

## Critical behavior and metastability of the magnetization in the random-field Ising system $\text{Fe}_{0.7}\text{Mg}_{0.3}\text{Cl}_2$

U. A. Leitão and W. Kleemann

*Angewandte Physik, Universität Duisburg, 4100 Duisburg 1, Federal Republic of Germany*

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Optical Faraday rotation was used to measure the uniform magnetization  $M$  vs  $T$  of the random-field Ising-model (RFIM) system  $\text{Fe}_{0.7}\text{Mg}_{0.3}\text{Cl}_2$  in fields  $65 \leq H \leq 500$  mT. In disagreement with previous specific-heat results, but in accordance with the "standard" RFIM system  $\text{Fe}_{1-x}\text{Zn}_x\text{F}_2$ , critical exponents  $\phi = 1.41 \pm 0.05$  and  $\tilde{\alpha} \sim 0$  are obtained from  $(\delta M/\delta T)_H$  and  $(\delta M/\delta H)_T$ . Dynamical rounding due to the finite measurement time scale,  $\tau \sim 100$  s, occurs below  $|t| \sim 0.003$ . Excess magnetization  $\Delta M$  is found in the field-cooled domain state in agreement with previous computer simulations. Scaling as  $\Delta M \propto H^2$  and the essential time independence are compatible with "opaque" potential-barrier models.

Many aspects of the random-field Ising model (RFIM)<sup>1</sup> have found their realization in Fishman-Aharony systems,<sup>2</sup> i.e., in diluted uniaxial antiferromagnets (AF's) in a uniform magnetic field  $H$  applied parallel to the easy  $c$  axis.<sup>3</sup> Despite the considerable progress made from both the theoretical and the experimental points of view, open questions are still quite numerous. Current interest is focused mainly on systems with dimension  $d=3$  with the goal a better understanding of (i) static critical behavior and scaling, both of which seem to hint at an effective dimensionality  $\tilde{d}=2$ ,<sup>3</sup> (ii) critical dynamics,<sup>4</sup> which seems to involve unusually large critical exponents  $\tilde{z}$ ,<sup>5</sup> or even "activated" dynamic scaling,<sup>6</sup> and (iii) metastability as evidenced by field-cooling- (FC) induced domain states,<sup>7</sup> whose interdependence with the critical dynamics<sup>5,6</sup> is still far from being clear.

Recently, studies on  $\text{Fe}_{1-x}\text{Zn}_x\text{F}_2$  of the ac susceptibility,  $\chi'(\omega)$ ,<sup>4</sup> and of the uniform magnetization,  $M$ ,<sup>8</sup> have proved to be particularly suitable for testing both static and dynamic scaling predictions. However, effects due to FC-induced metastability remained undetected in these investigations. This was surprising, since according to computer simulations<sup>9</sup> distinct differences between FC and zero-field-cooled (ZFC) magnetization should be observable at large enough RF. Presumably this condition was not fulfilled in the previous experiments<sup>4,8</sup> at moderate fields,  $H \sim 1$  T. A more favorable situation is met in our present investigation of  $\text{Fe}_{0.7}\text{Mg}_{0.3}\text{Cl}_2$ , which has been known to exhibit random field (RF) effects in both magnetic neutron scattering<sup>10</sup> and specific-heat,  $c_m$ , measurements.<sup>11</sup> For the first time we shall report on the expected<sup>9</sup> enhancement of  $M$  upon FC in comparison with that obtained after ZFC.

Under ZFC conditions both  $(\partial M/\partial T)_H$  and  $(\partial M/\partial H)_T$  diverge logarithmically as in the classical RFIM system  $\text{Fe}_{1-x}\text{Zn}_x\text{F}_2$ .<sup>8</sup> This signifies  $\tilde{\alpha} \sim 0$ , in sharp contrast with the specific-heat result,  $\tilde{\alpha} \sim -1$ .<sup>11</sup> Moreover, also our exponent  $\phi = 1.41 \pm 0.05$  describing the crossover from random-exchange to RF behavior, disagrees with that obtained from  $c_m$ -vs- $T$  data,<sup>11</sup> but confirms the value expected theoretically<sup>2,12</sup> and observed in  $\text{Fe}_{1-x}\text{Zn}_x\text{F}_2$ .<sup>3,8</sup> In our opinion the reliability of our

data is warranted by the local resolution of the Faraday-rotation (FR) technique<sup>8</sup> for the measurement of  $M$ , which avoids falsifying concentration gradients of the mixed-crystal samples to a large extent. Our new results are also of interest in the recent discussion<sup>11,13</sup> on Fisher renormalization of the critical exponents in the case of Fishman-Aharony systems.  $\tilde{\alpha} \sim 0$  would imply a trivial renormalization factor  $(1-\tilde{\alpha})^{-1} \sim 1$ , thus leaving unaffected the values of *all exponents* hitherto obtained on three-dimensional (3D) RFIM systems.<sup>3</sup>

