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Critical temperatures of superconductors with low dimensionality

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The superconducting state of a two-dimensional electron gas in the presence of a subgroup with a high density of states (DOS) is studied. It is shown that in this system a strong electron-phonon coupling arises; in addition, an important role is played by two-dimensional plasmons. The theory is applied to a description of the properties of the recently discovered class of materials with high T_c. Their low dimensionality and the presence of groups with high DOS make these materials unique.

INTRODUCTION

The present Brief Report is concerned with the superconducting state of a two-dimensional electron system. In addition to its low dimensionality, the system we shall be considering will be assumed to contain an additional electron subgroup with a high density of states.

Recently, high-T_c superconductivity has been discovered in the compounds Ba-La-Cu-O and Sr-La-Cu-O.[1-4] The question arises of what makes this class of materials unique. It is thought that the following analysis of the properties of the two-dimensional (2D) electron system has a direct relevance to this question. The main features of these systems, such as high T_c, the possibility of sometimes observing resistance minimum at higher temperatures, and 2D fluctuations can be understood from a united viewpoint.

PHONON MECHANISM WITH STRONG COUPLING AND PLASMON MECHANISM

Consider a 2D electron system such as in a single layer of a layered structure or a size-quantizing film. The Fermi surface may have a complicated shape. If we consider an isolated 2D layer, then we should introduce a Fermi curve.[5] Interlayer transitions take place due to a small overlap of electronic wave functions. (This small overlap is important for stabilization of the superconducting state.) Then the 3D Fermi surface is an ellipsoidal structure with strong anisotropy.

Consider a special case when the 2D electron system contains a subgroup with a high density of states near the Fermi level. In this case, the Fermi curve has sections which are almost linear. The presence of such sections affects noticeably the phonon spectrum.

Hence our model represents a 2D electron system with the Fermi curve containing linear (or quasilinear) sections. Then the Fermi curve has a number of nesting states. As a result, a structural transition to the charge-density-wave (CDW) state becomes favorable. Such a transition (at some temperature T_p) caused by the presence of linear sections of the Fermi curve has been studied by the author in Ref. 5.

The presence of nesting states results in the appearance of a low-phonon mode with finite momentum. This softening, along with the opening of an energy gap near the linear sections (if $T_p > T_c$) may lead to the occurrence of a resistance minimum.[5] Note that there are many states which do not belong to the linear sections of the Fermi curve.

Electron states interact with the low-phonon mode, and the smallness of the phonon frequency makes this interaction strong. The large value of the constant λ_{ph} describing the electron-phonon coupling may result in a relatively high T_c. But the superconducting state of the 2D electron system is affected also by a nonphonon mechanism which can make a noticeable contribution (see below). This nonphonon mechanism is connected with the interaction of the electrons with the plasmon mode. As is known (see, e.g., the review, Ref. 6), plasmons in the 2D case are characterized by a peculiar feature. Namely, their dispersion relation does not contain an energy gap (as the momentum $q \to 0$) and can be written in the form

$$\omega = aq^{1/2} , \tag{1}$$

where $a = (2\pi Ne^2/m^* k)^{1/2}$. Here N is the surface carrier concentration, m^* is the effective mass, and k is the dielectric constant. If L is the thickness of the layer (or of the film), then $N = nL$, where n is the usual electron concentration.

The nonphonon mechanism is connected with the exchange of virtual plasmons; this exchange also contributes to the superconducting state. As is known, in the 3D case the plasma frequency ω_0 usually is very high. In the 2D case the situation is radically different. It can be shown that the major contribution to attraction is made by the frequency range $\omega \sim \Omega_l$, where $\Omega_{ph} \ll \Omega_l \ll \omega_0$ (Ω_{ph} is the characteristic phonon frequency). The existence of this range is guaranteed by the dispersion law (1).

The effect of the plasmon mechanism on the properties of inversion layers has been studied in Ref. 7 using the method developed in Ref. 8. We are going to use a different approach. It is based on the method of the temperature Green's function and on the generalized Eliashberg equation. As a result, one can study the contributions of both (phonon and nonphonon) mechanisms. It is thought that the influence of both mechanisms makes the considered 2D system unique. The superconductor studied in Refs. 1-4 is an example of such a system.

