ, and η), together with the optimum lattice orientation θ. To calculate α, we introduce the Fourier series representation

$$D_1(y-y_0)/a=\sum_q \tilde{D}(q)e^{iq(y-y_0)} ,$$

where

$$\tilde{D}(q)=\frac{1}{L}\int_{-L/2}^{L/2}dyD_1(y)e^{-iqy} .$$

In terms of the Fourier coefficients $\tilde{D}(q)$, α is expressed via

$$\begin{aligned}\alpha=2a\sum_{q_{n,m}}&(-1)^{nm}f(q_{n,m})\tilde{D}(q_{n,m})\exp[-(\eta q_{n,m}^2/8\pi)]\\&\times\delta(q_{n,m}d\sin\theta,2\pi m)\\&\times\delta\left[q_{n,m}\left[\frac{\eta}{d}\sin\theta+\frac{\zeta}{d}\sin\theta\right],2\pi n\right] ,\end{aligned} \tag{21}$$

where

$$f(q)=\begin{cases}\cos(qy_0) & \text{for } D_1 \text{ even ,}\\ -i\sin(qy_0) & \text{for } D_1 \text{ odd ,}\end{cases}$$

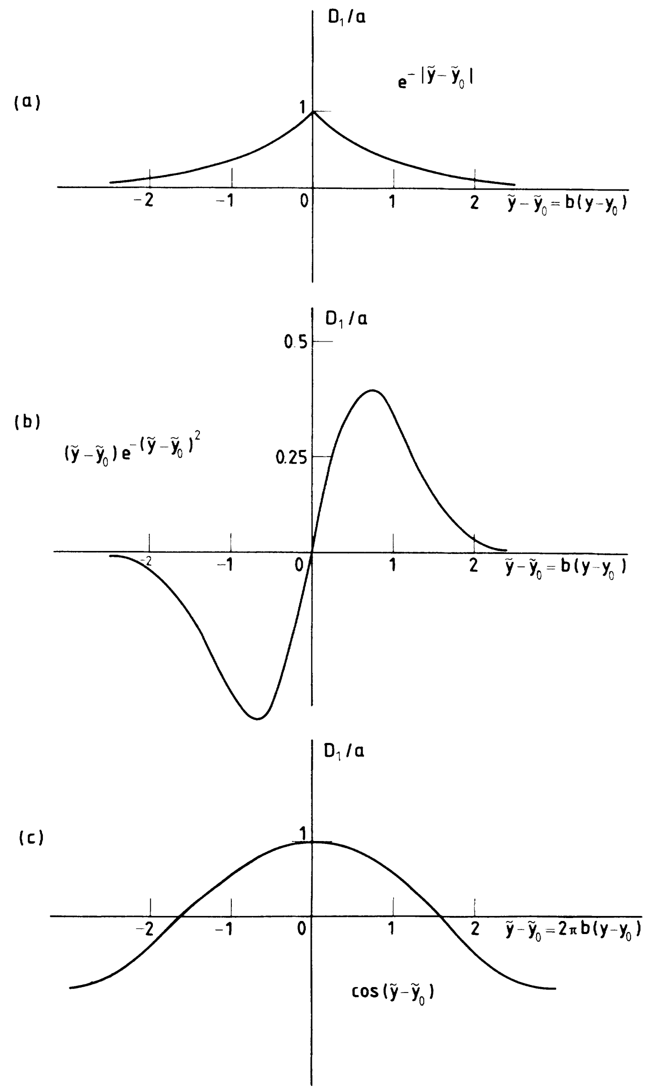

FIG. 2. Three types of spatial variation of the diffusion coefficient $D=D_0(1+D_1)$, where $D_1=af(y-y_0)$: (a) D_1/a even; (b) D_1/a odd; (c) D_1/a harmonic.