The RFIM regime of  $\text{Fe}_{0.7}\text{Mg}_{0.3}\text{Cl}_2$  is restricted to fields  $H \lesssim 0.4$  T owing to the occurrence of spin-flip transitions at about 0.6 T.<sup>14</sup> A full account of the  $H$  vs  $T$  phase diagram obtained from FR measurements along with a detailed study of the tricritical point will be given elsewhere.

All measurements were carried out on Bridgman-grown samples, which exhibit typical concentration gradients of  $\Delta x/\Delta l \sim 0.005 \text{ cm}^{-1}$  along the growth direction lying in the basal plane. Since the narrow HeNe laser beam (diameter 0.6 mm) probes the sample along its  $c$  axis (thickness  $\sim 0.2$  mm), rounding of the transition due to  $\Delta x$  is believed to be no larger than  $\delta t = 1.5 \times 10^{-3}$  ( $t = T/T_c - 1$ ). The FR angle  $\theta$ , induced by the magnetic field of a superconducting coil directed along the  $c$  axis, was measured with a resolution of  $\delta\theta = 0.006^\circ$  by use of a computer-controlled compensation method.<sup>8</sup> The temperatures were stabilized to within 1 mK prior to each measurement and measured with a carbon-glass resistor.

Typical  $\theta$ -vs- $T$  curves for  $H=130$  and 260 mT are shown in Fig. 1. As in  $\text{FeCl}_2$ ,<sup>15</sup> the FR is proportional to the uniform magnetization, as verified by comparison with recent<sup>16</sup>  $M$  vs  $T$  data obtained on  $\text{Fe}_{0.716}\text{Mg}_{0.284}\text{Cl}_2$ . Our curves were taken upon field heating (FH) after ZFC. The inset of Fig. 1 shows a FH-FC cycle with  $H=260$  mT around the maximum-slope temperature,  $T_c = 12.54$  K, after ZFC. It is seen that the FC magnetization exceeds the ZFC signal,  $\Delta\theta = \theta(\text{FC}) - \theta(\text{ZFC}) > 0$ , at temperatures  $T < T_{\text{eq}}$ , where  $T_{\text{eq}} \sim 12.65$  K. It was observed that the FC branch of the cycle is fully reversible on subsequent FH. This strongly hints at frozen-in metastability in the FC state, which will be recalled in more detail

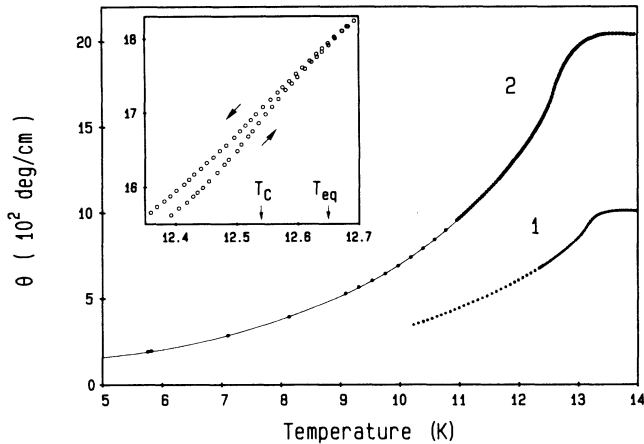


FIG. 1. Temperature dependence of the Faraday rotation measured with  $\lambda=633$  nm on  $\text{Fe}_{0.7}\text{Mg}_{0.3}\text{Cl}_2$  at (1)  $H=130$  mT and (2) 260 mT (with eye-guiding line) applied after ZFC. The inset shows a temperature cycle at  $H=260$  mT around  $T_c=12.54$  K and  $T_{eq}=12.65$  K (arrows) after ZFC.

below. Let us first discuss the equilibrium properties attributed<sup>3</sup> to the ZFC configuration.

