Conductivity relaxation time due to electron-hole collisions in optically excited semiconductors

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In an electron-hole plasma, electron-hole collisions conserve the total momentum but relax the electric current. The corresponding relaxation time $\tau_{e\text{-}h}$ is obtained from two coupled Boltzmann equations. We take into account multivalley structure and screening. An exact solution is given when the electron-hole plasma is in the quantum limit, while in the classical limit, we use a variational approach. We find that when the plasma density is increased at fixed temperature, $\tau_{e\text{-}h}$ goes through a minimum reached roughly when the plasma becomes degenerate. The size of $\hbar/\tau_{e\text{-}h}$ at the maximum is mainly controlled by the exciton binding energy.

Recent experiments[1] on the reflectivity of extremely dense electron-hole (e-h) plasmas created by femtosecond laser irradiations of silicon samples have shown a free-carrier absorption much larger than the usually quoted value resulting from electron-phonon collisions. This indicates the existence of a very short relaxation time (of the order of 10^{-15} s) for the e-h current. This free-carrier absorption was associated[1] with a new mechanism which can be seen only in dense e-h plasmas of semiconductors. In a metal, or a doped semiconductor, electron-electron collisions conserve the total momentum and the total current, whereas in an optically excited semiconductor the momentum is still conserved, in an e-h collision but the total current changes, even if the electron and hole masses are equal: e-h collisions produce a new relaxation mechanism for the current.

In this paper we investigate this relaxation mechanism by studying the conductivity of an e-h plasma at internal thermodynamical equilibrium, i.e., with a well-defined density n and temperature T. We *define* an effective relation time $\tau_{e\text{-}h}$ for the conductivity as

$$\mathbf{j}=\frac{ne^2\tau_{e\text{-}h}}{m}\mathbf{E}\ , \tag{1}$$

m being an appropriate reduced e-h mass for which we will make a definite choice later in the paper (the physical results will be of course independent of this choice). The conductivity is indeed controlled by e-h collisions if the e-h collision probability $\hbar/\tau_{e\text{-}h}$ is larger than the electron-phonon one $\hbar/\tau_{e\text{-ph}}$. The study of electron-phonon collisions, for all regimes of density and temperature, will be made in a separate work.[2]

The conductivity relaxation time $\tau_{e\text{-}h}$ depends *a priori* on the plasma temperature T, the plasma density n, and the Coulomb interaction, i.e., the electric charge e. In order to clarify the presentation of the results, it is convenient to express the density n in term of a temperature T_n defined as

$$k_B T_n=\frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} \tag{2}$$

(T_n is merely the temperature at which a gas of particles with mass m and fixed density n becomes degenerate; if this gas was cooled down to $T=0$, $k_B T_n$ would be its Fermi energy). In the same way the electrical charge is expressed in terms of an effective exciton energy

$$k_B T_{\text{ryd}}=\frac{me^4}{2\hbar^2\epsilon^2}\ , \tag{3}$$

where ϵ is the static dielectric constant. Because we deal with e-h Coulomb interaction, the e-h collision relaxation rate scales as $T_{\text{ryd}}\sim e^4$. Then from dimensional arguments, one has the following form[3] for the dependence of $\tau_{e\text{-}h}$ on e^2, T, and n:

$$\frac{\hbar}{\tau_{e\text{-}h}}=k_B T_{\text{ryd}} G\left[\frac{T}{T_n},\frac{T}{T_{\text{ryd}}}\right]. \tag{4}$$

The dependence of G on T_{ryd} (i.e., in e^2) will come from screening. The purpose of this paper is to calculate the function $G(T/T_n,T/T_{\text{ryd}})$.

We will consider a semiconductor with a multivalley band structure, an isotropic electron mass m_e and an isotropic hole mass m_h. We consider only a single heavy-hole band since the few light holes are not expected to change qualitatively or even quantitatively our results. The effective collision time $\tau_{e\text{-}h}$ is calculated explicitly for an e-h plasma in the degenerate limit as well as in the classical one. In both cases, the screening of the Coulomb interaction by the carriers is taken into account.

The e-h current results from the changes of the electron and hole distributions due to e-h collisions. These changes are described by a set of two coupled Boltzmann equations, one for the electron and one for the hole distribution. An *exact* solution is obtained in the quantum limit in a way similar to transport theory in Fermi liquids. Using the Born approximation for the e-h collision cross section, one finds

$$\frac{\hbar}{\tau_{e\text{-}h}}\sim k_B T_{\text{ryd}}\left[\frac{T}{T_n}\right]^2\left[\frac{T_n}{T_{\text{ryd}}}\right]^{1/4}. \tag{5}$$

The explicit temperature dependence of $\tau_{e\text{-}h}$ in the quantum limit comes from phase-space restrictions, while the last factor comes from the screening, $(T_n/T_{\rm ryd})^{1/4}$ being just the produce of the exciton Bohr radius by the screening wave vector.

