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Phase transitions of Josephson-tunnel-junction arrays at zero and full frustration

B. J. van Wees, H. S. J. van der Zant, and J. E. Mooij
Department of Applied Physics, Delft University of Technology, Delft, The Netherlands
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We have fabricated and studied square two-dimensional arrays of Josephson oxide tunnel junctions. Remarkable structure is observed in the longitudinal and transverse resistance as a function of a perpendicular magnetic field. The linear and nonlinear resistance have been measured for $f=0$ and $f=\frac{1}{2}$, where f is the flux in one cell normalized to the flux quantum. At $f=0$, for the nonlinear resistance, quantitative agreement is found with predictions for a Kosterlitz-Thouless transition, including the universal jump. At $f=\frac{1}{2}$ a different phase transition occurs, showing a jump with a nonuniversal value.

A two-dimensional array of superconducting islands,[1] weakly connected by Josephson junctions, provides a very interesting model system for the study of phase transitions from a fundamental point of view, as well as for the understanding of practical superconducting systems such as granular films. When a magnetic field is applied perpendicular to its plane, the array corresponds to the frustrated XY model which at present is the subject of intensive theoretical study.[2-9] The amount of frustration is indicated with the index f, which is equal to the ratio of the flux contained in one unit cell of the array and the flux quantum $\Phi_0=h/2e$. The fully frustrated case, $f=\frac{1}{2}$ for a square array, is of particular interest. Here several types of phase transitions may occur: a Kosterlitz-Thouless (KT) transition involving vortices of flux Φ_0 with weaker interaction than for $f=0$, an Ising-like transition involving formation of domains and/or possibly a Kosterlitz-Thouless-like transition involving vortices of flux $\Phi_0/4$ connected with corners in domain walls. Strong interaction between different processes is expected. Recent computer simulations by Berge, Diep, Ghazali, and Lallemand[6] indicate that Ising and Kosterlitz-Thouless transitions occur at the same temperature. Minnhagen[7] predicts a Kosterlitz-Thouless transition with a nonuniversal jump. No general agreement exists yet as to the dominant character of the transition as observed in Josephson-junction arrays. This is in contrast to the $f=0$ case where theory and experiment indicate a Kosterlitz-Thouless phase transition. We have performed detailed quantitative measurements on an array, comparing the phase transitions at $f=0$ and $f=\frac{1}{2}$. We then came to the conclusion that the transition at $f=\frac{1}{2}$ is not of the same character as at $f=0$. Our data at $f=\frac{1}{2}$ indicate a Kosterlitz-Thouless-like transition with a nonuniversal jump. The magnitude of that jump agrees with the prediction of Minnhagen.[7] Although some other experimental data have previously been reported on superconducting arrays at $f=\frac{1}{2}$, we believe that our data provide the first clear evidence as to the nature of the transition.

Our arrays contain Josephson tunnel junctions instead of proximity-effect junctions as used in most other studies.[10-13] In tunnel junctions the coupling energy, proportional to the critical current I_c, is much less strongly dependent on temperature. Also, for the same value of I_c, the impedance is orders of magnitude higher, which makes more accurate measurements possible. Only Voss and Webb previously reported on tunnel-junction arrays,[14,15] in their case having a transition temperature T_c of 15 mK. We have adjusted the critical current of our junctions to obtain values of T_c of several kelvin, enabling us to measure as a function of T both above and below T_c.

Our arrays are all-niobium; the junctions are fabricated using a shadow-evaporation method, described in Ref. 16. The junctions are $1\times0.2\ \mu\mathrm{m}^2$; the unit cell has an area of $50\ \mu\mathrm{m}^2$. Part of an array is shown in Fig. 1. Each niobium island is connected to four neighbors. We have investigated a considerable number of such arrays of different sizes and different values of the coupling strength. They show several novel features. Most of these results, as well as details of fabrication, will be published elsewhere.

