Topological disorder hierarchically trapped at frustration sites: Physical picture for a glass

Subodh R. Shenoy
School of Physics, University of Hyderabad, Hyderabad 500134, Andhra Pradesh, India
(Received 1 April 1986; revised manuscript received 26 September 1986)

A dynamical picture for glass is illustrated by a two-dimensional Josephson-junction-array model in an overall neutral flux distribution $\{\Phi_I\}$, $\sum_I \Phi_I=0$ that is of a self-similar, hierarchical type. A nonequilibrium ± 1 vortex population, trapped at low temperatures T on $\{\Phi_I\}$, can annihilate only in a sequential, slow $\sim t^{-T}$ manner, implying long-lived glassy behavior. Vortex accumulation sets in on cooling, at a temperature $T_G(\dot{T})$ logarithmically dependent on the cooling rate $\dot{T}$.

The glass transition is a problem both of long-standing[1,2] and of intense current interest.[3-6] The transition involves remarkable nonequilibrium features such as the cooling-rate dependence of the transition temperature and frozen-in entropy,[1,2] time-dependent specific heats,[7] and anomalously slow (e.g., power law, $\sim t^{-T}$) nonexponential[8-12] decays. To understand spin[2,13] and configurational[1,2] glasses one needs (a) well-defined variables to describe disorder and (b) a mechanism for trapping the disorder in the system, even over long observation times.

Topological excitations like dislocations, disclinations, or spin vortices are well-defined disorder variables in solids and magnets. They are extended clusters of atoms or spins with a definite center but no edges, circumventing the problem of cluster-boundary definition.[13] They play a central role in the (equilibrium) two-dimensional (2D) Kosterlitz-Thouless (KT) transition.[14,15] Nelson[3] has pictured the 3D glass transition as a random entanglement of such defects of a given sign, and suggested the study of simpler 2D glass models. Halsey[6] has found spin-glass-like behavior in Monte Carlo simulations of a 2D Josephson-junction network[15] in a uniform irrational flux distribution $\Phi=(3-\sqrt{5})/2$ per unit cell.

On the other hand, the recent idea of *hierarchy*[9] explains nonexponential decays in glasses through hopping models with hierarchically increasing barriers or, by implication, sequential relaxation of clusters.

Can a hierarchical pattern of frustration-produced barriers provide a mechanism for the long-time, glasslike trapping of (topological) disorder?

In this paper, the prototype used to illustrate such a mechanism is a 2D Josephson-junction-array (JJA) model or 2D XY model with a specified flux or frustration distribution $\{\Phi_I\}$ of an overall neutral ($\sum_I \Phi_I=0$) and self-similar type. The detailed microscopic dynamics of such a system under a cooling ramp $T(t)$ is a difficult problem that will not be attempted here. Instead (i) it is shown that for $T \ll T_{\rm KT}$ a nonequilibrium excess of trapped ± 1 vortices can undergo *hierarchical annihilation* over increasing nearest-neighbor frustration barriers, so the survival probability of the excess is $P(t) \sim t^{-T/T_h}$; (ii) I then summarize the slow decay of the trapped vortex excess $n_{\rm tr}(t)$ by the effective self-annihilation rate

$$k(t) \equiv -\dot{P}(t)/P(t)=(T/T_h)/t \ .$$

For $T>T_{\rm KT}$, $k(t)$ is used in kinetic equations for $n_{\rm tr}$ and for free vortices $n_{\rm free}$ that exist beyond the (screened) range of attraction $\xi_+(T)$, of $\{\Phi_I\}$. Cooling through $T_{\rm KT}$ at a rate $\dot{T}$ results in a slow-decaying accumulation $\tilde{n}_{\rm tr}(t)$, implying "glassy" behavior. (For the JJA this means dissipation even at $T=0$ from current-released vortices; for the 2D XY model this means apparently random spins.)

As elsewhere,[6] the $\{\Phi\}$ values cannot be achieved in real arrays; Monte Carlo tests are suggested. The ideas should also be applicable to 2D and 3D spin and configurational glasses, as commented on later.