$\delta(a,b)$ is the Kronecker symbol and m and n are integers. In both cases (a) and (b), the lowest energy configuration corresponds to α maximum, while β_A can be minimized independently. This gives $\theta=0$, $m=0$, and

$$q_n=nq_1=n\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]^{1/2} . \quad (22)$$

The equilibrium lattice consists, as in the homogeneous case,[9] of equilateral triangles of side d [$\zeta/d=d/2$, $\eta/d=(d\sqrt{3})/2$, $\beta_A=\beta_A^0=1.16$]. In the first order of the inhomogeneity-induced perturbation, the ground-state lattice structure is unchanged, but its orientation is strongly determined by the uniaxial modulation. (Notice that beside the stable configuration with $\theta=0$, there is one metastable, with $\theta=\pi/2$.) The corresponding energy gain (pinning energy density) is

$$\Delta F=-\frac{(\kappa_0-B)^2}{1+\beta_A^0(2\kappa_0^2-1)}\alpha(y_0) , \quad (23)$$

where

$$\alpha=2a\sum_{n=-\infty}^{+\infty} f(q_n)\tilde{D}(q_n)\exp[-(\eta q_n^2/8\pi)] \quad (24)$$

with q_n given by Eq. (22). In particular, for case (a) we have

$$\tilde{D}(q)=\frac{2}{Lb}\frac{1}{1+q^2b^2} ,$$

and

$$\tilde{D}(q)=\frac{\sqrt{\pi}}{2i}\frac{q}{Lb^2}\exp[-(q^2/4b^2)]$$

for case (b). In case (c), $\tilde{D}(q)=\frac{1}{2}[\delta(q_0-q)+\delta(q_0+q)]$, and α is different from zero only when the vortex lattice is commensurate with periodic modulation of D, i.e., when

$$q_0 d\sin\theta=2\pi m , \quad (25)$$

$$\frac{q_0}{d}(\eta\cos\theta+\zeta\sin\theta)=2\pi n , \quad (26)$$

where m and n are integers. In this case, the lattice consists[7] of isosceles triangles of vortices, with $\zeta/d=d/2$, while β_A varies with B, slightly deviating from β_A^0. For the commensurate (C) lattice we get[12]

$$\alpha=2a\exp[-(q_0^2/4\kappa_0 B)]\cos(q_0 y_0) \quad (27)$$

and

$$\Delta F=-\frac{(\kappa_0-B)^2(1+\alpha)}{1+\beta_A(2\kappa_0^2-1)}+\frac{(\kappa_0-B)^2}{1+\beta_A^0(2\kappa_0^2-1)} . \quad (28)$$

In the domain of B where the C lattice is stable, $\Delta F\le 0$, the energy gain is linear in modulation amplitude a. When $\Delta F>0$, the unpinned lattice of equilateral triangles is more favorable. The present linear theory does not give the pinning effects of order of a^2 or higher, nor the transition to weakly incommensurate (I) configurations, with large C domains separated by solitonic walls.[13] However, the range of stability of weakly pinned I structures is very narrow.[14] With increasing density of solitons, they become equivalent to harmonically distorted triangular lattices with position-independent energy, and unpinned in this sense.[13,14]

In the above expressions y_0 is still undetermined. Its equilibrium value y_0^e in the ground state is obtained, in all cases, from the requirement that $\alpha>0$ is maximum. When y_0 differs from y_0^e, there arises a pinning force of the density

$$\mathbf{F}_p=\frac{\partial\Delta F}{\partial y_0}\mathbf{e}_y \quad (29)$$

pushing the vortex lattice back to the equilibrium position.

When an external transport current of density $\mathbf{j}$ is supplied along the x direction into the specimen, its Lorentz driving force $\mathbf{F}_L=2\mathbf{j}\times\mathbf{B}$ produces a supplementary shift along the y axis. The new position of the vortex lattice results from the balance between $\mathbf{F}_p$ and $\mathbf{F}_L$. The critical current density, i.e., the maximum nondissipative current density corresponds to the maximum of the pinning force density $|\mathbf{F}_c|=|\mathbf{F}_p|^{\max}$:

$$2\mathbf{j}_c\times\mathbf{B}+\mathbf{F}_c=0 . \quad (30)$$

At the same time, the translated vortex lattice has still to be pinned: the energy gain $|\Delta F(y_0)|$ decreases with translation from its maximum value $|\Delta F(y_0^e)|$ and eventually vanishes at some critical distance y_0^c. Both y_0^e and y_0^c depend on the sign of the perturbation amplitude a, as shown below for case (a). The physical reason for this is evident, since $a>0$ corresponds to the increase of the electron mean free path l and to the decrease of κ, while for $a<0$ the situation is opposite. Keeping only the leading terms in the rapidly converging series for α and $F_p\propto\partial\alpha/\partial y_0$ [Eqs. (24) and (29)], we get

