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Nonequilibrium current fluctuations in semiconductors: A Boltzmann-equation—Green-function approach

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We present a method for calculating nonequilibrium current fluctuations (i.e., hot-electron noise) in semiconductors based on the Green function to the time-dependent Boltzmann equation. The method is valid for nondegenerate semiconductors for which the Boltzmann equation describes the normal (i.e., time-independent) transport properties. We illustrate the method in uniform systems within the relaxation-time approximation. We find that the noise can either increase or decrease from its equilibrium value, depending on the band structure and the energy dependence of the relaxation time.

I. INTRODUCTION

Hot-electron noise (i.e., nonequilibrium current fluctuations) in semiconductors is of great interest from both a fundamental and applied point of view. From a fundamental viewpoint, nonequilibrium noise provides information about a system that is not available from knowledge of the conductivity. This is not true in equilibrium where the "Callen-Welton-Kubo fluctuation-dissipation theorem" requires that the noise be proportional to the conductivity. From an applied viewpoint, many of the devices being developed operate under high electric fields and exploit nonequilibrium effects such as ballistic transport. A knowledge of nonequilibrium (hot-electron) noise is therefore essential to the design and performance of these devices.

In view of these facts, a method for calculating the nonequilibrium noise would be extremely useful. Presently, the fluctuations are calculated either from Monte Carlo simulations[1] or, by assuming that the distribution function is a heated, displaced Maxwellian and applying a generalized Einstein relation, which relates the noise to the electron temperature and the differential mobility.[2] In large electric fields, the distribution function can deviate substantially from Maxwellian and therefore the generalized Einstein relation is not applicable.

In this paper, we formulate a method for calculating the nonequilibrium noise based on the Green function for the time-dependent Boltzmann equation. The paper is organized as follows. The correlation functions and power spectra needed for describing the fluctuations are given in Sec. II and the method for calculating them is formulated in Sec. III. In Sec. IV, we illustrate the method with several examples, all within the relaxation-time approximation. The effect of a nonparabolic band structure is considered in Sec. V. Our results show that the nonequilibrium noise can either increase or decrease from its equilibrium Nyquist value depending on the band structure and the energy dependence of the scattering rate.

II. CORRELATION FUNCTIONS AND POWER SPECTRA

The current across a dc-biased semiconductor device fluctuates about its mean value $\langle I \rangle$. A measure of the current fluctuations $\delta I(t) \equiv I(t) - \langle I \rangle$ is the current-current correlation function

$$\Gamma(t_2,t_1) = \langle \delta I(t_2) \delta I(t_1) \rangle \ . \tag{2.1}$$

We consider only "stationary" fluctuations, that is, the correlation function depends only on the time difference $t = t_2 - t_1$:

$$\Gamma(t) = \langle \delta I(t) \delta I(0) \rangle \ . \tag{2.2}$$

If we assume that each electron in the sample fluctuates independently then the current-current correlation function (2.2), can be expressed in terms of the velocity-velocity correlation function for a single electron,

$$\Gamma(t) = \frac{Ne^2}{(2L)^2} \langle \delta v(t) \delta v(0) \rangle \ , \tag{2.3}$$

where N is the number of the electrons and $2L$ is the length of the sample.

Typically measured in a noise experiment, the "noise power spectrum" $S(\omega)$ is twice the cosine transform of the correlation function,

$$S(\omega) = 2 \int_{-\infty}^{\infty} dt \cos(\omega t) \Gamma(t) \ . \tag{2.4}$$

We will commonly refer to the noise power spectrum as the "noise."

The frequency dependence of the noise provides information about the fluctuations. Figure 1 shows a plot of three fluctuating signals and their corresponding power spectra, "white," $1/\omega^2$, and $1/\omega$. In addition to the frequency dependence the magnitude of the noise spectrum also provides information about the fluctuations. In this paper, the nonequilibrium spectra we calculate usually

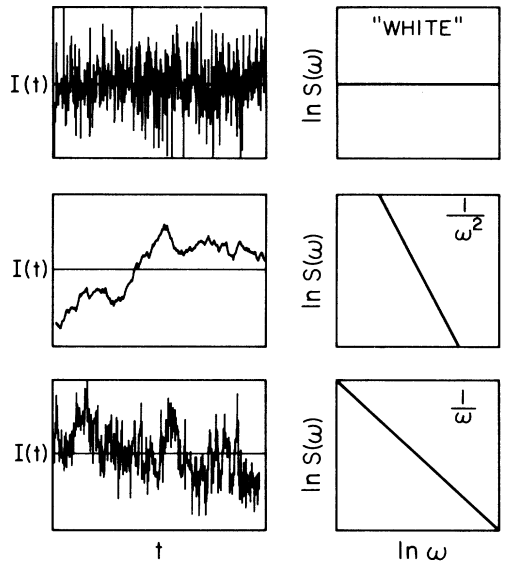

FIG. 1. Fluctuations and power spectra for three different random processes. The first signal has fluctuations which are completely uncorrelated in time and the resulting power spectrum is independent of frequency or "white" in analogy with white light. The second signal is extremely correlated and its power spectrum is $1/\omega^2$. The last signal has fluctuations which are not as uncorrelated as the first signal but less correlated than the second. This last spectrum is the famous $1/f$ spectrum ($f=\omega/2\pi$).

have a white power spectrum for frequencies below about 10^{12} Hz, and so we will be concerned primarily with the magnitude of the noise (below 10^{12} Hz) as a function of applied electric field and not the frequency dependence. We will call the noise the "low-frequency noise" since the frequency is low compared with the scattering rates but this should not be confused with $1/f$ noise which is also referred to as low-frequency noise and not considered in this paper.

III. A METHOD FOR DETERMINING NONEQUILIBRIUM CURRENT FLUCTUATIONS

Standard, time-independent transport properties in semiconductors such as velocity-electric field curves and thermopower are often calculated from the semiclassical Boltzmann transport equation[3–8] The current correlation function is a more complicated quantity since it involves time dependence. Still, we shall see that if the Boltzmann equation is valid for the time-independent quantities such as the velocity-field curve, then the current correlation function can be calculated by a Boltzmann-equation—Green-function approach.

A. The Boltzmann equation

The Boltzmann equation in nondegenerate semiconductors is usually written as[1,9]

$$D_t f(x,v,t)\equiv\left[\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}+\frac{eE}{m^*}\frac{\partial}{\partial v}\right]f(x,v,t)$$

$$=-\sum_{v'}[W_{v,v'}f(x,v,t)-W_{v',v}f(x,v',t)]\ , \qquad (3.1)$$

where $f(x,v,t)$ is the distribution function for the number of particles at (x,t) with velocity v. The left-hand side of Eq. (3.1) is the total derivative of f with respect to time t. Note that we use the convention that the electron has a charge $+e$. The second line of Eq. (3.1) is the collision integral. $W_{v,v'}$ is the transition rate for a collision *from* a state with velocity v *to* a state with velocity v'. The collision integral in Eq. (3.1) is valid for electron-phonon or electron-impurity scattering in nondegenerate semiconductors. For electron-electron scattering or degenerate semiconductors, a more complicated collision integral must be used.[9]

B. A method to determine the current fluctuations

The calculation reported here rests on the assumption that the individual electrons fluctuate independently. This assumption can break down in the presence of strong electron-electron scattering which introduces correlations between electrons.

Provided the above assumption holds, the current autocorrelation function then depends on the joint probability distribution for a single electron, $P(x_2,v_2,t_2\cap x_1,v_1,0)$, which gives the probability of finding an electron at $(x_1,t_1=0)$ with velocity v_1 and then having that electron arrive at (x_2,t_2) with velocity v_2. The joint probability can always be written as $R(x_2,v_2,t_2\,|\,x_1,v_1,0)P(x_1,v_1,0)$. Here $R(x_2,v_2,t_2\,|\,x_1,v_1,0)$ is the "conditional probability" (or "response function"), i.e., the probability of finding an electron at (x_2,t_2) with velocity v_2 *given* that the electron initially was at $(x_1,t_1=0)$ with velocity v_1. $P(x_1,v_1,0)$ is the "steady-state" probability of finding a particle at position x_1 with velocity v_1, that is, $P(x_1,v_1)$ is nothing more than the distribution function $f(x_1,v_1)$ (normalized to one since it is for a single particle) obtained by solving the time-independent Boltzmann equation (note that the steady-state distribution function is *not* the same as the equilibrium distribution).

