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Magnetoplasmons in a two-dimensional electron gas: Strip geometry

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We study the hydrodynamic magnetoplasma modes of a two-dimensional metallic strip deposited on a dielectric substrate. The "anomalous" dependence on the magnetic field, which has been predicted for different geometries, is also found in this case. We compare the effect of different equilibrium densities on the spectrum and analyze its dependence on the strip width. An analytical solution is reported for the case of the zero-field regime.

I. INTRODUCTION

The electromagnetic modes of a two-dimensional (2D) electron gas[1,2] have received much attention essentially in connection with the wide development of systems in which a very good approximation for a 2D electron gas is available. More recently the interest has been focused on the effect of boundaries on such systems. In fact, as in the 3D case, the presence of boundaries reduces the symmetry of the system and introduces new electromagnetic modes localized at the boundaries themselves.

In particular, recent experiments on electrons trapped on the surface of liquid ^{4}He in presence of a perpendicular magnetic field have been successfully explained in terms of magnetoplasmons localized to the boundaries of the 2D electron gas.[3,4] The main result of those studies was the detection of two sorts of magnetoplasma modes. One type shows the same behavior of the usual bulk modes whose frequency increases with the magnetic field, while the frequencies associated to the second set of modes decrease when the magnetic field increases. However, although these features seem not to depend on the shape of the boundaries (all the studied geometries show this behavior), the detail of the dispersion relations can be very sensitive to the shape of the boundaries.

The aim of this paper is to formulate in detail the problem for the special case of an infinitely long metallic strip deposited on a dielectric medium and compare the results with those obtained with different geometries.[5,6] The geometry used seems to us to be of increasing interest because these systems are becoming technologically available[7] and could be used in the future as a wave guide tunable with the magnetic field.

In the following we will neglect the effects of retardation on the field propagation. Thus the problem reduces to solving Laplace's equation with appropriate boundary conditions, since, as we will see, the electron dynamics will enter just as a boundary condition. We will take into account the electron motion on the basis of the hydrodynamic theory. In the general case we have found an infinite set of allowed frequencies labeled by an integer index. For fixed values of the magnetic field and of the wave number associated with the direction parallel to the strip, the self-induced density shows a localization at the edges which decreases for higher-order modes. We can speak, in this sense, of a quasi-1D electromagnetic mode analogous to those found for 3D wedge-like systems.[8] The spectrum also depends on the width of the strip and on the equilibrium density profile.

In Sec. II we set the basic equations for our problem taking into account the substrate. Then we give a quite general integral formulation (Sec. III) which allows an easy approximate solution in the case of a uniform equilibrium charge density. In Sec. IV a different approach is presented useful to investigate the effect of the equilibrium density on the dispersion relation. Finally, in Sec. V we discuss briefly our results.

II. PROBLEM FORMULATION

Our system is a 2D classical electron gas confined in a strip ($y=0$; $-c \le x \le c$) extending infinitely in the z direction. It is surrounded by a dielectric material with dielectric constant ϵ for $y<0$ and by vacuum for $y>0$. The system is immersed in a magnetic field $\mathbf{B}=(0,B,0)$ and the electron gas is also neutralized by a positive background (jellium model). However, this second condition can be relaxed because, as we will see later, the only equilibrium quantity that enter our problem is the equilibrium charge distribution which can be considered as an input. The basic equations in this linearized hydrodynamic model[9,10] are the equations of continuity, the equations of Euler and Poisson,

$$\frac{\partial n}{\partial t}+\nabla_2\cdot(n_0\mathbf{v})=0\ , \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t}=-\frac{s^2}{n_0}\nabla_2 n+\frac{e}{m}\nabla_2\phi+\omega_c\hat{\mathbf{y}}\times\mathbf{v}\ , \tag{2}$$

