Longitudinal magnetoresistance of potassium

M. L. Huberman
Department of Physics, Michigan Technological University, Houghton, Michigan 49931
(Received 5 January 1987)

Recently, Zhu and Overhauser showed that the Hall coefficient of potassium is anisotropic, depending on the angle between the applied magnetic field and the charge-density wave. It follows that the Hall coefficient of a polydomain sample is inhomogeneous. By means of effective-medium theory, the magnetoresistance of a domain structure has been evaluated. It is shown that both the longitudinal and transverse magnetoresistance increase with increasing field. The Kohler slope depends on the domain distribution. For a random distribution, the longitudinal and transverse Kohler slopes are about $\frac{1}{2}\%$ and $1\frac{1}{2}\%$, respectively.

I. INTRODUCTION

The nonsaturating magnetoresistance of potassium has been a long-standing puzzle.[1] All workers have found that both the longitudinal[2–4] and transverse[5] magnetoresistance increase linearly with field strength, showing no sign of saturation in the highest magnetic fields ($B \sim 100$ kG). Fear that this was an artifact caused by probes led to the development of inductive techniques,[6,7] which confirm the results of probe methods. The effect occurs in both polycrystalline and single-crystal samples. Although the nonsaturating magnetoresistance has been most extensively studied in potassium, it is observed in all the alkali metals.

The Kohler slope S is defined by

$$\Delta\rho = S\rho\omega_c\tau \,, \tag{1}$$

where $\Delta\rho$ is the resistivity change, ρ is the zero-field resistivity, $\omega_c = eB/mc$ is the cyclotron frequency, and τ is the electron relaxation time.[8] S is typically 10^{-3}–10^{-2} for both the longitudinal and transverse magnetoresistance. It depends upon sample preparation and history, varying by at least 50% for nominally identical samples.

It is especially hard to explain the nonsaturating longitudinal magnetoresistance. Semiclassical transport theorems[9] show that the longitudinal magnetoresistance of a homogeneous metal, whatever its Fermi surface, saturates when $\omega_c\tau > 1$. (Open orbits, which arise when the Fermi surface is multiply connected, cause the transverse magnetoresistance to increase, but do not affect the longitudinal magnetoresistance.[10]) Voids can cause a linear longitudinal and transverse magnetoresistance,[11] but the volume fraction required is known to be too high.[12,13] Surface imperfections can cause a linear transverse magnetoresistance,[14] but are not predicted to affect the longitudinal magnetoresistance (except as voids). A recent theory[15] of longitudinal and transverse magnetoresistance, based on magnetic-field corrections to the transport equation, has since been withdrawn[16] as an explanation for the linear magnetoresistance in potassium, since it requires that the Kohler slope depend strongly on temperature below 4.2 K, contrary to observation.[5]

The theory presented here is based upon the charge-density-wave structure of potassium,[17] which recently has been observed directly by neutron diffraction.[18] The new ingredient of this theory is the discovery[19] by Zhu and Overhauser of the Hall-resistivity relation in potassium. The proposed relation explains the amplitude and phase of the four-peak induced-torque patterns, observed in single-crystal spheres of potassium.[20,21]

The Hall coefficient is the ratio of the transverse, Hall electric field E_H to the product jB of the current density and the applied magnetic field (Fig. 1). In the usual, transverse Hall effect, **B** is perpendicular to **j**; in the longitudinal Hall effect, **B** is parallel to **j**.[22] Zhu and Overhauser found that both the transverse and longitudinal Hall coefficients of potassium are anisotropic, depending on the angle θ between **B** and the charge-density wave (CDW).

The CDW wave vector **Q** in potassium, determined by neutron diffraction,[18] is tilted 0.85° away from a [110] direction and lies in a plane rotated 47° away from the (001) plane. Because of the underlying cubic symmetry, there are 24 energetically equivalent **Q** directions, grouped in clusters of four about the six [110] axes. A single crystal is expected to be divided into **Q** domains, each having its **Q** along one of the 24 preferred axes. Since the Hall coefficient depends on the angle between **Q** and **B**, the Hall coefficient of a single crystal is then inhomogeneous.

