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Dynamical conductivity in the infrared from impurity scattering in a polar semiconductor

Bo E. Sernelius*
Solid State Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831
and Department of Physics, University of Tennessee, Knoxville, Tennessee 37996
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We derive an expression for the contribution to the dynamical conductivity from impurity scattering in a polar semiconductor. The derivation is based on the Kubo formalism and diagrammatic perturbation theory. Several different derivations of the dynamical conductivity and free-carrier absorption can be found in the literature. In the few cases where the impurity contribution has been considered, in the presence of coupling to optical phonons, the results which have been published disagree with each other. The purpose of this work is to resolve this problem and to find the cause of these discrepancies.

I. INTRODUCTION

A completely homogeneous one-component plasma, like a homogeneous electron gas, cannot absorb light. The reason is not only that the energy and momentum cannot be conserved in a process where an electron-hole pair is created and a photon is absorbed. Were this the only reason, processes involving multipair excitations could still take place. The reason is more fundamental than that. The perturbing potential, $(Ze/mc)\sum_i(\mathbf{p}_i\cdot\mathbf{A}+\mathbf{A}\cdot\mathbf{p}_i)$, that couples the light to the plasma and causes the absorption of light, commutes with the Hamiltonian because $\mathbf{P}=\sum_i\mathbf{p}_i$, the total momentum of the plasma, is a constant of motion. For absorption to take place, the Hamiltonian must contain terms not commuting with $\mathbf{P}$, such as the presence of impurities or coupling to phonons. Absorption can also occur in a multicomponent plasma where not all components have equal charge-to-mass ratios Z/m, because in that case the perturbing potential is no longer proportional to $\mathbf{P}$.

In a heavily doped semiconductor both couplings to phonons and impurities are present. In a many-valley semiconductor like Si or Ge, the conduction-band valleys are anisotropic and (in the n-doped case) the electrons can be regarded as a multicomponent plasma with different Z/m ratios. Thus, in these systems, absorption of light would occur even in the absence of phonons and impurities. In the present work we concentrate on the contribution from impurity scattering, which gives the dominating contribution at heavy doping. In particular, we focus on the effect the polar coupling has on this contribution.

The details of this absorption, apart from being of theoretical interest, are of great technological importance. In one field of semiconductor applications one uses heavy doping as a way to tailor the optical properties of semiconductor films. The reason one can do that is the following. A large-band-gap semiconductor is transparent for photon energies in the visible region and below. Heavy doping has two main effects. One is that the optical band gap is changed. The change depends on two competing effects. Many-body effects tend to reduce the gap. This reduction is counteracted by a band-gap widening due to blocking of the lowest states in the conduction band (for n-type doping). This widening is known as the Burstein-Moss effect. In the extreme high doping limit, the Burstein-Moss shift wins over the band-gap narrowing from many-body effects and the gap increases. This moves the upper boundary for the optical window upwards. The second effect from the doping is to introduce the free-carrier absorption (proportional to the dynamical conductivity) at the low-energy side, moving the lower boundary of the window upwards. Thus these heavily doped semiconductor films have interesting optical properties. They have the properties of "dirty" metals at the low-energy side and those of semiconductors on the high-energy side. They can, e.g., be used as coatings on highly energy-efficient windows, giving good solar transmittance and low thermal emittance.[1-3] The large-band-gap semiconductors are usually polar, and hence it is important to ensure that the effect from the polar coupling is correctly calculated.

The expression for the dynamical conductivity due to impurity scattering in a plasma in the absence of phonon interactions has been derived in several different ways. Ron and Tzoar[4] have derived it with a kinetic approach. An alternative, and in a way more physically transparent method is the so-called energy-loss method, which is described in detail in a very recent review article by Gerlach.[5] Ron and Tzoar[6] performed a derivation of the conductivity in quantum and classical multicomponent plasmas, based on the Kubo formula. They obtained a result for impurity scattering by letting the mass of one of the components go to infinity. Later Sirko and Mills[7] performed a similar derivation but obtained the ion contribution by treating the ions as frozen phonons. The results for the impurity contribution to the dynamical conductivity from all these different derivations agree, but the form of the result may seem somewhat confusing. The impurity potentials are assumed to be static, but in the final results they appear to be dynamically screened. When generalizing the result to include phonon couplings the question arises as to whether one should add the static or the dynamical phonon polarizability to the electronic one.

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This question bothers the experimentalists when they try to interpret their experimental data.[8] That there are discrepancies between the various published theoretical results does not make it easier. The answer to the question is that both the static and dynamical polarizabilities enter in a nontrivial way.

To our knowledge there have been four attempts to generalize the result to take polar coupling into account. Goettig,[9] using a kinetic approach, finds an expression that also seems to have been obtained by Gerlach and Mycielski,[10] using the energy-loss method. A different expression was given by Katayama and Mills[11] and a third expression was obtained very recently by Kleinert and Giehler.[12]

In the present work we derive the result in three different ways and, in each case, we obtain the same expression as was found in Refs. 9 and 10.

The basis of the derivation here is the dynamical conductivity for a multicomponent plasma in the presence of coupling to optical phonons. We give a brief derivation of this in Sec. II. This result is generalized to include impurity scattering in the following sections.

The impurity contribution can be derived in three ways. The first and most direct way is to add an electron-ion and phonon-ion term to the Hamiltonian from the start. We perform such a derivation in Sec. III. In the other two approaches one takes shortcuts from the result in the absence of impurities. In the second approach one calculates the conductivity for a one-component plasma in the presence of phonons, and generalizes the result to include impurities by treating them as frozen phonons. This approach will be taken in Sec. IV. In the third and last approach one uses the result for a two-component plasma and takes the limit when the mass of one of the components (which simulates the impurities) goes to infinity. This last approach will be described in Sec. V. Finally, in Sec. VI we make a summary and draw conclusions.

II. DYNAMICAL CONDUCTIVITY IN A MULTICOMPONENT PLASMA

In this section we will make a brief derivation of the dynamical conductivity of a multicomponent plasma coupled to optical phonons (the derivation is also similar in the case of coupling to acoustical phonons). We assume that the system is embedded in a dielectric medium and that the wave numbers and frequencies entering the results are small enough so that the background screening can be treated as a constant, ϵ_∞. The system under consideration could be an electron-hole plasma in a polar semiconductor, but what we really intend to do is to use the result as a basis for the following sections. In the next section we will use the result reduced to a one-component plasma and extend it to the case when impurities are present. In Sec. IV we will rederive the same result but using a shortcut, and will treat the impurities as frozen phonons. The multicomponent character of the plasma is needed first in Sec. V, where we obtain the impurity contribution by letting the mass of one of the components go to infinity.

