

Landau parameters of almost-localized Fermi liquids

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(Received 3 July 1986)

We derive partial sum rules for the intraband contributions to the charge and spin conductivities for almost-localized Fermi liquids in a lattice. From this we conclude that the $l=1$ Landau parameters have small values.

I. INTRODUCTION

Fermi liquids in which there are strong correlations so that they are almost localized have been the subject of renewed interest lately. Examples of such Fermi liquids are believed to be the metallic state of V_2O_3 close to the metal-insulator transition,¹ ^3He close to the crystallization transition,² and the f -electron bands of the heavy-electron systems.³⁻⁵ The features of this state are a strongly enhanced mass and spin susceptibility. The charge susceptibility depends on the particular form of the almost-localized Fermi liquid, i.e., whether there is an exactly integral number of fermions per site or not. The values of the isotropic Landau parameters are determined by these susceptibilities, but the higher-order Landau parameters require extra consideration. In this paper we show that from the sum rules it is possible to make general statements about the $l=1$ parameters ($F_1^{\nu a}$ in the standard notation).⁶

There has been in the literature some confusion over the values that should be expected, particularly for F_1^s . In a translationally invariant system (e.g., ^3He) the current-carrying mass m_c is the bare mass ($m_c = m$),⁶ so that in the case of a large mass enhancement, i.e., $m^*/m \gg 1$, the value of F_1^s must be large also. Some authors have proposed that in lattice systems [e.g., V_2O_3 (Ref. 7) on heavy-electron metals^{8,9}] the current-carrying mass is the band mass m_b and this type of cancellation should still approximately occur. Since experimentally m^*/m_b is large, this would imply that F_1^s should be large also. In this note we show, using the dipole sum rule on the conductivity for the Hubbard model, that a Fermi liquid in a lattice does not have such a cancellation in the almost-localized limit. Instead, $m_c \approx m^*$ and F_1^s is small.

The case of ^3He is special since here the lattice is fictitious, introduced to describe short-range order and therefore can move along with the Fermi liquid. In the true lattice case, the conductivity describes the motion of the Fermi liquid through the lattice and not the combined motion of the lattice and the Fermi liquid.

The result $m_c \approx m^*$ has a simple interpretation. A small number of effective carriers (empty or doubly occupied sites) carry the current and it is possible to reconcile these with the large Fermi surface (containing all the fermions due to Luttinger's theorem) only if m_c is large.

The result can be extended easily to the spin conductivity and therefore to a discussion of F_1^a . In fact, in the absence of spin-orbit coupling the results for the charge and spin conductivities are identical. In this case there is no distinction between real and fictitious lattice systems since a spin current, unlike a charge or mass current, is not carried through the motion of the lattice as a whole. We can therefore conclude quite generally that F_1^a is small in an almost-localized Fermi liquid—a result which is in agreement with experiment in ^3He .¹⁰

In Sec. II the sum rules are derived for the Hubbard model. The implications for the Landau parameters are discussed in Sec. III. The periodic Anderson model, which requires special treatment, is considered in Sec. IV. Some concluding remarks are made in Sec. V.

II. SUM RULES FOR THE HUBBARD MODEL

We consider the standard Hubbard model

$$H = -t_0 \sum_{\mathbf{n}, \alpha, s} [c^\dagger(\mathbf{n}, s)c(\mathbf{n} + \hat{\mathbf{n}}_\alpha, s) + \text{H.c.}] + U \sum_{\mathbf{n}} n(\mathbf{n}, \uparrow)n(\mathbf{n}, \downarrow), \tag{1}$$

where the vectors $\mathbf{n} = (n_x, n_y, n_z)$ number the sites of a simple-cubic lattice, $\hat{\mathbf{n}}_\alpha$ ($\alpha = x, y, z$) are the three unit vectors (1,0,0), (0,1,0), and (0,0,1), $c^\dagger(\mathbf{n}, s)$ and $c(\mathbf{n}, s)$ are creation and annihilation operators, respectively, for electrons at site \mathbf{n} with spin projection s , and $n(\mathbf{n}, s) = c^\dagger(\mathbf{n}, s)c(\mathbf{n}, s)$ is the electron-density operator at site \mathbf{n} with spin s . We assume periodic boundary conditions. Introducing the charge-density operator at \mathbf{n} ,