The JJA is modeled by[6]

$$\beta H=-K_0\sum_{\langle ij\rangle}\cos(\theta_i-\theta_j-A_{ij}) \ ,$$

where θ_i are superconductor phases defined on a square lattice of lattice constant a_0. For the bond vector potential $A_{ij}=0,\pi$ one recovers the 2D XY model, with plaquette frustration $\Phi=0,\frac{1}{2}$. The partition function can be mapped onto that of a Coulomb gas of thermal vortices $m_I=0,\pm 1$ on dual lattice sites I. The Hamiltonian is

$$\beta H=-\pi K_0\sum_{I,K}(m_I+\Phi_I)U_{IK}(m_K+\Phi_K) \ ,$$

with $U_{IK}=\ln(r_{IK}/a_0)$ and with $\{\Phi_I\}$ a background of externally determined flux points on dual lattice sites I. Overall "neutrality" holds, $\sum_I(\Phi_I+m_I)=0$, with a bare fugacity $y_0=\exp[-(\pi^2K_0/2)(m+\Phi)^2]$ governing the occupation. For $|\Phi|<\frac{1}{2}$, thermal vortices do not persist, as $T\rightarrow 0$.

Uniformly frustrated models $\Phi_I=\Phi\forall I$ have been studied elsewhere.[6] Here we consider a *non*uniform, neutral flux distribution $\sum_I\Phi_I=0$, implying $\sum_I m_I=0$. The simplest neutral 1D pattern is $+,-$ equally spaced and alternating. A nonequilibrium excess of $-1,+1$ vortices trapped on these would face a single barrier (time scale) and annihilate exponentially.

An explicit construction is now given for a *nonunique* 1D hierarchical distribution $\Phi_I=\pm|\Phi|$, in "quasi-neutral" triplets, each of net "charge" $\pm|\Phi|$ (with an overall neutralizing charge). This yields a hierarchy of barriers and time scales, and nonexponential annihilation.

Divide a line into three segments, further subdivide each into three, etc., until some smallest scale r_0. Some sites $1-x$ are systematically designated as empty, $\Phi=0$, with a fraction x of sites occupied with separation $r_0 \sim x^{-1/d}a_0$,

$d=1$. (We henceforth talk only about the $\Phi\neq 0$ sites.) (i) Put $+\Phi+\Phi-\Phi$ on the left-most elementary triplet of sites. Mirror-reflect (M) this as a unit, giving $-\Phi+\Phi+\Phi$ in the next triplet. Translate and charge-conjugate (CT) this second triplet to get $+\Phi-\Phi-\Phi$. (ii) Repeat the CTM procedure of (i), now treating the nine charges as an elementary unit, and so on. A 27-member sequence would be the following:

$$++--+++----+++$$
$$--++++---++-- .$$

Note that quasineutrality holds: Each of the above 3^n-member groupings, $n=1,2,\ldots$, has an excess charge density of $\pm 1/3^n$ that scales to zero. The mobile ± 1 vortices sit on the $\mp|\Phi|$ sites, as depicted in Fig. 1 in the $n=1$ generation. An analogous CTM-produced self-similar 2D square distribution for $n=1$ is given in the inset (background not shown).

The sequential decay is now manifest in Fig. 1. Succeeding generations of vortices see a hierarchy of energy barriers rising with separation, yielding a survival probability envelop $P(t)\sim t^{-T}=e^{-T\ln t}$, as now shown.

A notation[8] useful for labeling cell hierarchies[11] is illustrated on the Cayley tree of connectivity $C=3$ in Fig. 1. The largest-scale cells are 1,2,3, and their successive subdivisions are, e.g., $2\rightarrow 21,22,23$, etc. More generally, a given $n=1$ smallest-scale cell, one of C^N such cells, can be labeled as $\alpha^{(1)}\equiv(\alpha_N,\alpha_{N-1},\ldots,\alpha_2,\alpha_1)$ where $\alpha_i=1,2,\ldots,C$. This label can also be written in terms of the second generation $\alpha^{(2)}\equiv(\alpha_N,\ldots,\alpha_2)$ clumping of C subcells as $\alpha^{(1)}\equiv(\alpha^{(2)},\alpha_1)$.