The Hamiltonian for the system consists of three parts. One part, H_e, contains only particle operators; one part, $H_{\rm ph}$, only phonon operators; and the last part, $H_{e\text{-ph}}$, contains both operator types and describes the particle-phonon interaction.

Thus the Hamiltonian can be written as

$$H = H_e + H_{\rm ph} + H_{e\text{-ph}} , \tag{2.1}$$

where

$$H_e = \sum_{i,\mathbf{k},\sigma} \frac{\hbar^2 k^2}{2m_i} \hat{n}_{i,\mathbf{k},\sigma} + \frac{1}{2V} \sum_{\mathbf{q}} \frac{v(q)}{\epsilon_\infty} \sum_{i,j} Z_i Z_j (\rho^\dagger_{i,\mathbf{q}} \rho_{j,\mathbf{q}} - \rho_{i,0}\delta_{i,j}) , \tag{2.2}$$

$$H_{\rm ph} = \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}} C^\dagger_{\mathbf{q}} C_{\mathbf{q}} , \tag{2.3}$$

and

$$H_{e\text{-ph}} = -\frac{1}{\sqrt{V}} \sum_{\mathbf{q}} g(q)(C_{\mathbf{q}} + C^\dagger_{-\mathbf{q}}) \sum_i Z_i \rho_{i,\mathbf{q}} . \tag{2.4}$$

The indices i and j run over the components in the plasma, $\mathbf{q}$ and $\mathbf{k}$ over wave vectors, and σ over particle spin. The operators $C^\dagger$ and C are phonon creation and destruction operators, respectively. The particle density operators are defined as

$$\rho_{i,\mathbf{q}} = \sum_{\mathbf{k},\sigma} a^\dagger_{i,\mathbf{k},\sigma} a_{i,\mathbf{k}+\mathbf{q},\sigma} , \tag{2.5}$$

$$\rho^\dagger_{i,\mathbf{q}} = \sum_{\mathbf{k},\sigma} a^\dagger_{i,\mathbf{k}+\mathbf{q},\sigma} a_{i,\mathbf{k},\sigma} , \tag{2.6}$$

and the number operator as

$$\hat{n}_{i,\mathbf{k},\sigma} = a^\dagger_{i,\mathbf{k},\sigma} a_{i,\mathbf{k},\sigma} . \tag{2.7}$$

The operators $a^\dagger$ and a are creation and destruction operators for the particles, respectively. The function $v(q)$ is the Fourier transform of the Coulomb potential, V denotes the volume of the system, and $g(q)$ is the electron-phonon coupling constant:

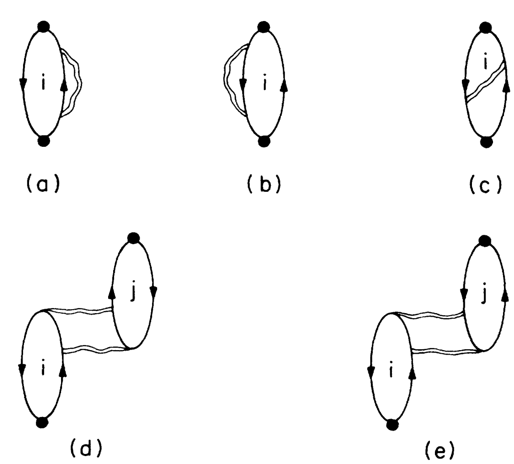

FIG. 1. The five infinite subsets of diagrams included in the derivation of the dynamical conductivity of a multicomponent plasma coupled to optical phonons. The wavy lines are the particle-particle interactions screened by the plasma and phonons.

$$g(q)=\left[\frac{v(q)}{2}\frac{\epsilon_0-\epsilon_\infty}{\epsilon_\infty\epsilon_0}\hbar\omega_L\right]^{1/2} . \quad (2.8)$$

The dynamical conductivity is given by the relation

$$\sigma(\omega)=\frac{ie^2}{\omega}\sum_j\frac{n_jZ_j^2}{m_j}+i\frac{\Pi(\omega)}{\hbar\omega} , \quad (2.9)$$

where $\Pi(\omega)$ is the retarded form of

$$\Pi(i\Omega_m)=-\frac{1}{V}\int_0^\beta d\tau\, e^{i\Omega_m\tau}\langle T_\tau j_\mu(\tau)j_\mu(0)\rangle . \quad (2.10)$$

We sum the subset of diagrams shown in Fig. 1, also used by Ron and Tzoar in Ref. 6. The index i or j inside the bubbles denotes the particle components. The particle-photon vertices indicated by the dots take on the values $eZ_i\hbar k_\mu/m_i$, where i denotes the particle type involved. k_μ is the projection of the wave vector, for the Green's functions surrounding the vertex, on the polarization vector of the light. We assume that the system has cubic symmetry so that Π can be represented by a scalar. Each of the five diagrams represents infinite subsets of diagrams, as the wavy lines represent $Z_iZ_j\Gamma(\mathbf{q},iw_m)$, where

$$Z_iZ_j\Gamma(\mathbf{q},i\omega_m)=Z_iZ_j\left[\frac{v(q)}{\epsilon_\infty}+g^2(q)D(\mathbf{q},i\omega_m)\right]\Bigg/\left[1-\sum_l\left[\frac{v(q)}{\epsilon_\infty}+g^2(q)D(\mathbf{q},i\omega_m)\right]Z_l^2\chi_l(\mathbf{q},i\omega_n)\right]$$
$$=Z_iZ_j\tilde{v}(\mathbf{q},i\omega_m)\Big/\left[1-\tilde{v}(\mathbf{q},i\omega_m)\sum_l Z_l^2\chi_l(\mathbf{q},i\omega_m)\right] , \quad (2.11)$$

where $D(\mathbf{q},i\omega_m)$ is the phonon Green's function

$$D(\mathbf{q},i\omega_m)=-\frac{2\omega_L}{\hbar(\omega_m^2+\omega_L^2)} . \quad (2.12)$$

The longitudinal optical phonon energy $\hbar\omega_L$ is assumed to be $\mathbf{q}$ independent in the q range of interest here. This makes the phonon Green's function $\mathbf{q}$ independent and we drop the first argument from here on. The particle-phonon interaction gives rise to an extra effective particle-particle interaction, and the net result for the particle-particle interaction can be expressed as

$$Z_iZ_j\tilde{v}(q,i\omega_m)=Z_iZ_jv(q)/\epsilon_L(i\omega_m) , \quad (2.13)$$

where the factors Z_i and Z_j can be regarded as vertex corrections and appear because we allow the particles to have charges different from e. The lattice dielectric function ϵ_L is given by

$$\epsilon_L(i\omega_m)=\epsilon_\infty\epsilon_0(\omega_L^2+\omega_m^2)/(\epsilon_\infty\omega_L^2+\epsilon_0\omega_m^2) , \quad (2.14)$$

and represents the combined screening from the background and from the optical phonons. For frequencies much higher than the optical phonon frequencies the phonons give no contribution to the screening, and ϵ_L tends to the background screening constant ϵ_∞. In the zero-frequency limit, ϵ_L tends to ϵ_0.