$$\alpha(y)\nabla^2\phi=4\pi en\,\delta(y)\Theta(x+c)\Theta(-x+c)\ , \tag{3}$$

where $\alpha(y)$ is equal to ϵ if $y<0$ and to 1 if $y>0$; Θ is the usual unit step function. In the preceding equations, e and m are the electron charge and mass, n_0 and n are, respectively, the equilibrium and the self-induced 2D densities, ϕ is the self-induced electrostatic potential, $\mathbf{v}$ is the local velocity in the x-z plane, and s is an effective wave speed that arises from the compressibility of the fluid (the terms containing s^2 describe the electron-electron interaction in the hydrodynamic approximation). The symbol ∇_2 is the operator $(\partial/\partial x, \partial/\partial z)$ and $\omega_c = eB/mc$ is the cyclotron frequency. We emphasize that the equilibrium properties are present in Eqs. (1)—(3) only through n_0 which depends in the general case on x and z. On principle we should compute n_0 by solving the proper equilibrium hydrodynamic equations[9] self-consistently but, in the following, we will only use n_0 as a parameter. Equations (1) and (2) describe the 2D dynamics of the electrons and couple electron motion to the self-induced potential ϕ. On the other hand, ϕ has to satisfy Laplace's equation everywhere but on the strip, where Eq. (3) becomes the boundary condition

$$\left.\frac{\partial\phi}{\partial y}\right|_{y=0+} - \epsilon \left.\frac{\partial\phi}{\partial y}\right|_{y=0-} = -e4\pi n(x,z) . \tag{4}$$

In this way the problem is reduced to solve Laplace's equation with boundary condition (4) where n depends self-consistently on the solution of (3) through Eqs. (1) and (2).

Translational invariance along z direction and in time implies that all the unknown quantities have a traveling wave solution of the form $e^{i(qz-\omega t)}$, where q and ω are, respectively, the wave number and the frequency associated to the solution. Putting together Eqs. (1) and (2) and assuming that n_0 depends only on x, we obtain

$$n(x)(\omega_c^2-\omega^2) = s^2\left[\frac{d^2}{dx^2}-q^2\right]n(x) - \frac{e}{m}n_0(x)\left[\frac{d^2}{dx^2}-q^2\right]\phi(x) + \frac{d}{dx}n_0(x)\left[\frac{s^2}{n_0(x)}\left[\frac{d}{dx}-\frac{\omega_c q}{\omega}\right]n(x) - \frac{e}{m}\left[\frac{d}{dx}-\frac{\omega_c q}{\omega}\right]\phi(x)\right] . \tag{5}$$

In our hydrodynamic model, Eq. (5) describes the intricate differential relation between the density and the potential. It takes into account both the dispersion and the effect of a nonuniform equilibrium density.

Here we want to emphasize that Eq. (5) includes a term proportional to $dn_0(x)/dx$. This means that we have to take some care in choosing $n_0(x)$ because, in order to have a nonsingular density $n(x)$, $n_0(x)$ should be continuous and vanish on the edges of the strip. In fact, if we introduce, for instance, the ansatz

$$n_0(x) = \rho_0\theta(x+c)\Theta(-x+c) , \tag{6}$$

Eq. (5) will contain terms proportional to $\delta(x\pm c)$. Hence, if we do not want such a singularity in the solution, the coefficients of the δ terms must vanish. From (1) and (3) it is easy to see that this condition is fulfilled if

$$v_x(\pm c) = 0 , \tag{7}$$

which means that no electron can escape from the strip. As we will see, the additional condition (7) can be satisfied only if $s\neq 0$, otherwise the self-induced density will contain δ singularities on the edges $x=\pm c$. Finally, we want to underline that, due to magnetic field, the solutions will not have a definite parity along the x axis.

III. INTEGRAL EQUATION FOR THE INDUCED DENSITY

Following essentially the scheme proposed by Fetter[5] for a different geometry we now present an integral formulation to our problem. This allows us, in connection with the ansatz (6), to solve the problem by using standard approximate method. With the help of Green functions, Poisson's Eq. (3) can be inverted[11] giving

$$\phi(x,y) = -\frac{2e}{1+\epsilon}\int_{-c}^{c} K_0\{q[(x-x')^2 + y^2]^{1/2}\}n(x')dx' , \tag{8}$$

where $K_0(z)$ is the modified Bessel function of the second kind of zero order. In order to get an integral equation we need to extract from the differential relation (5) an integral one connecting the potential $\phi(x,0)$ with $n(x)$. If we use the ansatz (6), this can be achieved easily with the help of an appropriate Green function. Unfortunately the inversion presents significant difficulties in the general case. We obtain

$$\phi(x) = \frac{m}{e\rho_0}\left[s^2 n(x) + (\omega^2-\omega_c^2) \times \int_{-c}^{c} G(x,x')n(x')dx'\right] , \tag{9}$$

where $\phi(x)$ is shorthand for $\phi(x,0)$ and $G(x,x')$ satisfies the following differential equation:

$$\left[\frac{d^2}{(dx')^2}-q^2\right]G(x,x') = \delta(x-x') , \tag{10}$$

for $|x'|<c$ and with the boundary conditions

$$\left[\frac{d}{dx'}+\frac{\omega_c q}{\omega}\right]G(x,x')\big|_{x'=\pm c} = 0 . \tag{11}$$