$$\rho(\mathbf{n}) = \sum_s n(\mathbf{n}, s), \tag{2}$$

and the spin-density operator at \mathbf{n} ,

$$\sigma_z(\mathbf{n}) = \sum_s s n(\mathbf{n}, s), \tag{3}$$

we define current operators by invoking the discretized version of the continuity equation,

$$\frac{\partial}{\partial t} \rho(\mathbf{n}, t) = -\frac{1}{a} \sum_\alpha [j^{(\rho)}(\mathbf{n} \rightarrow \mathbf{n} + \hat{\mathbf{n}}_\alpha, t) - j^{(\rho)}(\mathbf{n} - \hat{\mathbf{n}}_\alpha \rightarrow \mathbf{n}, t)], \tag{4}$$

where a is the lattice constant. This defines the current in the α direction,

$$j^{(\rho)}(\mathbf{n} \rightarrow \mathbf{n} + \hat{\mathbf{n}}_\alpha) = i \frac{at_0}{\hbar} \sum_s e [c^\dagger(\mathbf{n} + \hat{\mathbf{n}}_\alpha, s) c(\mathbf{n}, s) - c^\dagger(\mathbf{n}, s) c(\mathbf{n} + \hat{\mathbf{n}}_\alpha, s)] . \quad (5)$$

The corresponding equations for the spin current $j^{(\sigma_z)}$ are obtained by substituting ρ by σ_z and e by s . The current density in the α direction is then defined by

$$j_\alpha^{(i)} = \frac{1}{V} \sum_{\mathbf{n}} j^{(i)}(\mathbf{n} \rightarrow \mathbf{n} + \hat{\mathbf{n}}_\alpha) , \quad (6)$$

where V is the total volume, both for charge ($i = \rho$) and spin ($i = \sigma_z$) transport. It is easy to convince oneself that the time-dependent current operators can be written as derivatives:

$$j_\alpha^{(i)}(t) = \frac{\partial}{\partial t} q_\alpha^{(i)}(t) , \quad (7)$$

where

$$q_\alpha^{(i)} = \begin{cases} \frac{1}{V} \sum_{\mathbf{n}} n_\alpha \rho(\mathbf{n}) , & i = \rho \\ \frac{1}{V} \sum_{\mathbf{n}} n_\alpha \sigma_z(\mathbf{n}) , & i = \sigma_z . \end{cases} \quad (8)$$

Equation (7) is the lattice version of the relation between current and velocity in the continuum theory.

The Kubo formula for the conductivity at zero temperature is given by

$$\sigma_{\alpha\alpha}^{(i)}(\omega) = iV \int_0^\infty dt e^{-i\omega t} \langle [q_\alpha^{(i)}(t), j_\alpha^{(i)}] \rangle . \quad (9)$$

The spin conductivity can be interpreted as the response to a potential which acts oppositely on the two spin components, e.g., for $i = \sigma_z$ a spatially varying exchange field $\|\hat{\mathbf{z}}$. Since in the real part of the conductivity is an even function of ω , we find

$$\int_0^\infty d\omega \operatorname{Re} \sigma_{\alpha\alpha}^{(i)}(\omega) = -\frac{1}{2} \pi i V \langle [q_\alpha^{(i)}, j_\alpha^{(i)}] \rangle . \quad (10)$$

The evaluation of the commutator yields the sum rule for the charge conductivity,

$$\int_0^\infty d\omega \operatorname{Re} \sigma_{\alpha\alpha}^{(\rho)}(\omega) = -\frac{\pi e^2 a^2}{2\hbar V} \langle T_\alpha \rangle , \quad (11)$$

where T_α is the kinetic energy in the α direction [we write the first term of Eq. (1) as $\sum_\alpha T_\alpha$]. This equation has appeared earlier in the literature, at least for the one-dimensional case.^{11,12} It has been used for classifying quasi-one-dimensional conductors according to the strength of the effective electron-electron interaction (small versus large U).¹³ Proceeding in the same way for spin transport, we again obtain Eq. (11) with e^2 replaced

by 1. The sum rules on charge and spin transport are essentially identical.