For convenience we also introduce three more functions. The function Γ_0 is analogous to Γ in the absence of phonon coupling, i.e.,

$$\Gamma_0(\mathbf{q},i\omega_m)=v(q)\Big/\left[\epsilon_\infty-v(q)\sum_l Z_l^2\chi_l(\mathbf{q},i\omega_m)\right] . \quad (2.15)$$

The functions Λ and Λ_0 are defined as Γ and Γ_0, respectively, divided by their numerators, i.e.,

$$\Lambda(\mathbf{q},i\omega_m)=1\Big/\left[1-\tilde{v}(q,i\omega_m)\sum_l Z_l^2\chi_l(\mathbf{q},i\omega_m)\right] \quad (2.16)$$

and

$$\Lambda_0(\mathbf{q},i\omega_m)=1\Big/\left[1-\frac{v(q)}{\epsilon_\infty}\sum_l Z_l^2\chi_l(\mathbf{q},i\omega_m)\right] . \quad (2.17)$$

The function χ_l, appearing in Eqs. (2.11) and (2.15)–(2.17) is the polarizability for component l, and can be expressed as

$$\chi_l(\mathbf{q},i\omega_m)=\frac{2}{\hbar^2\beta V}\sum_{\mathbf{k},i\omega_n}G_l(\mathbf{k},i\omega_n)G_l(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m)$$
$$=\frac{2}{\hbar V}\sum_{\mathbf{k}}\frac{n_F^l(\mathbf{k})-n_F^l(\mathbf{k}+\mathbf{q})}{i\omega_m+\frac{\hbar}{2m_l}[k^2-(\mathbf{k}+\mathbf{q})^2]} , \quad (2.18)$$

where the functions G and n_F are particle Green's functions and Fermi occupation numbers, respectively.

We let the indices m and n on a frequency indicate that the frequency is an even and odd multiple, respectively, of $2\pi/\hbar\beta$, where β is $1/k_BT$.

Straightforward interpretation of the Feynman diagrams in Fig. 1, and the use of the fact that

$$G_i(\mathbf{k},i\omega_n)G_i(\mathbf{k},i\omega_n+i\Omega_m)$$
$$=\frac{1}{i\Omega_m}[G_i(\mathbf{k},i\omega_n)-G_i(\mathbf{k},i\omega_n+i\Omega_m)] , \quad (2.19)$$

gives the following results for the first three diagrams:

$$\Pi^{(a)}(i\Omega_m)=-\frac{2e^2\hbar^2}{(\hbar^2\beta Vi\Omega_m)^2}\sum_{\substack{\mathbf{k},\mathbf{q},i,\\ i\omega_n,i\omega_m}}\frac{Z_i^4}{m_i^2}k_\mu^2\Gamma(\mathbf{q},i\omega_m)G_i(\mathbf{k},i\omega_n)[G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m+i\Omega_m)-G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m)$$
$$-(i\Omega_m)G_i(\mathbf{k},i\omega_n)G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m)] , \quad (2.20)$$

$$\Pi^{(b)}(i\Omega_m)=-\frac{2e^2\hbar^2}{(\hbar^2\beta V i\Omega_m)^2}\sum_{\substack{\mathbf{k},\mathbf{q},i,\\ i\omega_n,i\omega_m}}\frac{Z_i^4}{m_i^2}k_\mu^2\Gamma(\mathbf{q},i\omega_m)G_i(\mathbf{k},i\omega_n)[G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m-i\Omega_m)-G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m)$$
$$+(i\Omega_m)G_i(\mathbf{k},i\omega_n)G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m)] , \tag{2.21}$$

and

$$\Pi^{(c)}(i\Omega_m)=-\frac{2e^2\hbar^2}{(\hbar^2\beta V i\Omega_m)^2}\sum_{\substack{\mathbf{k},\mathbf{q},i,\\ i\omega_n,i\omega_m}}\frac{Z_i^4}{m_i^2}(k_\mu^2+k_\mu q_\mu)\Gamma(\mathbf{q},i\omega_m)G_i(\mathbf{k},i\omega_n)\ [2G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m)-G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m+i\Omega_m)$$
$$-G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m-i\Omega_m)] . \tag{2.22}$$

These three contributions can be summed to give

$$\Pi^{(a)+(b)+(c)}_{(i\Omega_m)}=\frac{e^2\hbar^2}{2\hbar^2\beta V(i\Omega_m)^2}\sum_{i,j}\sum_{\mathbf{q},i\omega_m}\frac{Z_i^4Z_j^2}{m_i^2}q_\mu^2\Gamma(\mathbf{q},i\omega_m)\Gamma(\mathbf{q},i\omega_m-i\Omega_m)$$
$$\times[\chi_i(\mathbf{q},i\omega_m)-\chi_i(\mathbf{q},i\omega_m-i\Omega_m)][\chi_j(\mathbf{q},i\omega_m)-\chi_j(\mathbf{q},i\omega_m-i\Omega_m)]$$
$$+\frac{e^2\hbar^2}{2\hbar^2\beta V(i\Omega_m)^2}\sum_{\substack{\mathbf{q},i,\\ i\omega_m}}\frac{Z_i^4}{m_i^2}q_\mu^2 g^2(q)\Lambda(\mathbf{q},i\omega_m)\Lambda(\mathbf{q},i\omega_m-i\Omega_m)$$
$$\times[D(i\omega_m)-D(i\omega_m-i\Omega_m)][\chi_i(\mathbf{q},i\omega_m)-\chi_i(\mathbf{q},i\omega_m-i\Omega_m)] . \tag{2.23}$$

Some clarifications might be needed here. After direct addition of the three terms, we used the first equality in Eq. (2.18) and summed over $i\omega_n$. Then we kept half of that result, and then added the otherhalf, but this time with the dummy variable changes $\mathbf{k}\to-\mathbf{k}-\mathbf{q}$ and $i\omega_m\to-i\omega_m+i\Omega_m$. This eliminated the factor k_μ in favor of a factor q_μ, which allowed the summation over $\mathbf{k}$ to be performed according to the second equality of Eq. (2.18).

