Brief Reports

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Magnetoplasma modes of semiconductor superlattices with integral Landau-level filling

Ji-Wei Wu* and J. J. Quinn
Brown University, Providence, Rhode Island 02912
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The dispersion relation of magnetic excitons in a semiconductor superlattice is derived in the strong-magnetic-field limit. The Coulomb energy $e^2/\epsilon l_0$ is assumed to be smaller than the cyclotron energy $\hbar\omega_c$, and many-body correlation effects are treated exactly to lowest order in $(e^2/\epsilon l_0)/\hbar\omega_c$.

During the past decade, there have been many studies of the elementary excitations, transport, and optical properties of semiconductor superlattices.[1-6] Because of the quantization of the electronic motion along the superlattice axis, a type-I superlattice such as $GaAs/Al_xGa_{1-x}As$ can, for many purposes, be regarded as a periodic array of quasi-two-dimensional electron-gas layers. Many experimental techniques like far-infrared spectroscopy, cyclotron resonance, and inelastic light scattering, which have been used to investigate surface inversion layers, have proved to be powerful tools for studying superlattices as well. The collective charge-density excitations[7-9] of superlattices are particularly interesting in view of the absence of Landau damping of these modes. Experiments have verified[10] the quantitative prediction of theory for the dispersion of these plasma modes. In the presence of a dc magnetic field, the excitation spectrum is expected to change significantly. One recent example of such a modification is the magnetic field dependence of collective charge-density modes of the lateral surface of a superlattice.[11]

In this paper we investigate the properties of collective charge-density excitations of a superlattice in the presence of a strong dc magnetic field oriented parallel to the superlattice axis. Magnetoplasma modes of three-dimensional materials have been studied for many years. Some of the magnetoplasma modes are well described by a simple Drude-like local theory, while others depend upon nonlocal effects for their existence. Some excitations are strongly influenced by exchange-correlation effects, and the primary motivation for their experimental study has been to learn about these many-body effects. In strictly two-dimensional electron-gas systems the magnetoplasma modes[12] and their effect on optical properties was first investigated within the framework of the random-phase approximation (RPA). Recently a number of authors,[13-15] in particular Kallin and Halperin, have gone beyond the RPA for the case in which all Landau levels are either completely filled or completely empty. In this situation the Coulomb energy $e^2/\epsilon l_0$, where $l_0=(\hbar c/eB)^{1/2}$ is the magnetic length, and ϵ is the background dielectric constant, can be small compared to the magnetic energy $\hbar\omega_c$, and their ratio is a small dimensionless parameter which can be used to generate a valid peturbation expansion. Very recently MacDonald *et al.*[16] have investigated the effects of correlation on the magnetoplasma modes of a two-dimensional electron gas with a partially filled Landau level, where the simple perturbation expansion is not valid.

For superlattices intrasubband magnetoplasma modes have been studied in the RPA,[17] and with the inclusion of exchange-correlation effects via a local-energy-functional approach.[18] The RPA result yields the expected two-dimensional or three-dimensional behavior when the product qa of the wave vector along the layer and the layer spacing approaches the appropriate limiting value. The object of the present paper is to investigate the magnetoplasma modes of a periodic array of two-dimensional electron-gas layers beyond the RPA for the case when all Landau levels are either completely filled or completely empty. For large values of the layer separation $(qa\rightarrow\infty)$ the results are expected to reduce to the cyclotron modes studied by Kallin and Halperin. For smaller values of qa we expect to obtain a band of cyclotron modes. These modes are important for studying magnetic exciton spectra in semiconductor superlattices.

We consider an infinite superlattice consisting of two-dimensional electron-gas (2DEG) layers located at $z=la$, $l=0,\pm1,\ldots,\pm\infty$. Electrons in each plane have an equilibrium density n_s and mass m. In the presence of a magnetic field along the superlattice axis, the many-body Hamiltonian including the Coulomb interaction between electrons in the Landau gauge is

$$H = H_0 + V_{e\text{-}e} \,, \qquad (1)$$

$$H_0 = \sum_{n,l,k} \varepsilon_n C^\dagger_{nlk} C_{nlk} \,,$$

$$V_{e\text{-}e} = \frac{1}{2} \int \frac{d^2c}{2\pi^2} \sum_{l_1,l_2} V_q(l_1 l_2) \sum_{n_1,n_2,n_3,n_4} \sum_{k_1,k'} C^\dagger_{n,l_l,k+q_y} C_{n_2,l_2,k'-q_y} C_{n_3 l_2 k'} C_{n_4 l_1 k} \times \exp[iq_x l_0^2(k-k'+q_y)] C_{n_1 n_4}(\mathbf{q}) C_{n_2 n_3}(-\mathbf{q}) \,.$$