The key point in calculating this conditional probability is in realizing that it is the solution to the full time-dependent Boltzmann equation subject to the initial condition

$$R(x_2,v_2,t_2=0\,|\,x_1,v_1,0)=\delta(x_1-x_2)\delta(v_1-v_2)\ . \qquad (3.2)$$

Once $R(x_2,v_x,t_2\,|\,x_1,v_1,0)$ is known, the current autocorrelation function, Eq. (2.3), can be calculated from

$$\Gamma(t)=\frac{e^2}{(2L)^2}\int_{-L}^{L}dx_1\int_{-L}^{L}dx_2\int_{-\infty}^{\infty}dv_1\int_{-\infty}^{\infty}dv_2\,v_1v_2R(x_2,v_2,t\,|\,x_1,v_1,0)f(x_1,v_1)\ . \qquad (3.3)$$

In Eq. (3.3), the sample is assumed to run from $-L$ to L, and the distribution function is normalized to the total number of particles,

$$\int_{-L}^{L} dx \int_{-\infty}^{\infty} dv\, f(x,v)=N\ , \tag{3.4}$$

but $R(x_2,v_2,t_2 \mid x_1,v_1,0)$ is normalized to one (since it is a probability for finding a *single* electron),

$$\int_{-\infty}^{\infty} dv_2 \int_{-\infty}^{\infty} dx_2\, R(x_2,v_2,t_2 \mid x_1,v_1,0)=1\ . \tag{3.5}$$

To summarize, the method for calculating the correlation function and noise is given by

(i) Calculation of the steady state distribution function, $f(x,v)$ the solution to the time-independent Boltzmann equation, Eq. (3.1) with $\partial f/\partial t=0$.

(ii) Calculation of the response function, $R(x_2,v_2,t_2 \mid x_1,v_1,0)$, from the solution to the full time-dependent Boltzmann Eq. (3.1) with the initial condition (3.2).

(iii) Calculation of the current autocorrelation function $\Gamma(t)$ from Eq. (3.3) and the noise power spectrum, $S(\omega,E)$ from Eq. (2.4).

C. Equivalence to other methods

Van Vliet and Fasset in their classic article[10] on transport noise calculate diffusion noise based on a Green-function approach for the diffusion equation. They show in the article that the Green-function approach to calculating noise is equivalent to a Langevin approach of calculating the noise. Their proof of the equivalence was based on an expansion of the Green function in terms of its eigenfunctions and was done for an arbitrary linear operator $L(\partial/\partial r,\partial/\partial t)$. This should also hold true for the Boltzmann-equation—Green-function method approach (where the linear operator also depends on $\partial/\partial v$) and thus the Boltzmann-Langevin approach[9,11] for calculating current fluctuations and the Boltzmann-equation—Green-function method should produce equivalent results. Numerous other authors have discussed the equivalence of Green function methods to Langevin equations.[12,13]

While the Boltzmann-equation—Green-function method of calculating noise and the Boltzmann-Langevin method of calculating noise may be formally equivalent, the Boltzmann-equation—Green-function method has two distinct advantages over the Boltzmann-Langevin method.

(i) To determine the noise with the Boltzmann-Langevin approach, it is necessary to solve a complicated integro-differential equation for the flucation δf as a *functional* of the fluctuating random force. This is a difficult task. In the Boltzmann-equation—Green-function method, the equation for the Green function is not a functional equation and can thus be solved numerically or in some approximation. It is unclear how to solve a functional equation numerically or how to make approximations in the functional Langevin equation. (In principle, once approximations for the Green function are known, they then can be converted into the Langevin formalism.)

(ii) The correlations for the random fluctuations in the Langevin formalism are given in terms of the transition probabilities $W_{v,v'}$. If an approximation is made for the collision integral in Eq. (3.1), it is unclear what the correlation of the random fluctuation in the Langevin formalism is. In the Boltzmann-equation—Green-function method, any approximation for the collision integral for the steady-state distribution f can be consistently incorporated into the equation for the Green function.

Since the time-dependent Boltzmann equation is used to determine the current fluctuations, the Boltzmann-equation—Green-function method of calculating current fluctuations should be equivalent to Monte Carlo methods of determining the current fluctuations.

IV. EXAMPLES FOR UNIFORM, BULK SYSTEMS

In this section, we apply the Boltzmann-equation—Green-function method of calculating current fluctuations to uniform, bulk semiconductors with no spatial dependence so that the $v\partial f/\partial x$ term drops out of Eq. (3.1).

To obtain a tractable problem, we use the relaxation-time approximation for the collision integral. The models are one dimensional although extension to three dimensions is not difficult (and the results are qualitatively the same).

A. A constant relaxation time model

We first solve a simple model where the relaxation time is a constant. This model is too simple to describe real devices, but has the advantage that it is analytically solvable and illustrates many important features.

We assume that the electrons move in a single parabolic band with an effective mass m^*, and approximate the Boltzmann equation by

$$\frac{eE}{m^*}\frac{\partial f(v)}{\partial v}=-\left[\frac{f(v)-f_{\rm eq}(v)}{\tau_0}\right]\ . \tag{4.1}$$

Here, the equilibrium function $f_{\rm eq}(v)$ is a Maxwellian normalized to the density

$$f_{\rm eq}(v)=\frac{n\exp(-\beta m^* v^2/2)}{[2\pi/(\beta m^*)]^{1/2}} \tag{4.2}$$

with $\beta=1/(k_B T_0)$, T_0 being the lattice temperature. This approximation for the collision integral is not arbitrary but takes into account that particle number (and current) is conserved in the collision process.

The collision integral in Eq. (4.1) represents an inelastic scattering mechanism since

$$\int_{-\infty}^{\infty} dv\, v^2 \frac{\partial f}{\partial t}\bigg|_{\rm col}\neq 0\ , \tag{4.3}$$

i.e., energy is not conserved in the scattering process. This is essential since the energy the electrons gain from the electric field must be removed for a steady state to be reached. In this case, the energy is removed by the phonons. The phonons dissipate the energy they gain from the electrons through the boundary of the material. If the phonons cannot dissipate energy fast enough they will begin to heat. Then the heating of the phonons must be taken into account and a coupled set of Boltzman equations,

one for the electrons and one for the phonons, must be solved. While "hot phonons" have recently been shown to be of importance in quantum-well structures,[14,15] we assume the phonons are in equilibrium.

The solution to (4.1) is straightforward.[16,17] Using an integrating factor, $\exp[\int dv(m^*/eE\tau_0)]$, the differential equation can be directly integrated

$$f(v)=\int_{-\infty}^{v} dv' \frac{m^*}{eE\tau_0} e^{-m^*(v-v')/(eE\tau_0)} f_{\rm eq}(v') = \frac{n}{2v_d} \exp\left[-\frac{v}{v_d}+\frac{1}{2}\left[\frac{v_{\rm th}}{v_d}\right]^2\right] \times \operatorname{erfc}\left[\frac{1}{\sqrt{2}}\left[-\frac{v}{v_{\rm th}}+\frac{v_{\rm th}}{v_d}\right]\right] . \quad (4.4)$$

Here $v_{\rm th}$ is the thermal velocity, $v_{\rm th}=(k_B T_0/m^*)^{1/2}$, and v_d is the drift velocity, $v_d=eE\tau_0/m^*$, and erfc is the complementary error function.[18] From the asymptotic behavior of the complementary error function for large argument, it can be seen that the distribution function goes to the equilibrium Maxwell-Boltzmann distribution in the limit $E\rightarrow 0$.