It was first shown by Herring[23] that an inhomogeneous Hall coefficient produces a nonsaturating magnetoresistance. The inhomogeneity of the Hall electric field distorts the current paths, thereby increasing the magnetoresistance. Herring applied his theory to a material having a free-electron local conductivity, whose Hall coefficient varied due to fluctuations in carrier concentration. Then the transverse magnetoresistance increases indefinitely, but the longitudinal magnetoresistance saturates, since no Hall field arises when **B** is parallel to **j**. For a **Q**-domain structure, however, having the local conductivity proposed by Zhu and Overhauser, a nonuniform Hall field arises both when **B** is parallel to **j** and perpendicular to **j**, thus explaining the nonsaturating longitudinal and transverse magnetoresistance.

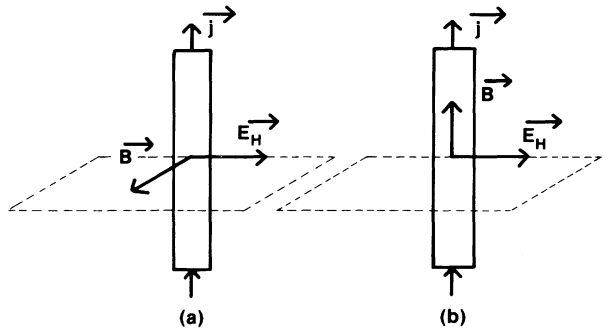

FIG. 1. Hall effect: (a) transverse and (b) longitudinal.

II. ANISOTROPIC HALL COEFFICIENT

The induced-torque anomalies in potassium require that the magnetoresistivity of a **Q** domain be[19]

$$\mathbf{E}=\rho[\mathbf{j}+t\omega_c\tau\mathbf{j}\times\hat{\mathbf{B}}+\alpha t\omega_c\tau(\hat{\mathbf{Q}}\cdot\hat{\mathbf{B}})\mathbf{j}\times\hat{\mathbf{Q}}] \ . \tag{2}$$

$\rho=m/ne^2\tau$ is the zero-field resistivity; t is the ratio of the Hall coefficient to the free-electron value, when **B** and **Q** are perpendicular; and α is the Hall-coefficient anisotropy. The new, third term is linear in **B** and even in $\hat{\mathbf{Q}}$. For simplicity, the zero-field resistivity anisotropy[24] and the open-orbit magnetoresistance[25] have been omitted. Since the component of **E** parallel to j in Eq. (2) is simply $E=\rho j$, the resistance of a **Q** domain is independent of magnetic-field strength and direction.

To evaluate the transverse Hall coefficient $R=E_H/jB$, let **B** be perpendicular to j. If $\hat{\mathbf{Q}}$ is perpendicular to **B**, the third term in (2) vanishes, yielding

$$R_\perp=tR_0 \ . \tag{3}$$

$R_0=-1/nec$ is the free-electron Hall coefficient. If $\hat{\mathbf{Q}}$ is parallel to **B**, the second and third terms in (2) add, yielding

$$R_\parallel=(1+\alpha)tR_0 \ . \tag{4}$$

In general, if $\hat{\mathbf{Q}}$ makes an angle θ with **B**, the Hall coefficient is

$$R=(1+\alpha\cos^2\theta)tR_0 \ . \tag{5}$$

From (3) and (4), the Hall-coefficient anisotropy, defined by $(R_\parallel-R_\perp)/R_\perp$, is equal to α.

In order to exhibit the longitudinal Hall effect, let **B** be parallel to j. For free electrons, the transverse electric field would vanish. If the relation (2) holds, however, this is not so. Taking $\hat{\mathbf{Q}}$ at an angle θ relative to **B** yields

$$E_H=\alpha t\rho\omega_c\tau j\sin\theta\cos\theta \ , \tag{6}$$

with the direction of the Hall field being parallel to $\hat{\mathbf{B}}\times\hat{\mathbf{Q}}$. The longitudinal Hall effect is a maximum when $\theta=45°$ and vanishes when $\theta=0°$ or 90°.