In this paper we report on results obtained with one array that is 128 islands wide and 384 islands long. Superconducting contact areas at the ends are connected over the width of the array. "Hall contacts" are placed at $\frac{1}{3}$ and $\frac{2}{3}$ of the length of the array; each of them is connected to four islands at the edge. The normal-state resistance

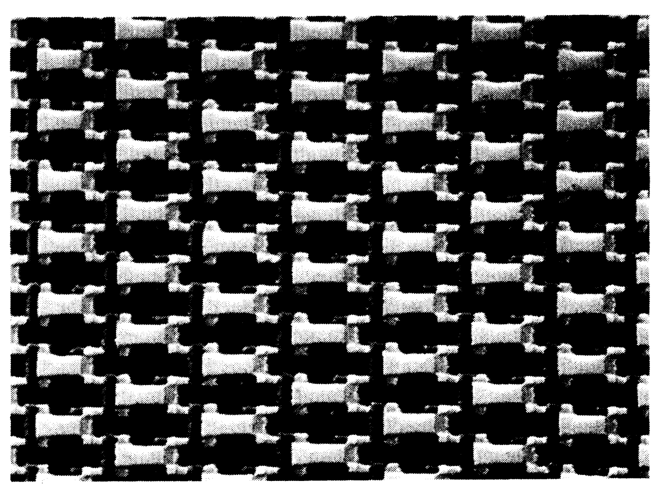

FIG. 1. Part of an array. Individual islands measure 7 by 4 μm, tunnel junctions are positioned at the corners where islands overlap.

R_n of the array is 850 Ω, corresponding to a single-junction value of 283 Ω. The critical current per junction I_c is extracted straightforwardly from the array current-voltage characteristic and is determined as a function of temperature. It varies from about 100 nA at 5.3 K, 300 nA at 4.6 K to 1 μA at 2.3 K.

As I_c is temperature dependent, the coupling-energy proportionality constant $J_0=(\hbar/2e)I_c$ is also temperature dependent. Knowing $I_c(T)$, we define the effective normalized temperature,

$$\tau=k_BT/J_0(T)=k_BT/[\hbar I_c(T)/2e] \ . \quad (1)$$

Over a wide temperature range, the resistance of our array shows a pronounced periodic structure in the resistance as a function of applied field. The period is 1 in the normalized flux parameter f. At about $f=15$ the structure has gradually disappeared. In Fig. 2 we show examples at several temperatures. In all graphs, R is about zero for zero field ($f=0$); the maximum value of the resistance near $f=0.5$ varies strongly with temperature, as indicated in the figure caption. At relatively high temperatures mainly a smooth variation with period 1 is seen, with very little additional structure. At around 4.5 K ($\tau=0.6$) this additional structure becomes more pronounced and, in particular, a clear dip appears at $f=\frac{1}{2}$ which distinguishes itself more and more at lower temperatures. One should note the symmetry of the plots around $f=0$ and, to a slightly lesser extent, around $f=0.5$. The symmetry and periodicity show that the apparently erratic structure is reproducible. All the dips are probably related to values of f equal to rational fractions p/q, but due to the large numbers it is difficult to assign p and q values unambiguously.

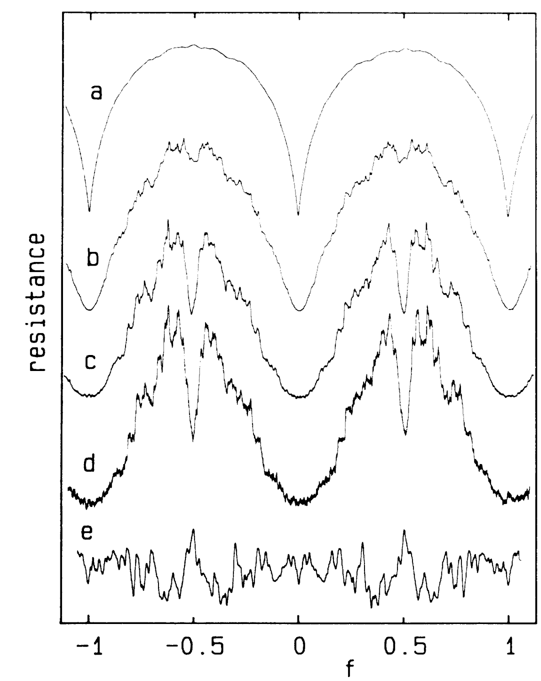

FIG. 2. Longitudinal (a–d) and transverse (e) magnetoresistance vs magnetic field at different temperatures. For all plots the resistance is about zero at $f=0$. Curve a: $T=5.22$ K, $\tau=2.0$, maximum resistance = 154 Ω. Curve b: $T=4.65$ K, $\tau=0.65$, $R_{max}=6.3$ Ω. Curve c: $T=4.30$ K, $\tau=0.42$, $R_{max}=0.26$ Ω. Curve d: $T=4.18$ K, $\tau=0.38$, $R_{max}=73$ mΩ. Curve e: $T=5.00$ K, $\tau=1.19$, R varies between -0.42 and $+1.0$ Ω.

