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Replica symmetry breaking in random-field systems

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The conventional mean-field theory of random-field systems is, for large but finite dimensionalities d, found to be in error. For larger applied fields, a separate glassy phase appears in the phase diagram of the bond-diluted antiferromagnet. For large d, the diluted antiferromagnet can be mapped on the antiferromagnet with Gaussian randomness and the phase diagram of this model is shown to have a de Almeida–Thouless line, marking the onset of the glassy phase by replica symmetry breaking. In the limit $d \to \infty$ we recover conventional mean-field theory.

The effect of quenched-in disorder on magnetic transitions is usually discussed either in terms of random exchange models, such as the Ising spin-glass, or models with a local random magnetic field, such as the random-field Ising model (RFIM). The relationship between these two classes is rather puzzling. On one hand, the diluted Ising antiferromagnet (AF) in a uniform magnetic field is the classical realization[1] of the RFIM. Furthermore, in $d=1$ the Ising spin-glass in a uniform magnetic field can be mapped[2] onto the RFIM and the same is true for a spin-glass on the Bethe lattice[3] (although in that case the boundary conditions should be treated with care[4]). The ordered phase of both classes is characterized by irreversibility[5] and long-time relaxation processes[6] due to multiple minima in the free-energy surface. However, despite the similarity between the RFIM and spin-glasses in a uniform field, there are profound differences in our description of these models. A spin-glass in a field exhibits a glassy phase even in the case of infinite range interaction, and the mean-field theory is quite nontrivial. The appearance of the low-temperature glassy phase is signaled by the onset of so-called replica symmetry breaking at the de Almeida–Thouless (AT) line.[7] This phase has a new order parameter, the Parisi function $q(x)$.[8] On the other hand, the RFIM with infinite range interaction has a normal paramagnetic to ferromagnetic transition[9] and there appears to be no need to introduce a new order parameter for random-field systems. Alternatively, if we neglect thermal fluctuations then the mean-field equations of the diluted Ising AF are

$$m(i)=\tanh\beta\left[H+\sum_j J_{ij}m(j)\right], \qquad (1)$$

with $m(i)$ the magnetization on site i, J_{ij} the exchange energy between nearest neighbors, and H the magnetic field. If we neglect quenched fluctuations and assume $m(i)$ to be uniform, then the resulting phase diagram[10] agrees with the infinite range result. This mean-field theory is commonly held to be valid above $d=6$. The low-temperature hysteresis in random-field systems is solely ascribed to slow relaxation due to impurity pinning of domain walls, which should only appear[5] below $d=5$. Thus there should be no hysteresis for higher dimensionalities, and, in general, there should be no real glassy phase.

In this paper we will show that this view is, in general, wrong. The first piece of evidence is from numerical solutions[11,12] of the mean-field equations [Eq. (1)] of the diluted AF in a field which indicated that the paramagnetic phase is separated from the AF phase by a glassy region (Fig. 1). In the region labeled LRO (long-range order) in Fig. 1, the zero-field cooled state has a lower free energy and in the shaded region the field cooled state does, although the zero-field cooled state remains metastable. It is, however, not clear whether this region has to be considered as a truly separate phase or merely as having long relaxation times due to domain-wall pinning, since these solutions were computed in $d=3$.

From the free energy of the short-range diluted AF in a field, we will derive a model Hamiltonian for random-field

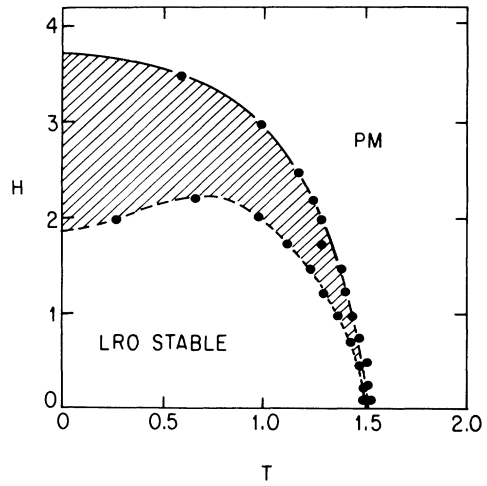

FIG. 1. Mean-field phase diagram for a $d=3$ diluted AF with 30% dilution, from Ref. 12. In the shaded region, the field cooled state has the lower free energy (no LRO) and in the unshaded region the zero-field cooled state (LRO) does.

systems, which for higher dimensions has a separate glassy phase. The onset of the glassy phase is indeed characterized by a line of replica symmetry breaking.[13] The maximum temperature for the onset of the glass depends on the number of nearest neighbors $z=2d$ as $(1/z)^{1/2}$, so in the limit $d\rightarrow\infty$, we do recover conventional mean-field theory.