We emphasize that, since the absolute value of the kinetic energy will decrease with increasing U , our sum rule depends on the strength of the interaction. This is a consequence of the model Hamiltonian which contains only a single state per site and thus allows only intraband transitions. The complete f -sum rule includes all allowed interband transitions and depends neither on the periodic potential nor on electron-electron interactions.⁶ In this sense Eq. (11) is to be interpreted as a partial sum rule. Experimentally the intraband contributions are indeed often well separated in energy from all the other contributions.¹³

III. CONSEQUENCES FOR FERMI-LIQUID PARAMETERS

In the following we discuss the sum rule in the limit of large U . We must distinguish the case of a half-filled band, where no empty sites are available, from all the other band fillings where the motion of electrons (or holes) remains possible even as $U \rightarrow \infty$. In the latter case there are two contributions to the kinetic energy. The first is associated with free motion and is proportional to $1 - n_0$, where n_0 is the mean density of electrons per site. The second contribution involves motion of electrons (holes) through high-energy ($\sim U$) intermediate states and is of order t_0^2/U . It is the only way transport can occur for a half-filled band. In general, free motion dominates the behavior for $\hbar\omega \ll U$, whereas the second contribution with weight t_0^2/U is responsible for transitions at $\hbar\omega \approx U$. In most cases, namely for all band fillings except for $n_0 \approx 1$, the high-frequency part is negligible and the sum rule is nearly exhausted by the low-frequency part. A connection to Fermi-liquid theory is then possible.

We consider first the case $n_0 < 1$ in the limit $U \rightarrow \infty$. All high-energy contributions with weight t_0^2/U vanish and only the low-energy part connected with electron motion to empty sites remains. (The treatment of hole transport for $n_0 > 1$ is completely analogous.) In Fermi-liquid theory the real part of the charge conductivity is written as⁶

$$\sigma_1(\omega) = \frac{4\pi N e^2}{m^*} \left[1 + \frac{F_1^s}{3} \right] \delta(\omega) = \frac{4\pi N e^2}{m_c} \delta(\omega) , \quad (12)$$

where N is the number of quasiparticles per unit volume.¹⁴ Equation (12) relates the current-carrying mass m_c to the Fermi-liquid parameters m^* and F_1^s . As shown by Landau, m_c would be identical to the bare mass if we were dealing with a translationally invariant system. In our lattice model, however, m_c can only be defined through Eq. (12). For $U \neq 0$ m_c has no specific relation to a band mass m_b , and m_c can be established through the sum rule (11). In fact, for $U \rightarrow \infty$ and $n_0 \neq 1$, the kinetic energy can operate only on empty sites and its expectation value must be proportional to $t_0(1 - n_0)$. Since $t_0 \sim m_b^{-1}$, it follows that

$$\frac{N}{m_c} \sim \frac{1 - n_0}{m_b} \rightarrow 0 \text{ for } n_0 \rightarrow 1 . \quad (13)$$

We know from Luttinger's theorem that the number of quasiparticles is the same as the number of particles. Therefore the vanishing of N/m_c for $n_0 \rightarrow 1$ cannot be attributed to a vanishing density of quasiparticles, but rather has to be associated with a diverging effective mass, $m_c \rightarrow \infty$.

A completely analogous discussion can be made for the spin current. In this case it is the spin-antisymmetric Landau parameter F_1^q which enters the analog of (12). We also define a spin-current-carrying mass $m_s \equiv m^*(1 + F_1^q/3)^{-1}$.

In order to obtain a more quantitative estimate of the mass m_c (or m_s), we use the Gutzwiller approximation (for the Hubbard model) where the mean energy per site is given by

$$\varepsilon = q(d, n_0)\varepsilon_0 + Ud \quad (14)$$

ε_0 is the mean kinetic energy per site for $U=0$ and d is the fraction of doubly occupied sites which tends to zero for $U \rightarrow \infty$. In the same limit the function $q(d, n_0)$ becomes^{1,2}

$$q(d, n_0) \rightarrow \frac{2(1-n_0)}{2-n_0}, \quad (15)$$

in agreement with the general arguments given above. Within the Gutzwiller approximate formula, q measures the discontinuity of the momentum distribution function at the Fermi surface and thus is related to the masses m_b and m^* as

$$q = m_b/m^* \quad (16)$$

On the other hand, in view of Eq. (14), q measures the ratio of the expectation values of the kinetic energy as $U \rightarrow \infty$ and for $U=0$, and therefore also between the corresponding conductivity sum rules. We use now Eq. (12) for the conductivity in the large- U limit. For $U=0$, $\sigma_1(\omega)$ is, of course, also given by Eq. (12), with m_c replaced by m_b . Therefore we find