The bottom trace of Fig. 2 shows the transverse resistance at one temperature. The voltage across one set of "Hall contacts" is shown with constant current through the regular end contacts. In a certain temperature range, roughly between 5.5 and 4.8 K, symmetric and periodic structure is found as a function of f, similar to the additional structure in the longitudinal resistance. The transverse resistance has both negative and positive values. We surmise that the rich structure observed in longitudinal and transverse resistance reflects the many special solutions existing for flux and current distribution in the array. The nonzero value of the transverse resistance indicates an asymmetry of the current distribution. Formation of domains may be involved.

At $f=0$ and at $f=\frac{1}{2}$ we have measured the linear resistance at low current and voltage levels as well as the nonlinear resistance at higher excitation levels. We have performed a detailed analysis, similar to that executed by Hebard and Fiory on continuous thin films.[17] In Fig. 3 the linear resistance is shown as a function of $1/\tau$ and in Fig. 4 the exponent a of the nonlinear resistance is shown following from the proportionality of V with I^a. We first discuss the results for $f=0$, next those for $f=\frac{1}{2}$. The behavior of the system for small integer values of f closely resembles that for $f=0$.

In zero magnetic field ($f=0$) the linear resistance (see Fig. 3) decreases exponentially as a function of $1/\tau$ over almost four orders of magnitude. This behavior can be expressed as

$$\ln(R/R_n)=-\alpha\tau^{-1} \ . \quad (2)$$

The resistance is expected to be proportional to the densi-

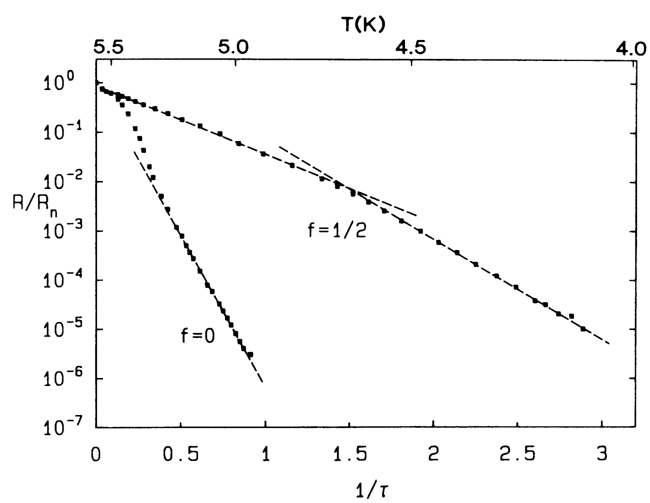

FIG. 3. Linear resistance at $f=0$ and $f=\frac{1}{2}$, as a function of inverse normalized temperature. Squares are measured data. Dashed lines indicate different regions of exponential decrease.

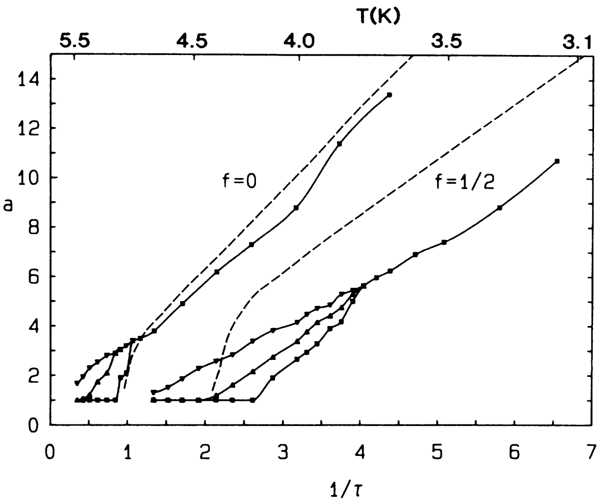

FIG. 4. Exponent a of nonlinear resistance as a function of inverse normalized temperature at $f=0$ and $f=\frac{1}{2}$. Squares: data at $V=3\times10^{-9}$ V, triangles: $V=10^{-7}$ V, inverted triangles: $V=10^{-5}$ V. Data coincide above $1/\tau=1.06$ for $f=0$ and above $1/\tau=4.03$ for $f=\frac{1}{2}$. Drawn lines connect data points. Dashed lines are Monte Carlo simulations (Ref. 2).

ty of free vortices, because their mobility is almost independent of temperature.[18] The observed exponential dependence on τ^{-1} suggests that the vortices are thermally activated. The energy of a single vortex in an array of size N is $\pi J_0 \ln N$. Consequently, if the vortices are independent, their density should be $\exp(-\pi J_0 \ln N/k_B T)$, leading to $\alpha=\pi \ln N$. For our array of 128×384 islands, α is expected to lie somewhere between 13 and 16. The actual slope of the data points corresponds to $\alpha=14.3$, in remarkably good agreement.