No exact solution of the Boltzmann equations exists to our knowledge for a classical gas. In this limit, one usually uses a variational principle to get a solution. Using again the Born approximation for the collision cross section, one finds

$$\frac{\hbar}{\tau_{e\text{-}h}} \sim k_B T_{\rm ryd} \left[\frac{T_n}{T} \right]^{3/2} \ln \left[\frac{T^4}{T_{\rm ryd} T_n^3} \right] . \tag{6}$$

Equation (6) shows that the effective e-h collision probability $\hbar/\tau_{e\text{-}h}$ decreases in the classical limit when, at a given temperature, the plasma density goes to zero—as there are no more particles to collide with. However, from Eq. (5) the same is true also in the quantum limit, when the plasma density becomes very large—because of quantum phase-space restrictions for collision processes—whereas by a naive argument, one would have though that the denser the e-h plasma, the stronger the effect of e-h collisions. Consequently, when the e-h density is increased at a fixed temperature, $\hbar/\tau_{e\text{-}h}$ shows a maximum for $T_n \sim T$, the amplitude of which is mainly controlled by the size of the Coulomb interaction, i.e., the effective exciton energy $k_B T_{\rm ryd}$.

In Sec. I we write the set of two coupled Boltzmann equations describing the electron and hole distributions. In Sec. II we give the exact solutions of these Boltzmann equations in the quantum limit and show how the conductivity is modified if one includes electron-electron and hole-hole collisions. In Sec. III we calculate the conductivity in the classical limit within a variational approach. In Sec. IV we discuss the behavior of the effective collision time as a function of the plasma density, and give orders of magnitude for $\tau_{e\text{-}h}$.

In this work we have, for clarity, left the electron-phonon collisions out of the Boltzmann equations. The comparison between the effective collision time due to e-h collisions with the one due to electron-phonon collisions will be made in an independent publication,[2] as well as a discussion[4] of the range of temperature and density where the conductivity is expected to be dominated by these e-h collisions. However, from now on it is clear that relaxation times of the order of 10^{-15} s observed experimentally are likely to require a process different from the usual electron-phonon interaction in order to be explained, and this justifies the present investigation. We will see that e-h collisions can indeed produce relaxation times as short as 10^{-15} s.

I. BOLTZMANN EQUATIONS

The electron and hole distributions of an e-h plasma in an electric field $\mathbf{E}$ satisfy the following Boltzmann equations[5] in the linear approximation:

$$\begin{aligned} \frac{\partial f_e^0(\mathbf{k}_1)}{\partial \epsilon^e} \frac{\hbar \mathbf{k}_1}{m_e} \cdot (-e\mathbf{E}) &= I_e(\mathbf{k}_1) , \\ \frac{\partial f_h^0(\mathbf{k}_1)}{\partial \epsilon^h} \frac{\hbar \mathbf{k}_2}{m_h} \cdot (e\mathbf{E}) &= I_h(\mathbf{k}_2) . \end{aligned} \tag{7}$$

The collision term I_e includes an e-h collision term $I_{e\text{-}h}$ and an electron-electron collision term $I_{e\text{-}e}$, similarly for I_h. As a first step, let us consider only e-h collisions:

$$\begin{aligned} I_{e\text{-}h}(\mathbf{k}_1) = -2(2\pi)^{-9} \int d^3k_2\, d^3k_3\, d^3k_4\, &\delta(\epsilon_1^e + \epsilon_2^h - \epsilon_3^e - \epsilon_4^h)\delta(\mathbf{k}_1^e + \mathbf{k}_2^h - \mathbf{k}_3^e - \mathbf{k}_4^h) \\ &\times [f_1^e f_2^h (1-f_3^e)(1-f_4^h) - (1-f_1^e)(1-f_2^h) f_3^e f_4^h] W(\mathbf{k}_1^e, \mathbf{k}_2^h, \mathbf{k}_3^e, \mathbf{k}_4^h) . \end{aligned} \tag{8}$$

$\mathbf{k}_1^e$ and $\mathbf{k}_3^e$ are the electron momentum before and after collision, $\mathbf{k}_2^h$ and $\mathbf{k}_4^h$ are the hole ones [see Fig. 1(a)]; $\epsilon^e = \hbar^2 k^2/2m_e$ and $\epsilon^h = \hbar^2 k^2/2m_h$ are electron and hole energies. f^e and f^h are the electron and hole distributions in the presence of the electric field while f_e^0 and f_h^0 are their equilibrium values; $f_1 \equiv f(\mathbf{k}_1)$. W is the scattering probability. For a screened Coulomb potential, within the Born approximation,[6] W is

$$\frac{\hbar}{2\pi} W(q) = \left[\frac{4\pi(e^2/\epsilon)a^2}{1+q^2a^2} \right]^2 , \tag{9}$$

where $\mathbf{q} = \mathbf{k}_3^e - \mathbf{k}_1^e$ is the momentum transfer during the collision and a the screening length. This Born formula is valid[5] only if the diffusing potential can be taken as a perturbation, i.e., for $a_0 \gg a$ or $\bar{k} a_0 \gg 1$, where $a_0 = \hbar^2/\epsilon/me^2$ is the Bohr radius and $\bar{k}$ is the average carrier momentum.

