PHYSICAL REVIEW B VOLUME 35, NUMBER 17 15 JUNE 1987-I

Cyclotron-resonance linewidth due to electron-phonon interaction in multi-quantum-well structures

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(Received 4 November 1986; revised manuscript received 24 December 1986)

In the present paper we have calculated the cyclotron-resonance linewidth due to the electron—acoustic-phonon interaction in multi-quantum-well structures. The deformation-potential approximation is used to calculate the cyclotron-resonance linewidth. The present results show that the cyclotron-resonance linewidth increases with an increase of the temperature and magnetic field. The cyclotron-resonance linewidth decreases with an increase of the quantum-well width, at constant quantum-well separation. It also decreases with increase of the quantum-well separation at constant quantum-well width. The decrease of the cyclotron-resonance linewidth with increase of the quantum-well height and the Landau-level number has also been observed. The present calculations are in qualitative agreement with the existing experimental results for GaAs-AlAs.

I. INTRODUCTION

The electron- (hole-)phonon interaction plays a very important role in the transport and optical properties of two-dimensional (2D) systems. Prasad and the present author[1] have calculated the relaxation rates due to the electron-phonon interaction in single quantum wells in the presence of a magnetic field. We found that the relaxation rates depend upon the density of states. Some part of the theory has been used to explain the different experimental results for the quantum Hall effect[2] and the electron relaxation rate.[3] Recently the author and Chaubey[4] have developed a theory for the magnetophonon oscillation damping (MPOD) due to the electron—optical-phonon interaction in multi-quantum-well structures (MQWS's). The effect of screening on the MPOD due to electron—optical-phonon interaction has been investigated on the basis of the Thomas-Fermi mean-field approximation. The Landau-level width (LLW) due to the electron-phonon interaction in MQWS's has also been calculated by Chaubey and Singh.[5] The present author has also investigated recently the effect of the hole-phonon interaction on the transport and optical properties of MQWS's.[6] The effect of internal strains due to lattice mismatch on the hole-phonon interaction has also been calculated.[6].

There are experimental results on the cyclotron resonance in 2D electron systems in the literature.[7–10] The cyclotron-resonance linewidth (CRLW) has been measured in two-dimensional systems including modulation-doped MQWS's. The broadening in the cyclotron-resonance peak was attributed mainly to electron-impurity scattering. Ando[11] has developed a theory for the CRLW due to electron-impurity scattering in the 2D electron gas. This theory is widely used to explain the experimental results. However, we know that in moduation-doped MQWS's electron-impurity scattering does not exist because of the separation of the mobile electrons and the impurity ions in different materials. Therefore the CRLW in these materials cannot be attributed only to electron-impurity interaction. The CRLW in modulation-doped material at nonzero temperatures should be attributed mainly to the electron-phonon interaction. Even in non-modulation-doped MQWS's one should include the electron-phonon interaction in addition to the electron-impurity interaction. In these materials both scattering mechanisms compete with each other as found experimentally by Lin *et al.*[12] and their magnetic field dependence is the same.[5]

In this paper we turn our attention to calculating the CRLW due to the electron—acoustic-phonon interaction in MQWS's. The deformation-potential approximation is used to calculate the electron-phonon interaction and the self-consistent approximation has been used to calculate the CRLW. It is found that the CRLW is proportional to $\sqrt{B}$ and proportional to $\sqrt{T}$ in the high-temperature limit. A similar magnetic and temperature dependence is also reported by us[4] in Landau-level-width and relaxation-rate calculations.[5,6] The theory predicts that the CRLW decreases with an increase of the quantum-well width at constant quantum-well separation and vice versa. We found also the CRLW decreases with increases of the quantum-well height and the Landau quantum number. The present calculations are compared with the experiments on a qualitative basis. Numerical results are obtained for GaAs-AlAs MQWS's in the extreme quantum subband limit where the energy difference between two Landau levels is very small compared to the energy difference between the first and second subband energy. The present theory is going to be useful in understanding the magneto-optical experiments.