The resulting phase diagram [Fig. 2(a)] resembles the numerical results in $d=3$ except that the glassy phase occupies a smaller part of the phase diagram. The glassy region increases if the dilution increases. A phase with broken replica symmetry carries its own order parameter, and if the $d=3$ phase diagram of Fig. 1 is indeed the extension of Fig. 2(a) to lower dimensions, then one would expect the critical behavior to be strongly affected. Even if for small d the glassy phase remains restricted to larger random fields, our results still have relevance, although not for magnetic realizations of the RFIM: Many nonmagnetic realizations of the RFIM, such as phase separation of binary mixtures of fluids in porous media,[14] require large random-field strengths. In any case, the infinite range RFIM cannot be taken as a representative mean-field theory for random-field systems, and at least for large d and large field strengths, interface arguments are insufficient to explain the hysteresis of random-field systems. Finally, we must mention that in the case of spin-glasses in a field, it has been questioned[15] whether there is an AT line at lower dimensions and similar reservations would apply to our case. Our starting point is the expression for the free energy of a bond-diluted AF using the replica method:

$$F=-\beta\lim_{n\rightarrow 0}\frac{\partial}{\partial n}\mathrm{Tr}_{\substack{\{S_i^\alpha,\sigma_j^\alpha\}\\ \alpha=1,\ldots,n}}\exp\left[\beta\left(H\sum_{i,\alpha}(S_i^\alpha+\sigma_i^\alpha)+\sum_{\langle i,j\rangle,\alpha}J_{ij}S_i^\alpha\sigma_j^\alpha\right)\right], \quad (2)$$

with α the replica index and σ_i^α and S_i^α Ising spins on the two sublattices with only nearest-neighbor exchange. The probability distribution of the J_{ij}'s is

$$P(J_{ij})=(1-x)\delta(J_{ij}+J_0/z)+x\delta(J_{ij}), \quad (3)$$

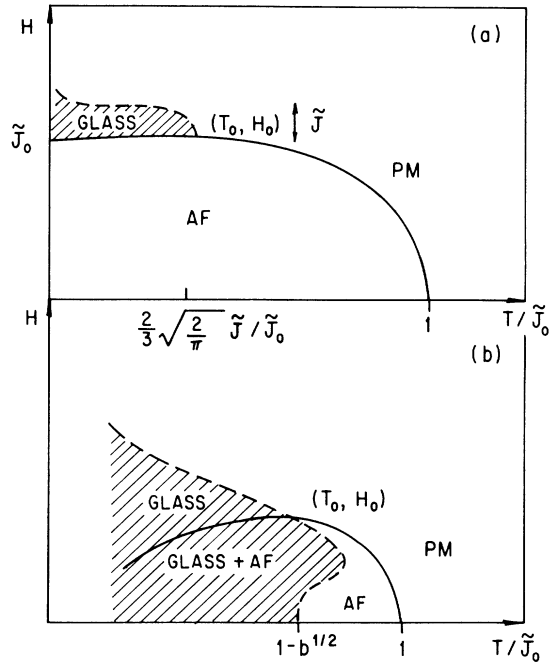

FIG. 2. Mean-field phase diagram of the random exchange AF in a field for large d. The mean exchange energy is $-\tilde{J}_0/z$ with z the coordination number. The width of the (Gaussian) distribution of the exchange is $\tilde{J}/\sqrt{z}$. The Néel temperature is shown as a straight line, the de Almeida–Thouless line as a dashed curve. (a) is the case of weak randomness ($\tilde{J}\ll\tilde{J}_0$), which also corresponds to a weakly diluted AF, and (b) is the case of strong randomness $\tilde{J}\simeq\tilde{J}_0$, so $b=1-\tilde{J}/\tilde{J}_0$ is small.

where x is the dilution and z the coordination number. The mean-field critical temperature for $x=1$ and $H=0$ is thus independent of z. Upon averaging over the $J_{i,j}$'s, one finds an expansion in $\varepsilon=\beta J_0/z$ in the exponent which, for high dimensionalities (large z), converges rapidly. To second order in ε

$$F=-\beta\lim_{n\rightarrow 0}\frac{\partial}{\partial n}\mathrm{Tr}\exp\left[-(1-x)\varepsilon\sum_{\langle i,j\rangle,\alpha}\sigma_i^\alpha S_j^\alpha+\tfrac{1}{2}x(1-x)\varepsilon^2\sum_{\langle i,j\rangle}\left(\sum_\alpha\sigma_i^\alpha S_j^\alpha\right)^2+\beta H\sum_{i,\alpha}(\sigma_i^\alpha+S_i^\alpha)\right]. \quad (4)$$