$$\alpha=2a\left\{\tilde{D}(0)+2\tilde{D}\left[\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]^{1/2}\right]\exp(-\pi/\sqrt{3})\times\cos\left[\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]^{1/2} y_0\right]\right\} \quad (31)$$

and

$$F_p=-\frac{(\kappa_0-B)^2\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]^{1/2}}{1+\beta_A^0(2\kappa_0^2-1)}\times 4a\tilde{D}\left[\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]^{1/2}\right]\exp(-\pi/\sqrt{3})\times\sin\left[\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]^{1/2} y_0\right] . \quad (32)$$

According to Eqs. (23) and (31), the energy gain in case (a) is largest when $a\cos q_1 y_0=|a|$. Thus for $a>0$ $y_0^e=0$, i.e., the lattice is centered at the inhomogeneity, with a row of vortices along the line $D=D^{\max}$. The pinning force is maximum, and $\Delta F=0$ when the distance y_0 reaches the critical value

$$y_0^c = -\frac{\pi}{2}\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]^{-1/2} = -\frac{h}{4}\,,$$

where $h = d\sqrt{3}/2$ is the vortex triangle height. For $a < 0$, $y_0^e = h/2$, the vortices are symmetrically placed to avoid the minimum of D and the critical distance is $y_0^c = h/4$. Actually, the above choice of equilibrium position y_0^e gives the largest value of the upper critical field H_{c2}. The inhomogeneity induced change $\delta H_{c2}/\kappa_0 = H_{c2}/\kappa_0 - 1$ can be approximated by the expression (see the Appendix)

$$\delta H_{c2}/\kappa_0 \cong 2a\left[\frac{2\pi}{\sqrt{3}} - 1\right] e^{-\pi/\sqrt{3}}\,\tilde{D}\left[\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]^{1/2}\right] \times \cos\left[\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]^{1/2} y_0\right] - a\tilde{D}(0)\,, \qquad (33)$$

which shows that the smallest decrease of H_{c2} (for $a > 0$) or the largest increase (for $a < 0$) are obtained with

$$a\cos\left[\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]^{1/2} y_0\right] = |a|\,.$$

Whereas in the ground state, where $F_p = 0$ and $\Delta = c\phi_0$, the lattice structure is essentially unchanged, this is not so for translated lattice. When $y_0 \neq y_0^e$, $\Delta = c(\phi_0 + \epsilon\phi_1)$, where ϵ [given by Eq. (A4) in the Appendix] increases with translation reaching maximum, together with F_p, at $y_0 = y_0^c$. Thus, pinning is clearly related to the distorsion of Δ and corresponding change of h. In the critical state, the current density $j_c = F_c/2B$ is obtained from

$$F_c = \frac{(k_0 - B)^2\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]^{1/2}}{1 + \beta_A^0(2\kappa_0^2 - 1)}\;\frac{8|a|\exp(-\pi/\sqrt{3})}{Lb\left[1 + \frac{4\pi}{\sqrt{3}}\frac{\kappa_0 B}{b^2}\right]} \qquad (34)$$

for case (a), and from

$$F_c = \frac{(\kappa_0 - B)^2}{1 + \beta_A(2\kappa_0^2 - 1)} 2|a|q_0 \exp(-q_0^2/2\kappa_0 B) \qquad (35)$$

for case (c). Similarly, for case (b) we find

$$F_c = \frac{(\kappa_0 - B)^2\left[\frac{4\pi}{\sqrt{3}}\kappa_0 B\right]}{1 + \beta_A^0(2\kappa_0^2 - 1)}\;\frac{2|a|\sqrt{\pi}}{Lb^2} \times \exp\left[-\frac{\pi}{\sqrt{3}}\left[\frac{\kappa_0 B}{b^2} + 1\right]\right]. \qquad (36)$$