Figure 2 shows the derivative  $(\delta\theta/\delta T)_H$  vs  $T$  in the vicinity of  $T_c(H)$  for different field strengths. Similarly as observed<sup>8</sup> in  $\text{Fe}_{1-x}\text{Zn}_x\text{F}_2$  sharp peaks appear thus defining  $T_c(H)$ . On increasing  $H$ , they become more symmetric and shift to lower temperatures according to the crossover scaling relation<sup>2</sup>

$$T_N - T_c(H) = bH^2 + cH^{2/\phi}, \quad (1)$$

where  $\phi$  is the crossover exponent and  $bH^2$  denotes the mean-field correction. Assuming the validity of the rela-

tion<sup>11</sup>

$$b(x) = b(0)T_N(0)/T_N(x),$$

we obtain  $b(0.3)=2.62$  K/T<sup>2</sup> inserting  $b(0)=1.49$  K/T<sup>2</sup>,<sup>11</sup>  $T_N(0)=23.60$  K, and  $T_N(0.3)=13.45$  K. Then a least-squares fit of the internal field  $H_i$  vs  $T_c(H_i)$  to Eq. (1) (Fig. 2, inset) for  $H_i \leq 320$  mT yields  $\phi=1.41 \pm 0.05$ . Very satisfactorily this value is near to that of the standard 3D RFIM system  $\text{Fe}_{1-x}\text{Zn}_x\text{F}_2$ ,  $\phi=1.42 \pm 0.03$ ,<sup>3,8</sup> in agreement with theoretical predictions.<sup>12</sup>

It is noted that our value of  $\phi$  lies well outside the error margins of that obtained<sup>11</sup> from  $c_m$  vs  $T$  data,  $\phi=1.25 \pm 0.11$ . One reason for the obvious discrepancy is suspected to lie in the lack of demagnetization correction in the former study.<sup>11</sup> Our data were corrected for this effect using the demagnetization factor achieved from FR data of the mixed phase in the spin-flip regime following the procedure of Dillon *et al.*<sup>15</sup> However, the correction turned out to be virtually constant,  $H_i \sim 0.75H$  for  $H_i \leq 320$  mT. Hence, the value of  $\phi$  does not change on replacing  $H_i$  by the applied field  $H$  in Eq. (1). More likely the reliability of the previous<sup>11</sup>  $\phi$  value may have suffered from severe drawbacks of the experimental method involved: Taken at a speed of 0.12 K/s the  $c_m$  vs  $T$  curves were *not* in equilibrium (e.g., peak shifts up to 0.1 K occur between FH and FC), whereas our data rely on point-by-point equilibration and subsequent waiting times of 10<sup>2</sup> s. It is, however, unclear how these uncertainties do change the measured  $H$ - $T$  phase diagram *systematically*. We rather believe that systematic errors originate from the unavoidable  $T_c$  spread due to the concentration gradients in relatively large sample volumes. Because of their asymmetry, the  $c_m$  vs  $T$  curves<sup>11</sup> peak at temperatures  $T_c$ , which are lower than the sample averages  $\bar{T}_c$ .<sup>17</sup> It is easily seen that the misfit,  $\bar{T}_c - T_c$ , in-

