Symmetry considerations and physical properties of the order parameter of the ^{3}He A-B interface

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The order parameter of the ^{3}He A-R interface is discussed in the light of a symmetry consideration, with particular attention paid to the separate role of spin and orbit spaces. The orientations of the order parameters of the bulk phases are shown explicitly. Four degenerate configurations, inter-related by time reversal and parity, are found. It is shown that the lack of symmetry within a given configuration leads to transport of transverse momentum across the interface when the phases are not in thermal equilibrium.

The physics of the ^{3}He A-B interface is now both of experimental[1,2] and theoretical[3–7] interest. Moreover, there is recent interest in understanding the A-phase texture obtained by slowly warming up from the B phase.[8,9] A knowledge of the orientation of the bulk order parameters at the interface will be the first step toward this understanding. Calculations have already been done by Cross[3] and Kaul and Kleinert (KK).[4] Unfortunately, neither of these works has included the dipole energy in their calculation, nor has considered completely and explicitly the relation between the spin and orbit spaces, and hence there is confusion about, for example, whether the Leggett configurations[10] in the bulk phases can both be satisfied. These questions shall be answered in the first part of this Brief Report. It will also be pointed out that there are four degenerate configurations for the interface, related to each other by time reversal and parity. Thus, there is a lack of symmetry (broken symmetry) within a single given configuration. An interesting consequence of this will also be considered. Some of these observations have already been mentioned in footnotes 15 and 16 of Ref. 5.

The order parameter of the A phase is defined by a triad $(\hat{\mathbf{w}}_1,\hat{\mathbf{w}}_2,\hat{\mathbf{l}})$ in orbital space and a vector $\hat{\mathbf{d}}$ in spin space. For the B phase we need a rotational matrix $R_{i\mu}(\hat{\boldsymbol{\omega}},\theta)$ relating the spin space (Roman indices) and orbit space (Greek indices) (here $\hat{\boldsymbol{\omega}}$ and θ are the rotational axis and angle, respectively), and the overall phase angle ϕ. The order parameter of the interface is obtained by minimizing the free energy. Since the overall phase angle of the entire system must be irrelevant, we shall hereafter choose $\phi=0$ for the B phase while rotating the triad $(\hat{\mathbf{w}}_1,\hat{\mathbf{w}}_2,\hat{\mathbf{l}})$ in the A phase appropriately.

First we note that the dipole interaction is weak compared to the other relevant energies. Therefore we can first minimize the free energy with this contribution dropped (and finally consider its effect by regarding it as a perturbation, see below). Since we now have complete rotation symmetry of the spin and orbit space separately, such a procedure will only lead to relations among vectors of the same subspace. Since $\hat{\mathbf{d}}$ is the only spin vector and $R_{i\mu}$ is the only other object that has a spin index, all relations can be expressed, in orbit space only, among the vectors $\hat{\mathbf{w}}_1,\hat{\mathbf{w}}_2,\hat{\mathbf{l}},\hat{d}^R{}_\mu\equiv d_iR_{i\mu}$, and $\hat{\mathbf{z}}$, the normal to the interface pointing from A to B. Both Cross and KK get, within their *Ansätze*, the relations

$$\hat{\mathbf{d}}^R=\hat{\mathbf{w}}_1=\pm\hat{\mathbf{z}}\ . \tag{1}$$

Hence $\hat{\mathbf{l}}$ is parallel to the boundary. Note, however, that at the present stage $\hat{\mathbf{l}}\cdot\hat{\mathbf{d}}$ and θ are not specified. Without loss of generality we choose $\hat{\mathbf{l}}=\hat{\mathbf{y}}$, and hence $\hat{\mathbf{w}}_2=\pm\hat{\mathbf{x}}$.