The two remaining diagrams are obtained as

$$\Pi^{(d)}=-\frac{4e^2\hbar^2}{(\hbar^2\beta V)^3(i\Omega_m)^2}\sum_{i,j}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}\sum_{\substack{i\omega_n,i\omega_m,\\ i\omega_{n'}}}\frac{Z_i}{m_i}\frac{Z_j}{m_j}Z_i^2Z_j^2k_\mu(k'_\mu+q_\mu)\Gamma(\mathbf{q},i\omega_m)$$
$$\times\Gamma(\mathbf{q},i\omega_m-i\Omega_m)G_i(\mathbf{k},i\omega_n)[G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m)-G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m-i\Omega_m)]$$
$$\times G_j(\mathbf{k}'+\mathbf{q},i\omega_{n'}+i\omega_m)[G_j(\mathbf{k}',i\omega_{n'}+i\Omega_m)-G_j(\mathbf{k}',i\omega_{n'})] \tag{2.24}$$

and

$$\Pi^{(e)}_{(i\Omega_m)}=-\frac{4e^2\hbar^2}{(\hbar^2\beta V)^3(i\Omega_m)^2}\sum_{i,j}\sum_{\mathbf{k}',\mathbf{k},\mathbf{q}}\sum_{\substack{i\omega_n,i\omega_{n'},\\ i\omega_m}}\frac{Z_i}{m_i}\frac{Z_j}{m_j}Z_i^2Z_j^2k_\mu k'_\mu\Gamma(\mathbf{q},i\omega_m)\Gamma(\mathbf{q},i\omega_m-i\Omega_m)G_i(\mathbf{k},i\omega_n)$$
$$\times[G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m)-G_i(\mathbf{k}+\mathbf{q},i\omega_n+i\omega_m-i\Omega_m)]G_j(\mathbf{k}'+\mathbf{q},i\omega_{n'}+i\omega_m)$$
$$\times[G_j(\mathbf{k}',i\omega_{n'})-G_j(\mathbf{k}',i\omega_{n'}+i\Omega_m)] , \tag{2.25}$$

and their sum gives

$$\Pi^{(d)+(e)}_{(i\Omega_m)}=-\frac{e^2\hbar^2}{2\hbar^2\beta V(i\Omega_m)^2}\sum_{i,j}\sum_{\mathbf{q},i\omega_m}Z_i^2Z_j^2\frac{Z_i}{m_i}\frac{Z_j}{m_j}q_\mu^2\Gamma(\mathbf{q},i\omega_m)\Gamma(\mathbf{q},i\omega_m-i\Omega_m)$$
$$\times[(\chi_i(\mathbf{q},i\omega_m)-\chi_i(\mathbf{q},i\omega_m-i\Omega_m)][(\chi_j(\mathbf{q},i\omega_m)-\chi_j(\mathbf{q},i\omega_m-i\Omega_m)] . \tag{2.26}$$

In Eqs. (2.24) and (2.25) the k_μ and k'_μ factors prevent the summations over $\mathbf{k}$ and $\mathbf{k}'$ and an elimination of the Green's functions in favor of the χ's. When the two contributions are added, the k'_μ factor drops out and the summation over $\mathbf{k}'$ and $i\omega_{n'}$ can be carried out. Then one can continue in exactly the same way as we did to obtain Eq. (2.23).

Adding the results of Eqs. (2.23) and (2.26) we obtain the following total contributions from the diagrams in Fig. 1:

$$\Pi(i\Omega_m)=\frac{e^2\hbar^2}{4\hbar^2\beta V(i\Omega_m)^2}\left[\sum_{\substack{i,j,\\ \mathbf{q},i\omega_m}} Z_i^2 Z_j^2\left[\frac{Z_i}{m_i}-\frac{Z_j}{m_j}\right]^2\Gamma(\mathbf{q},i\omega_m)\Gamma(\mathbf{q},i\omega_m-i\Omega_m)\right.$$
$$\times[\chi_i(\mathbf{q},i\omega_m)-\chi_i(\mathbf{q},i\omega_m-i\Omega_m)][\chi_j(\mathbf{q},i\omega_m)-\chi_j(\mathbf{q},i\omega_m-i\Omega_m)]$$
$$+2\sum_{\substack{i,\\ \mathbf{q},i\omega_m}} Z_i^2\left[\frac{Z_i}{m_i}\right]^2\Lambda(\mathbf{q},i\omega_m)\Lambda(\mathbf{q},i\omega_m-i\Omega_m)[\chi_i(\mathbf{q},i\omega_m)-\chi_i(\mathbf{q},i\omega_m-i\Omega_m)]$$
$$\left.\times[D(i\omega_m)-D(i\omega_m-i\Omega_m)]\right] . \tag{2.27}$$

In the absence of phonon coupling the second term vanishes and the first term gives the contribution from pure particle-particle scattering. As it should, this contribution vanishes in a one-component plasma.

We need the retarded form of Π. It is obtained by letting $i\Omega_m$ turn into $\Omega+i\delta$; but before one can take that limit one has to perform the summation over complex frequencies. This is done in the standard way, as described, e.g., in chapter 3.5 in Ref. 13, and one ends up with

$$\frac{1}{\hbar\beta}\sum_{i\omega_m} f(i\omega_m)\rightarrow \int_{\mathrm{Im}z=\Omega_m+\delta}\frac{dz}{2\pi i}f(z)n_B(z)-\int_{\mathrm{Im}z=\Omega_m-\delta}\frac{dz}{2\pi i}f(z)n_B(z)$$
$$+\int_{\mathrm{Im}z=\delta}\frac{dz}{2\pi i}f(z)n_B(z)-\int_{\mathrm{Im}z=-\delta}\frac{dz}{2\pi i}f(z)n_B(z) , \tag{2.28}$$

where f is the total summand in Eq. (2.27) and n_B is the function

$$n_B(z)=\frac{1}{e^{\beta z}-1} . \tag{2.29}$$

The integration paths are just above and just below the two branch cuts in the complex-frequency plane. The final result is, if we limit ourselves to the real part of the dynamical conductivity,