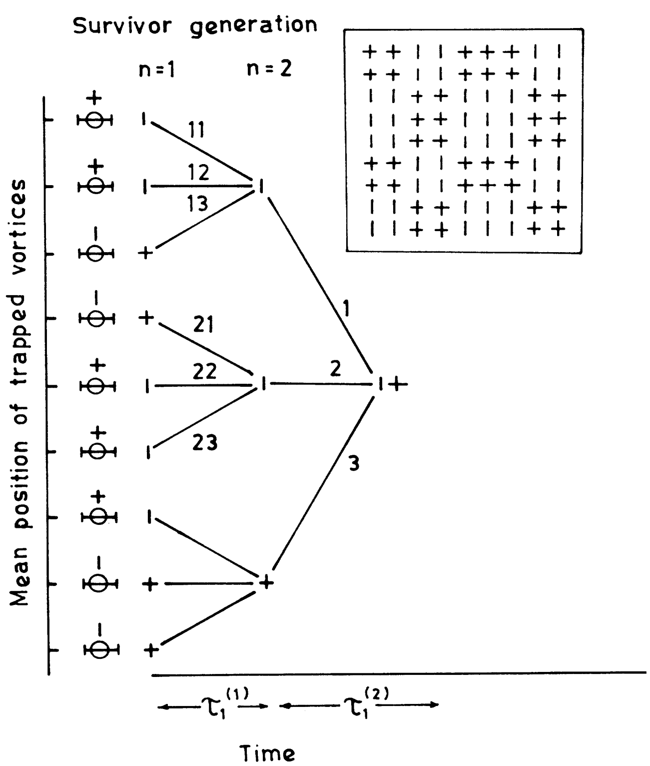

FIG. 1. Sequential annihilation, with time, of mobile ± 1 vortices trapped on 1D hierarchical $\mp\Phi$ background. The quasineutral clusters have one excess charge; the overall balancing excess vortex $+1$ is shown in the final generation. Inset: 2D version, also self-similar under nearest-neighbor annihilation.

The annihilation dynamics, in terms of the survival probabilities $P_{\alpha^{(1)}}$ of the (mobile) cell charges $q_{\alpha^{(1)}}=0,\pm 1$ is given by

$$\dot{P}_{\alpha^{(2)}\alpha_1}=-\sum_{\beta^{(1)}}\frac{Q_{\alpha^{(1)},\beta^{(1)}}P_{\alpha^{(1)}}P_{\beta^{(1)}}}{\tau(r_{\alpha^{(1)},\beta^{(1)}})} \tag{1a}$$

$$=-\sum_{\beta_1=1}\frac{Q_{\alpha^{(2)}\alpha_1,\alpha^{(2)}\beta_1}P_{\alpha^{(2)}\alpha_1}P_{\alpha^{(2)}\beta_1}}{\tau(r_{\alpha^{(2)}\alpha_1,\alpha^{(2)}\beta_1})}-\sum_{\substack{\beta^{(2)}\\(\neq\alpha^{(2)})}}\sum_{\beta_1=1}^{C}\frac{Q_{\alpha^{(2)}\alpha_1,\beta^{(2)}\beta_1}P_{\alpha^{(2)}\alpha_1}P_{\beta^{(2)}\beta_1}}{\tau(r_{\alpha^{(2)}\alpha_1,\beta^{(2)}\beta_1})} . \tag{1b}$$

Here the first (second) term describes nearest-neighbor (further-off) annihilations, with the projection factors

$$Q_{\alpha^{(1)},\beta^{(1)}}\equiv\tfrac{1}{2}q_{\alpha^{(1)}}q_{\beta^{(1)}}(q_{\alpha^{(1)}}q_{\beta^{(1)}}-1)$$

being nonzero only for sites $\alpha^{(1)},\beta^{(1)}$, both occupied by opposite charges, $q_{\alpha^{(1)}}+q_{\beta^{(1)}}=0$. The annihilation times $\tau(r_{\alpha^{(1)},\beta^{(1)}})$ depend on energy barriers that increase with the separation $r_{\alpha^{(1)},\beta^{(1)}}$ between the opposite-sign charges. The $n=1$ nearest-neighbor annihilation time is

$$\tau_1\simeq\omega_1^{-1}\exp[(T_0/T)\ln(r_0/a_0)] ,$$

where $2\pi K_0|\Phi|\equiv T_0/T$.