In helicon transmission experiments, Chimenti and Maxfield[26] found that the transverse Hall coefficient of single crystals of potassium depends on crystal orientation, R being about 4% larger than R_0 for **B** along a [100] direction and about 8% larger for **B** along a [110] direction. The induced-torque anomalies require that the Hall-coefficient anisotropy of a single **Q** domain be at least as large as 30%. $\alpha=0.3$ is therefore assumed. By Eq. (2), the Kohler slope and Hall coefficient of a domain structure scale with t, since **E** depends on $\omega_c\tau$ through the product $t\omega_c\tau$. For simplicity, $t=1.0$ is assumed.

III. EFFECTIVE-MEDIUM THEORY

The effective magnetoresistance of a CDW domain structure, having the local magnetoresistivity (2), is evaluated by means of the effective-medium approximation.[27–29] This is a mean-field type of approximation, which is expected to be valid when the domain size is greater than the electron mean free path but smaller than the sample dimensions. For $T<4.2$ K, the electron mean free path in high-purity potassium is about 0.1 mm; the domain size may vary, but is estimated[25] to be also about 0.1 mm.

The effective conductivity of a domain structure is then[25]

$$\overleftrightarrow{\sigma}_{\rm eff}=\overleftrightarrow{\sigma}_{\rm ext}+\left[\sum_{n=1}^{24}f_n(\overleftrightarrow{\sigma}_n-\overleftrightarrow{\sigma}_{\rm ext})\cdot\overleftrightarrow{\alpha}_n\right]\cdot\left[\sum_{n=1}^{24}f_n\overleftrightarrow{\alpha}_n\right]^{-1} \ . \tag{7}$$

f_n and $\overleftrightarrow{\sigma}_n$ are the volume fraction and conductivity of **Q** domain n. $\overleftrightarrow{\sigma}_{\rm ext}$ is the conductivity of the "host" surrounding each domain. $\overleftrightarrow{\alpha}_n$ is a tensor depending on $\overleftrightarrow{\sigma}_n$ and $\overleftrightarrow{\sigma}_{\rm ext}$

$$\overleftrightarrow{\alpha}_n=[\overleftrightarrow{1}-\overleftrightarrow{\Gamma}_{\rm ext}\cdot(\overleftrightarrow{\sigma}_n-\overleftrightarrow{\sigma}_{\rm ext})]^{-1} \ . \tag{8}$$

The tensor $\overleftrightarrow{\Gamma}_{\rm ext}$ depends only on the symmetric part $\overleftrightarrow{\sigma}^{(s)}_{\rm ext}$ of $\overleftrightarrow{\sigma}_{\rm ext}$, having the same principal axes. The eigenvalues Γ_i of $\overleftrightarrow{\Gamma}_{\rm ext}$, which satisfy the equation

$$\sum_{i=1}^{3}\lambda_i\Gamma_i=-1 \ , \tag{9}$$

are functions of the eigenvalues λ_i of $\overleftrightarrow{\sigma}^{(s)}_{\rm ext}$. If $\lambda_1<\lambda_2<\lambda_3$, then

$$\Gamma_1=-\frac{1}{(\lambda_2-\lambda_1)}\left[\frac{\lambda_2}{\lambda_3-\lambda_1}\right]^{1/2}(F-E) \ ,$$

$$\Gamma_3=-\frac{1}{(\lambda_3-\lambda_2)}\left[1-\left[\frac{\lambda_2}{\lambda_3-\lambda_1}\right]^{1/2}E\right] \ .$$

F and E are elliptic integrals of the first and second kinds, defined by

$$F(\delta,k)=\int_0^\delta(1-k^2\sin^2\phi)^{-1/2}d\phi \ ,$$

$$E(\delta,k)=\int_0^\delta(1-k^2\sin^2\phi)^{1/2}d\phi \ ,$$

having amplitude and modulus

$$\delta=\sin^{-1}(1-\lambda_1/\lambda_3)^{1/2} \ ,$$

$$k=\frac{(1-\lambda_1/\lambda_2)^{1/2}}{(1-\lambda_1/\lambda_3)^{1/2}} \ .$$

If $\lambda_1=\lambda_2<\lambda_3$, then $\Gamma_1=\Gamma_2$ and

$$\Gamma_3=-\frac{1}{\lambda_3-\lambda_1}\left[1-\frac{(\lambda_1/\lambda_3)^{1/2}\sin^{-1}(1-\lambda_1/\lambda_3)^{1/2}}{(1-\lambda_1/\lambda_3)^{1/2}}\right].$$

