rf quantum Hall effect in a superlattice

I. D. Vagner
Max-Planck-Institut für Festkörperforschung, Hochfeld Magnetlabor, 166X, F-38042, Grenoble, France

D. Bergman
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Science, Tel-Aviv University, Tel-Aviv 69978, Israel

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Helicon-wave propagation (rf Hall effect) in a model superlattice with a two-dimensional electron gas under quantum-Hall-effect conditions is studied in the framework of a Kronig-Penney-like dispersion relation. Numerical evidence for the occurrence of flat plateaus in the helicon resonance at low frequencies is presented. As the cyclotron frequency is approached, the plateaus are destroyed by the polarization and the displacement currents.

A well-established technique of magnetotransport measurements in the microwave and far-infrared (FIR) frequency domain is the observation of the helicon-wave resonance. The helicon wave is a circularly polarized electromagnetic wave with a quadratic dispersion.[1(a)] It arises from the suppression of the currents along the electric field of the wave as a result of a strong external magnetic field. The transverse currents generate a time-dependent magnetic field, which is sufficient to maintain self-sustaining oscillations. The helicon wave, therefore, can be thought of essentially as a rf Hall effect.

Recently, Maan *et al.*[2] observed the helicon resonance in a highly doped InAs-GaSb superlattice with a two-dimensional electron gas (2DEG). Tselis *et al.*[3] studied in detail elementary excitations in semiconductor superlattices and mention the idea that plateaus in the frequency of the helicon resonance should occur whenever there are quantum-Hall-effect (QHE) plateaus in the spectrum of the 2DEG of the superlattice, since at low frequencies the helicon dispersion simply depends on the Hall conductivity. The QHE in a superlattice with weak interlayer tunneling was predicted by Azbel[4] and observed experimentally by Stormer *et al.*[5]

The high precision of the QHE data, when $k_B T$ is much smaller than the energy gaps in the spectrum of the magnetized 2DEG caused by Landau quantization, is limited basically by the precision of the dc resistivity measurements. A high-frequency (in the GHz region) contactless method was used[6,7] to measure σ_{xy} via Faraday rotation of microwaves propagating in a GaAs/$Al_xGa_{1-x}As$ multi-quantum-well structure. Time-varying magnetic fields were used to study the response of the 2DEG in silicon inversion layers[8] and in the stage-2 Br_2-graphite intercalation compound.[9]

The theoretical work on the ac QHE[10] uses the concept of long quasiclassical electronic orbits in the 2DEG,[11] with potential fluctuations that are smooth on the scale of the Larmor radius. Because of the very long period of these quasiclassical orbits, they can exchange energy with an external field even at low frequencies (i.e., in the MHz range). Depinning of fractionally charged vortices[12,13] by time-varying fields may yield an enhancement of the fractional quantum Hall effect.[14]

Experimentally, in the GaAs/$Al_xGa_{1-x}As$ heterostructure in magnetic fields of about 8 T, at a temperature of 1.23 K, the upper limit on the electron scattering time is estimated in Ref. 15 as 1.5×10^{-3} sec. This means that the high-frequency ($\omega\tau>1$) regime, which in conventional semiconductors starts only in the FIR regime, may start at kiloherz frequencies in the QHE systems. In this regime, if we superimpose a fixed magnetic field $\mathbf{H}$ and a perpendicular time-dependent electric field $\mathbf{E}(t)$, electrons will have a drift component also in the direction of $\mathbf{E}$, in addition to the more familiar drift in the direction $\mathbf{E}\times\mathbf{H}$.[16] This additional drift, customarily called the polarization drift, leads to an imaginary contribution to the diagonal conductivity that is proportional to the frequency of $\mathbf{E}(t)$. This contribution is present even at the plateaus, where the real part of σ_{xx} vanishes. At frequencies approaching the cyclotron frequency this contribution will be as large as the Hall conductivity.[16–18]

The central point of our study is the existence of a nonvanishing (in the plateau regime) frequency-dependent imaginary contribution to the diagonal component of the resistivity tensor in a homogeneous 2DEG, and its influence on the helicon resonance in a superlattice under QHE conditions.

In what follows we consider the wave propagation in a model superlattice with QHE conditions in the 2DEG layers, which are separated by dielectric layers. We use a standard method[19] to reduce the Maxwell equations in a layered conductor with a perpendicular magnetic field to a Kronig-Penney-like problem. A numerical solution of the resulting dispersion equation exhibits flat plateaus in the magnetic field dependence of the low-frequency mode. This is the mode that is governed by the Hall currents in the 2DEG layers. At higher frequencies the in-plane polarization drift current as well as the displacement current in the conducting and insulating layers lead to strong deviations from the flat plateau structure.