The notations in the above equation[14] follow from the usual definition; we can write the density correlation function as

$$\chi(\mathbf{q},\omega;z,z') = \sum_{l,l'} \delta(z-la)\delta(z'-l'a)\chi_{ll'}(\mathbf{q},\omega) \,. \qquad (2)$$

One can introduce χ^0 as the irreducible part of χ; because of the translational invariance of the system, $\chi_{ll'} = \chi(l-l')$. The following RPA equation gives the relation between these two quantities:

$$\chi(l-l') = \chi^0(l-l') + \sum_{l_1,l_2} \chi^0(l-l_1) V_q(l_1-l_2) \chi(l_2-l') \,, \qquad (3)$$

where $V_q(z) = V_q \exp(-q|z|)$, and $V_q = 2\pi e^2/\epsilon q$ is the Fourier-transformed 2D Coulomb interaction. The dependence on q and ω is suppressed for simplicity.

By solving the Bethe-Salpeter equation for the vertex function shown in Fig. 1, the irreducible part of χ can be calculated to be $\chi^0_{ll'} = \chi^0 \delta_{ll'}$ where χ^0 is the irreducible part of the density correlation function of a single 2DEG. This result is obvious in the RPA approximation, but it is also true when all direct interactions between the electron-hole part are included, since the scattering of electrons (holes) between different layers are not allowed in our model. This fact can be seen schematically from the diagram of the irreducible vertex function in Fig. 1(b), where both ends of a continuous electron line should always carry the same layer index.

The above equations were solved by introducing the discrete Fourier transform

$$\chi(q_z) = \sum_{l=-\infty}^{\infty} e^{iq_z la} \chi(l) \,.$$

The magnetic exciton dispersion is determined from the singularity of $\chi(q_z)$, i.e., from the equation

$$1 - V_q S(q,q_z) \chi^0(q,\omega) = 0 \,, \qquad (4)$$

where $S(q,q_z) = \sinh(qa)/[\cosh(qa) - \cos(q_z a)]$ is a structure factor. This relation is formally identical to the expression for the bulk superlattice plasmon in the absence of a magnetic field. All effects due to the magnetic field are contained in χ^0.

We restrict our attention to the case where each layer has an integral value of the filling factor for the Landau levels. To lowest order in $(e^2/\epsilon l_0)/\hbar\omega_c$, $\chi^0(q,\omega)$ can be evaluated by including successive direct interactions between the electron and the hole (ladder diagrams). The excitation energy of the state with a particle in level n' and a hole in level n is then solved from Eq. (4):

$$E_{nn'}(q,q_z) = (n'-n)\omega_c + \sum_{n'} - \sum_{n} - \widetilde{V}_{nn'}(q,q_z) \,, \qquad (5)$$

where $\sum_n$ is the Hartree-Fock energy of an electron in level n and $\widetilde{V}_{nn'}(q,q_z)$ is the irreducible scattering amplitude consisting of contributions from the direct and exchange interactions between the members of the excited electron-hole pair.[13-15] The direct term arises from summing over the ladder diagrams in χ^0 and thus is the same as that of a single 2DEG. The exchange term is modified by the superlattice structure factor in Eq. (4). We obtain for $\widetilde{V}_{nn'}$