The annihilation barrier for $+1$ from competing nearest charges $-\Phi$, $-1+\Phi$ is peaked at Φr_0. The barrier height thus scales with separation as $\ln(r_0)$. The other, far-off $\{\Phi\}$ may marginally shift this, but cannot affect the essential point that for unscreened forces the barrier height scales as $\ln(r_0^{(n)})$ for nth generation surviving charges of separation $r_0^{(n)}$.

Defining a coarse-grained probability for the $n=2$ survivor charges by

$$P_{\beta^{(2)}}=\sum_{\beta_1=1}^{C}q_{\beta^{(2)}\beta_1}P_{\beta^{(2)}\beta_1}\Big/\left|\sum_{\beta_1=1}^{C}q_{\beta^{(2)}\beta_1}\right| ,$$

the first term in the $\dot{P}_{\alpha^{(2)}}$ equation vanishes from (1b). The rest can be written in the same form as (1) but with a new minimum time $\tau_1\rightarrow\tau_1^{(2)}$ in the first term. Here, as shown below for $n-1$ annihilations, $\tau_1^{(n)}=\tau(r_0^{(n)})$, where $r_0^{(n)}=(C^{1/d})^{(n-1)}r_0$ is the mean nth generation cell separation in d dimensions. The new projection operators $Q_{\alpha^{(2)},\beta^{(2)}}$ depend only on the $\alpha^{(2)},\bar{\alpha}_1$ survivor charge

$$q_{\alpha^{(2)}}=\sum_{\alpha_1=1}^{C}q_{\alpha^{(2)}\alpha_1}=q_{\alpha^{(2)}\bar{\alpha}_1} .$$

In the spirit of a multipole expansion, the total charge of the $\alpha^{(1)}$ cluster is placed at the center $\alpha^{(2)}$.

The equation for the coarse-grained $P_{\alpha^{(2)}}$ can be written as

$$\dot{P}_{\alpha^{(2)}}=-\sum_{\substack{\beta^{(2)}(\neq\alpha^{(2)})\\\beta_1,\alpha_1}}\frac{Q_{\alpha^{(2)}\alpha_1,\beta^{(2)}\beta_1}P_{\alpha^{(2)}\alpha_1}P_{\beta^{(2)}\beta_1}q_{\alpha^{(2)}\alpha_1}}{\left[\sum_{\alpha_1}q_{\alpha^{(2)}\alpha_1}\right]\tau(r_{\alpha^{(2)}\alpha_1,\beta^{(2)}\beta_1})}$$

$$=-\sum_{\beta^{(2)}}\frac{Q_{\alpha^{(2)},\beta^{(2)}}P_{\alpha^{(2)}}P_{\beta^{(2)}}}{\tau^{(2)}(r_{\alpha^{(2)},\beta^{(2)}})}+\delta , \tag{1c}$$

where the correction term δ is

$$\delta=+\sum_{\beta^{(2)}}\frac{Q_{\alpha^{(2)},\beta^{(2)}}P_{\alpha^{(2)}}P_{\beta^{(2)}}}{\tau^{(2)}(r_{\alpha^{(2)},\beta^{(2)}})} - \sum_{\substack{\beta^{(2)}(\neq\alpha^{(2)})\\ \beta_1,\alpha_1}}\frac{Q_{\alpha^{(2)}\alpha_1,\beta^{(2)}\beta_1}P_{\alpha^{(2)}\alpha_1}P_{\beta^{(2)}\beta_1}q_{\alpha^{(2)}\alpha_1}}{\tau(r_{\alpha^{(2)}\alpha_1,\beta^{(2)}\beta_1})\left[\sum_{\alpha_1}q_{\alpha^{(2)}\alpha_1}\right]} . \quad (1d)$$

