Analysis of giant quantum attenuation of sound waves due to spin-split Landau levels in bismuth

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In experiments of the giant quantum attenuation in bismuth, many unexplainable phenomena have been reported so far. We have made numerical calculations including the density-of-states dependence of the deformation potential in the theory of Gurevich, Skobov, and Firsov. The result shows that the deformation potential plays the most substantial role in peculiar behaviors of line shapes of the attenuation coefficient. We have obtained a satisfactory result which quantitatively agrees with the experimentally observed line shapes of attenuation peaks.

Figure 1 shows the experimental results of the giant quantum attenuation obtained by Fujimori,[1] in which the propagation directions of longitudinal sound waves and the magnetic field are along the binary axes of a bismuth single crystal. In this figure we show the configuration of the magnetic field and sound waves in the experiment for the trigonal plane of the Brillouin zone of bismuth. Here we also define the coordinate axes and the *a*-, *b*-, and *c*-electron pockets in the usual way. In these configurations only the attenuation peaks due to the electrons are observed. In the case (a) where both the sound waves and the magnetic field are applied along the same binary axis, an imbalance in the heights of the peaks due to a pair of spin-split Landau levels can be observed. On the other hand, in the case (b) where one of them is applied along a different binary axis from the other, the imbalance seen in (a) is not found. Later, the same result was obtained by Matsumoto *et al.*[2] for $Bi_{1-x}Sb_x$. They considered that this phenomenon is due to "complicated unknown effects." In Figs. 1(a) and 1(b), however, the directions of the magnetic field are quite the same and only the configurations of sound-wave propagation relative to the magnetic field are different. Therefore we can easily guess that this phenomenon is related to a deformation potential.

Giant quantum attenuation was theoretically predicted by Gurevich *et al.*[3] Applying their theory to the case of bismuth we get the following equation:

$$\alpha(H)=\sum_{j=a,b,c,h}\alpha_{0j}\frac{q}{\mathbf{q}\cdot\hat{\mathbf{e}}_H}\frac{\hbar\Omega_j}{8k_BT}\sum_{n,s}F^j_{n,s}\int_{-\infty}^{\infty}\frac{dy}{\pi}\frac{B_j}{1+B_j^2y^2}\cosh^{-2}\left[\frac{1}{2}\left[y^2-\frac{A^j_{n,s}}{k_BT}\right]\right], \quad (1)$$

where

$$\alpha_{0j}=\frac{(\det\tilde{m}^{\,j})^{1/2}(\hat{\mathbf{e}}_H\cdot\tilde{m}^{\,j}\cdot\hat{\mathbf{e}}_H)^{1/2}(\hat{\mathbf{e}}_p\cdot\tilde{D}^{\,j}\cdot\hat{\mathbf{e}}_q)^2\omega}{2\pi\rho v^2\hbar^3}, \quad (2)$$

$\tilde{m}^{\,j}$ is the mass tensor of the jth pocket, $\tilde{D}^{\,j}$ is the deformation potential tensor, $\mathbf{q}$ is the wave vector of the sound waves, $\hat{\mathbf{e}}_H$, $\hat{\mathbf{e}}_p$, $\hat{\mathbf{e}}_q$ are the unit vectors of the magnetic field direction, the polarization of the sound waves, and the sound propagation direction, respectively, v is the sound velocity, ρ is the mass density, ω is the angular frequency of the sound waves, Ω_j is the cyclotron frequency, and

$$F^j_{n,s}=\begin{cases}[1+(4n+2+2s\gamma_j)\hbar\Omega_j/E_g]^{1/2} & \text{for } j=a,b,c,\\ 1 & \text{for } j=h,\end{cases} \quad (3)$$

$$A^j_{n,s}=\begin{cases}[E_F(1+E_F/E_g)-E^j_{n,s}]/(1+2E_F/E_g) & \text{for } j=a,b,c,\\ E_0-E_F-E^j_{n,s} & \text{for } j=h,\end{cases} \quad (4)$$