It is straightforward to show that the solution of (10) with the condition (11) is

$$G(x,x') = \frac{-1}{4q\sinh(2qc)}[g_1(x+x') + g_2(|x-x'|)] ,$$

where,

$$g_1(u)=\alpha e^{g(u-2c)}+\frac{1}{\alpha}e^{-q(u+2c)}, \tag{12}$$

$$g_2(u)=2\cosh[q(2c-|u|)], \tag{13}$$

and $\alpha=(\omega+\omega_c)/(\omega-\omega_c)$. Introducing the dimensionless quantities $t=x/c$, $\Omega^2=\omega^2/\omega_p^2(c)$, $\Omega_c^2=\omega_c^2/\omega_p^2(c)$, $S^2=s^2/c^2\omega_p^2(c)$, and $\omega_p^2(c)=8\pi\rho_0e^2/mc(1+\epsilon)$, the density satisfies the integral equation

$$\int_{-1}^{1}\left[\frac{1}{\pi}K_0(qc\,|t-t'|)+(\Omega^2-\Omega_c^2)G'(t,t')\right]n(t')dt'=-S^2n(t)\,. \tag{14}$$

We have defined $G'(t,t')=(1/c)G(ct,ct')$. Equation (14) can be solved easily, by standard methods, substituting the function $K_0(|z|)$ in the kernel of Eq. (14) with $(\pi/\sqrt{2})\exp(-\sqrt{2}q\,|z|)$ which has equal area and second moment. Then the solution is

$$n(t)=Ae^{k_1ct}+Be^{k_2ct}+Ce^{-k_1ct}+De^{-k_2ct}, \tag{15}$$

where k_1 and k_2 are the two positive roots of

$$S^2k^4+k^2(3q^2S^2+q-\Omega^2+\Omega_c^2)+2q^2\left[S^2q^2+\frac{q}{2}-\Omega^2+\Omega_c^2\right]=0 \tag{16}$$

Inserting (15) in Eq. (14) it is easy to see that the four unknown coefficients of (15) must fulfill a 4×4 homogeneous linear system, whose solubility condition gives the dispersion relation $\Omega=f_n(\Omega_c,q,S)$. Unfortunately, the calculation of the determinant is very cumbersome so we report here only the limit $s\to0$:[12]

$$e^{-4k_1c}=\left[\left[\frac{\sqrt{2}k_1-1}{\sqrt{2}-k_1}\right]^2-\frac{\Omega_c^2}{\Omega^2}\right]\Bigg/\left[\left[\frac{\sqrt{2}k_1+1}{\sqrt{2}+k_1}\right]^2-\frac{\Omega_c^2}{\Omega^2}\right], \tag{17}$$

where $k_1^2=[qc+2(\Omega_c^2-\Omega^2)]/(qc+\Omega_c^2-\Omega^2)$.

We stress here that the limit $s=0$ has to be performed carefully. In fact the density $n(t)$ becomes singular in this limit at the edges of the strip because condition (7) is not more satisfied. Physically this can be understood since the electrons are not able to screen, when $s=0$, the charge accumulation at the edges due to the Lorentz force acting on the bounded electron gas.