V. DISCUSSION

In the above formulas, the first factor gives the familiar B dependence,[2,3] $F_c \propto (1 - B/B_{c2})^n (B/B_{c2})^m$, where $n = 2$ and m is between 0 and 1, while the second factor reflects the form of inhomogeneity. The temperature dependence becomes explicit when F_c is expressed in physical units. This is done below for the case (a) with $a < 0$ which we will discuss in more detail, since it could be relevant to the pinning by grain boundaries. As shown first by Zerweck,[15] the electron scattering at the grain boundary is an important source of flux pinning, causing reduction of the electron mean free path with respect to its bulk value. Its spatial variation, which is not strictly local but spreads over a long distance from the boundary, can be approximated by

$$l/l_0 = D/D_0 = 1 - |a|e^{-b|y - y_0|}$$

in a region of size L. The resulting critical force density (in physical units) is given by

$$F_c = \frac{|a|}{\pi L}\,\frac{(B_{c2})^2(1 - B/B_{c2})^2}{1 + \beta_A^0 - 1)}\left[\frac{\left[\Gamma\frac{B}{B_{c2}}\right]^{1/2}}{1 + \Gamma\frac{B}{B_{c2}}}\right] e^{-\pi/\sqrt{3}}\,, \qquad (34')$$

where

$$\Gamma = \frac{4\pi}{\sqrt{3}}\left[\frac{b^{-1}}{\xi_0(T)}\right]^2 \sim B_{c2}(T)\,.$$

The vortex lattice orientation $\theta = 0$ is implicit in the above result. The grain-boundary imposed orientation, also predicted by Thuneberg[16] in a quasiclassical theory of pinning due to the electron scattering, is observed by small-angle neutron scattering in Nb bicrystals.[17] The linear dependence of F_c on the inverse grain size $1/L$, explicit in Eq. (34'), has been often observed experimentally.[18,19] Another nice feature of Eq. (34') is that the temperature dependence (at given B) is completely determined by $B_{c2}(T)$ in the first factor; the term in parenthesis is temperature independent.[20] At given B/B_{c2}, F_c is only approximately proportional to $H_c^2 H_{c2}^{1/2}$; this is also observed in Pb/Bi alloys,[19] while in $A15$ superconductors[21] one finds that $F_c \propto (H_{c2})^2$. The temperature dependence of Γ could explain why the reduced pinning force F_c, relative to its value at a given reduced field B_0/B_{c2}, does not appear to be a universal function of B/B_{c2} when temperature is changed.[19]

The B dependence of F_c or j_c is usually discussed in terms of the so-called "Kramer plots" of $J_c^{1/2}B^{1/4}$ versus B.[3,21] In many cases one finds that while the form of Kramer's scaling relation (which implies a linear variation) is obeyed, the detailed predictions are not. In $A15$ superconductors, for example, a small amount of impurities may cause a downward curvature.[21] These deviations, for which various reasons may be invoked, appear quite naturally in our theory, reflecting the inhomogeneity shape. For the profile assumed in case (a) one gets

$$j_c^{1/2}B^{1/4} \propto \frac{(1 - B/B_{c2})}{\left[1 + \Gamma\frac{B}{B_{c2}}\right]^{1/2}}\,.$$

Another theory of grain-boundary flux pinning based on the electron scattering mechanism was developed[22] by Yetter, Thomas, and Kramer (YTK) and applied to pinning in Pb/Bi alloys by Yetter and Kramer.[19] Assuming a rigid vortex lattice, without perturbation of Δ or h, the authors calculated the change of the Ginzburg-Landau energy as due only to the modification of κ.[22] In the high field limit, the result of YTK is similar to ours: the main difference is linear, versus quadratic in our case, dependence of F_c on $(1-B/B_{c2})$. The comparison with experiment[19] shows that the specific pinning force (per unit area of grain boundary) $f=LF_c$ is, in the case of YTK, comparable to the experimental value f_{expt}, of the order of 10 N/m^2. However, this agreement may be fortuitous, since in calculation one assumes a small variation of κ (or l), even though in reality it may be quite large.[19] In this sense, our result is more acceptable: for the above system,[19] Eq. (34′) gives (with $\Gamma \sim 1$) $f/|a| \sim f_{\text{expt}}$, i.e., the experimental results would correspond to the pinning strength $|a| \sim 1$.