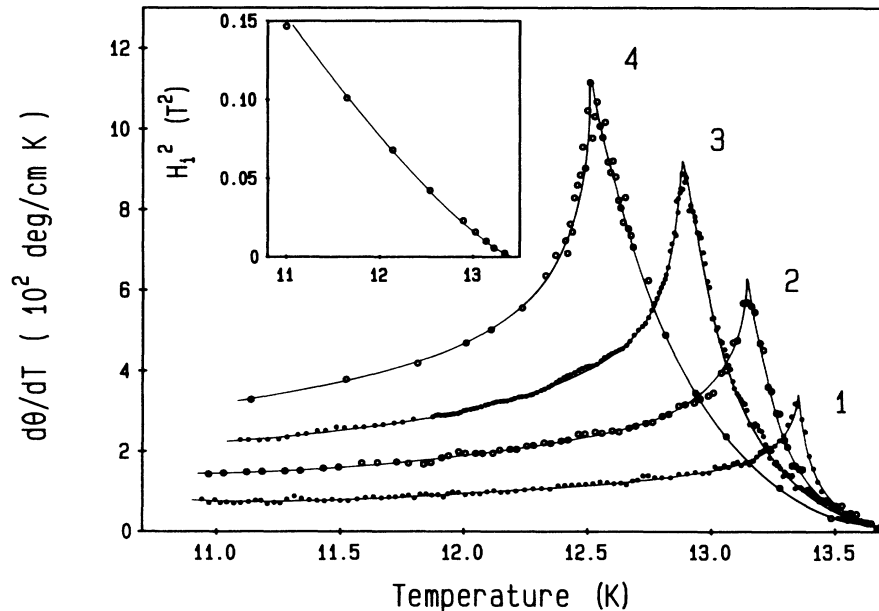


FIG. 2.  $(\delta\theta/\delta T)_H$  vs  $T$  for ZFC  $\text{Fe}_{0.7}\text{Mg}_{0.3}\text{Cl}_2$  measured at (1)  $H=65$ , (2) 130, (3) 195, and (4) 260 mT, respectively. The solid lines are guides for the eye. The inset shows  $H_i^2$  vs  $T_c(H_i)$  best fitted to Eq. (1) (solid line).

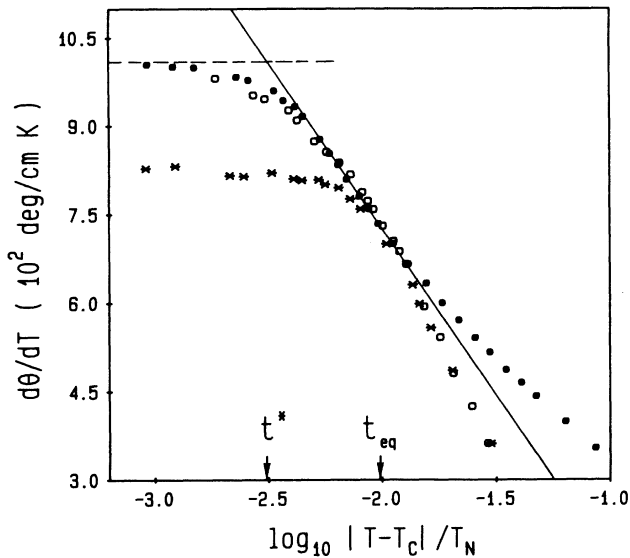


FIG. 3.  $(\delta\theta/\delta T)_H$  vs  $\log_{10}|t|$  for  $H=260$  mT (cf. Fig. 2), where  $t=(T-T_c)/T_N$  with  $T_c=12.54$  K and  $T_N=13.45$  K, after ZFC (filled circles:  $t < 0$ , open circles:  $t > 0$ ) and upon FC (asterisks:  $t > 0$ ), respectively.  $t_{eq}$  (arrow) corresponds to  $T_{eq}$  in Fig. 1. The horizontal line through the peak height and the  $\log_{10}|t|$  line intersect at  $t^*$  (arrow).

creases with increasing  $H$ . Hence, the emerging  $H$  vs  $\bar{T}_c$  phase line bends more strongly than would the  $H$  vs  $\bar{T}_c$  curve, thus yielding too small a value of  $\phi$ . Significantly smaller errors of this kind are expected in our FR measurements, since they probe about 30-times-smaller sample volumes and yield symmetric singularities of  $(\delta\theta/\delta T)_H$ , hence  $T_c \sim \bar{T}_c$ .

A plot of  $(\delta\theta/\delta T)_H$  vs  $\log_{10}|t|$  for  $H=260$  mT in Fig. 3 reveals a straight line for the ZFC data within  $t^*=0.003 \leq |t| \leq 0.02$  thus indicating a specific-heat exponent  $\bar{\alpha} \sim 0$  in close agreement with previous FR experiments.<sup>8</sup> Rounding occurs below  $t^*$ , which is significantly larger than the  $\Delta x$ -induced rounding interval  $\delta t \sim 0.0015$ . Albeit less clearcut than in the  $\text{Fe}_{1-x}\text{Zn}_x\text{F}_2$  study,<sup>8</sup> we believe the excess rounding to be due to the extreme slowing down of the critical dynamics of the RFIM system<sup>4-6</sup> on the time scale of our experiment ( $\tau \sim 10^2$  s).