$$E^j_{n,s}=(n+\tfrac{1}{2}+\tfrac{1}{2}s\gamma_j)\hbar\Omega_j, \quad (5)$$

$$B_j=(\mathbf{q}\cdot\hat{\mathbf{e}}_H)\tau[2k_BT/(\hat{\mathbf{e}}_H\cdot\tilde{m}^{\,j}\cdot\hat{\mathbf{e}}_H)]^{1/2}, \quad (6)$$

where the Fermi energy E_F is determined as a function of magnetic field by the charge neutrality, γ_j is the spin-splitting factor, τ is the relaxation time of the carriers, E_g is the energy gap between the valence band and the conduction band of the electron pocket, and E_0 is the overlapping energy between the electron band and the hole band. We use the Lax model[4] for the electrons. Here the deformation potential is considered to be constant. Goto *et al.*,[5] however, showed that when $\omega\tau \ll 1$ the deformation potential is given from the charge neutrality condition as

$$\tilde{D}^{\,j}=\frac{\sum_{k=a,b,c,h}(\tilde{C}^{\,j}-\tilde{C}^{\,k})n_k}{\sum_{k=a,b,c,h}n_k}, \quad (7)$$

where n_k ($k=a,b,c,h$) is the density of the number of particles on the Fermi level given as

$$n_k=\frac{\partial}{\partial E_F}\int_{-\infty}^{\infty}\rho_k(E)\frac{dE}{1+\exp[(E-E_F)/k_BT]}, \quad (8)$$

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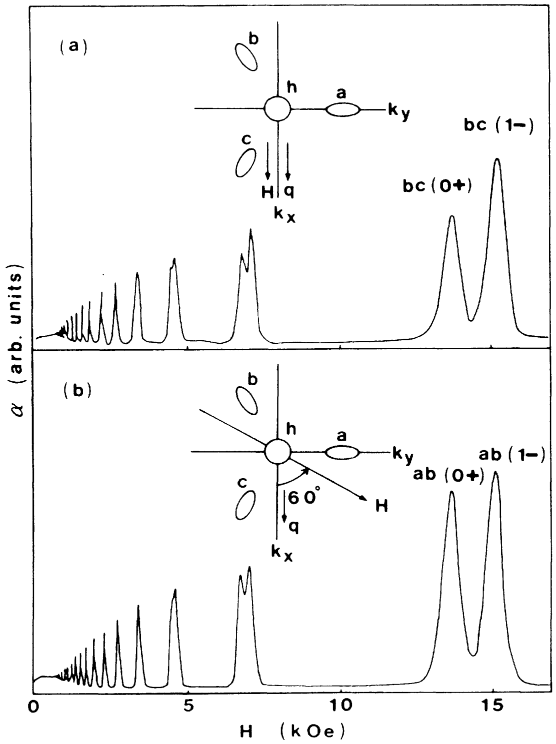

FIG. 1. Experimental results obtained by Fujimori at $T=1.2$ K and $\nu=100$ MHz. (a) $\mathbf{q}\|\mathbf{H}\|\hat{\mathbf{x}}$, (b) $\mathbf{q}\|\hat{\mathbf{x}}$ but $\mathbf{H}$ parallel to another binary axis.

$$\rho_k(E)=\mathrm{Re}\left[\rho_{0k}\left[E+i\frac{\hbar}{\tau}\right]\right], \tag{9}$$

where $\rho_{0k}(E)$ is the density of states without level broadening. When the temperature goes to absolute zero, n_k are equal to the density of states at the Fermi energy, $\rho_k\,(E_F)$.

On the other hand, $\tilde{C}^{\,j}$ is a bare deformation-potential tensor. The deformation-potential tensors for the a electrons and the holes are expressed as follows:

$$\tilde{C}^{\,a}=\begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & \lambda_4\\ 0 & \lambda_4 & \lambda_3\end{bmatrix}, \tag{10}$$

$$\tilde{C}^{\,h}=\begin{bmatrix}\Lambda_1 & 0 & 0\\ 0 & \Lambda_1 & 0\\ 0 & 0 & \Lambda_3\end{bmatrix}. \tag{11}$$

The other $\tilde{C}^{\,b}$ and $\tilde{C}^{\,c}$ are obtained by rotating $\tilde{C}^{\,a}$ by $\pm 2\pi/3$ around the z axis.