If $\lambda_1<\lambda_2=\lambda_3$, then $\Gamma_2=\Gamma_3$ and

$$\Gamma_1=\frac{1}{\lambda_3-\lambda_1}\left[1-\frac{(\lambda_3/\lambda_1)^{1/2}\sinh^{-1}(\lambda_3/\lambda_1-1)^{1/2}}{(\lambda_3/\lambda_1-1)^{1/2}}\right].$$

If $\lambda_1=\lambda_2=\lambda_3$, then $\Gamma_1=\Gamma_2=\Gamma_3=-\frac{1}{3}\lambda_3$.

The effective-medium approximation is obtained by requiring the conductivity $\vec{\sigma}_{\rm ext}$ of the host, surrounding each domain, to be just $\vec{\sigma}_{\rm eff}$. This self-consistent tensor,

$$\vec{\sigma}_{\rm eff}=\vec{\sigma}_{\rm ext}\ , \tag{10}$$

can be found easily by iteration of Eq. (7). Starting from a free-electron conductivity tensor, the solution typically converged after 20 iterations to within 0.1%.

IV. NONSATURATING MAGNETORESISTANCE

The domain structures of the potassium samples studied experimentally are not known. The domain distribution and size depend upon sample preparation and history, being affected, for example, by sample shape and size, surface orientation and facet size, etching procedure, annealing time, and thermal stress. The uncontrolled domain structure is responsible for the variation of the Kohler slopes for nominally identical samples.

In Fig. 2, the longitudinal magnetoresistance is plotted for a random domain distribution ($f_n=1/24$) for four field directions: **B** parallel to [100], [110], [111], and [123]. The longitudinal magnetoresistance is indeed nonsaturating. Even though the domain distribution is isotropic, the effective magnetoresistance is anisotropic, depending on field direction. The approximate Kohler slopes vary from about 0.3% to 0.6%.

In Fig. 3, the transverse magnetoresistance is plotted for a random domain distribution for the same four field directions and selected current directions. The transverse magnetoresistance is also nonsaturating and anisotropic. The approximate Kohler slopes vary from about 1% to 2%. This anisotropy of the high-field magnetoresistance has been seen in potassium.[7,30]

For an isotropic domain distribution, the transverse Hall coefficient R is isotropic. It is field independent for $\omega_c\tau>1$, having the calculated value $R=1.09R_0$. An inhomogeneous Hall coefficient is thus insufficient to explain the observed field-dependence of R.[30,31]

These calculations have been repeated with the Hall-coefficient anisotropy $\alpha=0.2$ and 0.4. The longitudinal and transverse Kohler slopes increase approximately in proportion to $\alpha^{4/3}$ and α^2, respectively. The deviation of the Hall coefficient from the free-electron value is proportional to α.

In order to illustrate the variety of possible behavior, the four domain distributions listed in Table I are considered. Each domain distribution is specified by the probabilities of the six domain groups corresponding to the six [110] axes; the four **Q** vectors within each domain group are assumed to occur with equal probability. The **Q** vectors in each distribution make approximately the same angle θ with the applied magnetic field. The magnetoresistance, plotted in Figs. 4 and 5, is strictly linear. The Kohler slopes and Hall coefficients are given in Table I.