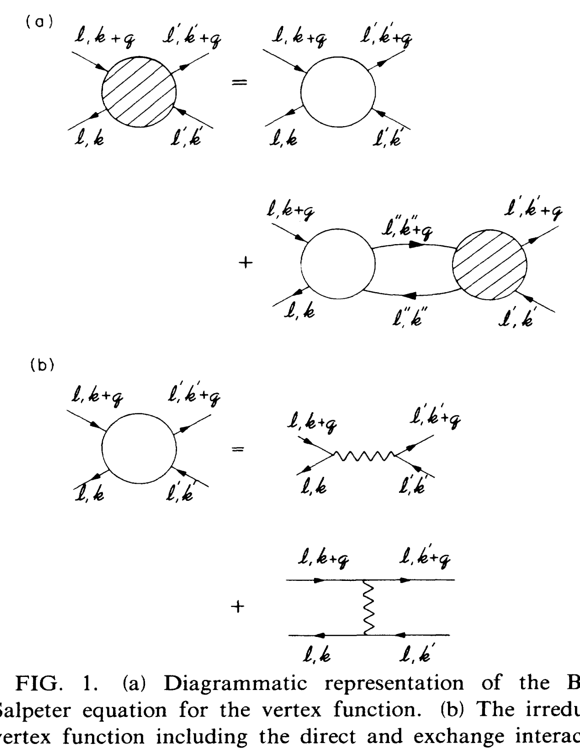

FIG. 1. (a) Diagrammatic representation of the Bethe-Salpeter equation for the vertex function. (b) The irreducible vertex function including the direct and exchange interactions between the members of the electron-hole pair. The Landau level indices of the electron (hole) states are omitted without causing confusion.

$$\widetilde{V}_{nn'}(q,q_z)=\frac{e^2}{\epsilon l_0}\int du\, \exp(-u^2/2)L_n(u^2 2)L_{n'}(u^2)J_0(qau)$$
$$-S(q,q_z)\frac{\nu_n}{2}\frac{e^2}{\epsilon}q\frac{n!}{n'!}\left[\frac{q^2l_0^2}{2}\right]^{n'-n-1}\left[L_n^{n'-n}\left[\frac{q^2l_0^2}{2}\right]\right]\exp(-l_0^2q^2/2)\,, \qquad (6)$$

where J_0 is the Bessel function, the L's are Laguerre polynomials, and $\nu_n=2$ or 1 depending on whether level n is filled for both spin orientations or not. When the neighboring layers are far apart, $S(q,q_z)\rightarrow 1$ as $qa\rightarrow\infty$. In that limit, Eq. (6) would reduce into Eq. (18) of Ref. 15, which is the result for a single 2DEG.

We calculate the exciton energy near ω_c ($n'=n+1$) for the system having its lowest Landau levels ($n=0$) filled in each layer, $\nu=\nu_0=2$. The explicit expression is

$$E_{01}(q,q_z)^{\hbar w_c}=\left[\frac{\pi}{2}\right]^{1/2}\frac{e^2}{\epsilon l_0}\left\{\frac{1}{2}-\frac{1}{2}\left[\left[1+\frac{q^2l_0^2}{2}\right]I_0(q^2l_0^2/4)-\frac{q^2l_0^2}{2}I_1(q^2l_0^2/4)\right]e^{-q^2l_0^2/4}\right.$$
$$\left.+S(q,q_z)\left[\frac{2}{\pi}\right]^{1/2}ql_0\exp(-q^2l_0^2/2)\right\}. \qquad (7)$$

There are two parameters with the dimension of length in this equation, the magnetic length l_0 and the layer spacing a. Under the condition of strong magnetic fields, a semiconductor with $\epsilon=10$ would have $l_0=50$—100 Å. The typical value for a is around 100—500 Å. So we take the ratio a/l_0 to be 1—10 in our calculation.