Using the definitions of $P_{\alpha^{(2)}},P_{\beta^{(2)}}$ and $Q_{\alpha^{(2)}\beta^{(2)}}$ we can "de-coarse-grain" the first term. [Note that $Q_{\alpha^{(2)}\beta^{(2)}}$ implicitly carries a factor $(1-\delta_{\alpha^{(2)},\beta^{(2)}})$]. Now, in a given cluster, $\alpha^{(2)}\alpha_1$ say, there is one survivor charge $\alpha^{(2)}\bar{\alpha}_1$ with all the rest of that generation annihilating, $\sum_{\alpha_1}q_{\alpha^{(2)}\alpha_1}=q_{\alpha^{(2)}\bar{\alpha}_1}$. Both in the 1D and 2D cases, for the hierarchical pattern considered, the separation between the survivor charges $r_{\alpha^{(2)}\bar{\alpha}_1,\beta^{(2)}\bar{\beta}_1}$ is the same as the separation between the centers of the next generation superclusters, $r_{\alpha^{(2)},\beta^{(2)}}$. Thus $\tau(r_{\alpha^{(2)}\bar{\alpha}_1,\beta^{(2)}\bar{\beta}_1})=\tau(r_{\alpha^{(2)},\beta^{(2)}})$ and we find that the survivor charge contributions to δ cancel out, provided we choose the next generation time scale as $\tau^{(2)}(r_{\alpha^{(2)}\beta^{(2)}})=\tau(r_{\alpha^{(2)}\beta^{(2)}})$. In general, for $n-1$ rescalings, the minimum time scale $\tau_1^{(n)}=\tau_1^{(n)}(r_0^{(n)})=\tau(r_0^{(n)})$, where $r_0^{(n)}$ is the minimum distance between cluster centers.

The contributions of the nonsurvivor or annihilating charges to the correction term δ can be estimated. Since the annihilating charges are initially equally populated, and annihilate with each other, $P_\alpha=P_\beta$ $(\alpha,\beta\neq\bar{\alpha},\bar{\beta})$, and $\dot{P}_\alpha=-1/\tau_1\,P_\alpha^2$ gives $P_\alpha(t)=\tau_1/(t+\tau_1)$. Thus for times at the next generation scale $\tau_1^{(2)}$, the relative error compared to the terms retained in Eq. (1a) is $\sim\exp[-2(T_0/T_d)\ln C]\ll 1$, a small correction that does not build up with generation. The decay error made in placing *next*-nearest $\pm$ charges at their respective cluster centers is also of relative order $\exp[-2(T_0/Td)\ln C]\ll 1$.

The overall decay envelope of the survivor charge probability density can now be estimated from the scale dependence of the minimum annihilation times. The logarithmic scale dependence[14] of K_0 is ignored throughout, as a higher-order correction. The time $\tau(r_0^{(n)})$ depends on the energy barriers, and energy barriers rise logarithmically, $\beta U_n=(T_0/T)\ln(r_0^{(n)}/a_0)$. Clearly, the minimum survival time for the nth generation scales as

$$\tau_1^{(n)}=\tau_1\exp[(T_0/T)\ln(r_0^{(n)}/r_0)] .$$

The survival probability density scales with the normalization factor $P\sim C^{-(n-1)}$, so in terms of $t\approx\tau_1^{(n)}$ one gets $P(t)=(t/\tau_1)^{-(T/T_h)}$. Here a hierarchical temperature T_h has been defined[11] $T_h/T\equiv 2\pi|\Phi|K_0/d$.

For $T>T_{KT}$ the (screened)[14] barriers level off beyond

$$\xi_+(T)=a_0\exp[b(T/T_{KT}-1)^{-1/2}] ,$$

with further annihilations exponential in time. The time $\tau_a(T)$ to annihilate to a generation n_{max} of scale ξ_+ is

$$\tau_a(T)=\tau_1^{(n_{max})}=\omega_1^{-1}\exp[(T_0/T)\ln(\xi_+/a_0)] .$$

For continuity, with the $T<T_{KT}$ unscreened result, we write

$$P(t)=(\tau_a/\tau_1)^{-T/T_h}\exp[-(T/T_h)(t/\tau_a-1)]$$

for $T>T_{KT}$.

Turning to model kinetics for n_{tr}, n_{free}, an effective n_{tr} annihilation rate that summarizes the essential physics is $k(t,T)\equiv-(\partial P/\partial t)/P$. With a τ_1 short-time cutoff, this rate is

$$k(t,T)=\begin{cases}\dfrac{(T/T_h)}{t+\tau_1(T)}, & t<\tau_a(T) ,\\[2ex] \dfrac{(T/T_h)}{\tau_a(T)+\tau_1(T)}, & t>\tau_a(T) ,\end{cases} \quad (2)$$

and first decreases $\sim t^{-1}$ (power-law decays), but then levels off (exponential decays).