The order parameter $\Delta(\omega_n,\boldsymbol{\kappa})$ is described by the equation (see Ref. 9):

$$\Delta(\omega_n,\boldsymbol{\kappa})=\frac{T}{(2\pi)^2 Z}\sum_{\omega_{n'}}\int d\boldsymbol{\kappa}'\Gamma(\omega_n-\omega_{n'},\boldsymbol{\kappa}-\boldsymbol{\kappa}')\times F^+(\omega_{n'},\boldsymbol{\kappa}') \ . \quad (2)$$

Here $\boldsymbol{\kappa}$ is the two-dimensional electron momentum, $\omega_n=(2n+1)\pi T$,

$$F^+(\omega_n,\boldsymbol{\kappa})=\Delta(\omega_n,\boldsymbol{\kappa})[\omega_n^2+\xi^2+\Delta^2(\omega_n,T)]^{-1}$$

is the anomalous Green's function describing the pairing, ξ is the electron energy referred to the Fermi level, Z is the renormalization function (I shall not write out the equation for Z), and Γ is the total vertex.

The total vertex Γ is a sum,

$$\Gamma=\Gamma_{\rm ph}+\Gamma_{\rm pl} \ , \quad (3)$$

where

$$\Gamma_{\rm ph}=\lambda_{\rm ph}D(\omega_n-\omega_{n'},\boldsymbol{\kappa}-\boldsymbol{\kappa}') \ . \quad (4)$$

Here

$$D=\Omega^2[\Omega^2+(\omega_n-\omega_{n'})^2]^{-1}$$

is the phonon Green's function and $\lambda_{\rm ph}$ is the electron-phonon coupling constant. The quantity $\Gamma_{\rm pl}$ describes the electron-plasmon interaction. Consider, at first, the usual vertex $\Gamma_{\rm pl}(\omega,\boldsymbol{\kappa})$. As is known, the vertex $\Gamma_{\rm pl}(\omega_n,\boldsymbol{\kappa})$ in the temperature technique can be obtained by the replacement $\omega\rightarrow -i\omega_n$. The quantity $\Gamma_{\rm pl}(\omega,\boldsymbol{\kappa})$ can be calculated in random-phase approximation (RPA) and is a function of the parameter $s=\omega/\kappa v_F$. One can show (a detailed analysis will be given elsewhere) that there is a region $(\omega,\boldsymbol{\kappa})$ corresponding to the effective attraction. In this region, $\Gamma_{\rm pl}$ can be written in the form (cf. Ref. 10):

$$\Gamma_{\rm pl}=\tilde{V}_c+\tilde{V}_c D_{\rm pl} \ . \quad (5)$$

The first term describes the usual Coulomb repulsion, and the second term is the product of $\tilde{V}_c$ [note that in 2D, the "bare" Coulomb interaction has the form $V^0(\boldsymbol{\kappa})=2\pi e^2/\kappa$] and an effective D function:

$$D_{\rm pl}=\omega_{\rm pl}^2(\kappa)[\omega^2-\omega_{\rm pl}^2(\boldsymbol{\kappa})]^{-1} \ .$$

Effective attraction corresponds to the region $\omega<\omega_{\rm pl}$.

CRITICAL TEMPERATURE

Let us evaluate T_c in the presence of both the phonon and the plasmon mechanisms. We consider the case of weak electron-plasmon coupling (electron-phonon coupling can be strong). Speaking of the phonon mechanism, we consider the interaction with the low-frequency phonon mode $\Omega_{\rm ph}$. Equation (2) can be written in the approximate form (at $T=T_c$):

$$\Delta(\omega_n,\boldsymbol{\kappa})=\frac{\lambda_{\rm ph}m^*\pi T}{Z}\sum_{\omega_{n'}}\frac{\Omega_{\rm ph}^2}{\Omega_{\rm ph}^2+(\omega_n-\omega_{n'})^2}\frac{\Delta(\omega_{n'})}{|\omega_{n'}|}+\frac{\lambda_{\rm pl}m^*\pi T}{Z}\sum_{\omega_{n'}}\frac{\Omega_{\rm pl}^2}{\Omega_{\rm pl}^2+(\omega_n-\omega_{n'})^2}\frac{\Delta(\omega_{n'})}{|\omega_{n'}|} \ . \quad (6)$$