II. THEORY

We consider a the well width, b the well separation, and W the well height in the MQWS. The electron-phonon interaction Hamiltonian in the deformation-potential approximation in the presence of a static magnetic field B applied perpendicular to the layers (i.e.,

$\mathbf{B}\|\hat{\mathbf{z}})$ is given by[5]

$$H_{e-\mathrm{ph}}=\sum_{\lambda}\sum_{\lambda'}\sum_{\mathbf{Q}}\langle\lambda'|V(\mathbf{Q})e^{-i\mathbf{Q}\cdot\mathbf{R}}|\lambda\rangle a_{\lambda}^{\dagger}a_{\lambda}[b_{\mathbf{Q}}^{\dagger}+b_{-\mathbf{Q}}]\ , \tag{1}$$

where

$$|V(\mathbf{Q})|^2=(\hbar E_u v_s \mathbf{Q}/2c_L)^{1/2},\ \ |\lambda\rangle\equiv|n,X,p\rangle$$

is the Landau wave function. Here n, X, and p are the Landau quantum number, center of the cyclotron orbit, and the subband quantum number, respectively. $a_{\lambda}^{\dagger}$ and $b_{\mathbf{Q}}^{\dagger}$ are the creation operators, respectively, for an electron and a phonon, whereas a_{λ} and $b_{\mathbf{Q}}$ are annihilation operators, respectively, for an electron and a phonon. $\mathbf{R}(r_{\perp},z)$ is the position vector of an electron in the MQWS and $\mathbf{Q}=(q_{\perp},q_z)$, $q_{\perp}$ and q_z being the components of the phonon's wave vector along the layer and perpendicular to the layer, respectively. E_u, ρ, v_s, and c_L are the deformation potential, the density, the phonon velocity, and elastic constant, respectively.

Following the work in Refs. 13 and 14, we obtained the expression for the CRLW with the help of Eq. (1) for the cyclotron transition between the Landau levels $|n,p\rangle\rightarrow|n+1,p\rangle$ in the MQWS. We denote the CRLW for this transition by $\Gamma_{\mathrm{CR}}(n,p)$.

$$\Gamma_{\mathrm{CR}}(n,p)=\sum_{n',p'}\sum_{\mathbf{Q}}\sum_{\pm}|V(\mathbf{Q})|^2|F_{pp'}(q_z)|^2N_{\mathbf{Q}}^{\pm}\times\left[\left[\frac{\Gamma_{\mathrm{CR}}(n',p)\left[J_{nn'}J_{n'n}-\left[\frac{n'+1}{n+1}\right]^{1/2}J_{nn'}J_{n+1,n+1}\right]}{\left[E_{n'p'}-E_{n+1,p}+\omega\mp\omega_{\mathbf{Q}}+\Delta_{n'p'}\right]^2+|\Gamma_{\mathrm{CR}}(n',p')|^2}\right]+\left[\frac{\Gamma_{\mathrm{CR}}(n',p')\left[J_{n+1,n'}J_{n',n+1}-\left[\frac{n'}{n+1}\right]^{1/2}J_{n,n'-1}J_{n',n+1}\right]}{(E_{np'}-E_{n'p'}+\omega\mp\omega_{\mathbf{Q}}+\Delta_{n'p'})^2+|\Gamma_{\mathrm{CR}}(n',p')|^2}\right]\right], \tag{2}$$

where E_{np} are Landau-level energies written as[5] $E_{np}=(n+\frac{1}{2})\hbar\omega+\varepsilon_p$. ε_p is the energy of the pth subband in the MQWS. Δ_{np} is the line shift, ω is the laser radiation frequency, and $\omega_{\mathbf{Q}}$ is the phonon frequency. $J_{n'n}(x)$ are well-known functions and are given in Ref. 5. $x=(q_{\perp}^2l^2/2)$, where l is the Landau radius and $q=\sqrt{\hbar/eB}$.

$$F_{pp'}(q_z)=\langle\phi_{p'}|\exp(iq_z z)|\phi_p\rangle\ ,$$

and its value is given in Ref. 5. Here ϕ_p is the wave function of the pth subband in the MQWS. $N_{\mathbf{Q}}^{\pm}=(N_{\mathbf{Q}}+\frac{1}{2}\pm\frac{1}{2})$ and $N_{\mathbf{Q}}$ is the phonon distribution function.