The model kinetic equations for the dissipation-causing variables are

$$\dot{n}_{tr}=-k(n_{tr}-\bar{n}_{tr})+x\frac{n_{free}-n_{tr}}{\tau_c}-\frac{n_{tr}}{\tau_e} , \quad (3)$$

$$\dot{n}_{free}=\frac{-\bar{n}_{free}(n_{free}-\bar{n}_{free})}{\tau_0}-x\frac{n_{free}-n_{tr}}{\tau_c}+\frac{n_{tr}}{\tau_e} . \quad (4)$$

Dipolar vortices and the KT transition[11] enter only indirectly. τ_c, $\tau_e(T)$, and $\tau_1(T)$ are the intrinsic capture, escape, and annihilation time scales, with

$$\tau_e=\omega_1^{-1}\exp[(T_0/T)\ln(\xi_+/a_0)] .$$

τ_0 is the time for (linearized) recombination of n_{free} imbalances. Neutrality in n_{free}, n_{tr} (separately) is assumed. At strict equilibrium ($\dot{T}=0$), from (3) and (4), $\bar{n}_{tr}=(1+\tau_c/x\tau_e)^{-1}\bar{n}_{free}\to 0$ as $T\to T_{KT^+}$.

The vortex kinetics is considered in an applied cooling ramp of constant and small slope $|\dot{T}|$,

$$T(t)=T_{KT}+\tfrac{1}{2}\Delta T-|\dot{T}|t,\ \Delta T/|\dot{T}|>t>0 . \quad (5)$$

Since $\tau_a(T)$ diverges as $T\to T_{KT^+}$, it becomes larger than the cooling time for $T<T_G(\dot{T})$, an accumulation onset temperature defined by $(T_{KT^+}\frac{1}{2}\Delta T-T_G)/|\dot{T}|=\tau_a(T_G)$. For $|\dot{T}|$ small it is easy to see that $T_G(\dot{T})\sim T_{KT}+(\ln|\dot{T}|)^{-2}$. (This is reminiscent of dynamic crossover temperatures T_ω in[15] JJA and superfluids and glass transition temperatures.[1])

In the regime

$$\tau_0^{-1}\gg\omega_1\gg\tau_1^{-1}\gg x\tau_c^{-1}\gg|\dot{T}|/T_{KT} ,$$

the cooling rate is slow enough so that dipolar pairs can define a common temperature $T(t)$. The times $\tau_e(T(t))$, $\tau_1(T(t))$ are swept, through (3), with

$$\bar{n}_{free}(T(t))=(\xi_+/a_0)^{-2}y_0^2\to 0$$

as $t\to(\Delta T/2|\dot{T}|)^+$. Using $\tilde{n}_{free},\tilde{n}_{tr}$ $(\tilde{n}\equiv n-\bar{n})$ as variables in (3) and (4) the rates $\dot{\bar{n}}_{free},\dot{\bar{n}}_{tr}$ enter as drive parameters on the right.

The "bath" $\tilde{n}_{free}$ is a fast mode[16] in the regime of interest, for $T>T_{KT}$, except very close to T_{KT} where there is little left to capture. The fast-mode condition $\dot{\tilde{n}}_{free}\approx 0$ eliminates $\tilde{n}_{free}$, with corrections[11,16] $|\dot{T}|\tau_0$, τ_0/τ_1, $\tau_0 x/\tau_c\ll 1$. For strict equilibrium $\dot{T}=0$, $\tilde{n}_{tr}$ decays exponentially for $T>T_{KT}$ and as t^{-T/T_h} for $T<T_{KT}$.