Here $\lambda_{\rm pl}=\tilde{V}_c\nu$, $\nu=m^*/(2\pi)^2$ is the density of states (see, e.g., Refs. 6 and 11), $\Omega_{\rm pl}$ is the characteristic plasmon frequency. One can show that in RPA $\Omega_{\rm pl}\simeq\beta^{1/2}\epsilon_F$, where $\beta=e^2/k\hbar V_F$ (k is the dielectric constant; in RPA $\beta\ll 1$). The major contribution to the electron-plasmon coupling comes from the short-wave region of the plasmon branch. We did not write out the usual term describing Coulomb repulsion. Let us point out that if a sample displays superconducting properties, this means that attraction overcomes Coulomb repulsion. In this case, every additional attraction mechanism will lead to an increase in T_c. This is how we view the role of plasmons.

T_c can be calculated by the Kellog method (see Ref. 12), by analogy with Ref. 13 (see also Ref. 14), and we obtain

$$T_c=1.14\Omega_{\rm ph}^{h_1}\Omega_{\rm pl}^{h_2}\exp\left[-\frac{1+\lambda_{\rm ph}}{\lambda_{\rm ph}+\lambda_{\rm pl}}\right] , \quad (7)$$

where $h_1=\lambda_{\rm ph}(\lambda_{\rm ph}+\lambda_{\rm pl})^{-1}$, and $h_2=\lambda_{\rm pl}(\lambda_{\rm ph}+\lambda_{\rm pl})^{-1}$.

Note that even if $\lambda_{\rm pl}$ is small, the plasmon mechanism can make a noticeable contribution. Indeed, expression (7) can be written in the form

$$T_c=T_c^{\rm ph}(\Omega_{\rm pl}/T_c^{\rm ph})^{h_2} \ , \quad (8)$$

or

$$T_c=T_c^{\rm ph}\exp[h_2\ln(\Omega_{\rm pl}/T_c^{\rm ph})] \ .$$

Here $T_c^{\rm ph}=1.14\Omega\exp(-1/\rho)$, and $\rho=\lambda_{\rm ph}(1+\lambda_{\rm ph})^{-1}$; $T_c^{\rm ph}$ is the critical temperature in the absence of the plasmon contribution (as was noted above, the Coulomb term μ can be introduced in the usual way).

One can see directly from Eq. (8) that the large magnitude of the ratio $\Omega_{\rm pl}/T_c^{\rm ph}$ may result in a noticeable increase of T_c with respect to $T_c^{\rm ph}$ even for relatively small values of $\lambda_{\rm pl}$. For example, if $\lambda_{\rm pl}=0.3$, $\lambda_{\rm ph}=2$, and $\Omega_{\rm pl}/T_c^{\rm ph}=15$, we obtain $T_c\simeq 1.4T_c^{\rm ph}$; that is, the plasmon mechanism is playing an essential role.

Let us estimate $\lambda_{\rm pl}$. One can see from Eq. (5) (see also, Ref. 6) that in the 2D case $\lambda_{\rm pl}\simeq e^2/k\hbar V_F$. The condition of the weak coupling corresponds to the applicability of RPA and the theory is thus self-consistent. The quantity $\lambda_{\rm pl}$ can be written in the form $\lambda_{\rm pl}=me^2/k\hbar^2(2\pi N)^{1/2}$. If, for example, $N\simeq 1.3\times 10^{15}$ cm^{-2} (e.g., $n=5.10^{21}$, $L\simeq 2.5\times 10^{-7}$ cm), $k=5$, we obtain $\lambda_{\rm pl}\simeq 0.5$. If $N=3.7\times 10^{15}$ cm^{-2}, then $\lambda_{\rm pl}\simeq 0.3$.