$$q = m_b/m_c \quad \text{and} \quad q = m_b/m_s \quad (17)$$

Comparing Eqs. (16) and (17), we conclude that in the Gutzwiller approach $m_c = m_s = m^*$ leading to the conclusion that $F_1^q = F_1^s = 0$. In a similar way the higher Landau parameters are expected to vanish. This result together with current estimates for the Landau parameters $F_0^{s,a}$ [$F_0^s \gg 1$, $F_0^a \leq -0.75$ (Refs. 8 and 9)] contradicts the forward-scattering sum rule (see, e.g., Vollhardt's review, Ref. 2) and indicates that the Gutzwiller approximate formulas are not fully self-consistent.

A key point in the above was the product form of the approximate kinetic energy. In the Appendix we show that this product form is exact for the Gutzwiller wave function with a fixed value of d , when a rigid displacement is made of the occupied \mathbf{k} states. However, we have not been able to prove that the product form continues to hold for the case of a single particle-hole excitation which determines the value of the Landau effective mass m^* . Therefore the result that follows from Gutzwiller's approximate kinetic energy (namely, $F_1^s = F_1^q = 0$) may not be exact for the Gutzwiller wave function and it is possi-

ble that F_1^s and F_1^q take small values. An analytical expansion for small U (Ref. 15) as well as numerical calculations for large U (Ref. 16) show that the Gutzwiller approximate formula is qualitatively correct, but that there are small corrections which imply that the reduction factor for the kinetic energy and the step in the momentum distribution function are not exactly equal. However, the differences are not large and can be accounted for by Landau parameters $F_1^{s,a}$ of the order of 1.

Similar arguments can be used for the half-filled band close to the metal-insulator transition, i.e., for $U < 8|\varepsilon_0|$. In this case the doubly occupied and empty sites are responsible for free motion. This part of the sum rule is therefore proportional to d . On the other hand, for $d \ll 1$, $q \approx 8d$.^{1,2} This suggests that the sum rule is almost exhausted by the free-motion part. Proceeding as above we conclude that also in the half-filled band case $m_c \approx m^*$, and the Landau parameters $F_1^{s,a}$ are small, at least in the Gutzwiller approximation as $U \rightarrow 8|\varepsilon_0|$, the critical value for the metal-insulator transition.

IV. APPLICATION TO THE PERIODIC ANDERSON MODEL

The derivation of Sec. III can be readily generalized to the periodic Anderson model, which we write as follows:

$$H = \sum_{\mathbf{k},s} \varepsilon(\mathbf{k}) c^\dagger(\mathbf{k},s) c(\mathbf{k},s) + E_f \sum_{\mathbf{n},l,s} n_f(\mathbf{n},l,s) + \frac{1}{2} \sum_{\mathbf{n}} \sum_{l,s \neq l',s'} U(l,l') n_f(\mathbf{n},l,s) n_f(\mathbf{n},l',s') + H' \quad (18)$$

where $\varepsilon(\mathbf{k})$ is the energy spectrum of the conduction band,

$$n_f(\mathbf{n},l,s) = f^\dagger(\mathbf{n},l,s) f(\mathbf{n},l,s)$$

is the number operator for f electrons at site \mathbf{n} , in the orbital l , and with spin s , and $U(l,l')$ are the on-site Coulomb interactions. H' describes the hybridization between the conduction band and the f levels,

$$H' = \sum_{\mathbf{n},l,s} [V_l f^\dagger(\mathbf{n},l,s) c(\mathbf{n},s) + V_l^* c^\dagger(\mathbf{n},s) f(\mathbf{n},l,s)] \quad (19)$$

where we have assumed that the matrix elements are diagonal in direct space, i.e., \mathbf{k} independent. The charge density at site \mathbf{n} , defined as

$$\rho(\mathbf{n}) = e \sum_s \left[c^\dagger(\mathbf{n},s) c(\mathbf{n},s) + \sum_l n_f(\mathbf{n},l,s) \right] \quad (20)$$

commutes with all terms in H except with the band term of conduction electrons. Therefore the current operator defined through the continuity equation (4) simply involves the conduction electrons. In a tight-binding description it is again given by Eq. (5). In this case we can proceed as in Sec. III. One easily verifies that Eqs. (7) and (8) still hold with $\rho(\mathbf{n})$ defined by Eq. (20). It follows that the sum rule (11) remains valid with T_α representing the kinetic energy of the conduction electrons. Therefore we do not expect strong modifications of the sum rule due to the f electrons as long as the hybridization is weak. However, it is the Drude term which we have to analyze in order to discuss the Landau parameters. It will be shown below that the f electrons contrib-

ute to this term. Unfortunately, in the case of the Anderson model it is not possible to simply consider a limiting case where the Drude term dominates the sum rule. Instead we proceed by examining renormalized band schemes which have been proposed for the Anderson model.^{5,17-19}