In order to facilitate the calculation, it is useful to write $J_{n'n}(x)$ functions occurring in Eq. (2) in terms of the K functions,[14] where K functions can be expressed in terms of Laguerre polynomials.[5,6] The summation over p' in Eq. (2) has only one term $p'=p$, since the cyclotron transition occurs within one subband and one can neglect the interaction between the subbands. Most of the cyclotron experiments are performed in a single subband in the extreme quantum limit, i.e., $p=1$ where the above approximation is valid. For summing over n' we use the high-magnetic-field approximation, where the coupling between Landau levels is neglected. Therefore only resonant terms which are characterized by matrix elements with states of the same oscillator quantum number are retained. Hence, Eq. (2) becomes

$$\Gamma_{\mathrm{CR}}(n,p)=\sum_{\mathbf{Q}}\sum_{\pm}|V(\mathbf{Q})|^2|F_{pp}(q_z)|^2N_{\mathbf{Q}}^{\pm}\left[\frac{\Gamma_{\mathrm{CR}}(n,p)(K_{n,n}+K_{n,n+1})}{(\omega_c-\omega\pm\omega_{\mathbf{Q}}+\Delta_{np})^2+\Gamma_{\mathrm{CR}}^2(n,p)}\right]. \tag{3}$$

Putting the cyclotron resonance condition $\omega=\omega_c+\Delta_{np}$ and making an elastic scattering approximation in which we may neglect $\omega_{\mathbf{Q}}$ occurring in the energy denominator,[5] the above equation after replacing summation over $\mathbf{Q}$ by integration and $q_z=z/l$ is reduced to

$$\Gamma_{\mathrm{CR}}^2(n,p)=A\int dx\int dz(2x+z^2)^{1/2}[K_{nn}(x)+K_{n,n+1}(x)]|F_{pp}(z)|^2\left[\frac{\exp[\alpha(2x+z^2)^{1/2}]+1}{\exp[\alpha(2x+z^2)^{1/2}]-1}\right], \tag{4}$$

where

$$A=\left[\frac{8E_u\hbar}{\pi^2\rho v_s l^4}\right],\ \ \alpha=\left[\frac{\hbar v_s}{k_B T l}\right],$$

$$x_p=[2m^*(W-\varepsilon_p)/\hbar^2]^{1/2},\ \ k_p=(2m^*\varepsilon_p/\hbar^2)^2\ ,$$

$$|F_{pp}(z)|^2=\beta\left[\frac{\sin(zb/2l)}{z}+\frac{t_1\cos(zb/2l)+z(t_2-z^2)\sin(zb/2l)}{(t_3-z^2)^2+4x_p^2z^2l^2}\right]^2,$$

$$\beta=\left[4\cos^2\left[\frac{k_pa}{2}\right]e^{-(x_pb)}\left[\frac{ax_p(x_p^2+k_p^2)}{x_pa(x_p^2+k_p^2)+2k_p^2}\right]\right]^2,$$

$$t_1=2x_p(k_p^2+x_p^2)l^2,\quad t_2=(k_p^2-3x_p^2)l^2,\quad t_3=(x_p^2+k_p^2)l^2 .$$

We substituted the value of $F_{pp}(qz)$ from Ref. 5 in the above equation. In the high-temperature approximation we replace

$$(2N_Q+1)=[2N_Q(q_\perp,q_z)+1]\approx[k_BT/\hbar v_s(q_\perp+q_z)^{1/2}] ,$$

and the above equation becomes

$$\Gamma_{\mathrm{Cr}}^2(n,p)=\left[\frac{k_BTE_u^2}{2\pi^2c_Ll^2}\right]\int dx\int dq_z\,|F_{pp}(q_z)|^2(K_{nn}+K_{n,n+1}) . \tag{5}$$

Note that the integrations over x and q_z are separable. Equation (5) gives $\Gamma_{\mathrm{CR}}\sim\sqrt{B}$ and $\Gamma_{\mathrm{CR}}\sim\sqrt{T}$. Similar results have also been obtained for the LLW.[5]

III. RESULTS AND DISCUSSIONS

In this section we present the numerical calculation for the CRLW of a GaAs-AlAs MQWS. The conclusions drawn for this system are also applied to other MQWS systems. We considered the extreme subband quantum limit (i.e., $p=1$) throughout the following calculations. We used Eq. (4) for the calculation of the CRLW. The parameters used in the calculations are given. The results for the CRLW versus magnetic field for three temperatures, $T=10$ K, $T=40$ K, and $T=70$ K, are shown in Fig. 1 by curves A, B, and C, respectively, for the extreme quantum Landau limit ($N=0$).