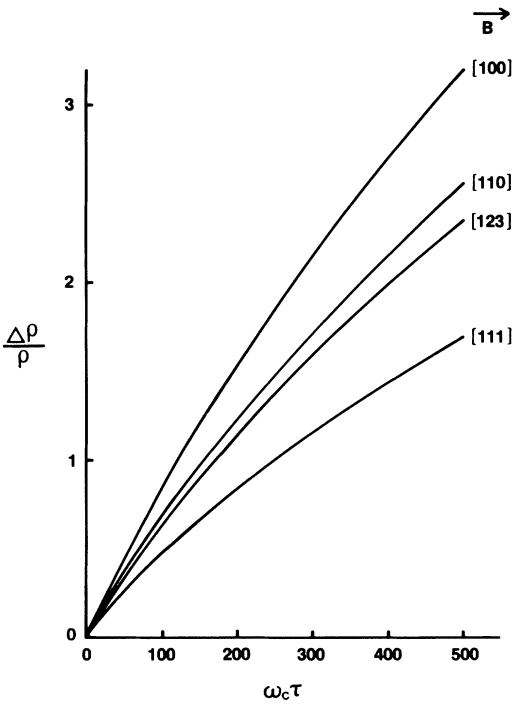

FIG. 2. Longitudinal magnetoresistance for a random domain distribution. Hall-coefficient anisotropy $\alpha=0.3$.

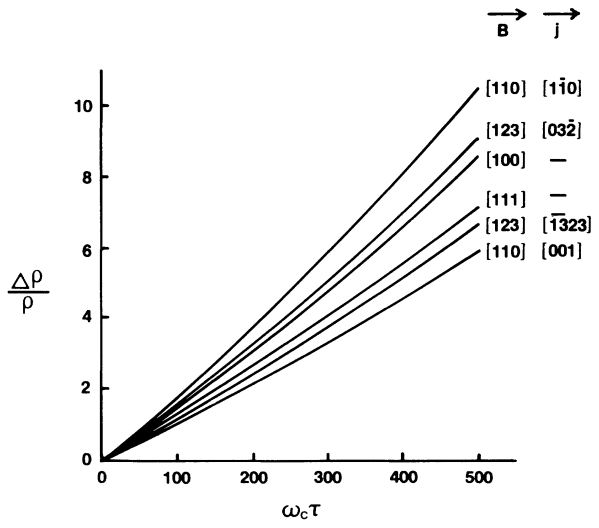

FIG. 3. Transverse magnetoresistance for a random domain distribution. Hall-coefficient anisotropy $\alpha=0.3$. The chosen current directions yield the maximum and minimum magnetoresistance. For **B** along the [100] or [111] directions, the magnetoresistance is the same for all perpendicular current directions.

TABLE I. Domain distribution, magnetic-field direction, angle θ between **Q** and **B**, longitudinal Kohler slope, current direction for the transverse magnetoresistance, transverse Kohler slope, and Hall coefficient. For I, III, and IV the transverse magnetoresistance is independent of the current direction.

	Domain-group probability											
	[110]	[1$\bar{1}$0]	[101]	[10$\bar{1}$]	[011]	[01$\bar{1}$]	**B**	θ	10^2S_L	j	10^2S_T	R/R_0
I	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	[100]	45°	1.5		1.5	1.15
II A	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	[110]	60°	1.2	[1$\bar{1}$0]	1.6	1.07
II B	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	[110]	60°	1.2	[001]	0.8	1.07
III	$\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	[111]	35°	1.3		1.3	1.20
IV	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$	[111]	90°	0		0	1.00

Open orbits, observed in potassium by the induced-torque method,[32,33] also contribute to the transverse magnetoresistance.[25] The magnitude of this effect is difficult to estimate, since the open-orbit electron fraction is unknown.

V. CONCLUSION

It is especially hard to explain a longitudinal magnetoresistance, since an electron in a magnetic field spirals about the field direction, its velocity parallel to the field being unaffected. An exotic mechanism is required. It has been shown that an inhomogeneous, longitudinal Hall coefficient produces a nonsaturating, longitudinal magnetoresistance. The calculated Kohler slopes agree with the observed values.

A dramatic test of this theory would be to field cool a potassium wire in a magnetic field large enough to induce formation of a single **Q** domain.[34] This theory requires that the longitudinal magnetoresistance would then disappear.

A nonsaturating longitudinal magnetoresistance is not unique to potassium and the other alkali metals, but also occurs in aluminum[35] and indium.[36] Since the phenomenon is still unexplained in these other metals, the theory presented here suggests that experimental searches be undertaken to determine whether they too have charge-density-wave ground states.

A microscopic theory of the Hall-coefficient anisotropy in potassium, caused by the CDW structure, is as yet undeveloped. This remains a challenging project for future research.

ACKNOWLEDGMENTS

It is a pleasure to thank A. W. Overhauser for advice and encouragement. Support by a grant from Research Corporation is gratefully acknowledged.