For ε of order 1, the expansion breaks down and Eq. (4) does not correspond to a diluted antiferromagnet anymore. It is easy to see that Eq. (4), in general, represents a random-exchange model with a Gaussian distribution of J_{ij} with $\langle J_{ij}\rangle=\bar{J}_0$ and $\langle(J_{ij}-\bar{J}_0)^2\rangle=2\bar{J}^2$, where $\bar{J}=[x(1-x)/2]^{1/2}J_0/z$ and $\bar{J}_0=-(1-x)J_0/z$. For $\bar{J}\lesssim-\bar{J}_0$, we should expect that Eq. (4) is in the "universality class" of the RFIM following the arguments of Ref. 1. We now will establish the mean-field phase diagram of Eq. (4). The fourth-order terms in the spins are, as usual, decoupled through the Hubbard-Stratonovich transformation with the following result:

$$F=-\beta\lim_{n\rightarrow 0}\frac{\partial}{\partial n}\int\prod_{i,\alpha,p}dx_{ip}^\alpha\int\prod_{i,\alpha,\beta,p}dy_{ip}^{\alpha\beta}\exp\left\{-\tfrac{1}{2}\sum_{p=1}^{3}\sum_{i,j,\alpha}x_{ip}^\alpha K_{ij}^{-1}x_{jp}^\alpha-\tfrac{1}{2}\sum_{p=1}^{3}\sum_{i,j,\alpha,\beta}y_{ip}^{\alpha\beta}K_{ij}^{-1}y_{jp}^{\alpha\beta}+\sum_i\ln\mathrm{Tr}_{\sigma^\alpha,S^\alpha}\right.$$
$$\times\exp\left[i\lambda_1\sum_\alpha x_{i1}^\alpha(\sigma_i^\alpha+S_i^\alpha)+\lambda_1\left(\sum_\alpha x_{i2}^\alpha\sigma_i^\alpha+\sum_\alpha x_{i3}^\alpha\sigma_i^\alpha\right)+\lambda_2\sum_{\alpha,\beta}y_i^{\alpha\beta}(\sigma_i^\alpha\sigma_i^\beta+S_i^\alpha S_i^\beta)\right.$$
$$\left.\left.+i\lambda_2\left(\sum_{\alpha\beta}y_{i2}\sigma_i^\alpha\sigma_i^\beta+y_{i3}^{\alpha\beta}S_i^\alpha S_i^\beta\right)+\beta H\sum_\alpha(\sigma_i^\alpha+S_i^\alpha)\right]\right\}, \quad (5)$$

where $\lambda_1[\frac{1}{2}(1-x)\varepsilon]^{1/2}$ and $\lambda_2=\{\frac{1}{2}[x(1-x)]\varepsilon^2\}^{1/2}$. The matrix K_{ij} is 1 if i and j refer to nearest neighbors, and is zero otherwise. For high dimensionalities, we can ignore fluctuations in $x_{i,p}$ and $y_{i,p}$ and evaluate the integrals by steepest descent[16] (tree approximation). The saddle-point equations are

$$x_1^\alpha = iz\lambda_1(m_1^\alpha + m_2^\alpha),\ x_{2,3}^\alpha = z\lambda_1 m_{1,2}^\alpha\ ,$$
$$y_1^{\alpha\beta} = z\lambda_2(q_1^{\alpha\beta}+q_2^{\alpha\beta}),\ y_{2,3}^{\alpha\beta} = iz\lambda_2 q_{1,2}^{\alpha\beta}\ , \tag{6}$$

where $m_{1,2}^\alpha$ and $q_{1,2}^{\alpha\beta}$ are the magnetization and Edwards-Anderson order parameter on the two sublattices:

$$m_1^\alpha = \langle\sigma^\alpha\rangle,\ q_1^{\alpha\beta} = \langle\sigma^\alpha\sigma^\beta\rangle\ , \tag{7}$$

and similarly for m_2 and q_2. The average is to be performed with the Hamiltonian H which is the part of F in Eq. (5) in square brackets. Assuming replica symmetry we set $q_{1,2}^{\alpha\beta}=q_{1,2}, m_{1,2}^\alpha = m_{1,2}$. The resulting mean-field equations are