Next we consider the effect of the dipole interaction. It is well known that in the bulk phases this gives the Leggett configurations $(\hat{\mathbf{l}}\cdot\hat{\mathbf{d}})^2=1$ and $\theta=\cos^{-1}-\frac{1}{4}$.[10] We shall argue that these will not be destroyed by the presence of the interface. First, we note that the A phase always has the ambiguity of $\hat{\mathbf{d}}\to-\hat{\mathbf{d}}$, $\hat{\mathbf{w}}_1\to-\hat{\mathbf{w}}_1$, $\hat{\mathbf{w}}_2\to-\hat{\mathbf{w}}_2$. Because of the $\pm$ signs in (1) cases with $\hat{\mathbf{d}}=-\hat{\mathbf{l}}$ will not lead to new configurations.

Now we choose $\hat{\mathbf{l}}=\hat{\mathbf{d}}$ for the A phase. For (1) to be satisfied we need the rotation matrix to rotate $\hat{\mathbf{d}}=\hat{\mathbf{l}}$ to a perpendicular direction $\hat{\mathbf{w}}_1=\pm\hat{\mathbf{z}}$. But this is always possible for $\theta=\cos^{-1}-\frac{1}{4}$ by suitably choosing $\hat{\boldsymbol{\omega}}$, thanks to the fact that $\cos^{-1}-\frac{1}{4}>\pi/2$. Moreover, for each of the cases in (1), there are two possible choices for the rotational axis $\hat{\boldsymbol{\omega}}$. Explicitly,[5] for the + sign in (1),

$$R=\begin{pmatrix} 1/2 & \pm\sqrt{3/2} & 0\\ 0 & 0 & 1\\ \pm\sqrt{3/2} & -1/2 & 0\end{pmatrix}\ , \tag{2a,b}$$

corresponding to

$$\hat{\boldsymbol{\omega}}=\sqrt{3/5}\,\hat{\mathbf{x}}\pm(1/\sqrt{5})\hat{\mathbf{y}}\pm(1/\sqrt{5})\hat{\mathbf{z}}\ , \tag{3a,b}$$

and for the − sign in (1),

$$R=\begin{pmatrix} 1/2 & \mp\sqrt{3/2} & 0\\ 0 & 0 & -1\\ \pm\sqrt{3/2} & 1/2 & 0\end{pmatrix}\ , \tag{2c,d}$$

corresponding to

$$\hat{\boldsymbol{\omega}}=-\sqrt{3/5}\,\hat{\mathbf{x}}\pm(1/\sqrt{5})\hat{\mathbf{y}}\mp(1/\sqrt{5})\hat{\mathbf{z}}\ . \tag{3c,d}$$

We shall refer to these four configurations as a, b, c, d, respectively.

We may worry that, though now we have also minimized the dipole energy in the bulk phases far from the boundary, we have not done so in the distorted region of the interface itself. This is true. However, even in cases where the interface region favors some other relative orientation between the spin and orbit space, the system shall not adjust to this: To do so we would gain a free energy (per unit area) of order $g_D\xi$ [since the thickness of the boundary is of order of the coherence length $\xi(T)$]; however, the system would then recover only in the dipole healing length R_D, and hence loses an energy of order $g_D R_D$. Unless we are very close to T_c such that $\xi(T) > R_D$, the system will just orient itself in the manner discussed in the last paragraph.

It is interesting that the configurations a–d are related by symmetry operations. Since we shall also be interested in the transmission coefficient of the excitations across the boundary, we shall first mention some salient features of it. In a superfluid the excitations can be specified by the momentum direction $\hat{n}$, the energy E, the particle-hole index ($k \gtrless k_F$), and the spin index σ. A particle (hole) has group velocity parallel (antiparallel) to its momentum. Since the excitations have wave vectors $|\mathbf{k}| \approx k_F \gg 1/\xi$, during their scattering with the interface, the deviations of $\hat{\mathbf{n}}$ can be ignored to the lowest order of approximation.[11] For given $\hat{n}$ and E, with $\hat{n}_z > 0$ say, there are two types of excitations (four if also counting the spin) incident on the boundary, namely, particles from the A phase and holes from the B phase. It can be shown, with a properly chosen spin index, that they have the same transmission coefficient.[7] Hence the quantity, namely, the spin-averaged transmission coefficient, $\bar{T}(\hat{n},E)$, is well defined. We shall also discuss the properties of this $\bar{T}$ below.