The distribution function $f(v)$ is plotted in Fig. 2 for several values of the electric field. In the figures throughout the paper, the distribution functions will be multiplied by $v_{\rm th}/n$ and plotted against $v/v_{\rm th}$. With this normalization convention, all distributions have unit area. Note: (1) The "high-velocity" tail decays as an exponential *linear* in the velocity, i.e., $f(v)\propto \exp(-m^*v/eE\tau_0)$. (2) The distribution is asymmetric; for *negative velocities*, $f(v)\propto \exp(-m^*v^2/2k_BT_0)$. (3) The width of the distribution function (electron temperature) increases with E and is given by

$$\langle (v-\langle v\rangle)^2\rangle = k_BT_0/m^* + (eE\tau_0/m^*)^2 .$$

Figures 3 and 4 show the exact distribution function $f(v)$ compared with two well-known approximations. Figure 3 shows the exact solution $f(v)$ for $v_d=2v_{\rm th}$ compared with a displaced Maxwellian, $f_{\rm DM}(v)$, which is a Maxwellian distribution with the same first two moments as the exact solution $f(v)$, i.e.,

$$f_{\rm DM}(v)=n\left[\frac{m^*}{2\pi k_B T_e}\right]^{1/2} e^{-m^*(v-\bar{v})^2/(2k_BT_e)} , \quad (4.5)$$

with

$$\bar{v}=\langle v\rangle=\frac{1}{n}\int_{-\infty}^{\infty} dv\, v f(v)\equiv v_d=\frac{eE\tau_0}{m^*} \quad (4.6)$$

and electron temperature

$$k_BT_e=\frac{m^*}{n}\int_{-\infty}^{\infty} dv (v-\bar{v})^2 f(v) . \quad (4.7)$$

The exact solution is asymmetric about its mean, peaks at a lower velocity, and decays faster in the negative velocity direction and slower in the positive velocity direction than the displaced Maxwellian. The equilibrium function (dotted line) is shown so that the increase in the width of $f(v)$ with electron field can be seen.

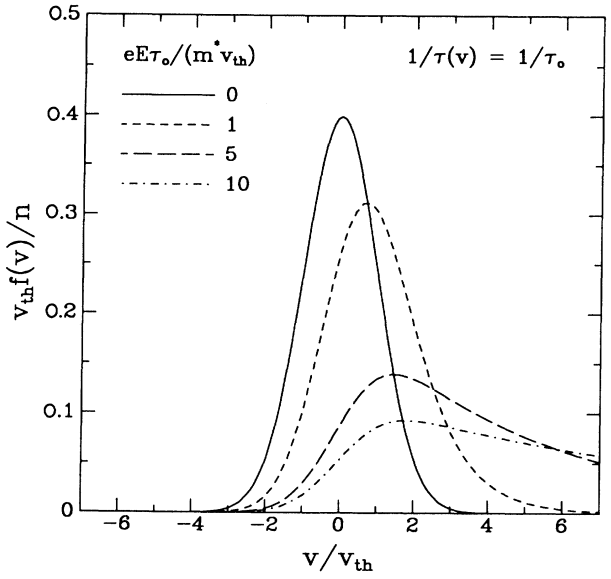

FIG. 2. The distribution function $f(v)$ for several values of the electric field. The distribution has several interesting features: (i) It is asymmetric about its mean. (ii) As the electric field increases, the number of electrons in the high positive velocity tail increases. (iii) The width of the distribution function increases as the electric field increases.

In Fig. 4, the exact distribution function $f(v)$ is compared with the "linear-in-E" solution $f_{\rm linear}(v)$. This solution is obtained by iterating Eq. (4.6) to linear order in E, and is given by

$$f_{\rm linear}(v)=f_{\rm eq}(v)-\frac{eE\tau_0}{m^*}\frac{\partial f_{\rm eq}(v)}{\partial v} . \quad (4.8)$$

As seen from the exact solution Eq. (4.4) $f(v)$ is nonanalytic in E so that an expansion in powers of E does not converge but is only asymptotic. For any finite E, the linear-in-E solution for the distribution function will actually become negative (which is unphysical) in the negative-velocity region. In spite of this shortcoming, the low-field current predicted by this approximation is still correct.

Current and Noise. The current is obtained by directly integrating over the distribution function. For the constant-τ model,

$$I=e\int_{-\infty}^{\infty} dv\, v f(v)=\frac{ne^2\tau_0}{m^*}E , \quad (4.9)$$

i.e., the current is strictly linear at all electric fields, which corresponds to an ohmic resistor. In realistic semiconductors, the scattering rates increase as a function of energy which leads to v-E curves which are nonlinear in E.

To calculate the noise, we solve the equation for the response function $R(v_2,t\mid v_1,0)$ with the relaxation time approximation for the collision integral. The solution is easily obtained by integration

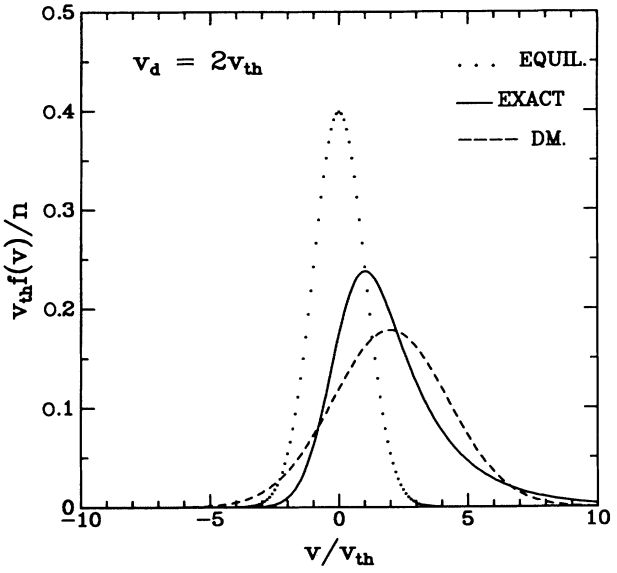

FIG. 3. The exact distribution function (solid line) compared to the displaced Maxwellian approximation (dashed line) for $v_d=2v_{\rm th}$. The displaced Maxwellian is a Maxwellian function with the same density, mean velocity, and effective temperature (variance) as the exact solution. The equilibrium distribution (dotted line) is shown for comparison.

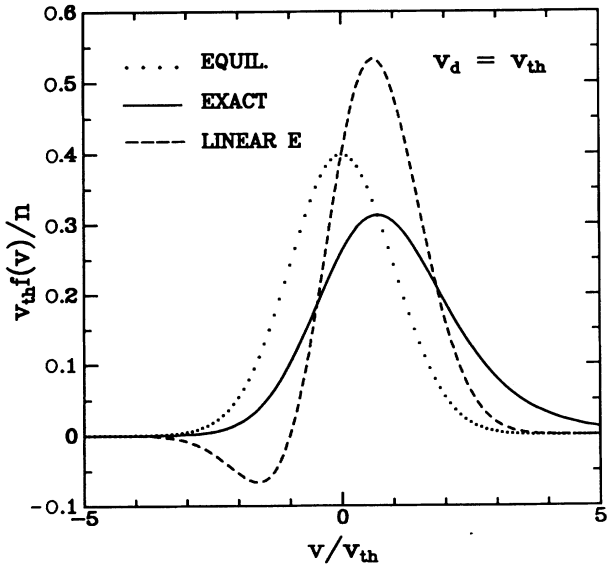

FIG. 4. The exact distribution function (solid line) compared to the linear-in-E approximation (dashed line) for $v_d=v_{\rm th}$. The linear-in-E solution becomes negative for large negative velocities no matter how small the electric field is. Since the exact distribution function is not analytic in E, an expansion of $f(v)$ in powers of E does not exist and the linear-in-E solution can only be an asymptotic expansion.

$$R(v_2,t\mid v_1,0)=\delta\left[v_2-\frac{eEt}{m^*}-v_1\right]e^{-t/\tau_0}+\int_0^t d\tilde{t}\,e^{-(t-\tilde{t})/\tau_0}\left[\frac{m^*}{2\pi k_B T_0}\right]^{1/2}\times\exp\left[-m\frac{[v-eE(t-\tilde{t})]^2}{2k_B T_0}\right]. \tag{4.10}$$

The current autocorrelation function is found from (3.3),

$$\Gamma(t)=N\left[\frac{e}{2L}\right]^2\langle(v-\langle v\rangle^2\rangle e^{-t/\tau_0}. \tag{4.11}$$

It decays exponentially from its $t=0$ value, $\Gamma(0)=N(e/2L)^2\langle(v-\langle v\rangle)^2\rangle$, with the correlation time being the relaxation time τ_0.