Let us first consider a pure band model [e.g., Eq. (18) with $U(l,l')=0$]. We assume that the f -level lies within the conduction band and that it pins the Fermi energy. The hybridization term leads to a spectrum with predominantly f -character in the region of the bare f level and conduction-band character outside. The conductivity $\sigma(\omega)$ will be dominated by two contributions, a Drude term at small frequencies due to transport at the Fermi energy and "interband" transitions at higher frequencies between band states across the hybridization gap. The Drude contribution originates from intersite transfer of f electrons which is mediated by the conduction electrons through H' . Therefore the effective hopping matrix elements for f electrons are given by

$$t(l,l') = V_l^* V_{l'} / [\epsilon(k_F) - E_f]. \quad (21)$$

In the weak-hybridization limit this expression is small and thus the effective mass will be large. The Coulomb interactions will reduce t in a way similar to the Hubbard model. Suppose that all Coulomb matrix elements are large and that, on the average, there is slightly less than one f electron per site, $n_f < 1$. The f electrons will then only move to unoccupied sites and the Drude term is multiplied by a factor $1 - n_f$. There is a second effect of interactions which acts in the opposite way. It can be viewed as a self-energy correction for the f level which reduces the energy denominator in Eq. (21) and thus increases t . The first effect proportional to $1 - n_f$ is, however, dominant if n_f is close to 1. This agrees with explicit calculations within renormalized band schemes,^{5,17-19} showing that the correlated system can be described in terms of an effective single-particle Hamiltonian with a renormalized hybridization term

$$\tilde{V}_l \propto (1 - n_f)^{1/2} V_l.$$

In view of Eq. (21) this yields a hopping matrix element

$$\tilde{t} \propto (1 - n_f) t.$$

The arguments given above indicate that heavy-electron systems, in particular certain Ce compounds where $n_f < 1$,²⁰ can be described in terms of an effective Hubbard-type Hamiltonian for the f electrons, at least with respect to properties which involve only the states near the Fermi energy. Proceeding as in Sec. III we conclude that the Landau parameters $F_1^{s,a}$ are small for these systems. Recent experiments²¹ on the plasma frequency associated with the Drude contribution in the heavy-electron system UPT₃ are consistent with this conclusion.

V. CONCLUSIONS

In this paper we examined the implications of sum rule on the charge (or mass) and the spin conductivities for the $l=1$ Landau parameters in the Hubbard model in the almost-localized limit.

In a certain limit the sum rule is dominated by the contribution of the Drude term of the Fermi liquid and in this limit one can show that F_1^s and F_1^a are small. This result has a simple interpretation, namely that in the almost-localized limit the effective number of carriers is small but they move with essentially the bare mass. The reconciliation of this result with the large Fermi surface required by the Luttinger theorem is made through a Fermi-liquid description with the usual number of carriers and a large effective mass (see also Ref. 3, p. 139). Thus Fermi liquids in the almost-localized limit are far from the Galilean-invariant limit. Within a renormalized band approximation we can extend the description to the periodic Anderson model and the heavy-electron system.

Note added in proof. A similar conclusion concerning the value of F_1^s in heavy-electron metals has been reached by C. M. Varma, K. Miyake, and S. Schmitt-Rink [Phys. Rev. Lett. **57**, 626 (1986)] using a different line of reasoning.

ACKNOWLEDGMENTS

We thank R. Joynt and K. Ueda for useful discussions. We also acknowledge support from the Schweizerischer Nationalfonds zur Förderung der wissenschaftlichen Forschung.

APPENDIX

We now discuss the exact calculation of the change in kinetic energy for a rigid displacement of the Fermi surfaces, for the Gutzwiller wave function.