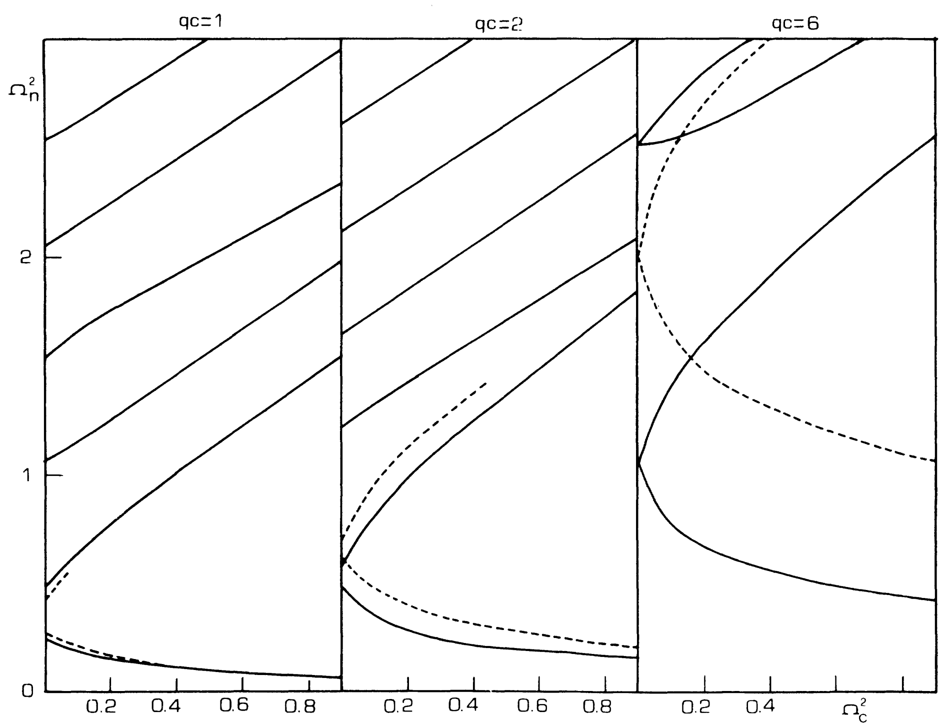

FIG. 1. The first few magnetoplasmonic squared frequencies as a function of Ω_c^2 for different values of qc. The solid lines correspond to the density profile $(1-x^2)^{1/2}$ [Eq. (22)] while the dashed lines correspond to the uniform one [Eq. (17)]. The upper dashed lines are stopped at those values of Ω_c where k_1 [see Eq. (17)] assumes imaginary values.

Equation (17) has two solutions which are plotted in Fig. 1 (dashed line) as a function of Ω_c for some specific values of qc. The first of the two solutions shows the anomalous behavior with the magnetic field already found for other geometries. For $qc \geq 5$ there are no relevant differences with the case of a half-plane:[5] it means that the two edges do not interact. When qc decreases the two edges interact, the frequencies repel and a frequency gap comes out at $\Omega_c = 0$.

In the zero-field regime, when c becomes smaller and smaller the allowed frequencies increase as $1/c$: a 1D electron gas, in fact, is not able to support electromagnetic modes. However, with increasing Ω_c, the frequency associated with the first mode decreases, so we have two competing effects which make it possible for a quasi-1D electron gas to support a self-induced traveling wave. We believe that this aspect could be very promising for a possible future use of this type of system as wave guides. Furthermore, the possibility of changing the frequency with the magnetic field seems to us also a very interesting feature.

IV. MATRIX EIGENVALUE PROBLEM

The integral formulation is a quite general approach to the problem and, as we have seen, it also leads to a very compact and manageable equation. Nevertheless, information on the solutions is easily extracted only in connection with the ansatz (6) for the equilibrium density. In order to study the effect of different electron density profiles on the allowed frequencies of the system, we develop a different scheme. From Eqs. (4) and (5), putting $s=0$, we obtain the following differential self-consistent condition for the potential ϕ:

$$\left.\frac{\partial\phi}{\partial y}\right|_{y=0+} - \epsilon \left.\frac{\partial\phi}{\partial y}\right|_{y=0-} = -\frac{8\pi\rho_0 e^2}{m(\omega_c^2-\omega^2)} \left[\sigma(x) \left[\frac{\partial^2}{\partial x^2} - q^2 \right] \phi(x) + \frac{1}{\omega}\frac{\partial\sigma}{\partial x} \left[\omega\frac{\partial}{\partial x} - q\omega_c \right] \phi(x) \right] , \tag{18}$$

where we have assumed $n_0(x) = \rho_0\sigma(x)$. This condition has to be fulfilled for $|x| < c$ and $y=0$; it is important to emphasize that Eq. (18) is not a differential equation for ϕ because the electrostatic potential has to satisfy Laplace's equation out of the strip. Furthermore, ϕ must be continuous everywhere and it means that we have to seek a solution whose y derivative is continuous when $|x| > c$ and discontinuous for $|x| < c$ in order to satisfy condition (18). This can be automatically achieved choosing as a solution of Laplace's equation a function of the following form:

$$\phi(\xi,\eta) = \sum_{n=0}^{\infty} [A_n \mathrm{ce}_{2n}(\eta,-Q)\mathrm{Fek}_{2n}(\xi,-Q + B_n \mathrm{ce}_{2n+1}(\eta,-Q)\mathrm{Fek}_{2n+1}(\xi,-Q)] , \tag{19}$$

where we have defined $Q = (qc/2)^2$. The potential in Eq. (19) is expressed in elliptic coordinates where $x = c\cosh\xi\cos\eta$ and $y = c\sinh\xi\sin\eta$, $(0 \leq \xi < \infty,\ 0 \leq \eta < 2\pi)$. The functions $\mathrm{ce}_m(\eta,-Q)$ and $\mathrm{Fek}_m(\xi,-Q)$ are Mathieu's and modified Mathieu's functions;[13,14] the former is a complete set of functions in the range $(0,\pi)$. The use of this coordinate system and of the complete system take into account in a "natural" way the necessary discontinuities of ϕ. In fact, because the segment $(-c,c)$ is defined by the equation $\xi=0$, the different conditions inside and outside $(-c,c)$ can be imposed separately on $\mathrm{ce}_m(\eta,-Q)$ and on $\mathrm{Fek}_m(\xi,-Q)$. In the Appendix we define all the quantities involved in Eq. (19) and discuss their main properties.

In elliptic coordinates Eq. (18) becomes

$$\left.\frac{\partial\phi}{\partial\xi}(\xi,\eta)\right|_{\xi=0} = \frac{-1}{2(\Omega_c^2-\Omega^2)\sin\eta} \left[\sigma(\eta) \left[\frac{\partial^2}{\partial\eta^2} - \frac{\cos\eta}{\sin\eta}\frac{\partial}{\partial\eta} - 4Q\sin^2\eta \right] + \frac{\partial\sigma}{\partial\eta}(\eta) \left[\frac{\partial}{\partial\eta} + qc\frac{\Omega_c}{\Omega}\sin\eta \right] \right] \phi(0,\eta) . \tag{20}$$

By using the orthonormality between Mathieu's functions and remembering the form assumed for ϕ we can reduce (20) to a linear system in the unknown A_n, B_n. For simplicity we consider, now, in detail only a particular density profile; that is $\sigma(\eta) = \sin\eta$. The general case is reported in the Appendix.

With this choice Eq. (20) becomes

$$A_m\alpha_m = \sum_{n=0}^{\infty} cq\frac{\Omega_c}{\Omega} B_n O_{m,n} \mathrm{Fek}_{2n+1}(0) , \tag{21a}$$

$$B_m\beta_m = \sum_{n=0}^{\infty} cq\frac{\Omega_c}{\Omega} A_n \frac{\hat{O}_{m,n}}{\pi} \mathrm{Fek}'_{2n}(0) , \tag{21b}$$

where

$$\alpha_m = (\Omega_c^2-\Omega^2)\mathrm{Fek}'_{sm}(0) - \mathrm{Fek}_{2m}(0) \left[Q + \frac{a_{2m}}{2} \right] ,$$

$$\beta_m = (\Omega_c^2-\Omega^2)\mathrm{Fek}'_{2m+1}(0) - \mathrm{Fek}_{2m+1}(0) \left[Q + \frac{b_{2m+1}}{2} \right] ,$$

and $O_{n,m}$, $\hat{O}_{n,m}$, a_{2m}, and b_{2m+1} are defined in the Appendix. The prime stands for derivation with respect to ξ.

Equation (20) couples the two kinds of coefficients A_n, B_n (A_n and B_n are associated, respectively, with even and odd functions in the variable x) in such a way that the potential has no definite parity along x; however, when $\Omega_c=0$, the equations for A_n and B_n are decoupled and we are left with an infinite number of allowed frequencies given by $\alpha_n=\beta_n=0$, whose associated electrostatic potential has definite parity. Inserting (21b) in (21a) we get the following linear system for the A_n:

$$A_m\alpha_m=4Q\frac{\Omega_c^2}{\Omega^2}\sum_{n=0}^{\infty}T_{m,n}\mathrm{Fek}_{2n}(0)A_n \ , \tag{22}$$

where

$$T_{m,n}=\sum_{l=0}^{\infty}O_{m,l}\hat{O}_{l,n}\mathrm{Fek}_{2l+1}(0)/\beta_l \ ,$$

whose solubility condition defines implicitly the magnetoplasmon frequencies.