$$\mathrm{Re}\sigma(\Omega)=-\frac{\mathrm{Im}\Pi(\Omega)}{\hbar\Omega}$$
$$=-\frac{e^2\hbar}{2\Omega^3}\sum_{i,j}Z_i^2Z_j^2\left[\frac{Z_i}{m_i}-\frac{Z_j}{m_j}\right]^2\int_0^\infty\frac{d\omega}{2\pi}\frac{\sinh\left[\frac{\hbar\beta\Omega}{2}\right]}{\sinh\left[\frac{\hbar\beta}{2}\left[\omega+\frac{\Omega}{2}\right]\right]\sinh\left[\frac{\hbar\beta}{2}\left[\omega-\frac{\Omega}{2}\right]\right]}\int\frac{d\mathbf{q}}{(2\pi)^3}q_\mu^2$$
$$\times\left\{\mathrm{Im}\left[\Gamma\left[\mathbf{q},\omega+\frac{\Omega}{2}\right]\right]\mathrm{Im}\left[\Gamma\left[\mathbf{q},\omega-\frac{\Omega}{2}\right]\chi_i\left[\mathbf{q},\omega-\frac{\Omega}{2}\right]\chi_j\left[\mathbf{q},\omega-\frac{\Omega}{2}\right]\right]\right.$$
$$+\mathrm{Im}\left[\Gamma\left[\mathbf{q},\omega-\frac{\Omega}{2}\right]\right]\mathrm{Im}\left[\Gamma\left[\mathbf{q},\omega+\frac{\Omega}{2}\right]\chi_i\left[\mathbf{q},\omega+\frac{\Omega}{2}\right]\chi_j\left[\mathbf{q},\omega+\frac{\Omega}{2}\right]\right]$$
$$-\mathrm{Im}\left[\Gamma\left[\mathbf{q},\omega+\frac{\Omega}{2}\right]\chi_i\left[\mathbf{q},\omega+\frac{\Omega}{2}\right]\right]\mathrm{Im}\left[\Gamma\left[\mathbf{q},\omega-\frac{\Omega}{2}\right]\chi_j\left[\mathbf{q},\omega-\frac{\Omega}{2}\right]\right]$$
$$\left.-\mathrm{Im}\left[\Gamma\left[\mathbf{q},\omega+\frac{\Omega}{2}\right]\chi_j\left[\mathbf{q},\omega+\frac{\Omega}{2}\right]\right]\mathrm{Im}\left[\Gamma\left[\mathbf{q},\omega-\frac{\Omega}{2}\right]\chi_i\left[\mathbf{q},\omega-\frac{\Omega}{2}\right]\right]\right\}$$
$$-\frac{e^2\hbar}{\Omega^3}\sum_i Z_i^2\left[\frac{Z_i}{m_i}\right]^2\int_0^\infty\frac{d\omega}{2\pi}\frac{\sinh\left[\frac{\hbar\beta\Omega}{2}\right]}{\sinh\left[\frac{\hbar\beta}{2}\left[\omega+\frac{\Omega}{2}\right]\right]\sinh\left[\frac{\hbar\beta}{2}\left[\omega-\frac{\Omega}{2}\right]\right]}$$
$$\times\int\frac{d\mathbf{q}}{(2\pi)^3}q_\mu^2 g^2(q)$$

$$\times\left\{\mathrm{Im}\left[\Lambda\left(\mathbf{q},\omega+\frac{\Omega}{2}\right)\right]\mathrm{Im}\left[\Lambda\left(\mathbf{q},\omega-\frac{\Omega}{2}\right)\chi_i\left(\mathbf{q},\omega-\frac{\Omega}{2}\right)D\left(\omega-\frac{\Omega}{2}\right)\right]\right.$$
$$+\mathrm{Im}\left[\Lambda\left(\mathbf{q},\omega-\frac{\Omega}{2}\right)\right]\mathrm{Im}\left[\Lambda\left(\mathbf{q},\omega+\frac{\Omega}{2}\right)\chi_i\left(\mathbf{q},\omega+\frac{\Omega}{2}\right)D\left(\omega+\frac{\Omega}{2}\right)\right]$$
$$-\mathrm{Im}\left[\Lambda\left(\mathbf{q},\omega+\frac{\Omega}{2}\right)\chi_i\left(\mathbf{q},\omega+\frac{\Omega}{2}\right)\right]\mathrm{Im}\left[\Lambda\left(\mathbf{q},\omega-\frac{\Omega}{2}\right)D\left(\omega-\frac{\Omega}{2}\right)\right]$$
$$\left.-\mathrm{Im}\left[\Lambda\left(\mathbf{q},\omega+\frac{\Omega}{2}\right)D\left(\omega+\frac{\Omega}{2}\right)\right]\mathrm{Im}\left[\Lambda\left(\mathbf{q},\omega-\frac{\Omega}{2}\right)\chi_i\left(\mathbf{q},\omega-\frac{\Omega}{2}\right)\right]\right\}, \tag{2.30}$$

where all functions are in retarded form.

III. DYNAMICAL CONDUCTIVITY IN A ONE-COMPONENT PLASMA IN THE PRESENCE OF IMPURITIES

In this section we derive the contribution to the dynamical conductivity of a polar semiconductor from impurity scattering. To obtain this contribution we add three terms to the Hamiltonian in Sec. II, viz., $H_{\text{imp-imp}}$, $H_{e\text{-imp}}$, and $H_{\text{imp-ph}}$ describing impurity-impurity, electron-impurity, and impurity-phonon interaction, respectively. These terms can be expressed as

$$H_{\text{imp-imp}}=\frac{1}{2V}\sum_{\mathbf{q}}\frac{v(q)}{\epsilon_\infty}(\rho^\dagger_{\text{imp},q}\rho_{\text{imp},\mathbf{q}}-\rho_{\text{imp},0}) , \tag{3.1}$$

$$H_{e\text{-imp}}=-\frac{1}{V}\sum_{\mathbf{q}}\frac{v(q)}{\epsilon_\infty}(\rho^\dagger_{\mathbf{q}}\rho_{\text{imp},\mathbf{q}}) , \tag{3.2}$$

$$H_{\text{imp-ph}}=-\frac{1}{\sqrt{V}}\sum_{\mathbf{q}}g(q)(C_{\mathbf{q}}+C^\dagger_{-\mathbf{q}})\rho_{\text{imp},\mathbf{q}} , \tag{3.3}$$

where we have restricted ourselves to a one-component plasma with particle charge $-e$ (e being a positive number), and assumed that each impurity has the charge $+e$. For simplicity we have assumed that the particle-impurity potential can be approximated by a Coulomb potential. We have introduced $\rho_{\text{imp},\mathbf{q}}$ to make the analogy between the impurities and particles as transparent as possible. It corresponds to the particle density operator in Eq. (2.5) but for the impurities is not a q-number but a c-number. This is because the impurities are assumed to be rigidly connected to the crystal, with no degrees of freedom. The entity $\rho_{\text{imp},\mathbf{q}}$ is the Fourier transform of the impurity density, i.e.,

$$\rho_{\text{imp},\mathbf{q}}=\sum_{j=1}^{N}e^{-i\mathbf{q}\cdot\mathbf{R}_j} , \tag{3.4}$$

where $\mathbf{R}_j$ is the position vector of impurity number j. The index j runs over the N impurities and $\rho_{\text{imp},0}$ equals N. It can sometimes be advantageous to treat all impurities as one big impurity with charge Ne and with the electron-impurity potential given by

$$U(\mathbf{q})=-\sum_{j=1}^{N}e^{i\mathbf{q}\cdot\mathbf{R}_j}v(q) . \tag{3.5}$$