Now we consider the symmetry operations. For definiteness we always start with the configuration a. Note that since a and b differ only in their relative spin-orbit rotation matrix R, they have the same $\bar{T}$ for given $\hat{n}$ and E. A similar statement holds for c and d. We first consider the time-reversal operator Θ,

$$\Theta a_{\mathbf{k}\sigma}\Theta^{-1} = e^{i\delta_\sigma} a_{-\mathbf{k},-\sigma}, \ \delta_\sigma = \frac{\pi}{2}\sigma ,$$

which reverses the momentum direction $\hat{\mathbf{n}}$, reverses $\hat{\mathbf{d}},\hat{\mathbf{l}},\hat{\mathbf{w}}_2$, changes $\phi=0$ to $\phi=\pi$, while keeping $\hat{\mathbf{w}}_1, R_{i\mu}$ unchanged. Since configurations a–d are defined with $\phi=0$ and $\hat{\mathbf{l}}=\hat{\mathbf{y}}$, for comparison we have to first perform a gauge transformation of π (which reverses $\hat{\mathbf{w}}_1$ and $\hat{\mathbf{w}}_2$) and a rotation of π about $\hat{\mathbf{z}}$. The last operation in particular changes R_a to $R_z^\pi R_a R_z^\pi = R_d$ and reverses the signs of the x and y momenta. Here R_z^π is rotation of π about the z axis, and the subscript a–d on R indicates which configuration it corresponds to as written in (2) and (3). Hence the time-reversed a (b) is d (c). It can also be shown that[7] the symmetry-related situation has the same $\bar{T}$ and, hence,

$$\bar{T}(\hat{n}_x,\hat{n}_y,\hat{n}_z;E)\,|_a = T(\hat{n}_x,\hat{n}_y,-\hat{n}_z;E)\,|_d \ . \tag{4}$$

Similarly we can consider the parity operation P,

$$P^+ a_{\mathbf{k}\sigma} P = e^{i(\pi/2)} a_{-\mathbf{k}\sigma} ,$$

which reverses $\hat{n}$ and keeps the order-parameter vectors unchanged but moves the $A(B)$ phase to the $z<0$ ($z>0$) side. To make a comparison we therefore need to rotate the whole system about $\hat{\mathbf{y}}$. The last operation reverses the signs of $\hat{n}_x$, $\hat{n}_z$ and changes R_a to $R_y^\pi R_a R_y^\pi = R_c$. Hence, the P operations change a (b) to c (d). Again, it can be shown that the symmetry operations leave $\bar{T}$ invariant and, hence,

$$\bar{T}(\hat{n}_x,\hat{n}_y,\hat{n}_z;E)\,|_a = \bar{T}(\hat{n}_x,-\hat{n}_y,\hat{n}_z;E)\,|_c \ . \tag{5}$$

Thus the operations Θ and P transform the configurations a–d among each other. Note also that these symmetry operations tell us nothing about $\bar{T}$ within a single given configuration (the symmetry of the "vacuum" is "broken").

Note that we do not have much restriction on $\bar{T}$. Recalling that $\bar{T}_a=\bar{T}_b$, $\bar{T}_c=\bar{T}_d$, (4) and (5) together imply $\bar{T}(\hat{n}_x,\hat{n}_y,\hat{n}_z)=\bar{T}(\hat{n}_x,-\hat{n}_y,-\hat{n}_z)$ within a single configuration. Supposing further that $\bar{T}$ is invariant under $\hat{n}_y \to -\hat{n}_y$ (see below),

$$\bar{T}(\hat{n}_x,\hat{n}_y,\hat{n}_z;E) = \bar{T}(\hat{n}_x,-\hat{n}_y,\hat{n}_z;E) \ . \tag{6}$$

Equations (4)–(6) then allow us to flip the signs of $\hat{n}_y,\hat{n}_z$ without affecting $\bar{T}$, but never the sign of $\hat{n}_x$. The transport across the interface is thus asymmetric with respect to the x direction.