The contribution $I_{e\text{-}h}$ of the e-h collisions to the hole collision integral I_h is obtained from $I_{e\text{-}h}$ by changing d^3k_2 into d^3k_1 and multiplying by v, the conduction-band degeneracy. Within the linear approximation for the

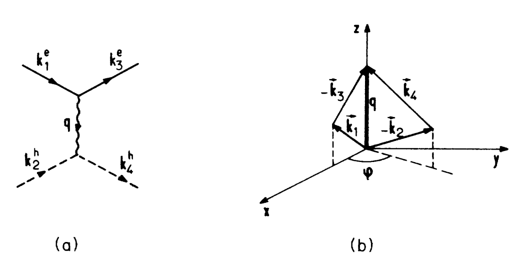

FIG. 1. (a) Collision between one electron $\mathbf{k}_1^e$ and one hole $\mathbf{k}_2^h$, with a momentum transfer $\mathbf{q}$. (b) Bipolar coordinates. The $\mathbf{z}$ axis is chosen along $\mathbf{q}$ and the plane xz is chosen to contain $\mathbf{k}_1$. φ is the angle between the planes $(\mathbf{k}_1, \mathbf{k}_3)$ and $(\mathbf{k}_2, \mathbf{k}_4)$.

change in Fermi distributions, one can define ψ^e and ψ^h by

$$f_1^e = f_{e1}^0 + f_{e1}^0(1-f_{e1}^0)\psi_1^e \,, \qquad f_2^h = f_{h2}^0 + f_{h2}^0(1-f_{h2}^0)\psi_2^h \,. \tag{10}$$

The collision integral reads in terms of ψ,

$$I_{e\text{-}h} = -2(2\pi)^{-9}\int d^3k_2\,d^3k_3\,d^3k_4\,\delta(\epsilon)\delta(\mathbf{k})W \times f_{e1}^0 f_{h2}^0(1-f_{e3}^0)(1-f_{h4}^0) \times(\psi_1^e+\psi_2^h-\psi_3^e-\psi_4^h) \,. \tag{11}$$

It is then easy to verify that the exact solutions, to first order in E, for the functions ψ verifying the Boltzmann equations (7) have the form

$$\psi^e(\mathbf{k}) = -\frac{e\mathbf{E}}{kT}\cdot\frac{\hbar\mathbf{k}}{m_e}\tau^e(k)\,, \qquad \psi^h(\mathbf{k}) = \frac{e\mathbf{E}}{kT}\cdot\frac{\hbar\mathbf{k}}{m_h}\tau^h(k)\,, \tag{12}$$

where $\tau^e(k)$ and $\tau^h(k)$ are unknown functions of the momentum moduli only [we want to stress that Eqs. (12) are not relaxation-time approximations but exact solutions]; from Eqs. (7) and (11), τ_e and τ_h should satisfy

$$f_{e1}^0(1-f_{e1}^0) = 2(2\pi)^{-9}\int d^3k_2\,d^3k_3\,d^3k_4\,\delta(\epsilon)\,\delta(\mathbf{k})W f_{e1}^0 f_{h2}^0(1-f_{e3}^0)(1-f_{h4}^0) \times\left[\tau_1^e - \tau_3^e\frac{\mathbf{k}_1\cdot\mathbf{k}_3}{k_1^2} - \frac{m_e}{m_h}\left[\tau_2^h\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1^2} - \tau_4^h\frac{\mathbf{k}_1\cdot\mathbf{k}_4}{k_1^2}\right]\right], \tag{13a}$$

$$f_{h2}^0(1-f_{h2}^0) = 2v(2\pi)^{-9}\int d^3k_1\,d^3k_3\,d^3k_4\,\delta(\epsilon)\,\delta(\mathbf{k})W f_{e1}^0 f_{h2}^0(1-f_{e3}^0)(1-f_{h4}^0) \times\left[\tau_2^h - \tau_4^h\frac{\mathbf{k}_2\cdot\mathbf{k}_4}{k_2^2} - \frac{m_h}{m_e}\left[\tau_1^e\frac{\mathbf{k}_2\cdot\mathbf{k}_1}{k_2^2} - \tau_3^e\frac{\mathbf{k}_2\cdot\mathbf{k}_3}{k_2^2}\right]\right]. \tag{13b}$$

One is now left with the resolution of Eq. (13). Before proceeding to discuss this resolution in the degenerate and in the classical limits as is done in Secs. II and III, one can look at the expression of the e-h current in terms of τ^e and τ^h. By definition, if the electric field $\mathbf{E}$ is along the x axis, the current $\mathbf{j}$ along this direction is

$$j_x = \sum_{k_1}(-e)\frac{\hbar k_{1x}}{m_e}f_1^e + \sum_{k_2} e\frac{\hbar k_{2x}}{m_h}f_2^h \,. \tag{14}$$

With use of the fact that for the unperturbed distribution f^0 there is no current, j_x is finally expressed from τ^e and τ^h as

$$j_x = e^2E\frac{4}{3}\left[\frac{v}{m_e}\int\frac{d^3k_1}{(2\pi)^3}\frac{\hbar^2k_1^2}{2m_ek_BT}\tau_1^e f_{e1}^0(1-f_{e1}^0) + \frac{1}{m_h}\int\frac{d^3k_2}{(2\pi)^3}\frac{\hbar^2k_2^2}{2m_hk_BT}\tau_2^h f_{h2}^0(1-f_{h2}^0)\right]. \tag{15}$$

The electric current is the sum of an electron contribution and a hole contribution as expected, but the collision times τ_e and τ_h result from a set of two coupled integrals equations (7).