The fact that the Hamiltonian does not contain a kinetic energy term for the impurities makes it possible to take care of the impurity-phonon interaction by the following unitary transformation:

$$U=\exp\left[\sum_{\mathbf{q}}f(\mathbf{q})(C_{\mathbf{q}}-C^\dagger_{-\mathbf{q}})\right],\quad f(\mathbf{q})=\frac{g(q)}{\hbar\omega_{\mathbf{q}}}\rho^\dagger_{\text{imp},\mathbf{q}} . \tag{3.6}$$

This transformation modifies only terms containing phonon operators. The phonon operators are transformed in the following way:

$$\begin{aligned}UC_{\mathbf{q}}U^\dagger&=C_{\mathbf{q}}+f(\mathbf{q}) ,\\ UC^\dagger_{\mathbf{q}}U^\dagger&=C^\dagger_{\mathbf{q}}+f^\dagger(\mathbf{q}) ,\end{aligned} \tag{3.7}$$

and the transformed Hamiltonian takes on the form

$$\begin{aligned}H=&\sum_{\mathbf{k},\sigma}\frac{\hbar^2k^2}{2m}\hat{n}_{\mathbf{n},\sigma}+\frac{1}{2V}\sum_{\mathbf{q}}\frac{v(q)}{\epsilon_\infty}(\rho^\dagger_{\mathbf{q}}\rho_{\mathbf{q}}-\rho_0)\\&+\frac{1}{\sqrt{V}}\sum_{\mathbf{q}}g(q)(C^\dagger_{\mathbf{q}}+C_{-\mathbf{q}})\rho_{\mathbf{q}}-\frac{1}{V}\sum_{\mathbf{q}}\frac{v(q)}{\epsilon_0}\rho^\dagger_{\mathbf{q}}\rho_{\text{imp},\mathbf{q}}\\&+\frac{1}{2V}\sum_{\mathbf{q}}\frac{v(q)}{\epsilon_0}(\rho^\dagger_{\text{imp},q}\rho_{\text{imp},\mathbf{q}}-\rho_{\text{imp},0})+\sum_{\mathbf{q}}\hbar\omega_{\mathbf{q}}C^\dagger_{\mathbf{q}}C_{\mathbf{q}}\\&+\frac{1}{2V}\sum_{\mathbf{q}}v(q)\left[\frac{1}{\epsilon_0}-\frac{1}{\epsilon_\infty}\right]\rho_{\text{imp},0} .\end{aligned} \tag{3.8}$$

The transformation has had the following effects on the Hamiltonian: The impurity-phonon interaction has been eliminated, and all impurity potentials are now screened by ϵ_0 instead of by ϵ_∞ as they were before the transformation. Apart from these changes, a new term, the last in Eq. (3.8), has appeared. It gives the change in the infinite self-energy of the ions from the interaction with the phonons. It has no effect on the conductivity. In obtaining Eq. (3.8) we have made use of the identity

$$\frac{v(q)}{\epsilon_\infty}-\frac{2g(q)^2}{\hbar\omega_L}=\frac{v(q)}{\epsilon_0} . \tag{3.9}$$

Using the transformed Hamiltonian and keeping contributions to second order in the impurity potential we obtain the diagrams in Fig. 2. Here the crosses denote $\rho^{\dagger}_{\text{imp},\mathbf{q}}$ and the double dashed lines connecting the crosses to the bubbles represent $v(q)V^{-1/2}\Lambda(\mathbf{q},0)/\epsilon_0$, i.e., an impurity potential screened by the total static dielectric function. If the momentum $\mathbf{q}$ is leaking out through one of these "dangling bonds," then $-\mathbf{q}$ is leaking out through the other one. No frequency is leaking out. This means that we can connect them to each other and replace the crosses by one cross, which represents $\rho^{\dagger}_{\text{imp},\mathbf{q}}\rho_{\text{imp},\mathbf{q}}$, and momentum and frequency are conserved around all closed loops of the diagrams. Now we see that the result for the impurity contribution can be obtained from the results of the previous section through replacing one of the functions $\Gamma(\mathbf{q},i\omega_m)$ by $\hbar\beta V^{-1}|U(\mathbf{q})/\epsilon(q,0)|^2\delta_{m,0}$ or $\hbar\beta V^{-1}|U(\mathbf{q})\Lambda(\mathbf{q},0)/\epsilon_0|^2\delta_{m,0}$ in each diagram. It can also be obtained by replacing either an electron-interaction line $v(q)/\epsilon_\infty$ or a phonon line on any one place in each of the infinite number of diagrams generated by the diagrams in Fig. 1 by $\hbar\beta V^{-1}|U(\mathbf{q})/\epsilon_0|^2\delta_{m,0}$. The $\hbar\beta$'s appear because one of the summations $(\hbar\beta)^{-1}\sum_{i\omega_m}$ is missing in our new diagrams. This last approach was essentially taken in Ref. 12, but to start with they included diagrams with more than one of these replacements. During the following derivation they dropped all diagrams with more than one replacement, by neglecting all replacements in the denominator of what correspond to our Γ's. In doing so they lost contributions. Their result corresponds to the omission of one of our Λ's above or, put in another way, the result one would get by performing the replacement not on any one position but on just one. As an example, assume that there are five possible replacements in a diagram. Then this diagram will generate five diagrams where each of these replacements has been performed one at a time. These five diagrams are all different but their numerical values are the same. In the procedure of Ref. 12 only one of these diagrams has been kept. Furthermore, the impurity-phonon interaction was neglected, and they have not started out with impurity potentials screened by ϵ_0.

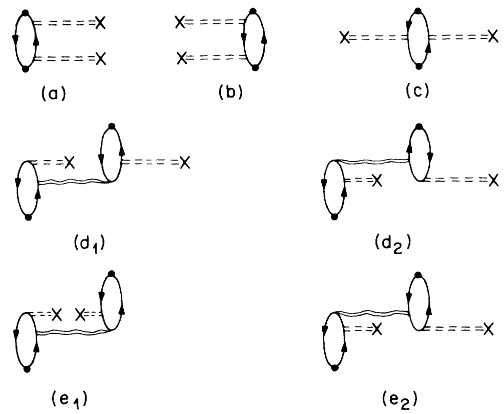

FIG. 2. The extra contributions to the dynamical conductivity due to impurity scattering to second order in the impurity potential. The double dashed lines are the particle-impurity interactions statically screened by the plasma and phonons.