This function has been obtained by standard numerical means cutting the infinite sum after N terms. The convergency has been tested by increasing N to $N+1$ and checking the variation on the frequencies obtained. We have found that, for the values of qc shown in the figures, $N=10$ is large enough, in the worst case, to obtain a variation of the frequency of about 10^{-3}, which seems to us very good. Furthermore, our numerical analysis shows that all the frequencies allowed are real although the matrix is not hermitian and hence, in principle, there are no prescriptions on the eigenvalues. In Fig. 1 we show the first allowed frequencies (solid line) as a function of the magnetic field. In the zero-field regime the frequencies arrange in an infinite set of pairs; the gap for each of them becomes smaller when qc increases (this happens at $qc>5$ for the first two pairs and for larger values of qc for higher-order pairs). With increasing magnetic field, all the frequencies increase except for the first which decreases toward zero. This confirms the presence of the "anomalous" magnetoplasma mode in such type of systems.

Another point of interest to us is to investigate the localization of the potential associated to the modes. In Fig. 2 we show $\phi(x,0)$ for $|x|<c$ for some typical values of the parameters. We note that only the first few modes are localized at the edges, while the potential (and hence the induced density) for the higher modes is spread out all over the strip. Furthermore, as we have seen in the approximate case, the modes of even order are mainly localized on the right side of the strip while the odd-order modes on the left side. We have also found that the localization increases quickly with increasing of qc or the magnetic field.

V. DISCUSSION

In the present work, we have studied the electromagnetic modes supported by a 2D metallic strip put on a dielectric substrate, with two different mathematical techniques, by using the hydrodynamic model to describe the motion of the electrons in the strip.

Our results confirm the existence of an anomalous magnetoplasma mode as predicted previously for different geometries. This is connected, so far as we can see, to the mixed effect of the Lorentz force and of the boundness of the system. In fact, an analogous dependence on the mag-

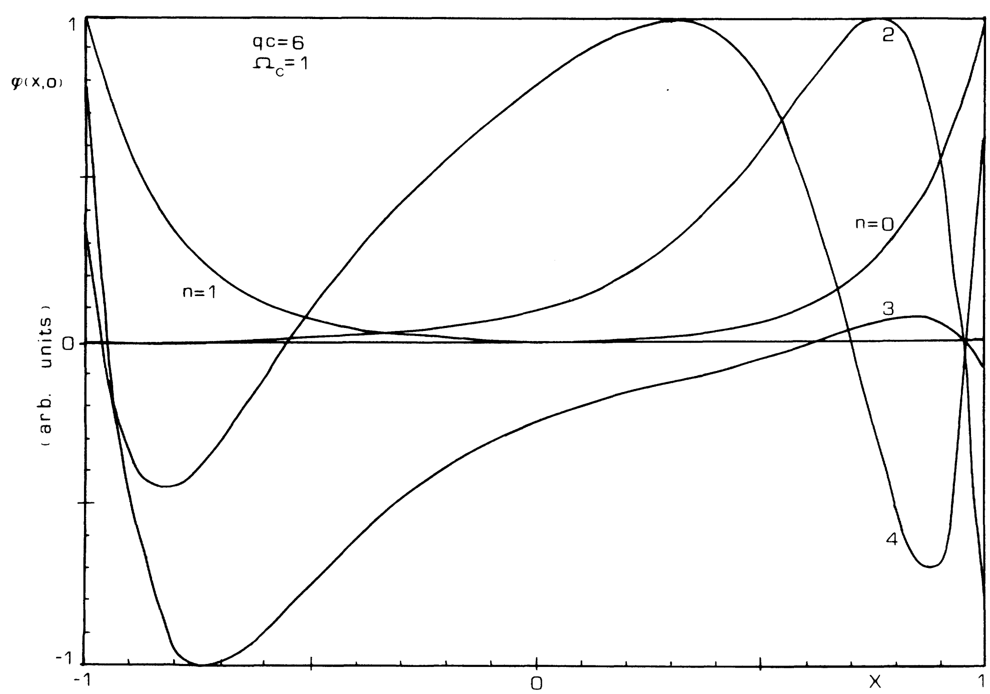

FIG. 2. The self-induced electrostatic potential $\phi(x,0)(|x|<1)$ associated to the first five modes. All the curves are normalized to their max value. n is the mode label.