The equation for $n_{\rm tr}$ is

$$\dot{\tilde{n}}_{\rm tr}=-\dot{\bar{n}}_{\rm tr}-\frac{\dot{\bar{n}}_{\rm free}}{1+\tau_c\bar{n}_{\rm free}/x\tau_0}-k(t)\tilde{n}_{\rm tr}-\frac{1/\tau_e+x/\tau_c}{1+(x/\tau_c)(\tau_0/\bar{n}_{\rm free})}\tilde{n}_{\rm tr}\ .$$

For $\dot{T}\neq0$, the solution is

$$\tilde{n}_{\rm tr}(t)=-\int_{-\infty}^{t}dt'\{\dot{\bar{n}}_{\rm tr}(T(t'))+[1+\gamma^{-1}(T(t'))]^{-1}\times\dot{\bar{n}}_{\rm free}(T(t'))\}e^{-W(t,t')}\ , \qquad (6)$$

where $W=W^{(1)}+W^{(2)}$,

$$W^{(1)}(t,t')\equiv\int_{t'}^{t}dt''k(t'',T(t''))$$

carries the hierarchy, and

$$W^{(2)}(t,t')\equiv\int_{t'}^{t}dt''[\tau_e^{-1}(T(t''))+x\tau_c^{-1}][1+\gamma(T(t''))]^{-1}$$

is the rapid relaxation to the $n_{\rm free}$ bath. Here $\gamma(T)\equiv x\tau_0/[\tau_c\bar{n}_{\rm free}(T)]$. After cooling stops, the accumulated fraction will decay as $\sim t^{-(T_{\rm KT}-\Delta T/2)/T_h}$.

If $k(t'')$ in $W^{(1)}$ is replaced by $1/\tau_1(T(t''))$, then $n_{\rm tr}$ just below $T_{\rm KT}$ is both exponentially small, and exponentially decaying.

Scaling all times in τ_0 and temperatures in $T_{\rm KT}$, the parameters chosen are[17] $\omega_1=0.1$, $\tau_c=100$, $\Delta T=2$, $\pi K_0(T)\approx\pi K_0(T_{\rm KT})\approx2.3256$, $x=0.7$, $b\approx\pi/2$, $\Phi=0.49$, and $d=2$, so $T_0=2.28, T_h=1.14$. A plot of $n_{\rm tr}$ vs T is given in Fig. 2 for various cooling rates $|\dot{T}|$, with $T_G(\dot{T})$ marked by an arrow.

The rise of $n_{\rm tr}$ and T_G with $|\dot{T}|$ is quite similar to the behavior of the frozen excess entropy-free volume and glass transition temperature in real glassy systems.[1,2]

It would be of great interest to do computer simulations to test this physical picture of glasses. The long-time thermal-history-dependent trapping of vortices at grain boundaries has been seen[18] in a 2D spin model for atoms on a substrate. In 3D, dislocation loops or disclination lines could play the role of m and Φ.

For a 2D Josephson array model, one could look for slow vortex annihilation in a prepared $\{m_l,\Phi_l\}$ structure $T\ll T_{\rm KT}$. Second, one could monitor the vortex population that gets trapped on $\{\Phi_l\}$ on cooling through $T_{\rm KT}$. Third, one could check that self-similar $\{\Phi_l\}$ structures appear spontaneously in 2D XY quenched averages.[19] Alternatively, the effects of irregularity added to the self-similar $\{\Phi_l\}$ structure could be investigated [see note (a) below].

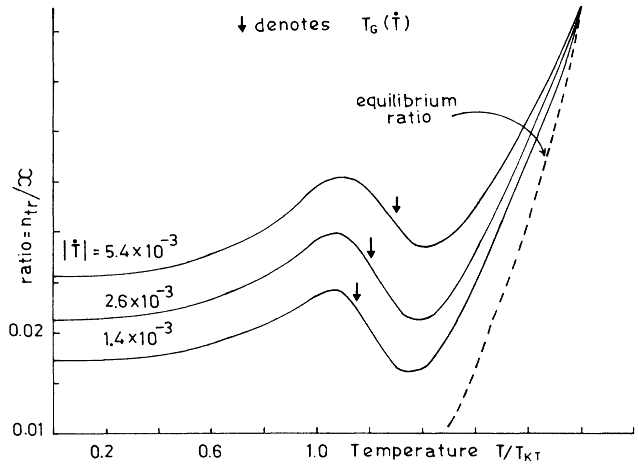

FIG. 2. Ratio of occupied trapping sites $n_{\rm tr}/x$ vs temperature for various cooling rates $|\dot{T}|$. Temperatures and quench rates are scaled in $T_{\rm KT}, T_{\rm KT}/\tau_0$.