When performing the derivation of the diagrams in Fig. 2, we do not have to perform a summation over complex frequencies but can obtain the retarded form of Π directly. The sum of the first three diagrams is

$$\Pi^{(a)+(b)+(c)}_{(\Omega)}=\frac{e^2}{V^2\Omega^2m^2}\sum_{\mathbf{q}}q_\mu^2\left|\frac{U(\mathbf{q})}{\epsilon_0}\right|^2\Lambda(\mathbf{q},0)^2 \times[\chi(\mathbf{q},0)-\chi(\mathbf{q},\Omega)]\ , \tag{3.10}$$

and the contribution from the remaining four gives

$$\Pi^{(d_1)+(d_2)+(e_1)+(e_2)}=-\frac{e^2}{V^2\Omega^2m^2}\sum_{\mathbf{q}}q_\mu^2\left|\frac{U(\mathbf{q})}{\epsilon_0}\right|^2\Lambda(\mathbf{q},0)^2v\frac{[\chi(\mathbf{q},0)-\chi(\mathbf{q},\Omega)]^2}{\epsilon_L(\Omega)-v\chi(\mathbf{q},\Omega)}\ . \tag{3.11}$$

The total result is

$$\Pi(\Omega)=-\frac{e^2}{V^2\Omega^2m^2}\sum_{\mathbf{q}}g_\mu^2\left|\frac{U(\mathbf{q})}{\epsilon_T(0)}\right|^2\frac{1}{v(q)}\frac{[\alpha(\mathbf{q},\Omega)-\alpha(\mathbf{q},0)][\epsilon_L(\Omega)+\alpha(\mathbf{q},0)]}{\epsilon_T(\mathbf{q},\Omega)}\ , \tag{3.12}$$

where ϵ_T denotes the total dielectric function

$$\epsilon_T=\epsilon_L+\alpha=\epsilon_L-v\chi\ . \tag{3.13}$$

Thus our final result for the dynamical conductivity is, after performing the angular integration,

$$\sigma(\Omega)=i\frac{ne^2}{m\Omega}-\frac{ie^4 2n}{m^2\Omega^3 3\pi}\int_0^\infty dq\,q^2S(q)\frac{[\alpha(q,\Omega)-\alpha(q,0)][\epsilon_L(\Omega)+\alpha(q,0)]}{\epsilon_T^2(q,0)\epsilon_T(q,\Omega)}\ , \tag{3.14}$$

where we have made use of Eq. (2.9) and have also done an ensemble average over the impurity positions resulting in the so-called liquid structure factor $S(q)$. $S(q)$ is identical to 1 for randomly distributed impurities. It is defined as

$$S(\mathbf{q})=\frac{1}{N}\langle\rho^{\dagger}_{\text{imp},\mathbf{q}}\rho_{\text{imp},\mathbf{q}}\rangle-(N-1)\delta_{\mathbf{q},0}\ , \tag{3.15}$$

where the angular brackets denote the ensemble average.

Our final result agrees with the results in Refs. 9 and 10. This completes this section. In the next section we will rederive the result using the analogy between the impurities and frozen phonons.

IV. FROZEN-PHONON APPROACH TO THE IMPURITY CONTRIBUTION TO THE DYNAMICAL CONDUCTIVITY

In this section we rederive the result obtained in the previous section by viewing the impurity scattering as due to static lattice disorder. We follow the derivation in Ref. 7 and extend it to be valid in a polar semiconductor.

Following the prescription in Ref. 7 for a nonpolar semiconductor, the following replacement should be performed in one place in each diagram:

$$g^2(q)D(\omega)\rightarrow\frac{i\pi\hbar\beta\eta}{2}\frac{1}{V}\left[\frac{v(q)}{\epsilon_\infty}\right]^2[\delta(\omega-\eta)-\delta(\omega+\eta)]\ , \tag{4.1}$$

and on the rest of the places where there are phonon lines, g^2D should be replaced by zero. Thus only diagrams that originally contained a single phonon line will survive. In Ref. 7 one actually started from the contribution to first order in the phonon interaction, and therefore only obtained diagrams with a single phonon line. Hence the requirement of two types of replacements which arises in the general case did not show up in this calculation. In deriving the result for the impurity contribution in the absence of coupling to phonons with the method in Ref. 12, one would also obtain the correct result because only the first-order diagrams will survive and no contributions are lost by dropping the impurity contributions in the denominator. We have here extended the result somewhat. In Ref. 7 one impurity was placed at the origin and the result was multiplied by the number of impurities. This gave a result which is only valid for a random distribution of impurities. We have here treated the N impurities as one big impurity by using $U(\mathbf{q})$ instead of $v(q)$ for the impurity potential. At the end we will take an ensemble average over the internal structure of this big impurity, to obtain a more general expression where correlation between the impurity positions can be accounted for.

Now we proceed to get the result from impurity scattering in the presence of coupling to phonons. The replacement according to Eq. (4.1), but with ϵ_∞ replaced by ϵ_0, should be performed on any one position in the diagrams, but no replacement of the second type should be performed.

The easiest way to obtain the result is to start from the one-component version of Eq. (2.30) and perform on it the following operation:

$$\frac{i\pi\hbar\beta\eta}{2}\frac{1}{V}\left[\frac{v(q)}{\epsilon_0}\right]^2[\delta(\omega-\eta)-\delta(\omega+\eta)]\frac{d}{d[g^2(q)D(\omega)]}\ , \tag{4.2}$$

where ω should take on the same value as the frequency argument of D in the term that is being operated on. One should note that D appears not only as separate factors in the expression but also in the expression for the Λ's.

If one is interested in the result for a multicomponent plasma one should use the full version of Eq. (2.30).

As our frequency integration is restricted to positive values, operations according to Eq. (4.2) will only give contributions which survive the integration from terms with frequency argument $\omega-\Omega/2$. The operation will give the result

$$\begin{aligned}\mathrm{Re}\sigma(\Omega)=&\frac{e^2\pi\hbar\beta\eta}{\Omega^3m^22}\int\frac{d\mathbf{q}}{(2\pi)^3}q_\mu^2\frac{1}{V}\left|\frac{v(q)}{\epsilon_0}\right|^2\\&\times\int_0^\infty\frac{d\omega}{2\pi}\frac{\delta\left[\omega-\frac{\Omega}{2}+\eta\right]-\delta\left[\omega-\frac{\Omega}{2}-\eta\right]}{\sinh\left[\frac{\hbar\beta}{2}\left[\omega-\frac{\Omega}{2}\right]\right]}\\&\times\{\mathrm{Im}[\Lambda(\Omega)]\Lambda(0)\chi(0)-\mathrm{Im}[\Lambda(\Omega)\chi(\Omega)]\Lambda(0)+\mathrm{Im}[\Lambda(\Omega)]\chi^2(0)g^2D(0)\Lambda^2(0)\\&+\chi(0)\Lambda^2(0)\mathrm{Im}[\Lambda(\Omega)\chi(\Omega)g^2D(\Omega)]-\mathrm{Im}[\Lambda(\Omega)\chi(\Omega)]\Lambda^2(0)\chi(0)g^2D(0)\\&-\mathrm{Im}[\Lambda(\Omega)g^2D(\Omega)]\chi^2(0)\Lambda^2(0)\}\ ,\end{aligned} \tag{4.3}$$