II. QUANTUM LIMIT

For a degenerate e-h plasma, Eqs. (13) can be solved exactly analytically by a procedure similar to the one used in Fermi liquids.[7] The quantum limit leads to a very crucial simplification in Eqs. (13): The energy and momentum conservation laws together with the Fermi form of the electron and hole distributions impose that all the electron and hole momenta stay close to their Fermi value within an energy range k_BT. For an e-h plasma density n and a conduction-band degeneracy v, these Fermi momenta K_e and K_h for electrons and holes are such that

$$n = vK_e^3/3\pi^2 = K_h^3/3\pi^2 \,. \tag{16}$$

Let us now examine how the calculation on the right-hand side of Eqs. (13) goes on. First, one performs the integration on $\mathbf{k}_4$ using the function $\delta(\mathbf{k})$. Then one makes a change of variables using bipolar coordinates [see Fig. 1(b)]. We introduce the vector $\mathbf{q}=\mathbf{k}_1-\mathbf{k}_3=-\mathbf{k}_2+\mathbf{k}_4$. Then

$$\int d^3k_2\,d^3k_3 = 2\pi\int\frac{k_2}{k_1}dk_2\,k_3\,dk_3\,k_4\,dk_4\,dq\,d\varphi \,, \tag{17}$$

φ being the angle between the planes $(\mathbf{k}_1,-\mathbf{k}_3)$ and $(-\mathbf{k}_2,\mathbf{k}_4)$ as shown in Fig. 1(b). The factor 2π comes from the integration on the azumuthal angle around $\mathbf{k}_1$. The introduction of the vector $\mathbf{q}$ is particularly appropriate since the scattering probability is expressed in terms of this momentum transfer [see Eq. (9)]. Moreover, this change of variable allows us to eliminate the angular part of the integrals. As a result the scattering probability appears only in parameters. The fact that the **k**'s stay close to the Fermi momentum implies that

$$q^2=(\mathbf{k}_1-\mathbf{k}_3)^2=k_1^2+k_3^2-2\mathbf{k}_1\cdot\mathbf{k}_3\simeq 2K_e^2-2\mathbf{k}_1\cdot\mathbf{k}_3 \ . \quad (18)$$

Consequently, the scalar products appearing in Eq. (13) are easily expressed in terms of q;

$$\mathbf{k}_1\cdot\mathbf{k}_3=K_e^2-q^2/2 \ ,$$
$$\mathbf{k}_2\cdot\mathbf{k}_4=K_h^2-q^2/2 \ , \quad (19)$$
$$\int \mathbf{k}_4\cdot\mathbf{k}_1\,d\varphi=\int -\mathbf{k}_2\cdot\mathbf{k}_1\,d\varphi=2\pi q^2/4 \ .$$

By referring back to Eqs. (13), one can rewrite them in a much simpler form:

$$n(-x)=\int dx'\,dx_3\,dx_4\,\delta(x+x'+x_3+x_4)n(x')n(x_3)n(x_4)\left[\alpha\left[\frac{\tau_e(x)}{m_e}-\frac{\tau_e(x')}{m_e}\right]+\beta\left[\frac{\tau_e(x')}{m_e}+\frac{\tau_h(x')}{m_h}\right]\right] , \quad (20a)$$

$$n(-x)=\int dx'\,dx_3dx_4\,\delta(x+x'+x_3+x_4)n(x')n(x_3)n(x_4)\left[\alpha\frac{K_h^2}{K_e^2}\left[\frac{\tau_h(x)}{m_h}-\frac{\tau_h(x')}{m_h}\right]+\beta\left[\frac{\tau_e(x')}{m_e}+\frac{\tau_h(x')}{m_h}\right]\right] , \quad (20b)$$

where $x=(\epsilon-\mu)/k_BT$ is the reduced energy. (x,x') corresponds to (x_1,x_2) in Eq. (12a) and (x_2,x_1) in Eq. (12b). x_3 and x_4 have also been changed into $-x_3a,d\,-x_4$. The functions $n(x)=(e^x+1)^{-1}$ come from the Fermi distributions f_0. We have used Eq. (16) to obtain Eq. (20b). The specific form of the scattering probability appears only in the coefficients α and β and does not enter elsewhere in the integral equations:

$$\alpha=\frac{2m}{\tau_0}\int_0^{2K_e}\frac{W(q)}{W(0)}\frac{dq}{K_e} \ ,$$
$$\beta=\frac{m}{\tau_0}\int_0^{2K_e}\frac{W(q)}{W(0)}\frac{q^2dq}{K_e^3} \ . \quad (21)$$