We start with the wave equation for a monochromatic complex electric field $\mathbf{E}$ of frequency ω propagating in a conducting medium:

$$\nabla \times \nabla \times \mathbf{E} + (\omega/c)^2 \epsilon \mathbf{E} = 0 , \tag{1}$$

where $\epsilon_{ij} = \epsilon_0 \delta_{ij} - (4\pi/i\omega)\sigma_{ij}(\mathbf{k},\omega)$ is the electric susceptibility tensor and σ_{ij} is the conductivity tensor. This equation is solved for the case of a plane electromagnetic wave incident along the superlattice periodicity axis $\mathbf{k}=(0,0,k_z)$, with a constant magnetic field H_0 also lying in the same direction. We assume zero conductance in that direction, $\sigma_{xz}=\sigma_{zx}=\sigma_{yz}=\sigma_{zy}=\sigma_{zz}=0$, and isotropy in the other two directions, $\sigma_{xx}=\sigma_{yy}$, and use the Onsager relations[1(b)] $\sigma_{ij}(\mathbf{H},\mathbf{k},\omega)=\sigma_{ji}(-\mathbf{H},-\mathbf{k},\omega)$ to obtain $\sigma_{xy}=-\sigma_{yx}$. Hence, for this simple geometry, we are left with only two independent components of the conductivity tensor: σ_{xy} and σ_{xx}. The wave equation can be reduced now to the form[19]

$$E''_+ + (\omega/c)^2 \epsilon_- E_+ = 0 , \tag{2}$$

where $E_+ = E_x + iE_y$ and $\epsilon = \epsilon_{xx} - i\epsilon_{xy}$.

For a wide class of periodic $\epsilon_-(z)$, Eq. (2) can be treated using Bloch's theorem (Mathiew or Hill equations). Note, however, that all the quantities, $E_{x'}$, $E_{y'}$, ϵ_{xx}, and ϵ_{xy}, are usually complex. We assume a model superlattice with 2DEG layers of width a, separated by insulating layers of width b with a dielectric constant ϵ_0. A standard procedure[19] then yields the Kronig-Penney-like dispersion relation

$$\cos(k_1 a)\cos(k_2 b) - (k_1^2 + k_2^2)\sin(k_1 a)\sin(k_2 b)/2k_1 k_2 = \cos(kd) , \tag{3}$$

where $d = a + b$,

$$k_{1,2}^2 = (\omega/c)^2 \epsilon_-^{(1,2)} , \tag{3a}$$

and $\epsilon_-^{(2)} = \epsilon_0$ and $\epsilon_-^{(1)} = \epsilon_{xx}^{\mathrm{el}} - i\epsilon_{xy}^{\mathrm{el}}$ refer to the dielectric and 2DEG layers, respectively.

The conductivity tensor in the 2DEG under strong magnetic fields[20] can be calculated from $\mathrm{j}(x,t) = (e/2)\mathrm{Tr}'\{\hat{f}[\hat{\mathbf{v}}\delta(x-x') + \delta(x-x')\hat{\mathbf{v}}]\} = \hat{\sigma}\mathbf{E}$, where the free-electron density matrix $\hat{f} = \hat{f}_0 + \hat{f}_1$ obeys the equation of motion: $\partial\hat{f}/\partial t + (i/\hbar)[\hat{H}\hat{f}] = 0$. Here $\hat{f}_0(\mathbf{H},\mu) = \{1 + \exp[(\hat{\mathbf{H}} - \mu)/kT]\}^{-1}$ is the free-electron equilibrium density matrix for the 2DEG under a strong magnetic field and $\hat{f}_1$ is a perturbation, linear with respect to the vector potential $\mathbf{A}$. The Hamiltonian is $\hat{H} = \hat{H}_0 + \hat{H}_1$ where $\hat{H}_0 = (1/2m)\hat{\mathbf{v}}_0^2$, $\hat{H}_1 = (e/2c)(\hat{\mathbf{v}}_0 \cdot \mathbf{A}_1 + \mathbf{A}_1 \cdot \hat{\mathbf{v}}_0)$ and the velocity operator for an electron in an external field is given by $\hat{\mathbf{v}}_0 = (\hat{\mathbf{p}} + e\mathbf{A}_0/c)/m$.

Following Refs. 20 and 3, one gets

$$\mathrm{Re}\sigma_{xy}^{2\mathrm{D}} \approx n_s ec/H , \tag{4a}$$

$$\mathrm{Im}\sigma_{xy} \approx 0 , \tag{4b}$$

$$\mathrm{Re}\sigma_{xx} \approx 0 , \tag{4c}$$

$$\mathrm{Im}\sigma_{xx} \approx \sigma_{xy}(\omega/\omega_c) , \tag{4d}$$

where $\omega_c = eH/mc$ is the cyclotron frequency, m^* is the effective mass, and n_s is the areal density of the 2DEG. In the frequency regime $\omega < \omega_c < \omega_p$ we obtain, from Eq. (3a) and Eq. (4),

$$k_1^2 = (\omega/c^2) 4\pi\sigma_{xy}^{3\mathrm{D}}(1 + \omega/\omega_c) \tag{5}$$

which is the standard 3D helicon dispersion[2,3,21] with $\sigma_{xy}^{3\mathrm{D}} = \sigma_{xy}^{2\mathrm{D}}/d$. Equation (5) corresponds to the dotted lines in Fig. 1.