Figure 1 shows that as the magnetic field increases, the CRLW increases. The CRLW is proportional to $\sqrt{B}$ as pointed out in Eq. (5). Recently Voison *et al.*[11] have measured the CRLW in modulation-doped GaAs-AlAs heterojunctions where the interaction between electron and impurity is reduced in the modulation-doped materials. They observed that the CRLW increases with an increase of the magnetic field and is proportional to $\sqrt{B}$. Their results are consistent with the present theory on a qualitative basis. As far as quantitative comparison between theory and experiments is concerned, the present theory gives a value for the magnitude about 20% to 30% less than the experimental results. There are also experimental results on other systems where the increase of the CRLW with magnetic field has also been observed. For example Brummel *et al.*[9] measured the CRLW as a function of the magnetic field in GaAs-InP and $\mathrm{Ga}_x\mathrm{In}_{1-x}\mathrm{As}$-$\mathrm{Al}_x\mathrm{In}_{1-x}\mathrm{As}$ heterojunctions and superlattices. They observed the increase of the CRLW with the increase of the magnetic field and it varies $\sim\sqrt{B}$.

Ando[11] also found the $\sqrt{B}$ dependence of the CRLW due to electron-impurity scattering systems. It means that both the electron-phonon and electron-impurity scattering mechanisms are competing with each other in nonmodulated doped systems too. The effect of this interplay between the two scattering mechanisms in zero magnetic field has been observed recently by Lin, Tsiu, and Weimann.[12] There are no such experimental studies in presence of magnetic field.

Figure 1 shows that the CRLW increases as temperature increases. At high temperatures the CRLW $\sim\sqrt{T}$ as pointed out in Eq. (5). The present results are con-

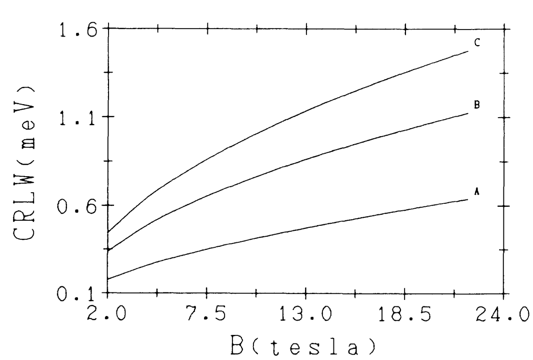

FIG. 1. The CRLW vs magnetic field. Curves A, B, and C are for the temperatures $T=10$ K, $T=40$ K, and $T=70$ K, respectively. The other parameters are $a=b=30$ Å, $W=0.12$ eV, and $N=0$.

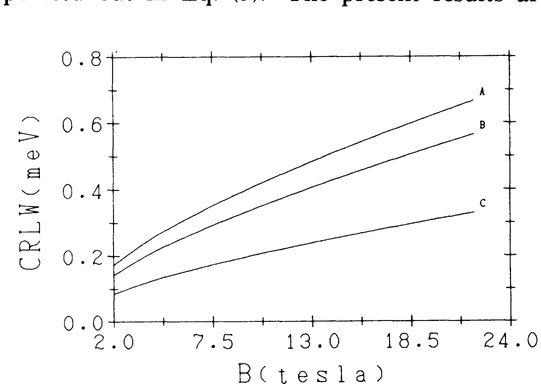

FIG. 2. The CRLW vs magnetic field. Curves A, B, and C represent the Landau level number $N=1$, $N=2$, and $N=3$, respectively. The other parameters are $a=b=30$ Å, $W=0.12$ eV, $B=10$ T, and $T=10$ K.