The constant τ_0 has the dimension of a time

$$\frac{\hbar}{\tau_0}=\frac{1}{(2\pi)^4}\frac{m_e^2m_h^2}{m}(k_BT)^2\frac{W(0)}{\hbar^5}$$

(Note that the momentum transfer q varies from zero to twice the shortest of the two Fermi momenta K_e and K_h; here this is K_e as it is always the case in semiconductors). In order to decouple the two integral equations (20), let us introduce two new functions defined as

$$J(x)=\frac{\tau_e(x)}{m_e}+\frac{\tau_h(x)}{m_h} \ ,$$
$$L(x)=\frac{\tau_e(x)}{m_e}-\frac{K_h^2}{K_e^2}\frac{\tau_h(x)}{m_h} \ . \quad (22)$$

The function $J(x)$ is the one entering the expression for the current; as from Eq. (15) one sees that

$$j=ne^2E\int J(x)n(x)n(-x)dx \ . \quad (23)$$

Then the Eqs. (20) can be rewritten in terms of $J(x)$ and $L(x)$ as

$$\frac{n(-x)}{\alpha s}=\int dx'\,dx_3\,dx_4\,\delta(x+x'+x_3+x_4)n(x')n(x_3)\times n(x_4)[J(x)-(1-\beta/\alpha s)J(x')] \ , \quad (24a)$$

$$0=\int dx'\,dx_3\,dx_4\,\delta(x+x'+x_3+x_4)n(x')n(x_3)n(x_4)\times[L(x)-L(x')] \ , \quad (24b)$$

where s is a constant, independent of the density

$$s=K_h^2/(K_e^2+K_h^2)=(1+v^{-2/3})^{-1} \ . \quad (25)$$

The integral equation (24a) is now similar to the one found in Fermi liquids and will be solved using a similar procedure.[7] Equation (24b) will be considered later. The integrations over x_3 and x_4 are performed and leads to

$$\frac{2}{\alpha s}=(x^2+\pi^2)J(x)-(1-\beta/\alpha s)\cosh\frac{x}{2}\times\int dx'\frac{(x+x')J(x')}{\sinh[(x+x')/2]\cosh(x'/2)} \ . \quad (26)$$

Then one expands $J(x)$ (which is easily seen to be an even function of x) over a set of eigenfunctions $\Phi_\nu(x)$,

$$J(x)=2\cosh\frac{x}{2}\sum_{\nu=1}C_\nu\Phi_\nu(x) \ , \quad (27)$$

where the $\Phi_\nu(x)$ are defined by

$$\int dx'\,\Phi_\nu(x')\frac{x-x'}{\sinh[(x-x')/2]}=\frac{2}{\nu(\nu+1)}(x^2+\pi^2)\Phi_\nu(x) \ ,$$
$$\int \Phi_\mu(x)\Phi_\nu(x)(x^2+\pi^2)dx=\delta_{\mu\nu} \ . \quad (28)$$

One can show that

$$A_\nu=\int\frac{\Phi_\nu(x)}{\cosh(x/2)}dx=-\frac{2}{\pi}\frac{(2\nu+1)^{1/2}}{\nu(\nu+1)} \ . \quad (29)$$

From Eqs. (26)–(29) one obtains easily the coefficient C_ν as

$$C_\nu=-\frac{1}{\pi}\frac{(2\nu+1)^{1/2}}{\beta+\alpha s[\nu(\nu+1)/2-1]} \ . \quad (30)$$

By referring back to expression (23) for the current, the integration is straightforward using Eq. (27), and one finally gets the exact expression for the current of an e-h plasma in the quantum limit

$$j=ne^2E\sum_{\nu}\tfrac{1}{2}C_\nu A_\nu$$
$$=n\frac{e^2}{\beta}E\sum_{\nu_{\rm odd}}\frac{(2\nu+1)}{\pi^2\nu(\nu+1)\{1+(\alpha/\beta)s[\nu(\nu+1)/2-1]\}}\;. \tag{31}$$

The two coefficients α and β, defined in Eq. (21), depend on the explicit form of W. The calculation of α and β is straightforward if one uses the Born approximation (9). Noting that this approximation can be used only if the screening length a is smaller than the Bohr radius a_0 or smaller than $(Ka)a_0$, the Born approximation implies $K_e a \gg 1$ (i.e., $T_n \gg T_0$), because the Thomas-Fermi screening length is given by

$$\left[\frac{4\pi e^2 a^2}{\epsilon}\right]^{-1}=\frac{\partial n_e}{\partial\mu_e}+\frac{\partial n_h}{\partial\mu_h}=\frac{K_e(vm_e+v^{1/3}m_h)}{\pi^2\hbar^2}\;. \tag{32}$$