The noise, calculated from Eq. (4.11) by Fourier transformation, is

$$S(\omega,E)=\frac{4ne^2\tau_0}{1+(\omega\tau_0)^2}\left[\frac{A}{2L}\right]\langle(v-\langle v\rangle)^2\rangle, \tag{4.12}$$

with A being the cross-sectional area of the sample. Evaluating the variance, we obtain the final expression for the noise power spectrum

$$S(\omega,E)=\frac{4ne^2\tau_0}{1+(\omega\tau_0)^2}\left[\frac{A}{2L}\right]\left[\frac{k_B T_0}{m^*}+\left[\frac{eE\tau_0}{m^*}\right]^2\right]. \tag{4.13}$$

For $E=0$, the noise is the thermal Nyquist noise, as it must be. When $E\neq 0$, there are corrections to the thermal noise that vary as E^2. These corrections represent nothing more than the heating of the electron gas by the electric field. The formula for the noise could have been written as

$$S(\omega,E)=\frac{4ne^2\tau_0}{1+(\omega\tau_0)^2}\left[\frac{A}{2L}\right]\frac{k_B T_e}{m^*}, \tag{4.14}$$

with T_e the electron temperature. Applying an electric field causes the electron gas to heat and increase its variance. This increase in the variance leads to an increase in the noise.

The frequency dependence of the noise is Lorentzian. For frequencies $\omega\ll 1/\tau_0$ the noise is white (frequency independent), while for $\omega\gg 1/\tau_0$ the noise varies as $1/(\omega\tau_0)^2$. Typical τ_0 are in the range 10^{-13} sec so that the noise is white at the usual frequencies measured.

The current (solid line) and noise (dashed line) as a function of electric field are shown in Fig. 5 for a constant τ_0. The noise throughout this paper is normalized by $4k_B T_0/R_0$ where R_0 is a "characteristic resistance" given by $1/R_0=ne^2\tau_0 A/(m^*2L)$. The current is linear for all fields, while the noise is constant for small fields and varies as the square of the field for large fields.

The main results for a constant—relaxation-time model are as follows: (1) The distribution function (a) is nonana-

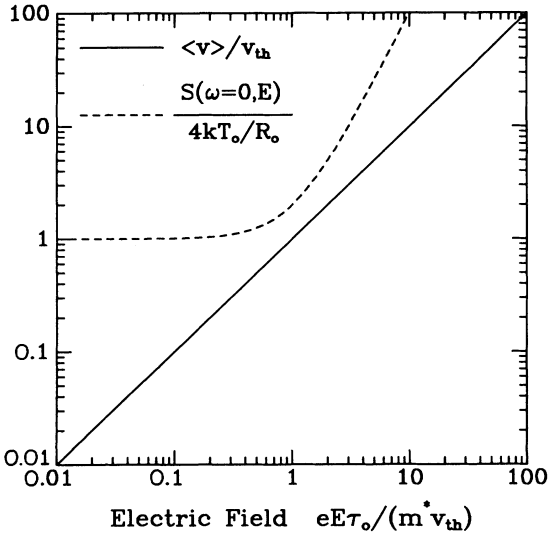

FIG. 5. The average velocity and noise versus the electric field for a constant relaxation time. The average velocity is given in units of $v_{\rm th}$ and the noise is given in units of $4kT_BT_0$ where R_0 is a characteristic resistance given by $1/R_0 = ne^2\tau_0 A/(2Lm^*)$ and the electric field is given in units of $m^*v_{\rm th}/e\tau_0$. The current is linear in the electric field for all values of E. The noise is constant and given by the thermal Nyquist relation for small E but increases quadratically for large E due to the heating of the electron gas.

lytic in the electric field, (b) is not a displaced Maxwellian, and (c) has a high positive velocity velocity tail that decays as $f(v) \propto \exp(-m^*v/eE\tau_0)$; (2) the current is linear for all values of the electric field; (3) the noise calculated by the Boltzmann-equation—Green-function method (a) has a Lorentzian frequency dependence, (b) for $E=0$, is thermal and satisfies the fluctuation-dissipation theorem, and (c) for $E\neq 0$, has an E^2 field-dependent correction which results from the increased width of the distribution function caused by the heating of the electrons by the electron field.

B. A constant—mean-free-path model

In the preceding section, we looked at a constant relaxation time model and found that the current was linear in E at all fields, and that there were quadratic in E corrections to the thermal noise due to the heating of the electrons by the electric field. Real semiconductors however have nonlinear velocity-field curves and the energy dependence of the scattering rates is important. In this section we look at energy-dependent relaxation-times and their effect on the distribution function, current, and noise. We find that a scattering rate that increases with energy tends to decrease the noise. This effect opposes the increase in noise due to the electron heating and therefore the noise versus electric field curve can be quite complicated, either increasing or decreasing with E.

The Boltzmann equation for an energy-dependent relaxation-time is written

$$\frac{eE}{m^*}\frac{\partial f(v)}{\partial v} = -\left[\frac{f(v)-A(E)f_{\rm eq}(v)}{\tau(v)}\right] . \tag{4.15}$$

Note the factor $A(E)$ before the equilibrium distribution function. $A(E)$ is determined by the constraint that the collision operator must vanish when integrated over v (this insures that the current continuity equation is obeyed), and implies that

$$A(E) = \frac{\int_{-\infty}^{\infty} dv\, f(v)/\tau(v)}{\int_{-\infty}^{\infty} dv\, f_{\rm eq}(v)/\tau(v)} . \tag{4.16}$$

For the constant relaxation time approximation of the first section, $A(E)=1$ for all values of E and so $A(E)$ was not explicitly in the equation. Note that $A(E=0)=1$ for any $\tau(v)$ since the collision integral must vanish for the equilibrium distribution function.

This model is formally equivalent to writing the transition rates in the full Boltzmann equation as $W_{v,v'}=S(v,v')f_{\rm eq}(v')$ where $S(v,v')=S(v',v)$ and choosing S to have a separable form,[17]

$$S(v,v') = \frac{[1/\tau(v)][1/\tau(v')]}{\overline{(1/\tau)}} ,$$
$$\overline{(1/\tau)} = \int_{-\infty}^{\infty} dv \frac{f_{\rm eq}(v)}{\tau(v)} . \tag{4.17}$$

The solution to this model is obtained by multiplying Eq. (4.15) by an integrating factor and then directly integrating the differential equation in much the same way the constant relaxation-time model was solved,[17]

$$f(v) = A(E)\int_{\infty}^{v} dv' \left[\exp\left[-\int_{v'}^{v} d\tilde{v}\frac{m^*}{eE\tau(\tilde{v})}\right]\right]\frac{m^* f_{\rm eq}(v')}{eE\tau(v')} . \tag{4.18}$$

The normalization of $f(v)$ is used to determined $A(E)$. The average velocity is then calculated from Eq. (4.6).

It is straightforward though tedious to solve the equation for the response function. The expression then obtained for the power spectrum is quite complicated. Details of the solution are outlined in the Appendix [where the problem is solved for a general band structure $\epsilon(k)$]. The resultant integrals are numerically evaluated in the low-frequency limit to give the results obtained here.