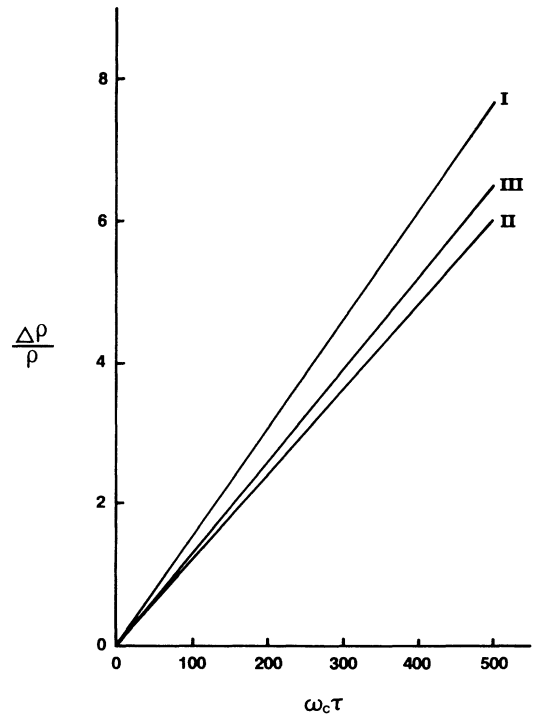

FIG. 4. Longitudinal magnetoresistance for the domain distributions in Table I. Hall-coefficient anisotropy $\alpha=0.3$.

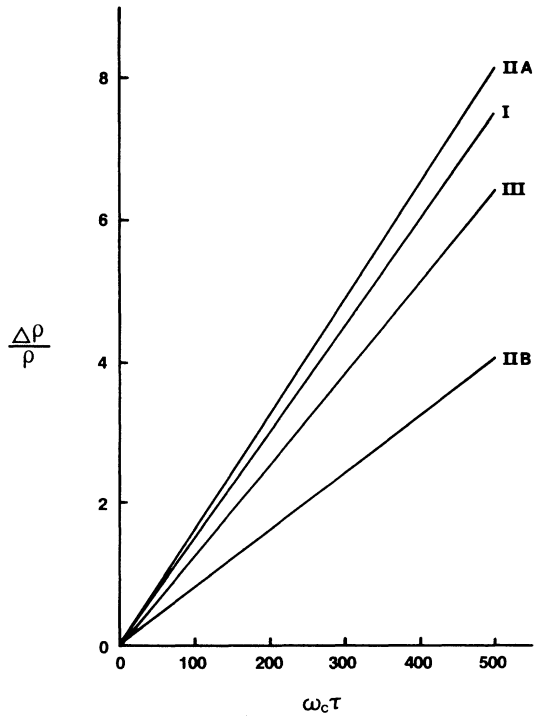

FIG. 5. Transverse magnetoresistance for the domain distributions in Table I. Hall-coefficient anisotropy $\alpha=0.3$.

[1]For two views of the galvanomagnetic properties of potassium, see R. Fletcher, Can. J. Phys. **60**, 679 (1982); and A. W. Overhauser, *ibid.* **60**, 687 (1982).

[2]B. K. Jones, Phys. Rev. **179**, 637 (1969); J. S. Lass, J. Phys. C **3**, 1926 (1970).

[3]A. M. Simpson, J. Phys. F **3**, 1471 (1973). The interpretation of this experiment may need revision when the Hall coefficient is anisotropic.

[4]R. Fletcher, Phys. Rev. Lett. **45**, 287 (1980).

[5]H. Taub, R. L. Schmidt, B. W. Maxfield, and R. Bowers, Phys. Rev. B **4**, 1134 (1971).

[6]F. E. Rose, Ph.D. thesis, Cornell University, 1965.

[7]P. A. Penz and R. Bowers, Phys. Rev. **172**, 991 (1968).

[8]For high-purity potassium, the residual resistivity is typically about 4000 times smaller than the room-temperature resistivity $\rho=7.2\ \mu\Omega\,\mathrm{cm}$, implying $\tau\approx 1.5\times 10^{-10}$ s for $T<4.2$ K. Assuming this value, $\omega_c\tau=50$ at $H=20$ kG.