In this limit Eqs. (9) and (21) give $\beta=\pi m/4\tau_0(K_e a)^3$, while α/β is very large and on the order of $(K_e a)^2$. Physically, this happens because only collisions with small momentum transfer are important. Consequently the $\nu=1$ term of the expansion (29) gives the result with an excellent accuracy. Physically, this is because the limit $\beta\to 0$ corresponds to a situation where we have only forward scattering. The collisions do not destroy the current and the conductivity is infinite. The corresponding divergence comes from the $\nu=1$ term, and therefore this term is a very good approximation for small β. Finally,

$$j=\frac{ne^2E\tau_0}{m}\frac{6}{\pi^3}(K_e a)^3\;. \tag{33}$$

For completeness, let us look at the solution of Eq. (24b) for the function $L(x)$, the knowledge of both $J(x)$ and $L(x)$ being necessary to find $\tau_e(x)$ and $\tau_h(x)$, i.e., the Fermi distributions in presence of an electric field. A procedure similar to the one used for $J(x)$ shows that the solution of Eq. (24b) is

$$L(x)=2c\cosh\frac{x}{2}\Phi_1(x)=-c\frac{\sqrt{3}}{2\pi} \tag{34}$$

when c is an undetermined constant. This undetermination comes from the translational invariance of the problem. The proper choice for c has to be imposed physically. It comes from the conservation of momentum during collisions:

$$\sum_{k_1}\mathbf{k}_1 f_1^e+\sum_{k_2}\mathbf{k}_2 f_2^h=0 \tag{35}$$

if we assume that the total momentum is initially zero. Using Eqs. (10) and (12), this conservation law becomes

$$\int[\tau_e(x)-\tau_h(x)]n(x)n(-x)dx=0 \tag{36}$$

and the constant c can be easily deduced from it.

Up to now, we have only considered e-h collisions and neglect e-e collisions in the collision integral I_e. e-e collisions cannot by themselves relax any current (if only one spherical electron mass is considered) since they conserve the total momentum. However, they modify the electron distribution created by e-h collisions as we will see below. Electron-electron collisions add a new term $I_{e\text{-}e}$ to the collision integral I_e defined in Eq. (7). One has $I_e=I_{e\text{-}e}+I_{e\text{-}h}$ with

$$I_{e\text{-}e}(k_1)=-2v(2\pi)^{-9}\int d^3k_2\,d^3k_3\,d^3k_4\,\delta(\epsilon)\,\delta(\mathbf{k})W$$
$$\times[f_1f_3(1-f_2)(1-f_4)$$
$$-(1-f_1)(1-f_3)f_2f_4]\;. \tag{37}$$

Similarly, a new integral $I_{h\text{-}h}$ for hole-hole collisions appears in I_h. Then we proceed as before using Eqs. (10) and (12), in order to get a set of two integral equations for τ_e and τ_h similar to Eqs. (13). Using the simplification of the quantum limit, one finds that electron-electron collisions add a new term

$$v\alpha\frac{m_e^2}{m_h^2}\left[\frac{\tau_e(x)}{m_e}-\frac{\tau_e(x')}{m_e}\right] \tag{38}$$

to the large square brackets of Eq. (20a), and that hole-hole collisions induce a new term

$$\alpha'\frac{m_h^2}{m_e^2}\left[\frac{\tau_h(x)}{m_h}-\frac{\tau_h(x')}{m_h}\right] \tag{39}$$

in the large square brackets of Eq. (20b), α' being obtained by changing K_e into K_h in α. As one can see, the introduction of e-e and h-h collisions does not change the structure of the integral equations (20) for $\tau_e(x)$ and $\tau_h(x)$, but only modifies the numerical coefficients: $\alpha\tau_e(x)$ becomes $\alpha_e\tau_e(x)$ with $\alpha_e=\alpha(1+vm_e^2/m_h^2)$ and $\alpha\tau_h K_h^2/K_e^2$ becomes $\alpha_h\tau_h(x)$ with $\alpha_h=\alpha K_h^2/K_e^2+\alpha' m_h^2/m_e^2$. The calculation goes on similarly, introducing two new functions, $J(x)$ defined as in Eq. (22) and $\tilde{L}(x)=\alpha_e\tau_e(x)/m_e-\alpha_h\tau_h(x)/m_h$. The solution $J(x)$ is just obtained from Eq. (24) by changing α into $\tilde{\alpha}=(\alpha_e+\alpha_h)/\alpha_e\alpha_h$ and similarly for the expression (31) of the e-h current.

Since e-e and h-h collisions do not change β (this is linked to the fact that the current is not changed by these collisions), they do not change the $\nu=1$ term of expression (31) for j and our final result Eq. (33) remains valid.