A decrease in electron concentration leads to an increase of $\lambda_{\rm pl}$ (see above; this has been noted in Ref. 7) and to a decrease of the pre-exponential factor. At the same time, the electron density of states in the 2D case does not depend on p_F (see above). Hence, the low dimensionality allows to obtain high T_c even for systems with relatively small electron concentration (and, consequently, with a small value of p_F).

We have studied the effect of the plasmon branch (1), which exists even for a single 2D electron group. Additional contribution can be due to the presence of other electron groups and the corresponding "demon" states (see, e.g., Ref. 15). This problem will be considered elsewhere.

Ba-La-Cu-O AND Sr-La-Cu-O SYSTEMS —CONCLUDING REMARKS

The appearance of a superconducting state with high T_c is determined in our model by two factors: (1) low dimensionality (2D electron system) and (2) the presence of an additional electron subgroup with a high density of states. As a result, the Fermi curve has sections which are almost linear. A high value of T_c is due to the strong interaction of the major set of electronic states with the low-phonon mode and to the contribution of the plasmon mechanism.

It is believed that the high values of T_c discovered in the systems Ba-La-Cu-O and Sr-La-Cu-O (Refs. 1–4) are determined by the factors described here. Of course, a detailed numerical analysis should be based on calculations of the band structure, considerations of hybridization, etc.

According to Ref. 16, the system possesses a 2D structure. The metallic state is due mainly to the presence of Cu atoms and to the oxygen deficiency. The presence of a subgroup with a high density of states (DOS) is due to the mixed-valence state of Cu (Ref. 17) or to the contribution of La atoms. Note that the recent measurements of the heat capacity[4] which indicate a large DOS are consistent with the picture described here.

The question of the resistance minimum is of interest. It has to be pointed out that its presence is not universal and is dependent upon the relationship between T_p and T_c. If $T_p > T_c$, there appears a minimum, due basically to the appearance of a gap on a portion of the Fermi line (see, e.g., Ref. 5). Apparently this is observed in Refs. 1 and 3. However, if $T_p < T_c$, the picture is different, although phonon softening affects the temperature dependence of the resistance. In this case, the presence of an electron group with high DOS is also favorable for intensifying the electron-phonon interaction. In both cases the appearance of low-phonon modes leading to strong coupling is essential.

These factors described above make the model above-developed applicable. All of the observed phenomena can be explained by the features of our model. T_c is determined by a number of parameters (see above). Varying them in the desired direction may result in further increase of T_c. It would be important to carry out electron tunneling and neutron scattering experiments. The plasmon peak, in our case, lies at a relatively low-frequency Ω_{pl}. This frequency would show up as an additional (compared to the neutron data) peak in the function $g(\Omega)=\alpha^2(\Omega)F(\Omega)$ [$F(\Omega)$ is the phonon DOS and $\alpha^2(\Omega)$ describes the electron-phonon interaction], obtained by means of tunneling measurements and inversion of the Eliashberg equation. This would be a manifestation of the plasmon mechanism.

Thus, the unique properties of Ba-La-Cu-O and similar compounds are related to the 2D character of their electronic systems and the presence of an electron subgroup with a high DOS. The high T_c is determined by the action of both mechanisms; phonon (strong coupling to the low mode) and plasmon, specific to the 2D case.

Note added. After submitting this article, I became aware of papers[18,19] in which the phonon softening and the appearance of strong electron-phonon coupling have been studied. A very important experimental observation has been carried out in Ref. 20; namely, the presence of low-phonon modes has been observed experimentally by neutron scattering.

I would like to mention also the interesting result obtained in Ref. 21. The authors produced single crystals of high-T_c oxides and found that their behavior in a magnetic field indicates a 2D character of the electron transport. The elimination of such factors as inhomogeneities and the proximity effect will result in a value of $\beta=\Delta(0)/T_c > \beta_{BCS}=1.76$. Such large values have been observed in Ref. 22; this is a manifestation of strong electron-phonon coupling.

Recently a new high-T_c supercondcuting transition has been achieved in a Y-Ba-Cu-O system.[23] A structural analysis carried out in Ref. 24 shows the presence of a layered structure (implying reduced dimensionality) with the interlayer distance even greater than in oxides studied earlier.

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