$$m_{1,2} = \frac{1}{\sqrt{2\pi}}\int dx\, e^{-(1/2)x^2} \times\tanh[\beta(H-\tilde{J}_0 m_{2,1}+\tilde{J}x\sqrt{q_{2,1}})]\ , \tag{8a}$$

$$q_{1,2} = \frac{1}{\sqrt{2\pi}}\int dx\, e^{-(1/2)x^2} \times\tanh^2[\beta(H-\tilde{J}_0 m_{2,1}+\tilde{J}x\sqrt{q_{2,1}})]\ , \tag{8b}$$

where $\tilde{J}_0 = -\bar{J}_0 z$ and $\tilde{J}=\bar{J}\sqrt{z}$. To discuss replica symmetry breaking we substitute Eqs. (8) back into F. Remarkably, the same free energy was studied previously by Korenblit and Shender[17] (KS) in a somewhat different context, and by translating their results to our problem we can read off the AT line:

$$T_F^A(H) = \tilde{J}^4\frac{1}{2\pi}\int dz_1\int dz_2 \exp[-\tfrac{1}{2}(z_1^2+z_2^2)] \times\operatorname{sech}^4[(H-\tilde{J}_0 m_1+\tilde{J}z_1\sqrt{q_1})/T_F] \times\operatorname{sech}^4[(H-\tilde{J}_0 m_2+\tilde{J}z_2\sqrt{q_2})/T_F], \tag{9}$$

where q_1, q_2, m_1, and m_2 are taken from Eq. (8) Again, using the results of KS we can discuss the phase diagram. We first assume that x is very small and that $z\gg 1$, and so $\tilde{J}\ll\tilde{J}_0$. In that limit, the sublattice magnetizations are those of a pure antiferromagnet with exchange constant $\tilde{J}_0$. Using steepest descent in Eq. (9),

$$T_F(H)\cong\tfrac{2}{3}\left(\frac{2}{\pi}\right)^{1/2}\tilde{J}\exp\left[-\frac{1}{4\tilde{J}^2}[(H-\tilde{J}_0 m_1)^2 + (H-\tilde{J}_0 m_2)^2]\right]. \tag{10}$$

For low temperatures and $H>\tilde{J}_0$, $m_1=m_2$, so from Eq. (10)

$$T_F(H)\cong\tfrac{2}{3}\left(\frac{2}{\pi}\right)^{1/2}\tilde{J}\exp\left[-\frac{1}{2\tilde{J}^2}(H-\tilde{J}_0)^2\right]. \tag{11}$$

If $H<\tilde{J}_0$, then $m_1=-m_2$, and $T_F(H)$ is of order $\exp(-z)$, which is negligible for large z and below the validity of range of our expansion. There is a multicritical point at $H_0=J_0$ and $T_0=T_F(H_0)$. The phase diagram is shown in Fig. 2(a). Because T_F is, through $\tilde{J}$, proportional to $(1/z)^{1/2}$ there is no replica symmetry breaking in the limit of infinite z.

For large dilution and smaller coordinate numbers, $\tilde{J}$ becomes comparable to $\tilde{J}_0$. Although our expansion in ε is not valid anymore, we can still use Eq. (4) to gain qualitative insight into the phase diagram. If $b=1-\tilde{J}/\tilde{J}_0$ is small compared to one, then the multicritical point is located at

$$T_0=\tilde{J}(1-b^{1/2}),\ H_0=\tilde{J}(4b^{3/4}/3^{1/2})\ . \tag{12}$$

For field strengths in excess of H_0, the AT line is the phase boundary

$$T_F(H)=\tilde{J}[1-b^{1/2}(H/H_0)^{2/3}]\ . \tag{13}$$

The AF order parameter is zero. For field strengths below H_0 there is a rapid increase by an amount $\tilde{J}b^{1/2}$ in $T_F(H)$ until it reaches a maximum value. The AF order parameter is now finite so the glassy phase is mixed. If H is further reduced, then the AT line reaches the $H=0$ axis at $T_F(0)=T_0$. The small-b phase diagram is shown in Fig. 2(b). In the limit of zero H the diluted AF should have, at $T=0$, perfect AF order so that the AT line should not intersect the $H=0$ axis in that case. The fact that the AT line of Eq. (4) does reach the $H=0$ axis is due to the fact that ε is not small for $b\sim 0$, so including higher order terms in ε in Eq. (4) must, for small H, reduce $T_F(H)$.

In conclusion, the model Hamiltonian we propose as typical for random-field systems, Eq. (4), in general, has replica symmetry breaking and thus a true glassy phase which can occupy a substantial part of the phase diagram, especially for larger effective random fields.

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