III. CLASSICAL LIMIT

To our knowledge the Boltzmann equations (7) or (13) cannot be solved exactly for an e-h plasma in the nondegenerate limit. The usual technique[8,9] is to replace $\tau_e(x)$ and $\tau_h(x)$ by trial functions which can be taken as

$$\tau_e(x)=\sum_{n=0}\tau_{en}x^n\;,\qquad \tau_h(x)=\sum_{n=0}\tau_{hn}x^n\;, \tag{40}$$

and to use a variational principle in order to find the best choice for these trial functions. This variational principle[4] states that $J_{e\text{-}h}/(J_e+J_h)^2$ should be minimum with

$$J_{e\text{-}h}=\frac{1}{2}\sum_{1,2,3,4} W\delta(\epsilon)\,\delta(\mathbf{k})(\psi_1^e+\psi_2^h-\psi_3^e-\psi_4^h)^2 \times f_1^e f_2^h(1-f_3^e)(1-f_4^h)\;,$$

$$J_e=\sum_1 \psi_1^e\left[-\frac{\partial f_{e1}^0}{\partial\varepsilon^e}\right]\frac{\hbar\mathbf{k}_1}{m_e}\cdot(-e\mathbf{E})\;, \tag{41}$$

$$J_h=\sum_2 \psi_2^h\left[-\frac{\partial f_{h2}^0}{\partial\varepsilon^h}\right]\frac{\hbar\mathbf{k}_2}{m_h}\cdot(e\mathbf{E})\;.$$

The variational principle fixes the shape of the τ function up to a multiplicative constant; this constant is determined going back to the Boltzmann equation: it should be such that[2]

$$J_e+J_h=J_{e\text{-}h}\;. \tag{42}$$

The calculation is considerably simplified by keeping only the lowest-order term in the expansion of the τ; this simplification turns out to be the approximation mostly used as it already gives a good result; we will also employ it in this paper. It is then straightforward to show that J_e and J_h are simply

$$J_e=n\frac{e^2E^2}{k_BT}\frac{\tau_e}{m_e}\;,\qquad J_h=n\frac{e^2E^2}{k_BT}\frac{\tau_h}{m_h}\;, \tag{43}$$

while the quantity $J_{e\text{-}h}$ can be written as

$$J_{e\text{-}h}=n\frac{e^2E^2}{k_BT}\left[\frac{\tau_e}{m_h}+\frac{\tau_h}{m_h}\right]^2\frac{m}{\tau_0'}J_0\;. \tag{44}$$

τ_0' is a constant having the dimension of a time

$$\frac{\hbar}{\tau_0'}=\frac{1}{9\pi^8}\frac{m^2}{\hbar^5}\frac{m_em_h}{m_e+m_h}W(0)(k_BT_n)^{3/2}(k_BT)^{1/2}\;, \tag{45}$$

while J_0 depends explicitly on the form of $W(q)$:

$$J_0=\int\frac{W[(2mk_BTu^2/\hbar^2)^{1/2}]}{W(0)}u^2d^3u\,d^3u_1\,d^3u_2e^{-u_1^2-u_2^2}\delta\left[u^2+\frac{2}{m_e+m_h}\left[\frac{m_em_h}{m}\right]^{1/2}\mathbf{u}\cdot(\mathbf{u}_1m_h^{1/2}+\mathbf{u}_2m_e^{1/2})\right]\;, \tag{46}$$

where $u=\hbar^2k^2/2mk_BT$. The integrations over $\mathbf{u}_1$ and $\mathbf{u}_2$ are straightforward if one replaces the δ function by its standard integral representation

$$J_0=\pi^{7/2}\int_0^\infty 2u^3\exp\left[-\frac{u^2m(m_e+m_h)}{4m_em_h}\right]\times\frac{W[(2mk_BTu^2/\hbar^2)^{1/2}]}{W(0)}du\;. \tag{47}$$

In order to explicitly get J_0, one needs to have W. Before doing so, one can note that the electric current defined in Eq. (9) depends on J_0 as

$$j=ne^2E\left[\frac{\tau_e}{m_e}+\frac{\tau_h}{m_h}\right]=\frac{ne^2E}{m}\frac{\tau_0'}{J_0} \tag{48}$$

when we have used the normalization condition (42) with the expressions (42) and (43) for $J_e,+J_h$, and $J_{e\text{-}h}$.

The use of the Born approximation for W relies on either one of the two conditions[3] $a_0\gg a$ or $\bar{k}a_0\gg1$. Equation (32) for the screening length gives in the case of a nondegenerate e-h plasma

$$4\pi e^2a^2=k_BT/2n\;, \tag{49}$$

so that the first condition $a\ll a_0$ reads $T_{\rm ryd}T^2\ll T_n^3$, while the second one reads $T_{\rm ryd}\ll T$. Since for the nondegenerate limit, $T_n\ll T$, one finds that the Born approximation is valid for a classical e-h plasma if $T_{\rm ryd}\ll T$ while it is never valid if $T\ll T_{\rm ryd}$. We will restrict ourselves in the following to the range of validity of the Born formula. The coefficient J_0 is then $J_0=\pi^{7/2}j(u_0^2)$ with

$$j(u_0^2)=\int_0^\infty\frac{2u^3\exp[-u^2m(m_e+m_h)/4m_em_h]}{(1+u^2/u_0^2)^2}du \tag{50}$$

and

$$u_0^2=\frac{\hbar^2}{2ma^2k_BT}\equiv\frac{16}{3\pi}\frac{T_{\rm ryd}^{1/2}T_n^{3/2}}{T^2}\;. \tag{51}$$