Equation (7) shows that the deformation potential itself reveals quantum oscillations of the density-of-states type. Then a kind of correlation among the attenuation peaks will appear since the attenuation peaks due to one pocket will be amplitude modulated not only by its density of states but also by the density of states of all other pockets.

Figure 2 shows the result of the numerical calculations of the attenuation coefficient in the same configuration of the magnetic field and the sound waves as in Fig. 1 using the expression of deformation potentials in Eq. (1). The band parameters used in the calculation are those of Smith *et al.*,[6] the values of the bare deformation-potential tensor components for the $\tilde{C}^{\,a}$ and $\tilde{C}^{\,h}$ given by Katsuki[7] are used and the relaxation time is chosen to be 5.0×10^{-10} sec. The result shown in Fig. 2 reproduces the experimental result of Fig. 1. Here we consider the essential conditions for these phenomena. In Fig. 1 the longitudinal waves are applied along the x axis, therefore, the (xx)th components of the deformation-potential tensors contribute to the attenuation. Considering the crystal symmetry of bismuth we get the relation $C^b_{xx}=C^c_{xx}=(\lambda_1+3\lambda_2)/4$. In the case of Fig. 1(a) or Fig. 2(a), the attenuation peaks are due to the b and c electrons. With this magnetic field configuration and magnitude the density of states of the a electrons and that of the holes are considered nearly constant as shown in Fig. 2(c) and the deformation potentials of the b and c electrons are approximately given as $D^b_{xx}=D^c_{xx}\propto 1/(n_b+\mathrm{const})$. Due to the asymmetric character of the density of states, n_b in the denominator at the magnetic field of the $(0+)$ peak is larger than at the magnetic field of the $(1-)$ peak as shown in Fig. 2(c). Therefore the deformation potential at the magnetic field of the $(1-)$ peak is larger than at the

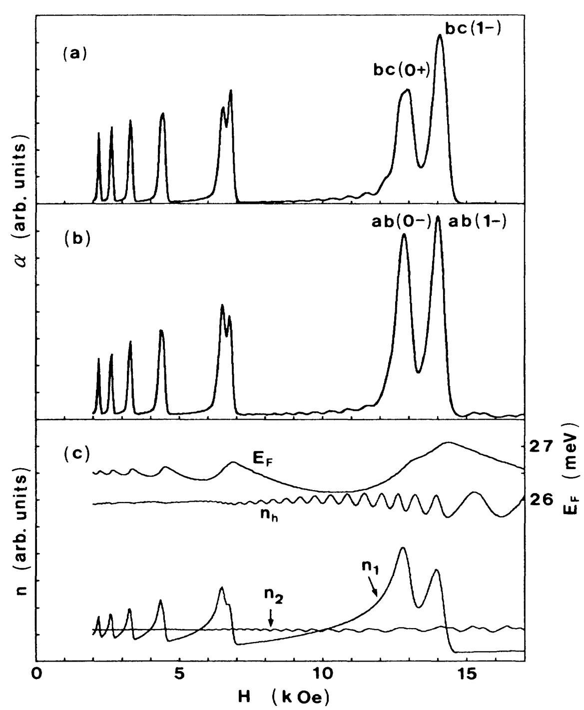

FIG. 2. Calculated results at $T=1.2$ K, $\nu=100$ MHz, and $\tau=5.0\times 10^{-10}$ sec. (a) $\mathbf{q}\|\mathbf{H}\|\hat{\mathbf{x}}$, (b) $\mathbf{q}\|\hat{\mathbf{x}}$ but $\mathbf{H}$ parallel to another binary axis, (c) the magnetic field dependencies of the Fermi energy and the density of the number of particles. n_1 corresponds to n_b and n_c in (a) and to n_a and n_b in (b). n_2 corresponds to n_a in (a) and to n_c in (b).

magnetic field of the $(0+)$ peak. On the other hand, in the case of (b), the attenuation peaks due to the a and b electrons appear. This time the density of states of the c electrons and that of the holes are nearly constant. In the numerators and denominators of the expressions of D_{xx}^a and D_{xx}^b, n_b and n_a appear, respectively. In this case n_b and n_a are equal, therefore, the effect of the densities of states on the deformation potentials is lessened and the imbalance in the heights of the attenuation peaks disappears.