Much stronger rounding, starting below  $|t| \sim 0.01$  for  $H=260$  mT, appears upon FC (Fig. 3). This temperature is close to the equilibrium boundary,  $t_{eq} \sim 0.009$  (Fig. 1, inset), below which the system is believed to condense into a domain state.<sup>3</sup> RFIM critical behavior is, hence, restricted to the range  $t_{eq} < t < 0.02$  under FC conditions, where the upper boundary presumably indicates crossover into the random exchange regime.<sup>3</sup>

The "dc" susceptibility corresponding to  $(\delta\theta/\delta H)_T$  is determined as the difference of two  $\theta$  vs  $T$  curves taken at  $H \pm \delta H/2$  with  $H=260$  mT and  $\delta H=8$  mT (Fig. 4). It shows similar critical behavior as does  $(\delta\theta/\delta T)_H$  (Fig. 2) in its singular part. This is obtained by subtracting a smooth regular background,<sup>8</sup> which would correspond to  $\chi'(\omega \rightarrow \infty)$ . A logarithmic plot reveals a symmetric straight line in the range  $0.003 < |t| < 0.06$ , thus

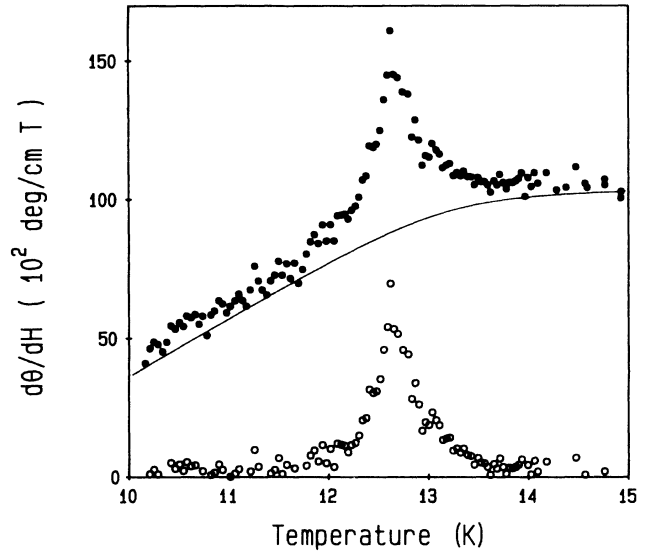


FIG. 4.  $(\delta\theta/\delta H)_T$  vs  $T$  for ZFC  $\text{Fe}_{0.7}\text{Mg}_{0.3}\text{Cl}_2$  measured at  $H=260$  mT with  $\delta H=8$  mT (filled circles) and its singularity (open circles) after subtracting a standard background function (solid line, see text).

confirming the critical exponent  $\bar{\alpha} \sim 0$ . The onset of dynamic rounding below  $t^*=0.003$  is, again, believed to be due to the finite "frequency" of the experiment,  $\omega = \tau^{-1}$  with  $\tau \sim 10^2$  s.

As shown previously,<sup>8</sup> both  $(\delta\theta/\delta T)_H$  and  $(\delta\theta/\delta H)_T$  are related to second derivatives of the free energy  $F$ . They are expected to scale as  $H^\gamma |t|^{-\bar{\alpha}}$  with different amplitude exponents  $\gamma_1$  and  $\gamma_2$ , respectively. Whereas for  $\text{Fe}_{1-x}\text{Zn}_x\text{F}_2$  the experimental values  $\gamma_1 \sim 0.6$  and  $\gamma_2 \sim 1.0$  are close to the theoretical predictions,<sup>8</sup> the values found for  $\text{Fe}_{0.7}\text{Mg}_{0.3}\text{Cl}_2$ ,  $\gamma_1 = 1.1 \pm 0.2$  and  $\gamma_2 = 1.3 \pm 0.5$ , seem to differ significantly. As outlined above, demagnetization effects are not expected to account for this discrepancy. We rather believe that the proximity of the tricritical point ( $T_t \sim 8.5$  K,  $H_t \sim 0.6$  T) plays a crucial role. Presumably the exponents  $\gamma$  do change, if the ordinary field-induced crossover from antiferromagnetic to spin-flip behavior would properly be accounted for in the scaling form of  $F$ . Unfortunately, to the best of our knowledge theoretical predictions for the scaling properties of weak-anisotropy randomly diluted AF's in a uniform field  $H$  are lacking up to now.