One of the simplest energy-dependent models is given by $1/\tau(v) = |v|/v_{\rm th}\tau_0$, which corresponds to the electrons having a constant mean free path given by $l = v_{\rm th}\tau_0$. Such a model could arise from having a constant matrix element and a $\sqrt{\epsilon}$ density of states. For this scattering rate, the integrals in Eq. (4.18) can be done analytically. The resulting expression for the distribution function is given by

$$f(v) = \begin{cases} \left[\frac{1-u^2}{1-u^3}\right]\frac{n\exp(-\beta m^* v^2/2)}{\sqrt{2\pi/(\beta m^*)}} , & v<0 \\ \left[\frac{1+u^2}{1-u^3}\right]\frac{n\exp(-\beta m^* v^2/2)}{\sqrt{2\pi/(\beta m^*)}} & \\ \quad -\frac{2u^2}{1-u^3}\left[\frac{ne^{-\beta m^* v^2/2u^2}}{\sqrt{2\pi/(\beta m^*)}}\right], & v>0 , \end{cases} \tag{4.19}$$

where

$$u^2=\frac{eE\tau_0}{m^*v_{\rm th}} . \tag{4.20}$$

The calculated distribution functions are shown in Fig. 6 for several values of the electric field. The features to note are the following: (1) The nonequilibrium distribution functions are still asymmetric about the mean and are not displaced Maxwellians. (2) The hot-electron tail, present in the constant relaxation time model is no longer present at low fields. At larger fields, there is a substantial amount of electrons with high velocities. In fact, from the analytic expression for $f(v)$, one sees that high-velocity behavior is given by $f(v)\propto \exp(-\beta m^* v^2/2)$ for small fields and by $f(v)\propto \exp(-v^2/2u^2)$ for large fields. This behavior is not too surprising. The energy dependence of the scattering rate is $\sqrt{\epsilon}$. For small energies, the change in scattering rate with change in energy increases rapidly and this makes it difficult for electrons to reach velocities in the tail before they scatter. At higher energies, the change in scattering rate now increases slowly with change in ϵ and more electrons attain higher velocities before scattering and thus the number of electrons with high velocities decays slower than at low fields. (3) There is structure at $v=0$. This structure is associated with the fact that the scattering rate goes to 0 at $v=0$.

The average velocity for this model is given by

$$\langle v\rangle=(2/\pi)^{1/2}\left[u^2\left[\frac{1-u^2}{1-u^3}\right]\right]v_{\rm th} , \tag{4.21}$$

with u given by Eq. (4.20). This is plotted in Fig. 7. For low fields, the average velocity is linear in the field while for large fields the dependence is $\sqrt{E}$,

$$\langle v\rangle=\begin{cases}\dfrac{2}{\sqrt{2\pi}}\left[\dfrac{eE\tau_0}{m^*}\right], & \dfrac{eE\tau_0}{m^*}\ll v_{\rm th}\\[2ex] \dfrac{2}{\sqrt{2\pi}}\left[\left[\dfrac{eE\tau_0}{m^*}\right]v_{\rm th}\right]^{1/2}, & \dfrac{eE\tau_0}{m^*}\gg v_{\rm th} .\end{cases} \tag{4.22}$$

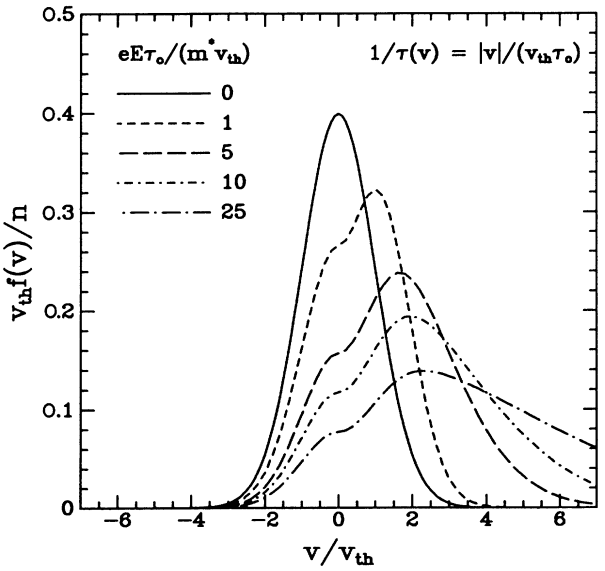

FIG. 6. The distribution function for several values of electric field for a scattering rate given by $1/\tau(v)=|v|/v_{\rm th}\tau_0$. The distribution functions are as follows, (i) asymmetric about their mean, (ii) do not have as long a high-velocity tail as the constant τ case at low fields, (iii) show structure at $v=0$ due to the fact that the scattering rate vanishes there. Note that as the electric field increases, the widths become wider.

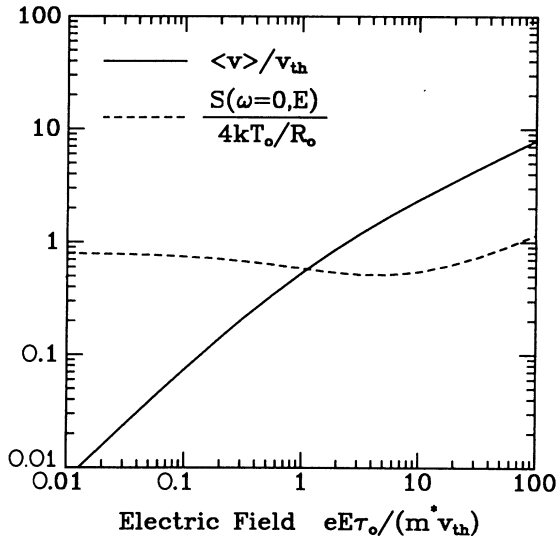

FIG. 7. Noise and average velocity for the constant mean-free-path model. The average velocity (solid line) is linear in the electric field for small values of the electric field but for large fields varies as the square root of the electric field. The noise (dashed line) decreases from its thermal equilibrium value at small electric fields but at large electric fields increases.

The noise for this model is plotted in Fig. 7 on a log-log plot. Note that the noise initially decreases and then finally increases. How can this occur if the electric field is heating up the electron distribution and causing the electron temperature to rise? The answer is that the noise depends not only on the electron temperature but also on the scattering rate. The low-frequency noise is the area under the autocorrelation function, i.e.,

$$S(\omega=0,E)=\int_{-\infty}^{\infty} dt\,\Gamma(t) . \tag{4.23}$$

The area under the autocorrelation function is roughly proportional to its height times its width. The height of the autocorrelation function is

$$\Gamma(0)=\langle(v-\langle v\rangle)^2\rangle , \tag{4.24}$$

i.e., the variance or temperature of the distribution function. As the electric field is increased, the electrons gain energy and the variance increases. This increases the height of the distribution and tends to increase the noise. The noise also depends on the width of the correlation function which is determined by the correlation time, i.e., the time on which an electron's velocity is correlated and is approximately given by the average collision time. If

the electron scattering rates increase as a function of energy, then when the electric field is increased, the average energy of the particles is increased and so is the average scattering rate. This means the correlation time decreases which causes the noise to decrease. Increased scattering rates also have a secondary effect which reduces the noise. A scattering rate that increases with electric field causes the variance or width of the distribution to increase more slowly with field than would a constant scattering rate and thus electron heating is smaller with scattering rates that are stronger functions of energy.

To summarize the main effects:

(i) Electric field increase $\Rightarrow$ heating or increased variance of the velocity distribution $\Rightarrow$ noise increase.

(ii) Electric field increase can increase the effective scattering rates $\Rightarrow$ correlation time decrease $\Rightarrow$ noise decrease.

(iii) Increased scattering rates $\Rightarrow$ reduced electron heating $\Rightarrow$ reduced noise.

C. An optic-phonon model

The final energy dependence we consider (more energy dependences are considered in Ref. 19) is a scattering rate given by

$$\frac{1}{\tau(v)}=\{1+K\theta[v^2-(Cv_{\rm th})^2]\}/\tau_0 . \tag{4.25}$$

Here, K and C are constants. This represents a scattering rate that is one constant below $Cv_{\rm th}$ and then another scattering channel opens and the scattering rate is another constant above $Cv_{\rm th}$. This could represent some type of optic-phonon scattering since an electron can always scatter by absorbing an optic phonon but can't scatter by emitting an optic phonon unless it has an energy greater then $\hbar\omega_0$, the optic-phonon energy. This type of scattering rate would most likely represent some sort of deformation-potential optic-phonon scattering since polar optic-phonon scattering highly favors forward scattering (small q transfer).