netic field is also found for the proper frequencies of a 3D dielectric slab when the field is parallel to the slab. On the other hand, the existence of an infinite and discrete number of allowed frequencies, as we have seen, seems to us to be essentially related to the two-dimensional nature of the problem. In fact, the electrostatic modes of wedge-like systems[8] (which can be always reduced to a 2D problem taking into account their translational invariance) also show a spectrum of frequencies which can be labeled by an integer number.

The two different approaches allowed us to investigate different density profiles and to study the effect of a pressure term on the excitation spectrum.

The integral formulation has been used to study, in an approximate way, a system characterized by a uniform equilibrium electron density and by a pressure term. We want to emphasize that the use of our approximate method should provide a qualitative and quantitative good fit for the magnetic field dependence of the first two modes, but it is not able to provide any information about higher-order modes, which, on the other hand, we have found with the exact differential formulation. The accuracy of this approximate method has been discussed recently by Fetter[15] for the half-plane geometry with the conclusion that this approximation does not introduce any qualitative correction in the spectrum. Another interesting aspect is connected with the pressure term. We have checked that for realistic values of S^2 the charge density shows up as a strong localization on the edges, while for larger values of S^2 the density (and also the electrostatic potential associated) is less localized and the dependence of the frequencies on the magnetic field becomes flatter. For simplicity, we have only reported the limit $S=0$, in Sec. III, because there are no differences with the realistic case $S\sim 10^{-3}$. However, unlike the $(s\neq 0)$ case, the charge density becomes singular on the edges in the limit stated above.

The differential formulation of the problem is exact from the mathematical point of view, but s is taken to be zero. Although the calculations can be done for an arbitrary profile of the equilibrium electronic density we have chosen, in the case studied, $n(x)=\rho_0[1-(x/c)^2]^{1/2}$. The property $n_0(\pm c)=0$ assures us of obtaining a finite value for the charge density at the edges. We have found an infinite number of allowed frequencies labeled by an integer number and they show the same dependence on the magnetic field obtained previously, suggesting that this feature is, qualitatively, insensitive to modifications of both the geometry and the equilibrium density.

However, a more accurate comparison of the results for the different profiles (see Fig. 1), shows that for small qc there are no significant differences between the two spectra while when qc increases we find a large frequency shift. This can be understood because when qc is small the electrostatic potential associated to the mode is not very localized at the edges and, hence, the results do not depend very much on the details of the $n_0(x)$ at the edges.

APPENDIX

The Mathieu's and associated modified Mathieu's functions with even index, introduced in Eq. (19), satisfy the equations

$$\left[\frac{d^2}{d\eta^2}+a_{2m}(Q)+2Q\cos(2\eta)\right]\mathrm{ce}_{2m}(\eta,-Q)=0 \tag{A1}$$

and

$$\left[\frac{d^2}{d\xi^2}-[a_{2m}(Q)+2Q\cosh(2\xi)]\right]\mathrm{Fek}_{2m}(\xi,-Q)=0\ , \tag{A2}$$

where a_{2m} are called characteristic numbers. Similar equations are satisfied by the odd Mathieu's functions ce_{2m+1} and the associated modified Mathieu's functions Fek_{2m+1} with the only substitution of the characteristic numbers a_{2m} with b_{2m+1}.[16]

For any positive integer n, the product $f_n(\xi,\eta)=\mathrm{ce}_n(\eta,-Q)\mathrm{Fek}_n(\xi,-Q)$ is the solution of the equation $(\nabla_2^2-q^2)f_n(\xi,\eta)=0$, satisfying the boundary conditions $f_n(\xi,\eta)\to 0$ as $\xi\to\infty$ and the periodicity condition $f_n(\xi,\eta)=\pm f_n(\xi,\eta+\pi)$. The functions ce_n have the following uniformly convergent expansion:

$$\mathrm{ce}_{2m}(\eta,-Q)=\sum_{r=0}^{\infty}A_{2r}^{(2m)}(Q)(-1)^{r+m}\cos(2r\eta)\ , \tag{A3}$$