Recently<sup>11,13</sup> it was argued that Fisher renormalization might affect the RFIM critical behavior of Fishman-Aharony systems. Since the RF varies like  $M(T)$  at constant applied field  $H$ , the phase transition is not observed along a path parallel to the  $T$  axis in the RF vs  $T$  phase diagram. Renormalized exponents, e.g.,  $\bar{\alpha}' = -\bar{\alpha}/(1-\bar{\alpha})$ , are expected to appear, if  $\bar{\alpha} \geq 0$ .<sup>13</sup> However, in view of  $\bar{\alpha}' = 0$  emerging from this and previous<sup>3,8</sup> investigations, one readily obtains the trivial renormalization  $\bar{\alpha} = \bar{\alpha}' = 0$ ,  $\bar{\gamma} = \bar{\gamma}'$ , etc. Nontrivial renormalization is thus presently a mere speculation, unless future very low frequency measurements will reveal asymptotic exponents  $\bar{\alpha} > 0$  as recently predicted.<sup>18</sup>

Let us finally discuss the  $\Delta\theta$  effect appearing upon a ZFC-FH-FC cycle around  $T_c(H)$  and  $T_{eq}(H)$  (Fig. 1, inset). Figure 5 shows  $\Delta\theta$  vs  $t$  for different values of  $H$ . The reversibility temperatures  $t_{eq}$  are of the order  $10^{-2}$  and increase with increasing  $H$ . Scaling with  $t_{eq} \propto H^{2/\nu}$  (after mean-field correction) is expected,<sup>3</sup> but cannot safely be confirmed because of the large scatter of our data points. Maximum values  $\Delta\theta_m$  are obtained at  $t_m \sim -t_{eq}$ . On further lowering  $T$  we find a monotonous decrease of  $\Delta\theta$ , closely resembling that of  $\theta$  (ZFC) vs  $T$  and yielding residual values  $\sim \Delta\theta_m/10$  at liquid-helium temperature. Most remarkable is the proportionality of  $\Delta\theta_m$  with  $H^2$  (Fig. 5, inset). Furthermore, virtually *no* time dependence of  $\Delta\theta$  was found within  $5\text{ s} < \tau < 7200\text{ s}$  after quenching from above  $t_{eq}$ ,  $t \sim 0.05$ , into metastable states at constant temperatures  $|t| \lesssim 0.02$ . All of these properties were reproduced on different samples for  $H \lesssim 0.4\text{ T}$ , reasonable signal-to-noise ratios requiring  $H > 50\text{ mT}$ .

The  $\Delta\theta$  effect is qualitatively compatible with recent mean-field computer experiments on randomly diluted Ising AF's:<sup>9</sup> (i) The  $M$  vs  $T$  curves smoothly split just below  $M_{max}$ , where  $M(FC) > M(ZFC)$  in the low-field limit; (ii) the excess magnetization of FC samples is reversible at all  $T$ ; (iii)  $\Delta M$  is preferentially concentrated at the AF domain walls, which mainly nucleate at vacancies of the spin system. Sufficiently far from  $T_c$  (in our experiments tentatively at  $t \lesssim t_m$ ) the walls become smooth and well defined. Hence, assuming a system of domains with average radius  $R$  and constant excess surface magnetization density, one readily finds  $\Delta M \propto R^{-1}$ . Since  $R$  generally depends on both the RF and the time  $\tau$ ,  $R(H, \tau) \propto H^{-\nu_H} r(\tau)$ ,<sup>7</sup> we thus expect

$$\Delta\theta \propto H^{\nu_H} / r(\tau). \quad (2)$$

Experimentally, we find  $\nu_H = 2.0 \pm 0.1$  (Fig. 5, inset) in

perfect agreement with  $\nu_H = 2$  predicted for broad domain walls in  $d=3$ .<sup>7</sup> Time independence,  $r(\tau) \sim \text{const}$ , as observed for  $\tau > 5\text{ s}$ , seems to agree with Bruinsma and Aeppli's<sup>7</sup> low-temperature prediction of frozen domains with size  $R$  and "opaque" potential barriers. Within experimental accuracy, however, very weak time dependence  $r(\tau) \propto \ln(\tau/\tau_0)$  with  $\tau_0 \sim 10^{-13}\text{ s}$ , as predicted by Villain<sup>7</sup> for thermal depinning of domain walls, may not completely be ruled out. This would better agree with our experimental condition,  $T \lesssim T_c$ , and with recent neutron data<sup>10</sup> on  $\text{Fe}_{0.7}\text{Mg}_{0.3}\text{Cl}_2$  showing weak time dependence of the FC state. Systematic studies to check very weak time dependence of  $\Delta\theta$  are presently under way. The final discussion should also account for possible failure of our basic assumption  $\Delta M \propto R^{-1}$  under isothermal growth conditions.