We write the Gutzwiller wave function for the Hubbard model, in three dimensions, as

$$|\psi_G\rangle = g^D |\psi_0\rangle = \sum_{\{\alpha\uparrow\}, \{\alpha\downarrow\}} g^D \Gamma_0\{\alpha\uparrow\} \Gamma_0\{\alpha\downarrow\} | \{\alpha\uparrow\}, \{\alpha\downarrow\} \rangle,$$

where

$$|\psi_0\rangle = \prod_{k < k_{F\sigma}, \sigma} c_{k,\sigma}^\dagger |0\rangle$$

is the Fermi sea of noninteracting spin- $\frac{1}{2}$ fermions,

$$\{\alpha_\sigma\} = (\mathbf{n}_{1,\sigma}, \dots, \mathbf{n}_{N,\sigma})$$

are configurations on the lattice,

$$| \{\alpha\uparrow\}, \{\alpha\downarrow\} \rangle = \prod_{\sigma, \mathbf{n}_\sigma \in \{\alpha_\sigma\}} c_{\mathbf{n}_\sigma, \sigma}^\dagger |0\rangle,$$

and $\Gamma_0\{\alpha_\sigma\}$ is the corresponding Slater determinant,

$$\Gamma_0\{\alpha_\sigma\} = \det(\{e^{i\mathbf{n}_\sigma \cdot \mathbf{k}_\sigma}\}),$$

where \mathbf{n}_σ runs over $\{\alpha_\sigma\}$ and \mathbf{k}_σ over the Fermi sea. D counts the doubly occupied sites.

Now we displace the up and down Fermi seas by \mathbf{p}_\uparrow and \mathbf{p}_\downarrow , respectively. The Slater determinants transform as

$$\begin{aligned} \Gamma_{\mathbf{p}_\sigma}\{\alpha_\sigma\} &= \det(\{e^{i\mathbf{n}_\sigma \cdot (\mathbf{k}_\sigma + \mathbf{p}_\sigma)}\}) \\ &= \prod_{\mathbf{n}_\sigma \in \{\alpha_\sigma\}} e^{i\mathbf{n}_\sigma \cdot \mathbf{p}_\sigma} \det(\{e^{i\mathbf{n}_\sigma \cdot \mathbf{k}_\sigma}\}) \\ &= \left[\prod_{\mathbf{n}_\sigma \in \{\alpha_\sigma\}} e^{i\mathbf{n}_\sigma \cdot \mathbf{p}_\sigma} \right] \Gamma_0\{\alpha_\sigma\}. \end{aligned}$$

Notice that

$$|\Gamma_{\mathbf{p}_\sigma}\{\alpha_\sigma\}|^2 = |\Gamma_0\{\alpha_\sigma\}|^2$$

and

$$\langle \psi_G\{\mathbf{p}_\sigma\} | \psi_G\{\mathbf{p}_\sigma\} \rangle = \langle \psi_G | \psi_G \rangle .$$

The corresponding transformation of the expectation value of the kinetic energy is

$$\begin{aligned} \langle \psi_G\{\mathbf{p}_\sigma\} | T_{ij\uparrow} | \psi_G\{\mathbf{p}_\sigma\} \rangle &= \cos[(\mathbf{n}_i - \mathbf{n}_j) \cdot \mathbf{p}_\uparrow] \\ &\quad \times \langle \psi_G | T_{ij\uparrow} | \psi_G \rangle , \quad (\text{A1}) \\ T_{ij\uparrow} &= c_{i\uparrow}^\dagger c_{j\uparrow} + c_{j\uparrow}^\dagger c_{i\uparrow} . \end{aligned}$$

We want to stress the fact that this transformation law is valid for all values of g . For the case of nearest-neighbor hopping only and no magnetization, we get, from (A1),

$$\langle T \rangle_{|\mathbf{p}_\sigma|} = \frac{1}{6} \left[\sum_{\sigma, \alpha=x,y,z} \cos(ap_\sigma^\alpha) \right] \langle T \rangle_0 ,$$

where a is the lattice constant of a simple-cubic lattice. Since the effective mass is spin independent, it follows that

$$F_1^a = F_1^s$$

exactly for all g , or $F_1^{a,-\sigma} = 0$. As the transformation law is the same for the interacting and the noninteracting cases, and since $\langle T \rangle_0$ scales with m/m^* , F_1^a and F_1^s should be both small, but for any more quantitative statement one would need a good estimate of $\langle T_0 \rangle (m/m^*)$.

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