where for simplicity we have left out the argument $\mathbf{q}$ in all functions. The expression can be reduced to a much simpler form and becomes, after integration over angles and ensemble averaging,

$$\mathrm{Re}\sigma(\Omega)=\frac{e^42n}{m^2\Omega^33\pi}\int_0^\infty dq\,q^2S(q)\mathrm{Im}\left[\frac{[\alpha(q,\Omega)-\alpha(q,0)][\epsilon_L(\Omega)+\alpha(q,0)]}{\epsilon_T^2(q,0)\epsilon_T(q,\Omega)}\right] \tag{4.4}$$

which exactly agrees with the real part of Eq. (3.14) (we limited ourselves to the real part, here).

The expression does not agree with the result of Ref. 11, also derived using the frozen-phonon approach. We argue that the deviations are not due to a deficiency in the frozen-phonon approach but to some mistake made in Ref. 11. It is not quite clear how they arrived at their expression so we cannot pinpoint where the mistake was made. We merely note that if we in our expression omit all static particle polarizabilities the result transforms into theirs. We point out that the derivation of the formula for the impurity contribution was just a peripheral result in that reference. It discussed the magnetic field dependence of the high-frequency scattering rates in lead salts, and found that this dependence was dominated by the contributions from the phonon scattering.

V. IMPURITY CONTRIBUTION TO THE DYNAMICAL CONDUCTIVITY BY LETTING THE MASS OF ONE COMPONENT IN A TWO-COMPONENT PLASMA GO TO INFINITY

In this section we will rederive the results obtained in the previous sections by using an alternative approach. We start from the result in a two-component plasma in the presence of phonon coupling and simulate the effect from the impurities by letting the mass of one of the components go to infinity. This method was used earlier in Ref. 6 in the absence of phonon coupling and in Ref. 9 in the presence of phonon coupling. We start from the two-component version of Eq. (2.30). In order to get a general expression valid for correlations in the impurity distribution we treat the impurities as one giant impurity and the particle impurity potential is given by $U(\mathbf{q})$ of Eq. (3.5).

At finite temperature the susceptibility for the impurity takes, in the infinite-mass limit, the form

$$\chi_i(\mathbf{q},\omega)=-\frac{i\pi\beta}{V}\left|\frac{U(\mathbf{q})}{v(q)}\right|^2\omega\delta(\omega)\ . \tag{5.1}$$

In the infinite-mass limit terms containing photon vertices with the impurity mass vanish, and only terms where the frequency argument in the impurity polarizability is equal to $\omega+\Omega/2$ survive the frequency integration. The result is

$$\begin{aligned}\mathrm{Re}\sigma(\Omega)=&-\frac{e^2\hbar}{\Omega^3 m^2}\int\frac{d\mathbf{q}}{(2\pi)^3}q_\mu^2\left[-\frac{i\beta\pi}{V}\left|\frac{U(\mathbf{q})}{v(q)}\right|^2\right]\\&\times\int_0^\infty\frac{d\omega}{2\pi}\frac{\omega\delta(\omega)}{\hbar\omega\beta/2}\{\mathrm{Im}[\Gamma(\Omega)]\Gamma(0)\chi(0)-\mathrm{Im}[\Gamma(\Omega)\chi(\Omega)]\Gamma(0)\\&+\mathrm{Im}[\Lambda(\Omega)]\frac{v\Lambda^2(0)}{\epsilon_L(0)}\chi(0)g^2D(0)+\frac{v\Lambda^2(0)}{\epsilon_L(0)}\mathrm{Im}[\Lambda(\Omega)\chi(\Omega)g^2D(\Omega)]\\&-\frac{v\Lambda^2(0)}{\epsilon_L(0)}g^2D(0)\mathrm{Im}[\Lambda(\Omega)\chi(\Omega)]-\frac{v\Lambda^2(0)}{\epsilon_L(0)}\chi(0)\mathrm{Im}[\Lambda(\Omega)g^2D(\Omega)]\}\ .\end{aligned} \tag{5.2}$$

After integration over angles and ensemble averaging we arrive at exactly the same result as in Eq. (4.4).

VI. SUMMARY AND CONCLUSIONS

We have derived an expression for the high-frequency dynamical conductivity due to impurity scattering in a polar semiconductor. The derivation was based on the Kubo formula and diagrammatic perturbation theory and was performed in three different ways. In Sec. III it was derived in a direct way and in Sec. IV by treating the impurities as frozen phonons. In Sec. V we derived the result for a two-component plasma and obtained the impurity contribution by letting the mass of one of the components go to infinity. In the last derivation we let all the impurities be regarded as one giant impurity and performed at the end an ensemble averaging over the internal structure of this impurity. By doing so the result was generalized so as to be valid for a nonrandom distribution of impurities, and the limitation of this approach, as discussed in Ref. 9, was eliminated.

The results obtained from the three derivations were identical, showing that the variations in the earlier published results were not approach dependent. We found that the published results in Ref. 9 and the unpublished results in Ref. 10 were correct while those in Refs. 11 and 12 were erroneous.

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*Permanent address: Department of Physics and Measurement Technology, University of Linköping, S-581 83 Linköping, Sweden.

[1]I. Hamberg and C. G. Granqvist, Appl. Phys. Lett. 44, 721 (1984).

[2]I. Hamberg, C. G. Granqvist, K.-F. Berggren, B. E. Sernelius, and L. Engström, Vacuum 35, 207 (1985).

[3]I. Hamberg, C. G. Granqvist, K.-F. Berggren, B. E. Sernelius,

and L. Engström, Phys. Rev. B **30**, 3240 (1984).
[4]A. Ron and N. Tzoar, Phys. Rev. **131**, 1943 (1963).
[5]E. Gerlach, J. Phys. C **19**, 4585 (1986).
[6]A. Ron and N. Tzoar, Phys. Rev. **131**, 12 (1963).
[7]R. Sirko and D. L. Mills, Phys. Rev. B **18**, 5637 (1978).
[8]I. Hamberg (private communications).
[9]S. Goettig, J. Phys. C **17**, 4443 (1984).
[10]E. Gerlach and J. Mycielski (unpublished, but the result was mentioned in Ref. 5).
[11]S. Katayama and D. L. Mills, Phys. Rev. B **19**, 6513 (1979).
[12]P. Kleinert and M. Giehler, Phys. Status Solidi B **136**, 763 (1986).
[13]G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1981).