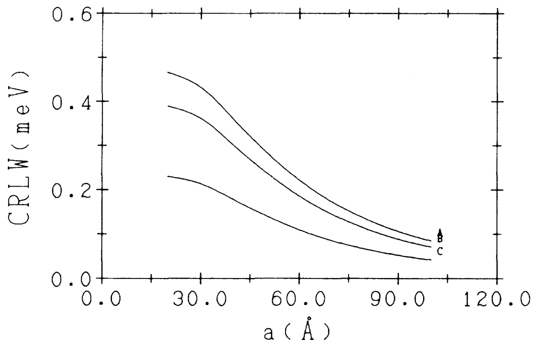

FIG. 3. The CRLW vs a for fixed $b=30$ Å. The curves A, B, and C are for $N=1$, $N=2$, and $N=3$, respectively. The other parameters are $B=10$ T, $T=10$ K, and $W=0.10$ eV.

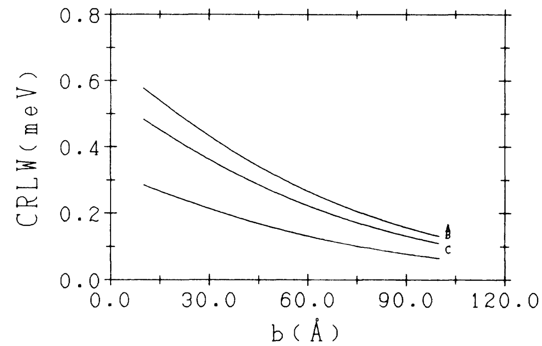

FIG. 4. The CRLW vs b for fixed $a=30$ Å. The curves A, B, and C are for $N=1$, $N=2$, and $N=3$, respectively. The other parameters are $B=10$ T, $T=10$ K, and $W=0.10$ eV.

sistent with the experimental results of Voison *et al.*[10] They found the increase of the CRLW with the increase of temperature. For example, the experimental value of the CRLW increased almost two times between temperatures 10 and 100 K. Theory gives also very similar temperature dependence.

Of course the variation of the CRLW with temperature does depend on the magnetic field too. The experimental results of Muro *et al.*[8] in GaAs-AlAs hetrostructures also indicate the increase of the CRLW with temperature.

We have also calculated the variation of the CRLW with N, a, b, and W. In Fig. 2 we presented the CRLW versus magnetic field for three Landau levels $N=1$, 2, and 3 which are represented by curves A, B, and C, respectively. The CRLW decreases with the increase of N for fixed magnetic fields. A similar result has also been found by Ando[11] for the electron-impurity scattering. In Figs. 3 and 4, we presented the CRLW versus a for fixed b and the CRLW versus b with fixed a, respectively. In both cases the CRLW decreases with the increase of the a and b, respectively. We will try to understand these results as follows.

The increase of the quantum-well width a, for fixed quantum-well separation b means moving slowly from 2D and 3D systems. The CRLW decreases with the increase of a, means the electron-phonon interaction is more important in 2D than in 3D. This is consistent with experimental predictions of Holenyak *et al.*[15] where they showed that electron-phonon interaction is more important in 2D than 3D. Increasing b for fixed a means the increasing of the distance between the quantum wells. The CRLW decreases with the increase of b. Increasing b means decreasing the interaction between the quantum wells. In other words we are considering the single-quantum-well system (SQWS) instead of MQWS. The theory predicts the enhancement of the CRLW in MQWS due to the interaction between the quantum wells with respect to SQWS. This point is supported by the other calculation of CRLW with the quantum well height W. The results are not presented here. The CRLW decreases with the increase of the quantum-well height.

In conclusion, we have presented here the theoretical and numerical calculations of CRLW in the MQWS due to electron-phonon interactions. The CRLW increases with the increase of B and T and decreases with the increase of a, b, and w. The theory is in qualitative agreement with the existing experiments. And even in some cases a quantitative agreement between theory and experiment is also found.

ACKNOWLEDGMENTS

The author is thankful to Dr. M. P. Chaubey for helpful discussions and to Professor J. Nuttall and his students for computational help. The author thanks The National Sciences and Engineering Research Council (NSERC) Canada for financial support.

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