If we assume that the scattering rate in Eq. (4.25) is due to optic-phonon scattering, then K is equal to the ratio of the probability of scattering by optic-phonon emission to the probability of scattering by optic-phonon absorption. Since the probability of optic-phonon absorption is proportional to n (the number of optic phonons present) and the probability of emission is proportional to $n+1$, we see (ignoring density of states and matrix element factors) that K should be given by

$$K\approx\frac{n+1}{n}=e^{\beta\hbar\omega_0} . \tag{4.26}$$

The constant C is determined by the optic-phonon energy,

$$C^2=\frac{2\hbar\omega_0}{m^*v_{\rm th}^2} . \tag{4.27}$$

For Si, Ge, and GaAs, the optic-phonon energy is 63, 37, and 35 meV, respectively.[20]

The effect of this scattering rate on the distribution function is shown in Fig. 8. As an electron's energy becomes greater than the optic-phonon emission threshold, there is a steep change in the slope of the distribution function associated with the increase in the scattering rate due to optic-phonon emission. For high electric fields, a fair number of electrons have velocities greater than the threshold velocity and since the scattering rate above the threshold is constant, a high-velocity tail begins forming in the direction of the electric field.

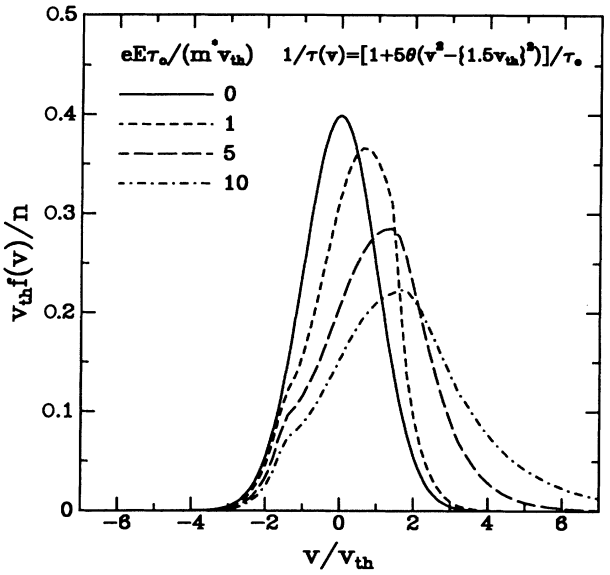

FIG. 8. The distribution function for an optic-phonon model. The slope of the distribution function changes discontinuously at the velocities $v=\pm\sqrt{2\hbar\omega_0/m^*}=1.5v_{\rm th}$ due to the fact that an additional channel for scattering now opens up—electrons can now scatter by emitting optic phonons. At high electric fields, the average scattering rate is no longer increasing with the electric field and a high velocity tail begins to appear.

The current and noise calculated numerically for this model are shown in Fig. 9. For simplicity, we take $C=1.5$ which corresponds at room temperature to an optic-phonon energy of about 56 meV, somewhere between Si and Ge. The constant K is chosen to be 5. If the scattering were due to just optic phonons, then according to Eq. (4.26), K should be about 9.4. K could be less than 9.4 if there were some other scattering mechanism (such as acoustic phonon scattering) which was also present. In the first graph, the current is plotted against the electric field on a linear scale. The current is linear in E at small fields, (with a slope of 0.567 as shown in the graph by the dashed line) increases less rapidly than linearly at intermediate fields, and then becomes linear again at high fields but with a smaller slope (the dashed line has a slope of $\frac{1}{6}$, which corresponds to an effective $\tau\approx 6\tau_0$) than it had at small fields. This behavior is shown more clearly in the second plot which is on a log-log scale and shows both the current and noise versus the electric field. It is seen that the current is linear at low fields and high fields but at intermediate regions, there is a crossover region where the slope of the two linear regions changes. The key feature of the optic phonon is this crossover region.

The noise is also shown on the log-log plot. For small fields, the noise is constant at a value given by the fluctuation-dissipation theorem (0.567, the value of the slope in the first linear region for the current). At intermediate fields, the noise decreases from its $E=0$ value due to the fact that the scattering rate abruptly increases as the threshold for optic-phonon scattering is reached. Beyond this threshold, the scattering rate becomes effectively constant and the noise begins to increase at high fields due to the heating of the electrons by the electric field. Thus we have an example of a scattering rate were the noise decreases at first due to increasing scattering rates but finally the rates saturate and the noise then begins to increase, due to heating.

V. NONPARABOLICITY

Here, we consider the effect of a nonparabolic band structure on the shape of the distribution function, the average velocity, and the noise. We find that for typical values of the nonparabolicity parameter α, the effect on the distribution function (in **k** space) is minimal both in equilibrium and in the presence of high electric fields. On the other hand, nonparabolicity can cause the current to saturate and the noise to decrease even if the scattering rates are constant. This later effect is associated with the fact that the energy is linear in k for large k; accordingly fluctuations in the k states do not lead to velocity fluctuations since the different k states have the same velocity.

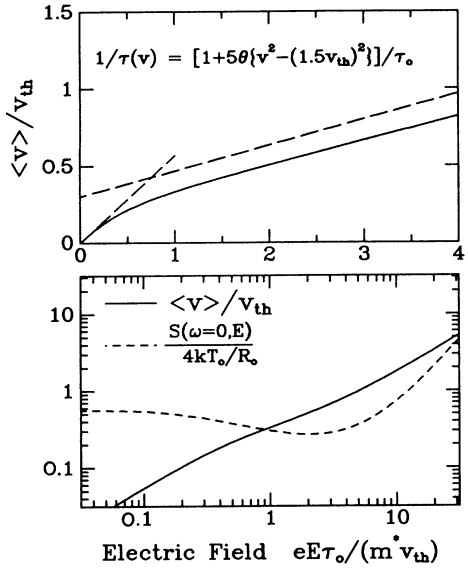

FIG. 9. The average velocity and noise for the optic phonon scattering rate. The top graph shows the average velocity plotted against electric field on a linear scale. At small fields, the average velocity is linear in the electric field with a slope of ≈ 0.567 (long-dashed line). At high fields, the average velocity is again linear in the electric field but the slope has decreased (the long-dashed line shows a slope of $\frac{1}{6}$, which corresponds to a scattering rate of $6/\tau_0$). The bottom graph shows the average velocity and noise on a log-log plot. The linearity of the average velocity for high and low fields can now be more easily seen. For the noise, we see an initial decrease in the noise from its thermal value at low fields. This decrease is associated with the fact that many of the electrons are accelerated by the electric field to velocities above the optic-phonon emission threshold and thus see a sudden sharp increase in the effective scattering rate. At high electric fields, most of the electrons are already above the optic phonon emission threshold, and therefore the scattering rate no longer increases with the electric field and the heating of the electrons causes the noise to increase.

Nonparabolicity is easily incorporated into the Boltzmann equation. Consider a band with a dispersion $\epsilon(k)$ (we take $\hbar=1$). For concreteness, we take

$$\epsilon(k)=[(1+2k^2\alpha/m^*)^{1/2}-1]/2\alpha \ .$$

For small k, the energy depends quadratically on wave vector while for large k the dependence is linear. Typical values of α are in the range of 0.5—1.0 $(\text{eV})^{-1}$.[11] In the figures, we give α in units of $(k_B T_0)^{-1}$ at room temperature, (i.e., in units of $\frac{1}{25}$ meV). The value of α is typically in the range of 0.01—0.02 in these units, so that the amount of nonparabolicity in real systems is less than is shown. The velocity is given by

$$v(k)=\frac{\partial\epsilon(k)}{\partial k}=\frac{k}{m^*(1+2k^2\alpha/m^*)^{1/2}} \ , \tag{5.1}$$

which is linear for small k but constant for large k.