The nonlinear resistance is also measured with a lock-in technique. At fixed temperature the current I is varied and the voltage V is determined. A plot of $\log V$ vs $\log I$ is made at each T. At low temperatures these plots are completely straight lines over more than four orders of magnitude of V. At a certain temperature, which is T_c, curvature sets in.[17] In this way T_c can be identified; we find it to be 4.88 K, corresponding to $\tau_c=0.94$. This is a very reasonable value, a factor of 1.67 lower than the theoretical unrenormalized value $\pi/2$. The slopes in the $\log V$-$\log I$ plot provide values for the exponent a, assuming $V\propto I^a$. As long as the plots are straight, a is independent of the excitation level. In Fig. 4 we show a as a function of $1/\tau$. Below $1/\tau_c$ different values are found at different voltages, as determined from the derivative. For the smallest voltage the exponent a decreases fast from 3.2 to 1.0 at T_c.

The exponent a is a quantity that is very useful in studying the phase transition of two-dimensional superconducting systems. It is directly related to the helicity modulus Γ, which is used in the evaluation of numerical simulations of the XY model. The relation between Γ and a is given by

$$a-1=\pi\Gamma/\tau\ . \tag{3}$$

At τ_c the quantity Γ/τ jumps from $2/\pi$ to zero according to the theory for Kosterlitz-Thouless transitions. This is one way in which the "universal jump" expresses itself. In superconductors, a is expected to jump from 3 to 1 when T surpasses T_c. The transition is less steep for systems of smaller size and also for higher measuring current or voltage as visible in Fig. 4. The value of a at T_c is 3.2, slightly higher than the theoretical value of 3. The same was found by Hebard and Fiory in thin films;[17] they explained that it is very difficult to observe the critical behavior that occurs in an extremely narrow temperature region below T_c. In Fig. 4 the dashed line is the behavior of Γ as found by Teitel and Jayaprakash in a Monte Carlo calculation.[2] Equation (3) has been used for conversion.

For $f=0$, the nonlinear resistance shows excellent agreement with predictions for a KT transition, and with computer experiments. We stress that to obtain Figs. 3 and 4 no fitting has been applied at all. Values of τ, R/R_n, and a all follow straightforwardly from measured data. It is surprising that the linear resistance gives no indication of the expected increase of free vortices above the Kosterlitz-Thouless unbinding temperature.

We have performed exactly the same measurements of linear and nonlinear resistance in a magnetic field yielding $f=\frac{1}{2}$. The results turn out to be very different. Figure 3 shows that $\log_{10}(R/R_n)$ plotted against $1/\tau$ exhibits two separate portions which are both distinctly straight. The change occurs near $1/\tau=1.4$. The slope is lower at the highest temperatures. Assuming a relation between R/R_n and $1/\tau$ as in Eq. (2), the slope at high temperatures ($1/\tau$ below 1.4) corresponds to $\alpha=3.1$. The slope at low temperatures is parallel to $\alpha=4.7$. In the magnetoresistance curves, of which examples are given in Fig. 2, the change in behavior occurring at $\tau=0.7$ is visible because below that temperature the $f=\frac{1}{2}$ dip is prominent with respect to other features. At higher temperatures the ordering at $f=\frac{1}{2}$ is apparently not stronger than at other f values. For arbitrary values of f between 0.25 and 0.75 the slope α is close to 3.1 for all temperatures above $\tau=0.6$.

In the nonlinear resistance we find behavior that is qualitatively similar but quantitatively very different from the behavior at $f=0$. Again straight lines are found in the log V-log I plot over more than four orders of magnitude in V up to a certain temperature. This T_c is equal to 3.80 K, corresponding to $\tau_c=0.25$. In Fig. 4 the exponent a is shown as function of $1/\tau$. Below $1/\tau_c$ the exponent varies with voltage, at lower temperatures the curves coincide. The drop of the exponent above τ_c is less steep than for $f=0$, but well pronounced. The exponent a falls from 5.5 at T_c to 1 at higher temperatures. In the figure a second dashed line indicates the results of Teitel and Jayaprakash for $f=\frac{1}{2}$. Their curve is now distinctly different from ours.