The decrease of  $\Delta\theta$  for  $t < t_m(H)$  is primarily due to the decrease of the RF being proportional to  $M(T)$ , hence the close correspondence with  $\theta(\text{ZFC})$  vs  $T$  (Fig. 1). It remains to be shown if a strict proportionality  $\Delta\theta \propto \theta^{\nu_H}$  holds, as suggested by relation (2). This test would shed more light on the very origin of the excess *uniform* magnetization in the FC state, which seems to be related to the simultaneous decrease of the *staggered* magnetization.<sup>10</sup> Future discussions should also be extended to other dilute AF, showing either very large ( $\text{Mn}_{1-x}\text{Zn}_x\text{F}_2$ , Ref. 19) or nearly negligible ( $\text{Fe}_{1-x}\text{Zn}_x\text{F}_2$ , Ref. 20)  $\Delta M$  effects.

In conclusion, our FR data have provided evidence that solid solutions of  $\text{Fe}_{1-x}\text{Mg}_x\text{Cl}_2$  fit well with many predictions for the RFIM in both the ZFC equilibrium and the metastable FC domain state. Strong similarity with the only hitherto undisputed model system,  $\text{Fe}_{1-x}\text{Zn}_x\text{F}_2$ , is now apparent. It will be interesting to test and compare other important properties, e.g., dynamical scaling observed via  $\chi'(\omega)$  or  $(\delta\theta/\delta H)_T$  vs  $\omega$ . It is hoped that ex-

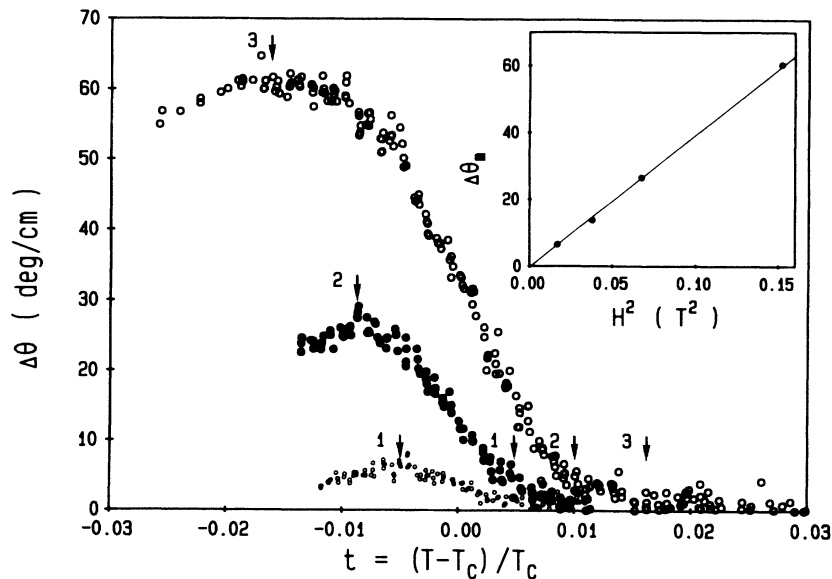


FIG. 5. Excess Faraday rotation  $\Delta\theta$  vs  $t = T/T_c - 1$  of  $\text{Fe}_{0.7}\text{Mg}_{0.3}\text{Cl}_2$  cycled around  $T_c(H)$  (see Fig. 1) measured at (1)  $H = 130\text{ mT}$  ( $T_c = 13.15\text{ K}$ ), (2)  $260\text{ mT}$  ( $T_c = 12.54\text{ K}$ ), and (3)  $390\text{ mT}$  ( $T_c = 11.62\text{ K}$ ). The respective temperatures  $t_{eq} > 0$  and  $t_m < 0$  (see text) are indicated by arrows. The inset shows  $\Delta\theta_m$  vs  $H^2$  together with a best linear fit (solid line).

periments of this kind will eventually help to find the true asymptotic exponents predicted theoretically.<sup>18,21</sup> Furthermore, in  $\text{Fe}_{1-x}\text{Mg}_x\text{Cl}_2$  a detailed study of the competition between RF and spin-flip processes near to the tricritical point will be of special interest.

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