The details of the ideas would have to be separately explored for different physical systems. But the picture of topological excitations trapped on self-similar frustration distributions seems worth pursuing, as a possible unifying framework for glassy systems.

Note added in proof. (a) The Ogielski-Stein model slow decays persist for irregular Cayley trees [D. Kumar and S. R. Shenoy, Phys. Rev. B **34**, 3547 (1986)]. (b) The quasineutral condition whereby lower-level charges determine the higher cluster charge patterns is similar to a hierarchical memory model [V. Dotsenko, Physica A **140**, 410 (1986)].

It is a pleasure to thank D. Visveswara Rao for help with the numerical work and the Department of Atomic Energy (India) and Department of Science and Technology (India) for partial support, and to acknowledge helpful suggestions regarding the presentation by H. R. Krishnamurthy.

[1]W. Kauzmann, Chem. Rev. **48**, 219 (1948); A. E. Owen, in *Amorphous Solids and the Liquid State,* edited by N. H. March, R. A. Street, and M. P. Tosi (Plenum, New York, 1985), and references therein.

[2]P. W. Anderson, in *Ill-Condensed Matter, Proceedings of the Les Houches Summer School XXX,* edited by R. Balian, R. Maynard, and G. Toulouse (North-Holland, Amsterdam, 1979), and references therein.

[3]D. R. Nelson, in *Topological Disorder and Condensed Matter,* edited by F. Yonezawa and T. Ninomiya (Springer, New York, 1983); Phys. Rev. Lett. **50**, 982 (1983).

[4]D. M. Duffy and N. Rivier, J. Phys. (Paris) **43**, 293 (1982); G. Venkataraman and D. Sahoo, Contemp. Phys. **27**, 3 (1986).

[5]J. P. Sethna, Phys. Rev. Lett. **51**, 2198 (1983).

[6]T. Halsey, Phys. Rev. Lett. **53**, 1018 (1985); C. Jayaprakash and S. Teitel, Phys. Rev. B **27**, 598 (1983).

[7]J. Zimmerman and G. Weber, Phys. Rev. Lett. **46**, 661 (1981).

[8]See, for example, articles by R. G. Palmer, K. Binder, and W. Kinzel, in *Heidelberg Colloquium on Spin Glasses,* edited by J. L. Van Hemmen and I. Morgenstern (Springer-Verlag, New York, 1983).

[9]R. G. Palmer, D. L. Stein, E. Abrahams, and P. W. Anderson, Phys. Rev. Lett. **53**, 958 (1984).

[10]B. A. Huberman and M. Kerszberg, J. Phys. A **18**, L331 (1985); S. Teitel and E. Domany, Phys. Rev. Lett. **55**, 2176 (1985).

[11]D. Kumar and S. R. Shenoy, Solid State Commun. **57**, 927 (1986).

[12]A. T. Ogielski and D. L. Stein, Phys. Rev. Lett. **55**, 1634 (1985).

[13]D. Choudhury and A. Mookerjee, Phys. Rep. **114**, 1 (1984).

[14]J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973); J. M. Kosterlitz, *ibid.* **7**, 1046 (1974).

[15]Ch. Leeman, Ph. Lerch, G. A. Racine, and P. Martinoli, Phys. Rev. Lett. **56**, 1291 (1986).

[16]H. Haken, *Advanced Synergetics* (Springer-Verlag, New York, 1983), see especially Chap. 7.

[17]The regime here is $T_{\mathrm{BCS}} > T_h > T_G > T_{\mathrm{KT}}$. T_{BCS} is the grain transition temperature.

[18]S. Tang and S. D. Mahanti, Phys. Rev. B **33**, 3419 (1986).

[19]E. Fradkin, B. A. Huberman, and S. H. Shenker, Phys. Rev. B **18**, 4789 (1978). This shows that the quenched-frustration average can be written as a Boltzmann-like factor with the same long-range potential as thermal vortices. Quasineutral self-similar clusters should be favored over charge-segregated clusters, dominating long-time behavior even in random distributions.