Using k_1 from Eq. (5) and k_2 from Eq. (3a) the dispersion equation, Eq. (3), can be reduced to a simple algebraic equation in the long-wavelength limit $kd \ll 1$:

$$\omega^2 \epsilon_0 + 4\pi\alpha\sigma_{xy}\omega(1 + \omega/\omega_c) - c^2 k^2 = 0 . \tag{6}$$

Here $\alpha = a/d < 1$. At very low frequencies, the term ω/ω_c in parentheses can be neglected, and we get the following, heliconlike solution:

$$\omega = c^2 k^2 / 4\pi\alpha\sigma_{xy} = c^2 k^2 h / \alpha n_F e^2 . \tag{7}$$

The second equality assumes that we are in the QHE regime, namely, that there are n_F completely filled Landau levels and that the Fermi energy is in a magnetic energy gap. This simplified approximate solution, which exhibits completely flat plateaus, was already obtained in Ref. 3. Note that it neglects the nonzero and frequency-dependent value of σ_{xx} as exhibited in Eq. (4d) above, which is due to the polarization drift.

In order to investigate the effect of this nonzero σ_{xx} we solved the dispersion equation (3) numerically for layered structures with various values of the total number of layers N. The solutions are exhibited in Fig. 1, and they show clearly that at higher frequencies the plateaus are distorted. This is caused by the displacement currents [the third term in Eq. (6)] and by the nonvanishing imaginary part of diagonal conductivity, Eq. (4d) [the factor

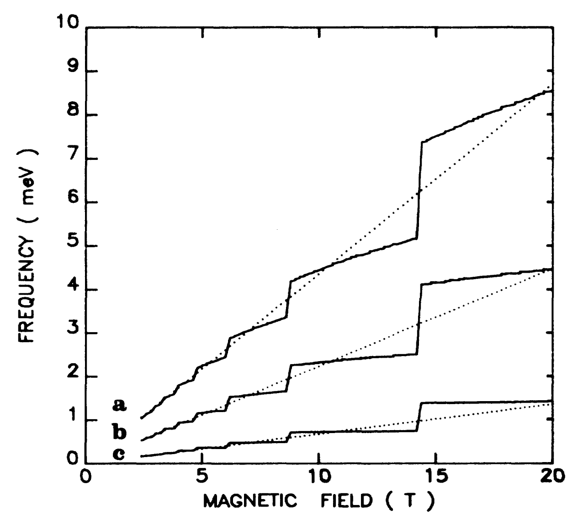

FIG. 1. Numerical evidence for the occurrence of plateaus in the frequency of the helicon resonance in a GaAs/$Al_xGa_{1-x}As$ superlattice with parameters: the electron concentration is $n_0 = 5\times10^{11}$ cm^{-2}, the GaAs layers are 100 Å thick, the $Al_xGa_{1-x}As$ barriers are 400 Å thick. N is the number of 2DEG layers in a superlattice: (a) $N=300$; (b) $N=500$; (c) $N=1000$.

$1+\omega/\omega_c$ in the linear term of Eq. (6)]. We note that for a superlattice with a large number of periods N (e.g., low frequencies) the behavior is adequately described by the simplified dispersion equation (7).

The physical origin of $\mathrm{Im}\sigma_{xx}$, Eq. (4d), can be understood as follows. In crossed ac electric and dc magnetic fields the charge carriers experience an acceleration along the electric field which is proportional to the frequency:[16]

$$d\mathbf{v}/dt = -i\omega c(\mathbf{E}\times\mathbf{H})/H^2 \ . \tag{8}$$

The corresponding inertial force $\mathbf{F}_{\mathrm{in}}=m\,d\mathbf{v}/dt$ causes an additional drift velocity $\mathbf{v}_p = c(F_{\mathrm{in}}\times\mathbf{H})/eH^2$ and a corresponding current $j_p = nev_p = nmc^2E/H^2$ in the direction of the time-varying electric field $E(t)$. This contributes to the diagonal conductivity:

$$\sigma_{xx} = i\omega nmc^2/H^2 \tag{9}$$

which is our Eq. (4d). Note that σ_{xx} does not depend on the relaxation time τ and remains finite also in the plateau regime, in contrast with the vanishing diagonal dc conductivity.

To summarize, we studied the magnetic field dependence of the frequency of a heliconlike wave in a superlattice of 2DEG layers under QHE conditions, using a Kronig-Penney-like dispersion relation. We find that at sufficiently low frequencies the helicon resonance exhibits flat plateaus simultaneously with the σ_{xy} plateaus of the QHE. As the resonance frequency grows, the plateaus acquire a finite slope which increases with frequency and are eventually destroyed by the in-plane polarization currents. It is clear, therefore, that experimental studies of the helicon resonance in a superlatice of 2DEG layers may shed new light on the frequency dependence of the conductivity tensor in the QHE regime.

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