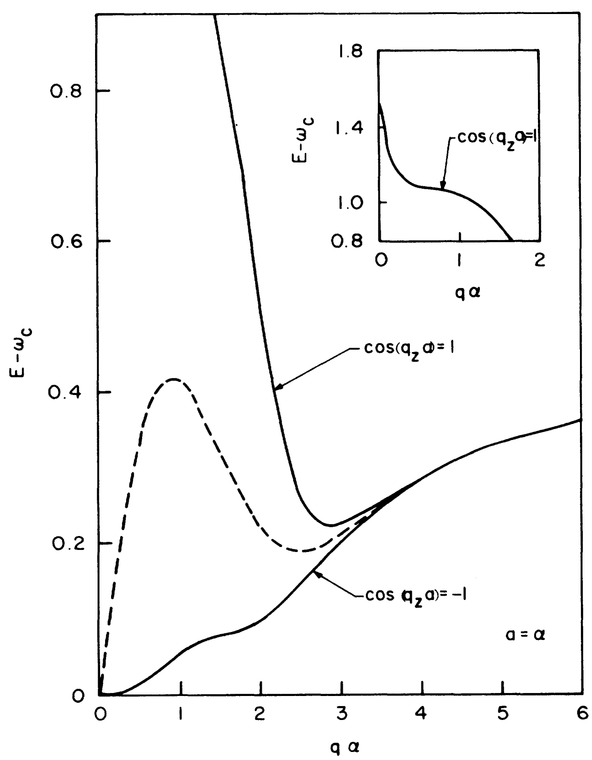

FIG. 2. Dispersion relation of magnetic exciton in a superlattice for transition $0\rightarrow 1$. The superlattice periodicity is chosen to be $a=l_0$. The dashed line is the dispersion for a single 2D electron layer. The top part of the diagram is shown in the inset with reduce scale. The vertical scale is in units of $(\pi/2)^{1/2}(esp2/\epsilon l_0)$.

Figures 2 and 3 show the exciton energy as a function of ql_0. For fixed value of q, the possible choices of q_z in the range $(0,\pi/a)$ form a band. For $q_z\neq 0$, each dispersion curve is quadratic in the long-wavelength limit ($ql_0\ll 1$). When ql_0 is sufficiently large, the band converges into the dispersion curve for a single layer. This is due to the fact that the RPA term becomes negligible for large ql_0, and therefore the modification of the structure factor is insignificant in that limit. At $qa=0$, the $q_z=0$ mode has a finite interception $E_{01}-\hbar\omega_c=2e^2/\epsilon a$ on the vertical axis. This seemingly unfamiliar result actually re-

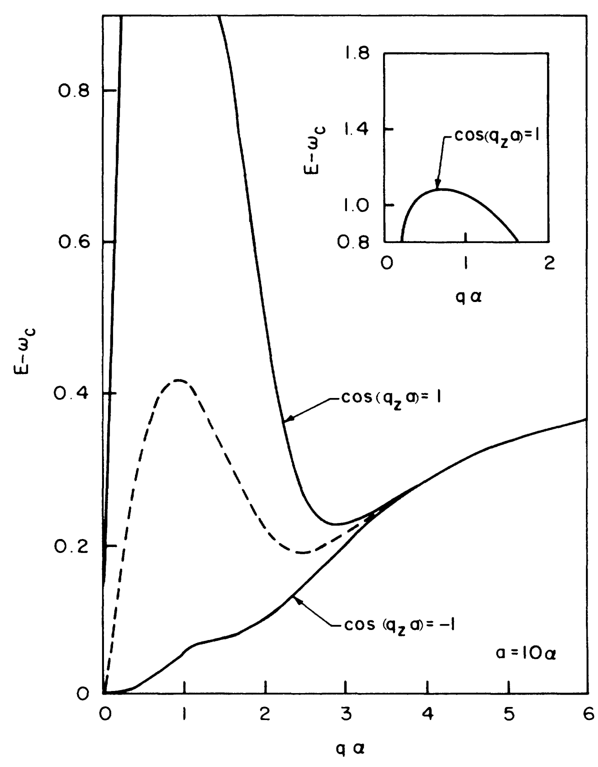

FIG. 3. Magnetic exciton dispersion with $a=10l_0$.

veals the equivalence of the system with a three-dimensional (3D) uniform electron gas with the same mean density. Note that $\nu=\nu_0=2$ implies $n_s=2/2\pi l_0^2$. Our result is

$$E_{01}(q=0,q_z=0)=\hbar\omega_c+2e^2/\epsilon l_0=\omega_c+\Omega_p^2/2\omega_c$$

with $\Omega_p^2=4\pi n_s e^2/m\epsilon a$ being the well-known 3D plasma frequency. This simply is an approximate expression of $E=(\omega_c^2+\Omega_p^2)^{1/2}$ to the accuracy of our many-body calculation.

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*Present address: Department of Physics, Indiana University, Bloomington, Indiana 47405.

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