Another example of pinning corresponding to our case (a) can be found in lamellar eutectics: due to the variation of concentration of the one-phase atoms within the other phase layers,[23] D_1 has presumably a Lorentzian shape. In the case of periodic inhomogeneity distribution, case (c), large peaks on the $j_c(B)$ curve, characteristic of the commensurability effect, have been observed experimentally in superconducting alloys with periodically modulated composition[24] and in some superconducting super lattices.[25] In the latter case, however, the origin of pinning may be quite different from the present one. The present pinning mechanism could be relevant, beside the modulated alloys, to the "hard" superconductors with a cellular structure of dislocations.[26] The density of dislocations in the "walls" of cells being higher than inside the cells, l and D vary periodically. However, a commensurate lattice can be formed only if the period of the cellular structure is comparable to the vortex spacing d. For large periods, the pinning at cell walls[26,27] could be described as above. As an example, we apply case (a) with $L \sim 1$ μm and $\Gamma \sim 0.1$ to the surface pinning by cell walls in the Pb/Tl superconductor.[27] Neglecting the B dependence of the factor $1/1+\Gamma(B/B_{c2})$ in Eq. (34′) and taking the bulk characteristics from Ref. 27, we get, in units (10^4 A/cm^2),

$$j_c/|a| = \frac{F_c B}{|a|} \sim 10(j_c)_{\text{expt}}$$
$$= 10 \left[4.51(1-B/B_{c2})^2 \left[\frac{B}{B_{c2}} \right] \right]^{-1/2} .$$

This would imply a pinning strength an order of magnitude smaller than for the grain-boundary pinning,[19] since we reproduce the experimental result[27] with $|a| \sim 0.1$.

To conclude, we have developed a theory of pinning by uniaxial variation of the mean free path in dirty superconductors near H_{c2}. The method could be easily applied to the case of two-dimensional variation; however, we have chosen to work with the one-dimensional case, since it takes into account the anisotropy present in many experimental situations.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor J. R. Clem for valuable discussions. This research is supported in part by Grant No. JFP-520 under the National Science Foundation's US-Yugoslavia Research Program.

APPENDIX: EVALUATION OF THE UPPER CRITICAL FIELD H_{c2}

Similar to the homogeneous case, H_{c2} is determined from the linearized first GL equation [here Eq. (4)], which can be put into the form

$$(\Pi_0 D \Pi_0)\Delta = \Delta \tag{A1}$$

where $\Pi_0 = \nabla/i\kappa_0 - \mathcal{A}_0$, $\mathcal{A}_0 = -By\mathbf{e}_x$, and $\Delta = c\phi$. To obtain H_{c2}, one can solve (1.1) variationally,[7] looking for the lowest eigenvalue E_0 of

$$(\Pi_0 D \Pi_0)\phi = E_0 \phi \ . \tag{A2}$$

By definition, the condition $E_0 = 1$ determines H_{c2}. Expanding ϕ in terms of Eilenberger's functions,[7] $\phi = \phi_0 + \epsilon\phi_1$, where ϕ_0 is given ϕ_0 by Eq. (23) and

$$\phi_1 = -\left[\frac{2}{\kappa_0 B} \right]^{1/2} \frac{\partial \phi_0}{\partial y} ,$$

we find

$$E_0 = \frac{\langle D\,|\Pi_0 \phi|^2 \rangle}{\langle |\phi|^2 \rangle} = \frac{B}{k_0}(1 + \langle D_1 |\phi_1|^2 \rangle) \tag{A3}$$

and

$$\epsilon = \frac{1}{4} \left[\frac{2}{\kappa_0 B} \right]^{1/2} \frac{\partial}{\partial y_0} (\langle D_1 |\phi_1|^2 \rangle + \alpha/2) , \tag{A4}$$

where α is given by Eq. (24). The variation of the upper critical field, calculated explicitly, is given by

$$\frac{\delta H_{c2}}{k_0} = -\langle D_1 |\phi_1|^2 \rangle$$
$$= a \sum_{n=-\infty}^{\infty} (q_n^2 \eta/4\pi - 1)\exp[-(q_n^2 \eta/8\pi)]$$
$$\times \tilde{D}(q_n) f(q_n) , \tag{A5}$$

where we have put $\theta = 0$ and $q_n = nq_1$. Keeping only the leading terms in Eq. (A5), we obtain, with $q_1 = [(4\pi/\sqrt{3})\kappa_0 B]^{1/2}$ and $f(q) = \cos q y_0$, the result for case (a) given by Eq. (33) in the text.

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