Since the transmission probability depends on the sign of the x momenta, this raises the possibility of a transfer of x momentum across the interface when the two sides are not in thermal equilibrium. For simplicity, we assume (6) and, hence, $\bar{T}$ is independent of the signs of $\hat{n}_y$ and $\hat{n}_z$. For the excitations incident on the interface from A we parametrize them by the polar angle θ measured from the $\hat{\mathbf{y}}=\hat{\mathbf{l}}$ axis and azimuthal angle ϕ measured from the $+\hat{\mathbf{x}}$ axis as $(\hat{n}_x,\hat{n}_y,\hat{n}_z)=\pm(\sin\theta\cos\phi,\cos\theta,\sin\theta\sin\phi)$ for particles and holes, respectively ($n_z \gtrless 0$), $0 \le \theta,\phi \le \pi$. The rate at which x momentum is delivered from A to B is given by

$$\dot{P}_{A\to B} = 2N(0)\hbar v_F k_F \int_> \frac{d\Omega}{4\pi} \int_{|\Delta_A(\theta,\phi)|}^{\infty} dE\, f(E,T_A)\sin^2\theta\sin\phi\cos\phi(\bar{T}-\bar{T}') \ , \tag{7}$$

where the factor 2 comes from the spin sum, $N(0)$ is the density of states at the Fermi surface for one spin, the $>$ sign reminds us that Ω is restricted to the half-sphere as mentioned, f is the Fermi function, T_A the temperature of A,

$$\bar{T}(\Omega) = \bar{T}(\sin\theta\cos\phi,\cos\theta,\sin\theta\sin\phi)$$

and

$$\bar{T}'(\Omega) = \bar{T}(-\sin\theta\cos\phi,-\cos\theta,-\sin\theta\sin\phi).$$

For the excitations incident from B, we parametrize them as $(\hat{n}_x,\hat{n}_y,\hat{n}_z)=\pm(\sin\theta\cos\phi,\cos\theta,-\sin\theta\sin\phi)$ for particles and holes ($\hat{n}_z \gtrless 0$), respectively, thus again restricting θ,ϕ to the previously mentioned half-sphere. Using the fact that $\bar{T}$ is independent of the sign of $\hat{n}_z$, the quantity $\dot{P}_{B\to A}$ is exactly the same as in (7) except replacing $A \to B$ throughout. The force per unit area is $F_x = \dot{P}_{A\to B} - \dot{P}_{B\to A}$. If $T_A = T_B$, $F_x = 0$ as expected. However, if $\Delta T \equiv T_A - T_B \neq 0$, then we have

$$F_x = 2N(0)\hbar v_F k_F \int_> \frac{d\Omega}{4\pi} \int_{\Delta_{\max}}^{\infty} dE \sin^2\theta \sin\phi \cos\phi \times \frac{\partial f}{\partial T}(\bar{T} - \bar{T}')(\Delta T) \ , \quad (8)$$

giving a force across the interface if $\bar{T} \neq \bar{T}'$ (if $\bar{T}$ depends on the sign of $\hat{n}_x$).

For illustration we consider the case where the order parameter of the interface is a linear combination of the bulk phases

$$d_{i\mu}(z) = \lambda(z) d_{i\mu}^A + \kappa(z) d_{i\mu}^B \ . \quad (9)$$

$\lambda(-\infty) \to 1$, $\lambda(+\infty) \to 0$, $\kappa(-\infty) \to 0$, $\kappa(+\infty) \to 1$. It follows that, for suitable choice of ($\hat{\mathbf{n}}$-dependent) spin axis,[5]

$$\Delta^{\hat{\mathbf{n}}}(z) = \lambda(z)\Delta_A(-\hat{n}_x + i\hat{n}_z) + \kappa(z)\Delta_B[\pm(1-\hat{n}_z^2)^{1/2} + i\hat{n}_z] \ , \quad (10)$$