[9]I. M. Lifshitz, M. Ia. Azbel, and M. I. Kaganov, Zh. Eksp. Teor. Fiz. **31**, 63 (1956) [Sov. Phys.—JETP **4**, 41 (1957)].

[10]This has been experimentally verified in Cu. See P. M. Martin, J. B. Sampsell, and J. C. Garland, Phys. Rev. B **15**, 5598 (1977).

[11]J. B. Sampsell and J. C. Garland, Phys. Rev. B **13**, 583 (1976); D. Stroud and F. P. Pan, *ibid.* **13**, 1434 (1976); K. D. Schotte and D. Jacob, Phys. Status Solidi A **34**, 593 (1976); M. R. Stinson, J. Phys. F **10**, L133 (1980).

[12]D. R. Schouten and C. A. Swenson, Phys. Rev. B **10**, 2175 (1974).

[13]S. A. Werner, E. Gurmen, and A. Arrott, Phys. Rev. **186**, 705 (1969).

[14]G. J. C. L. Bruls, J. Bass, A. P. van Gelder, H. van Kempen, and P. Wyder, Phys. Rev. B **32**, 1927 (1985).

[15]G. D. Mahan, J. Phys. F **13**, L257 (1983); J. W. Wu and G. D. Mahan, Phys. Rev. B **30**, 5611 (1984).

[16]G. D. Mahan, Phys. Rep. **145**, 251 (1987).

[17]For a theoretical and experimental review, see A. W. Overhauser, in *Highlights in Condensed Matter Theory,* Proceedings of the International School of Physics "Enrico Fermi," Course 89, Varenna, Italy, 1983, edited by F. Bassani, F. Fumi, and M. P. Tosi (North-Holland, Amsterdam, 1985), p. 194.

[18]T. M. Giebultowicz, A. W. Overhauser, and S. A. Werner, Phys. Rev. Lett. **56**, 1485 (1986).

[19]Xiaodong Zhu and A. W. Overhauser, Phys. Rev. B **30**, 622 (1984); *ibid.* **30**, 6864 (1984).

[20]J. A. Schaefer and J. A. Marcus, Phys. Rev. Lett. **27**, 935 (1971).

[21]F. W. Holroyd and W. R. Datars, Can. J. Phys. **53**, 2517 (1975); P. G. Coulter and W. R. Datars, J. Phys. F **14**, 911 (1984); M. Elliott and W. R. Datars, *ibid.* **13**, 1483 (1983).

[22]The longitudinal Hall effect has been observed recently in $SrAs_3$. See M. Mollendorf and W. Bauhofer, Phys. Rev. B **30**, 1099 (1984); W. Bauhofer, *ibid.* **32**, 1183 (1985).

[23]C. Herring, J. Appl. Phys. **31**, 1939 (1960).

[24]Marilyn F. Bishop and A. W. Overhauser, Phys. Rev. B **18**, 2447 (1978).

[25]M. L. Huberman and A. W. Overhauser, Phys. Rev. B **25**, 2211 (1982).

[26]D. E. Chimenti and B. W. Maxfield, Phys. Rev. B **7**, 3501 (1973).

[27]R. Landauer, J. Appl. Phys. **23**, 779 (1952).

[28]D. Stroud and F. P. Pan, Phys. Rev. B **20**, 455 (1979).

[29]M. L. Huberman and A. W. Overhauser, Phys. Rev. B **23**, 6294 (1981).

[30]M. J. O'Shea and M. Springford, Phys. Rev. Lett. **46**, 1303 (1981).

[31]P. A. Penz, Phys. Rev. Lett. **20**, 725 (1968).

[32]P. G. Coulter and W. R. Datars, Phys. Rev. Lett. **45**, 1021 (1980).

[33]P. G. Coulter and W. R. Datars, Can. J. Phys. **63**, 159 (1985).

[34]Mark L. Boriack, Phys. Rev. Lett. **44**, 208 (1980).

[35]F. R. Fickett, Phys. Rev. B **3**, 1941 (1971).

[36]J. C. Garland and R. Bowers, Phys. Rev. **188**, 1121 (1969).