The distribution function. The distribution function is now conveniently written as a function of k and not the velocity. The equation for the distribution function is the

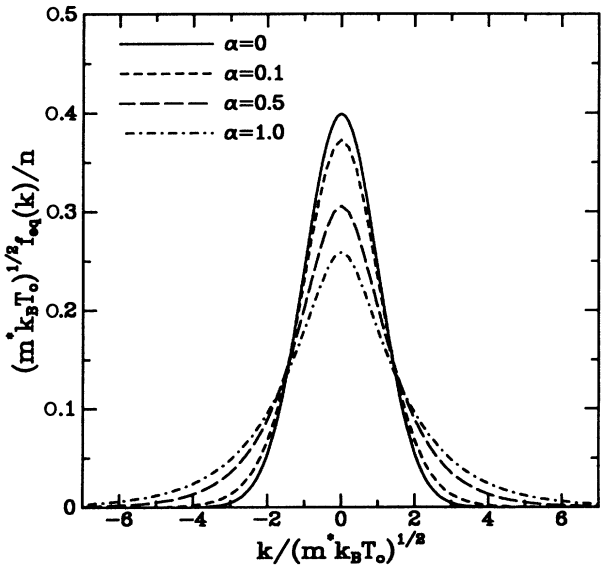

FIG. 10. The equilibrium distribution function $f_{\text{eq}}(k)$ for values of the nonparabolicity parameter α. The equilibrium distribution function is given by $f_{\text{eq}}(k)=Ce^{-\beta\epsilon(k)}$. The effects of nonparabolicity is seen to be a slight increase in the number of electrons with high-k values. This increase results from the fact that the energy is lower in the nonparabolic model than it is in the parabolic model and thus the higher k states are more populated. Realistic values of α are smaller than those given here and the actual differences between the nonparabolic and parabolic equilibrium functions is small.

same as Eq. (4.15) except that v is replaced by k/m and the equilibrium distribution function $f_{eq}(k)$, is no longer Gaussian but is

$$f_{eq}(k)=Ce^{-\beta\epsilon(k)} . \tag{5.2}$$

The constant C is determined by normalization. For $\epsilon(k)=(1/2\alpha)[(1+2k^2\alpha)^{1/2}-1]$, C is given by $n\sqrt{\alpha/2}\exp(1/2\alpha)/K_1(1/\alpha)$ with K_1 a modified Bessel function of the second kind. Figure 10 shows the equilibrium distribution function $f_{eq}(k)$.

The solution for $f(k)$ is clearly the same as (4.18) except that v is replaced by k/m^* and the equilibrium function is modified as described above. For typical α's, the effect on the resulting distribution functions, even for large electric fields is small.

Average velocity and noise. While nonparabolicity does not change the shape of the distribution function appreciably, it can change the average velocity and noise substantially. Since $v(k)=\partial\epsilon(k)/\partial k$, the current is given by

$$\langle v\rangle=\int_{-\infty}^{\infty}dk\frac{\partial\epsilon(k)}{\partial k}f(k) \tag{5.3}$$

and the noise is given by

$$S(w)=2e^2\int_0^{\infty}dt\cos(\omega t)\int_{-\infty}^{\infty}dk_2\int_{-\infty}^{\infty}dk_1\frac{\partial\epsilon(k_1)}{\partial k_1}\frac{\partial\epsilon(k_2)}{\partial k_2}R(k_2,t\mid k_1)f(k_1) . \tag{5.4}$$

These equations together with the expressions for $f(k)$ and $R(k_2,t\mid k_1)$ solve the problem of a k-dependent scattering time in a nonparabolic energy band $\epsilon(k)$. In general, the integrals cannot be done analytically even for a constant relaxation time and must be evaluated numerically. In the Appendix, we write down the equations for the current and noise in a simplified form from which the integrals can be numerically evaluated. The results for the simple constant relaxation time model with energy dispersion $\epsilon(k)=(1/2\alpha)[(1+2k^2\alpha)^{1/2}-1]$ are shown in Fig. 11. The average velocity is linear in the field for small field but saturates as the field increases.

The noise $S(\omega=0,E)$ is shown in Fig. 12 for the constant relaxation time model. In the parabolic case, the noise is thermal then increases as E^2 with an applied electric field. As the band becomes more nonparabolic, the noise decreases at large fields. The decrease is due to the fact that even though there are fluctuations in the different k states, the velocities associated with these k states are the same so that fluctuations in the k states do not lead to velocity fluctuations. For $\alpha=0.01$, the noise is thermal at low values, begins to increase with field due to the heating of the electrons and then decreases when the electrons get to the linear part of the band. As α is increased even further, the noise no longer increases above its thermal value because the heated electrons are in the linear part of the band. Finally, one should note the drop

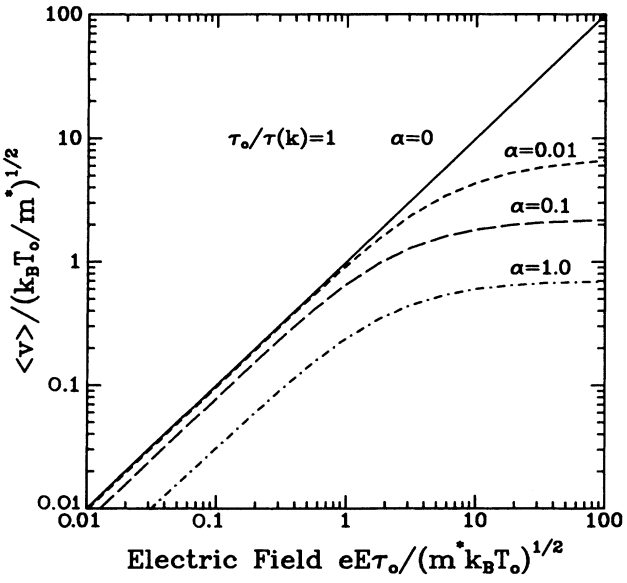

FIG. 11. Average velocity for a constant relaxation time in a nonparabolic band. The solid line corresponds to $\alpha=0$, the dashed line to $\alpha=0.01$ (in units of $1/k_BT_0$), the long-dashed line to $\alpha=0.1$ and the dotted-dashed line to $\alpha=1.0$. The most realistic value of α corresponds to the short-dashed line. Although for this value of α, the distribution function does not change appreciably, there is a significant change in the current associated with the velocity saturating in the linear part of the energy band.

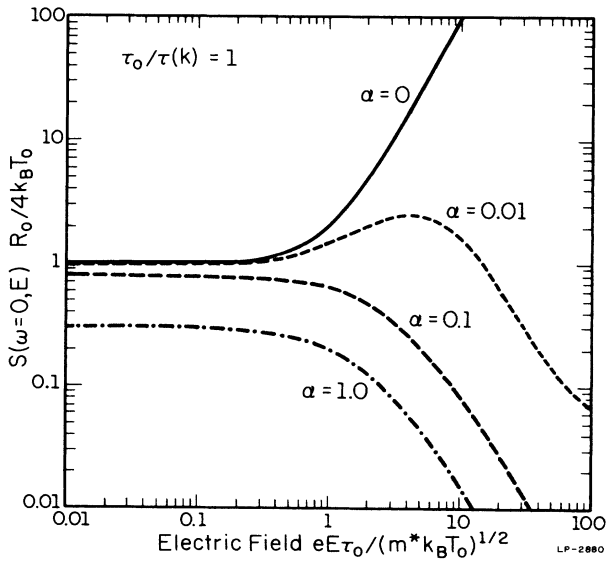

FIG. 12. The noise for a constant relaxation time in a nonparabolic band. As the electrons move up the band to the linear region, the velocity fluctuations are suppressed. This occurs not because there are fewer fluctuations between the various k states in the band but because these different k states have the same velocity.

in the low-field (thermal) noise as α increases. This results from the fact that the linear conductivity decreases as α increases as is evident from Fig. 11.

To summarize the results of this section, we have considered a k-dependent scattering rate in a nonparabolic band given by $\epsilon(k)$. For the special case $\epsilon(k)=(1/2\alpha)[(1+2k^2\alpha)^{1/2}-1]$, we found that the average velocity saturates as a function of the electric field and that the noise decreases at high fields because fluctuations in the k states no longer lead to velocity fluctuations.