the sign depending on the spin direction. This presents a transparent case of why $\bar{T}$ depends on the sign of $\hat{n}_x$: this sign tells us how the phase angle of $\Delta^{\hat{\mathbf{n}}}(z)$ varies with z when we go from A to B phase. This, being like a superfluid current (along $\hat{\mathbf{z}}$ and different for each $\hat{\mathbf{n}}$), affects the transition coefficient. Thus, what we are discussing is a quantum-mechanical effect. In particular, it would vanish if the ballistics of Greaves and Leggett[12] were used, for then only the gap magnitude matters.[13] From (10) we also see that $\bar{T}$ is independent of the sign of $\hat{n}_y$.[14] Interestingly, the case (9) with the restriction $\lambda + \kappa = 1$ (KK)[4] can be shown to be a special case where $\bar{T} = \bar{T}'$ and $F_x = 0$ by pure mathematical coincidence.[7] For illustration we use

$$\lambda = 0.5[1 - \tanh(z/R_0)] + h \ ,$$

$$\kappa = 0.5[1 + \tanh(z/R_0)] + h \ ,$$

where $h = (a/2)e^{-z^2/b^2\xi^2}$. Setting $h = 0$ reduces to the KK *Ansatz*.[4] We shall take $R(T) = 1.267\xi(T)$, $a = 0.037$, $b = 1.98$ from minimization of the Ginzburg-Landau energy giving an energy $\approx 1\%$ lower than KK. For this order parameter we get, at melting pressure, for ΔT in mK and F_x in dyne/cm^2,

$$F_x = 0.43(\Delta T) \text{ at } T = 0.3\Delta_B \ ,$$

$$F_x = 0.05(\Delta T) \text{ at } T = 0.2\Delta_B \ .$$

The F_x for configurations b–d are identical.

To avoid misunderstanding a few comments will be made. The order parameter of the interface is a function of z only. Hence, when an excitation is scattered by the interface, its total momentum in the plane of the interface is conserved. However, the broken symmetry of the interface (and the associated supercurrents) causes an overall preferential transmission of excitations with positive x momenta over the negative ones. Therefore, when $T_A > T_B$ there is a delivery of x momentum from the normal component of the A phase to that of the B phase. The A-phase normal component acquires a negative x momentum. However, the total x momentum of the excitations is unchanged.

The order parameter used here is almost definitely not accurate enough to give a realistic estimate. However, assuming the above values a power of 10 μW per cm^2 will result in a force of order 10^{-2} dyne/cm^2. This effect may be extremely interesting to investigate experimentally.[15]

Note added in proof. Recently I became aware of a more accurate calculation of the order parameter of the interface.[16] This order parameter and that of KK fall in the same symmetry class, and the orientation of the order-parameter vectors is the same. The absence of some symmetry mentioned in this paper is likewise common to the results of Schopohl.[16] Hence, all the discussion in this paper is unaffected except (i) the intuitive explanations in Refs. 13 and 14 become less transparent, and (ii) the precise value of F_x is affected.

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[13]In this case sign flip of $\hat{n}_x$ in $\Delta^{\hat{\mathbf{n}}}$ can be compensated by the $\pm$ sign in (10); thus the spin-averaged $\overline{T}$ is unchanged. For the quantum-mechanical case this cannot be done because of the definite sign of the imaginary part of $\Delta^{\hat{\mathbf{n}}}$.

[14]Flipping the sign of $\hat{n}_z$ changes the direction of the "flow." However, it also changes an excitation moving from A to B to one moving from B to A. This provides an intuitative explanation of the independence of $\overline{T}$ on the sign of $\hat{n}_z$ mentioned earlier.

[15]In the realistic case both the heat flux and F_x are linear functions of temperature difference and normal velocity difference across the interface. See Ref. 7.

[16]N. Schopohl (unpublished).