Noting that u_0^2 is small when the Born formula is valid, one simply finds $j(u_0^2)\sim u_0^4\ln u_0^{-2}$. Going back to the expression (48) for the electric current, one finally finds in the Born approximation

$$J=\frac{ne^2E}{m}\frac{\tau_0'}{\pi^{7/2}u_0^4\ln u_0^{-2}}\;. \tag{52}$$

IV. DENSITY DEPENDENCE OF THE EFFECTIVE e-h COLLISION TIME

We have obtained in Sec. II the exact solution for the conductivity in the quantum limit ($T\ll T_n$) and have given its explicit form [Eq. (33)] within the Born approximation. In terms of T, $T_{\rm ryd}$, and T_n, one finds

$$\frac{\hbar}{\tau_{e\text{-}h}}=k_BT_{\rm ryd}\left[\frac{T}{T_n}\right]^2\left[\frac{T_n}{T_{\rm ryd}}\right]^{1/4}\times\left[\frac{\pi^{5/2}}{12}\frac{vm_e^2m_h^2}{m^{7/2}(m_ev^{2/3}+m_h)^{1/2}}\right]\;. \tag{53}$$

The factor $(T/T_n)^2$ is due to the usual phase-space restriction in the quantum limit, while the factor $(T_n/T_{\rm ryd})^{1/4}$ comes from screening. As $T_n \sim n^{2/3}$, we conclude that the effective e-h collision time diverges at large density, as $n^{7/6}$, mainly due to quantum restriction. We have introduced an arbitrary reduced mass m in the definitions of J, $T_{\rm ryd}$, and T_n; one can easily check that m disappears from the electric current J as it should. The band-structure effect appears only within the last set of large parentheses in Eq. (53). If we choose $m^{-1}=m_e^{-1}+m_h^{-1}$, this term equals 12 for $m_e=m_h$ and $v=1$, while it is 35 for silicon.

The nondegenerate limit ($T_n \ll T$) studied in Sec. III, gives for the effective e-h collision time within the Born approximation ($T_0 \ll T$),

$$\frac{\hbar}{\tau_{e\text{-}h}}=k_B T_{\rm ryd}\left[\frac{T_n}{T}\right]^{3/2}\left[\ln\frac{T^2}{T_n^{3/2}T_{\rm ryd}^{1/2}}\right]\times\left[\frac{16}{9\pi^{3/2}}\frac{m_e m_h}{m(m_e+m_h)}\right] \quad (54)$$

to leading order in T/T_n. One finds that the collision probability is proportional to the e-h density ($T_n^{3/2}\sim n$) and goes to zero when there are no more carriers, as expected (see Fig. 2). The ln term is a slowly varying function which comes from screening. The band-structure part, placed with the last set of large parentheses, is simply $32/9\pi^{3/2}\sim 0.6$ with our choice $m^{-1}=m_e^{-1}+m_h^{-1}$; it does not depend on v as expected for independent classical carriers.

To our knowledge there is no analytical way to solve the set of two coupled Boltzmann equations in the intermediate regime when $T\sim T_n$. We have seen in the previous results Eqs. (53) and (54), that $\hbar/\tau_{e\text{-}h}$ goes to zero when n is very small and also when n is very large. Therefore one can reasonably expect for $\hbar/\tau_{e\text{-}h}$, as a function of n, a smooth curve with a maximum in the intermediate region $T\sim T_n$. In this range the low- and the high-density results both lead to $\hbar/\tau_{e\text{-}h}$ proportional to the exciton binding energy $k_B T_{\rm ryd}$ [noting that $(T_n/T_{\rm ryd})^{1/4}$ or $\ln(T_n/T_{\rm ryd})$ vary very slowly with $T_n/T_{\rm ryd}$]. The numerical factor has to be estimated from an interpolation between the two limiting results. It could possibly be of the order of 10. For silicon this gives a relaxation time in the 10^{-14}-to-10^{-15}-s range.

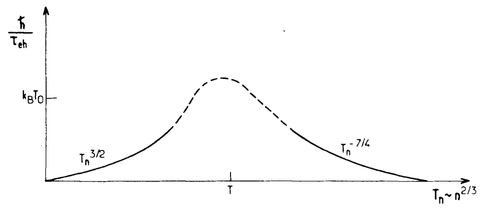

FIG. 2. Density dependence of the e-h collision time in the degenerate and the classical limit. T_n varies with density as $n^{2/3}$. $k_B T_0$ is the effective exciton binding energy ($T_0 \equiv T_{\rm ryd}$ in the text).

CONCLUSION

We have shown that the conductivity relaxation time $\tau_{e\text{-}h}$ due to e-h collisions increases as $n^{7/6}$ when the e-h plasma density becomes very large, due to quantum phase-space restrictions. It also diverges, as n^{-1}, for a very dilute system because there is no more collision. Consequently, one expects for $\hbar/\tau_{e\text{-}h}$ a maximum, at intermediate density, when the e-h plasma becomes degenerate, the size of the collision probability $\hbar/\tau_{e\text{-}h}$ at the maximum being mainly controlled by the Coulomb interaction, i.e., the exciton binding energy. This is in contradiction with the naive idea that the effect of e-h collisions should be more important in densest plasmas.

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