VI. CONCLUSIONS

In this paper, we have formulated the Boltzmann-equation—Green-function method for calculating nonequilibrium current fluctuations and have demonstrated the applicability of the method to uniform, bulk semiconductors. We have calculated the distribution function, average velocity and noise in several relaxation-time models as a function of electric field.

Unlike the velocity-field curve which usually is a monotonic increasing function of the electric field, the noise can increase and decrease above its thermal equilibrium value when an electric field is applied. There are two nonlinear effects that change the noise from its thermal equilibrium value. The first is the heating of the electron gas by the electric field which increases the electric temperature and increases the noise. The second is the energy dependence of the scattering rates. Scattering rates that increase with energy decrease the correlation time and tend to decrease the noise as a function of electric field. Also, increasing scattering rates slow the heating of the electron gas.

We have also looked at the effect of band structure and nonparabolicity on the noise spectrum. There we found that a band that becomes linear at high k will lead to a saturation of the average velocity and a decrease in the noise spectrum. This decrease comes about not because there are no longer fluctuations in the various k states but because these fluctuations no longer lead to velocity fluctuations.

ACKNOWLEDGMENTS

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APPENDIX: EXPRESSIONS FOR THE NOISE AND AVERAGE VELOCITY

In this appendix we provide expressions for the average velocity and noise that are numerically evaluated to get the results in the text. We assume that the band structure is given by $\epsilon(k)$ so that the velocity depends on k through $v(k)=\partial\epsilon(k)/\partial k$. The current is given by a double integral

$$\langle v\rangle = A\int_{-\infty}^{\infty} dk \int_{-\infty}^{k} dk' \frac{1}{eE\tau(k')} v(k) \exp\left[-\int_{k'}^{k} d\tilde{k} \frac{1}{eE\tau(\tilde{k})}\right] f_{\rm eq}(k') \,. \tag{A1}$$

To obtain an expression for the noise, it is easier to deal with the equation for the response function in Laplace space. The equation for $\tilde{R}(k_2,s\mid k_1)$, the Laplace transformed Green function is

$$s\tilde{R}(k_2,s\mid k_1)+eE\frac{\partial\tilde{R}(k_2,s\mid k_1)}{\partial k_2} = -\left[\frac{\tilde{R}(k_2,s\mid k_1)-A(s,E;k_1)f_{\rm eq}(k_2)}{\tau(k)}\right]+\delta(k_2-k_1) \,. \tag{A2}$$

This equation can be solved by using an integrating factor,

$$\tilde{R}(k_2,s\mid k_1)=\int_{-\infty}^{k_2} dk' \frac{1}{eE\tau(k')} \exp\left[-\frac{s(k_2-k')}{eE}-\int_{k'}^{k_2} d\tilde{k} \frac{1}{eE\tau(\tilde{k})}\right][A(s,E;k_1)f_{\rm eq}(k')+\delta(k'-k_1)] \,, \tag{A3}$$

with $A(s,E;k_1)$ determined by integrating this equation over k_2 and using the normalization condition on $\tilde{R}(k_2,s\mid k_1)$. The result is

$$A(s,E;k_1)=\left[\frac{1}{s}-\int_{-k_1}^{\infty} dk' \frac{1}{eE\tau(k')} \exp\left[-\frac{s(k'-k_1)}{eE}-\int_{k_1}^{k'} d\tilde{k} \frac{1}{eE\tau(\tilde{k})}\right]\right] \times\left[\int_{-\infty}^{\infty} dk_2 \int_{-\infty}^{k_2} dk' \frac{1}{eE\tau(k')} \exp\left[-\frac{s(k_2-k')}{eE}-\int_{k'}^{k_2} d\tilde{k} \frac{1}{eE\tau(\tilde{k})}\right] f_{\rm eq}(k')\right]^{-1} \,. \tag{A4}$$

The expression for $A(t,E;k_1)$ is the inverse Laplace transform of Eq. (A4). In general, the inverse Laplace transform cannot be done analytically. If we are interested in the noise however, we do not have to invert the Laplace transform but can analytically continue the transform into real frequency ω. In doing this, we exploit the fact that the autocorrela-

tion function is an even function of time (for a stationary process), and take twice the real part of $\tilde{R}(k_2,s \mid k_1)$ evaluated at $s=i\omega$. The expression for the noise is then

$$S(\omega,E)=4\,\mathrm{Re}\left[e^2\int_{-\infty}^{\infty}dk_2\int_{-\infty}^{\infty}dk_1\,v(k_2)v(k_1)\tilde{R}(k_2,s=i\omega\mid k_1)f(k_1)\right]. \tag{A5}$$

From these equations, after a bit of algebraic manipulations, one obtains an expression for the noise

$$S(\omega,E)=\left\{\frac{1}{C^2+S^2}\left[\left[\frac{\langle v\rangle}{\omega}(V_cS-V_sC)\right]-C(V_cX_c-V_sX_s)-S(V_sX_c+X_cV_c)\right]\right.$$
$$\left.+A\int_{-\infty}^{\infty}dk_2\int_{-\infty}^{k_2}du\frac{1}{eE\tau(u)}v(k_2)f_{\mathrm{eq}}(u)\exp\left[-\int_u^{k_2}d\tilde{k}\frac{1}{eE\tau(\tilde{k})}\right]\left[\int_u^{k_2}dk'\frac{1}{eE}v(k')\cos\left[\frac{\omega(k_2-k')}{eE}\right]\right]\right\}, \tag{A6}$$

with A given by Eq. (4.16) and C, S, X_c, X_x, V_c, and V_s by

$$\left.\begin{matrix}C\\S\end{matrix}\right\}=A\int_{-\infty}^{\infty}dk\int_{-\infty}^{k}dk'\frac{1}{eE\tau(k')}\exp\left[-\int_{k'}^{k}d\tilde{k}\frac{1}{eE\tau(\tilde{k})}\right]f_{\mathrm{eq}}(k')\times\left\{\begin{matrix}\cos\left[\dfrac{\omega(k-k')}{eE}\right]\\ \sin\left[\dfrac{\omega(k-k')}{eE}\right]\end{matrix}\right\}, \tag{A7}$$

$$\left.\begin{matrix}V_c\\V_s\end{matrix}\right\}=A\int_{-\infty}^{\infty}dk\int_{-\infty}^{k}dk'\frac{1}{eE\tau(k')}\exp\left[-\int_{k'}^{k}d\tilde{k}\frac{1}{eE\tau(\tilde{k})}\right]f_{\mathrm{eq}}(k')v(k)\times\left\{\begin{matrix}\cos\left[\dfrac{\omega(k-k')}{eE}\right]\\ \sin\left[\dfrac{\omega(k-k')}{eE}\right]\end{matrix}\right\}, \tag{A8}$$

$$\left.\begin{matrix}X_c\\X_s\end{matrix}\right\}=A\int_{-\infty}^{\infty}dk\int_{-\infty}^{k}dk'\frac{1}{eE\tau(k')}\exp\left[-\int_{k'}^{k}d\tilde{k}\frac{1}{eE\tau(\tilde{k})}\right]f_{\mathrm{eq}}(k')\left[\int_{k'}^{k}d\tilde{k}\frac{v(\tilde{k})}{eE}\times\left\{\begin{matrix}\cos\left[\dfrac{\omega(k-\tilde{k})}{eE}\right]\\ \sin\left[\dfrac{\omega(k-\tilde{k})}{eE}\right]\end{matrix}\right\}\right]. \tag{A9}$$

In the low-frequency limit the noise is

$$S(\omega=0,E)=A\int_{-\infty}^{\infty}dk\int_{-\infty}^{k}dk'\frac{1}{eE\tau(k')}\exp\left[-\int_{k'}^{k}d\tilde{k}\frac{1}{eE\tau(\tilde{k})}\right]f_{\mathrm{eq}}(k')\left[[v(k)-\langle v\rangle]\left[\frac{\epsilon(k)-\epsilon(k')}{eE}-\langle v\rangle\frac{k-k'}{eE}\right]\right]. \tag{A10}$$

It is this quantity which is numerically evaluated to produce the results given in the text.

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