When we evaluate our results at $f=\frac{1}{2}$, it is clear that the phase transition is not the same as at $f=0$ where a Kosterlitz-Thouless transition with a universal jump is seen. The linearity of the log V-log I plot in a large region, for $f=\frac{1}{2}$ as for $f=0$, makes it likely that the underlying mechanism, unbinding of vortex-antivortex pairs induced by the current, might be the same. A Kosterlitz-

Thouless-like transition with a nonuniversal jump has been suggested by Minnhagen[7] for fully frustrated arrays. When we use his relation between the value of τ_c and the size of the jump, we find that for $\tau_c = 0.25$ the jump in a should be about 4.6. Our jump of 4.5 is in good agreement with that prediction.

Our data leave open certain questions. The value of the transition temperature for $f=0$, $\tau_c = 0.94$, agrees well with results for thin films and computer simulations. Our value for $f=\frac{1}{2}$, $\tau_c = 0.25$, can only be compared to results from computer simulations. They indicate a τ_c around 0.5. It may be that the transition for $f=\frac{1}{2}$ is much more sensitive to the disorder that is inevitably present in a fabricated array. The transverse resistance, bottom trace in Fig. 2, needs further study to understand its significance.

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[1]C. J. Lobb, Physica B **126**, 319 (1984).
[2]S. Teitel and C. Jayaprakash, Phys. Rev. B **27**, 598 (1983).
[3]W. Y. Shih and D. Stroud, Phys. Rev. B **30**, 6774 (1984).
[4]M. Y. Choi and S. Doniach, Phys. Rev. B **31**, 4516 (1985).
[5]T. C. Halsey, J. Phys. C **18**, 2437 (1985).
[6]B. Berge, H. T. Diep, A. Ghazali, and P. Lallemand, Phys. Rev. B **34**, 3177 (1986).
[7]P. Minnhagen, Phys. Rev. B **32**, 7548 (1985).
[8]E. Granato and J. M. Kosterlitz, Phys. Rev. B **33**, 4767 (1986).
[9]S. E. Korshunov, J. Stat. Phys. **43**, 17 (1986).
[10]D. Kimhi, F. Leyvraz, and D. Ariosa, Phys. Rev. B **29**, 1487 (1984).
[11]D. W. Abraham, C. J. Lobb, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **26**, 5268 (1982).
[12]R. K. Brown and J. C. Garland, Phys. Rev. B **33**, 7827 (1986).
[13]C. Leemann, P. Lerch, G.-A. Racine, and P. Martionoli, Phys. Rev. Lett. **56**, 1291 (1986).
[14]R. F. Voss and R. A. Webb, Phys. Rev. B **25**, 3446 (1982).
[15]R. A. Webb, R. F. Voss, G. Grinstein, and P. M. Horn, Phys. Rev. Lett. **51**, 690 (1983).
[16]B. J. van Wees, L. E. M. de Groot, K. L. Hagemans, J. Kortlandt, and J. E. Mooij, in *Microcircuit Engineering 84,* edited by A. Heuberger and H. Beneking (Academic, New York, 1985), p. 257.
[17]A. F. Hebard and A. T. Fiory, Phys. Rev. Lett. **50**, 1603 (1983).
[18]C. J. Lobb, D. W. Abraham, and M. Tinkham, Phys. Rev. B **27**, 150 (1983).

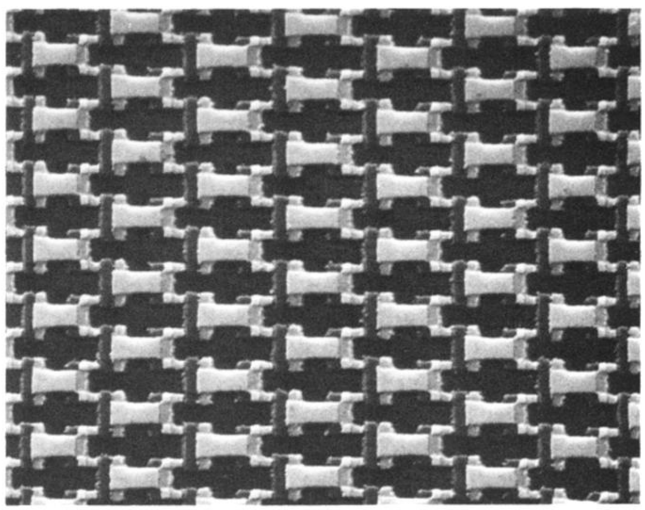

FIG. 1. Part of an array. Individual islands measure 7 by 4 μm, tunnel junctions are positioned at the corners where islands overlap.