$$\mathrm{ce}_{2m+1}(\eta,-Q)=\sum_{r=0}^{\infty}B_{2r+1}^{(2m+1)}(Q)(-1)^{r+m}\times\cos[(2r+1)\eta]\ , \tag{A4}$$

from which we see that $\mathrm{ce}_{2m}(\eta,-Q)$ is even and $\mathrm{ce}_{2m+1}(\eta,-Q)$ odd when x is changed into $-x$ on the strip $(|x|<c)$. The coefficients $A_n^r(-Q)$ are given by recurrence relations; they have been well studied in literature together with the characteristic numbers: in any case, their numerical calculation is not very difficult.

Writing the general electronic equilibrium density profile as

$$\sigma(\eta)=\sum_{p=0}^{\infty}\sin(2p+1)\eta \tag{A5}$$

we obtain that the function (19) satisfies the Laplace equation and the boundary condition (18), if the coefficients A_n and B_n are a solution of the linear set of equations

$$A_m\mathrm{Fek}'_{2m}(0)(\Omega_c^2-\Omega^2)=\sum_{n=0}^{\infty}A_nD_{m,n}\mathrm{Fek}_{2n}(0)+B_n\frac{cq}{\pi}\frac{\Omega_c}{\Omega}O_{m,n}\mathrm{Fek}_{2n+1}(0)\ , \tag{A6}$$

$$B_m\mathrm{Fek}'_{2m+1}(0)(\Omega_c^2-\Omega^2)=\sum_{n=0}^{\infty}B_nF_{m,n}\mathrm{Fek}_{2n+1}(0)+A_n\frac{cq}{\pi}\frac{\Omega_c}{\Omega}\hat{O}_{m,n}\mathrm{Fek}_{2n}(0)\ , \tag{A7}$$

where we have defined

$$\pi D_{m,n} = a_{2n} P_{m,n} + 2QR_{m,n} + S_{m,n} + 4QZ_{m,n} + Y_{m,n} \ , \tag{A8}$$

$$\pi F_{m,n} = b_{2n+1} \hat{P}_{m,n} + 2Q\hat{R}_{m,n} + \hat{S}_{m,n} + 4Q\hat{Z}_{m,n} + \hat{Y}_{m,n} \ . \tag{A9}$$

All the quantities in the right-hand sides of (A7) and (A8) and the quantities $O_{m,n}$ and $\hat{O}_{m,n}$ are overlap integrals obtained by projecting on the complete basis $\mathrm{ce}_n(\eta)$. They are

$$P_{m,n} = \int_0^\pi \mathrm{ce}_{2m}(\eta)\sigma(\eta)\mathrm{ce}_{2n}(\eta)\frac{d\eta}{\sin\eta} \ ,$$

$$R_{m,n} = \int_0^\pi \mathrm{ce}_{2m}(\eta)\sigma(\eta)\cos(2\eta)\mathrm{ce}_{2n}(\eta)\frac{d\eta}{\sin\eta} \ ,$$

$$S_{m,n} = \int_0^\pi \mathrm{ce}_{2m}(\eta)\sigma(\eta)\cos\eta \, \mathrm{ce}'_{2n}(\eta)\frac{d\eta}{\sin^2\eta} \ ,$$

$$Z_{m,n} = \int_0^\pi \mathrm{ce}_{2m}(\eta)\sigma(\eta)\sin\eta \, \mathrm{ce}_{2n}(\eta)d\eta \ ,$$

$$Y_{m,n} = -\int_0^\pi \mathrm{ce}_{2m}(\eta)\sigma'(\eta)\mathrm{ce}'_{2n}(\eta)\frac{d\eta}{\sin\eta} \ ,$$

$$O_{m,n} = \int_0^\pi \mathrm{ce}_{2m}(\eta)\sigma'(\eta)\mathrm{ce}_{2n+1}(\eta)d\eta \ .$$

In the preceding expressions the prime means derivative with respect to η. The quantities $\hat{P},\hat{R},\hat{S},\hat{Z},\hat{Y},\hat{O}$ can be obtained from the preceding ones interchanging ce_{2n} with ce_{2n+1}. Using (A3), (A4), and (A5) the above integrals can be reduced to sums of products of the well known quantities $A_{2r}^{2n}(Q)$ and $B_{2r+1}^{2n+1}(Q)$.

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