Formerly, Matsumoto *et al.*[8] analyzed the attenuation peaks due to the electrons. They reported the following contradiction: The tail of the lower-field side of the attenuation peak becomes too large if the full width at half maximum of the attenuation peak is fitted to the experiment, while if the relaxation time is chosen long so that the line shape of the lower-field side of the peak might accord with the experiment, the full width at half maximum becomes too small. To solve this paradox, Nagai *et al.*[9] gave a new theory which takes account of the vertex correction due to the impurity effect, analyzed the result of Fujimori, and claimed that this problem was solved. But the paradox of the relaxation time occurred because the magnetic field dependence of the Fermi energy was not taken into account. Therefore, if we correctly take into account the magnetic field dependence of the Fermi energy and that of the deformation potentials the experimental result can be quantitatively explained with the theory of Gurevich *et al.* Figure 3 shows the comparison of the results of the numerical calculation with $\mathbf{q}\|\mathbf{H}\|x$ axis. In Fig. 3(a) the magnetic field dependence of the Fermi energy and that of the deformation potentials are correctly taken into account, in 3(b) E_F is taken as constant, in 3(c) E_F and deformation potentials are taken as constant. Both the changes of the Fermi energy and the deformation potentials with the magnetic field affect the full width at half maximum of peaks. If we neglect these changes the full width at half maximum will become too much narrower than the experimental results. Applying the magnetic field dependence of the Fermi energy and the density of states dependence of the deformation potentials into the equation by Gurevich *et al.*, we could completely reproduce the experimental results, such as the full width at half maximum, the line shape at the lower side of the peaks and even the heights of a pair of spin-split peaks which could not be explained so far.

Previously, Goto *et al.* presented Eq. (7) for the deformation potential of bismuth and qualitatively explained the correlation between the attenuation peaks due to the electrons and those due to the holes discovered experimentally by Fujimori.[1] At that time, however, there was argument about the relaxation time of bismuth and it was pointed out that the condition $\omega\tau \ll 1$ under which Eq. (7) is derived might be broken.[10] Since the value which we used in the calculation to reproduce the experimental results is 5.0×10^{-10} sec, the condition mentioned above is fully satisfied.

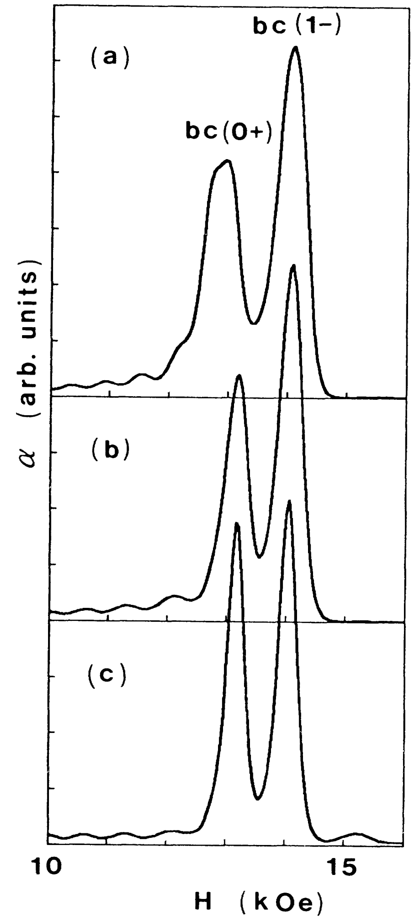

FIG. 3. Comparison of the calculated results at $T=1.2$ K, $\tau=5.0\times10^{-10}$ sec. (a) Correctly calculated, (b) calculated with the Fermi energy taken as constant, (c) calculated with the Fermi energy and the deformation potentials taken as constant.

Gurevich *et al.* predicted that the quantum oscillations of the attenuation coefficient of the sound waves become spikelike because of the conservation of the momentum and energy, and the oscillations are quite different from those of the de Haas type seen in the electric resistance and the specific heat. But the deformation potential also exhibits de Haas–type quantum oscillations. These two types of quantum oscillations make the line shape of the attenuation coefficient of the sound waves even more complicated than the quantum oscillations such as in the magnetoresistance.

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