Directed percolation: "field" exponents and a test of scaling in two and three dimensions

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Directed percolation is a simple model which is believed to fall into the same universality class as Reggeon field theory. A large body of series data in both "field" and "thermal" (concentration) variables is now available for both of these models, and they provide an ideal framework for studying the relationship between field and thermal corrections to scaling. In particular it is of interest to test whether the measured nonanalytic confluent correction in the field direction, Ω, is equal to $\Delta_1/\beta\delta$ in any system. This relation between Ω and Δ_1, the thermal correction and β, and $1/\delta$, the exponents of the percolation probability in the thermal and field directions, has been proposed but never successfully confirmed for isotropic percolation. We make direct estimates of $\Omega=0.45\pm0.15(2d)$, $\Omega=0.4\pm0.2(3d)$, $1/\delta=0.111\pm0.003(2d)$, $1/\delta=0.285\pm0.35(2d)$, and $\Delta_1=0.75\pm0.10(3d)$, and invoke extant estimates of $\Delta_1(2d)$ and β to confirm the scaling relation.

I. INTRODUCTION

There is at present a great deal of interest in the nature of higher-order corrections to the dominant critical behavior in many different systems. A large amount of effort has been invested in the study of these corrections for Ising models[1,2] and isotropic percolation,[3] both with series expansions and field theoretic or analytic approaches. For percolation, the percolation probability $P(p,\lambda)$ is believed to take the forms

$$P(p,\lambda=1)=P_0(p-p_e)^{\beta}[1+a(p-p_e)^{\Delta_1}+\cdots]$$

and

$$P(p_c,\lambda)=P_h(1-\lambda)^{1/\delta}[1+a(1-\lambda)^{\Omega}+\cdots],$$

where $\lambda=e^{-H}$, Δ_1 is the thermal correction and Ω is the first "field" correction. For two-dimensional (2D) isotropic percolation the Δ_1 measured in series analyses is inconsistent with the measured Ω if the scaling relation[4]

$$\Omega=\Delta_1/\beta\delta \tag{1}$$

is used for their comparison. The measured $\Delta_1\sim1.2$, gives $\Omega\sim0.5$ via (1). The measured Ω [$=0.64\pm0.08$ (Ref. 5) $=0.63\pm0.05$ (Ref. 6)] is inconsistent with $\Delta_1\sim1.2$ and (1) but is consistent with (1) and the $\Delta_1=\frac{5}{3}$ recently conjectured by Saleur,[7] and with an origin in the nonlinear scaling field (the Aharony-Fisher correction, $\Omega=0.604$). It is also quite consistent with the magnetic correction proposed by den Nijs,[8] $\Omega=0.73$; this possibility gives in fact the best $1/\delta$ value but requires us to revise the interpretation of Fig. 1 of Ref. 6. At any rate, the plethora of possible explanations still does not confirm or otherwise the correctness of relation (1) since conclusive proof would require $\Omega\sim0.5$, and despite all the confusion Ω is clearly >0.6. We can only conclude that if the predicted $\Omega\sim0.5$ correction is present its amplitude must be very small.

It is thus of interest to determine whether (1) can be shown to hold in any system. The obvious candidate for such a study is the Ising model, but the available field series are too short to draw any conclusions from their analysis. Another candidate is directed percolation. Here extensive field series have recently been generated, and the field theoretic results for Reggeon field theory (RFT), which is in the same universality class,[9] mean that a large body of data is available for purposes of comparison. We note that the problems that occurred in confirming (1) for isotropic percolation, videlicit the presence of additional irrelevant operators originating either in the nonlinear scaling field or in nonleading magnetic singularities could also occur in directed percolation. If they are present with large amplitudes in directed percolation then we could arrive at a similar stalemate to that occurring in isotropic percolation and be unable to confirm or otherwise the correctness of relation (1). Likewise numerical support for (1) would not completely exclude the possibility of additional nearly relevant magnetic eigenvalues, since there could be an accidental degeneracy, or there could be a nonleading singularity that is more relevant with an extremely small amplitude.

We analyze both 2D and 3D directed percolation and Reggeon quantum-spin (RQS) series. The development of the $P(p,\lambda)$ series from the perimeter polynomials is presented in Sec. I. As discussed in Secs. III and IV, we obtain the first series estimates for $1/\delta$ in directed percolation:

$$1/\delta=0.111\pm0.003\ (2D)\ ,$$

$$1/\delta=0.285\pm0.035\ (3D)\ ,$$

and greatly improved estimates for

$$\Omega = 0.45 \pm 0.15 \text{ (2D)} ,$$

$$\Omega = 0.4 \pm 0.2 \text{ (3D)} .$$

Excellent Δ_1 estimates for 2D have recently been made,[10] and in Sec. V we reanalyze the RQS series and some directed percolation susceptibilities in D=3 to obtain a Δ_1 estimate of

$$\Delta_1 = 0.75 \pm 0.10 \text{ (3D)} .$$

Throughout the calculations we have attempted to reevaluate previous analyses of all the series studied, and this has led to some revised γ, ν, and p_c estimates. We suggest

$$p_c = 0.7053 \pm 0.0010 \text{ (square, site)} ,$$

$$p_c = 0.5953 \pm 0.0008 \text{ (triangular, site)} ,$$

$$\nu = 1.280 \pm 0.015 \text{ (3D, RQS)} ,$$

$$p_c = 0.3825 \pm 0.001 \text{ (sc, bond)} ,$$

$$\gamma = 1.60 \pm 0.04 \text{ and } \gamma_r = 2.74 \pm 0.07 \text{ (sc, bond)} .$$

II. SERIES DEVELOPMENT FOR DIRECTED PERCOLATION

Extensive listings of perimeter polynomials are now available for directed percolation in all relevant dimensions: the data[11,12] extend through size $s=24$ (square site), $s=18$ (triangular site), $s=15$ (simple cubic site). The use of recursive relations for site directed percolation is explained in detail in Refs. 11 and 12. $P(p,\lambda)$ can be obtained from the polynomials $D_s(p)=\sum_t g_{st}(1-p)^t$, by the relation

$$P(p,\lambda) = 1 - \sum D_s(p) p^{s-1} \lambda^s .$$

The g_{st} summarize the partition of directed site clusters at fixed size s, according to the number of isolating neighbors t. As explained by Duarte,[11] clusters in directed problems are rooted at the origin, so the g_{st} play the role of sg_{st} in undirected percolation. They can be used to study several dominant critical exponents like τ (from the cluster numbers),[13] γ (from the first moment of the cluster size distribution),[14] and $\sigma = 1/\Delta = 1/(\gamma+\beta)$ (from the correction to the average perimeter-to-size ratio at the percolation threshold).[12]

III. TWO-DIMENSIONAL ANALYSIS FOR δ AND Ω

We consider the $P(p_c,\lambda)$ as $\lambda \to 1$ series for both the square (S) and triangular (T) site percolation problems. We have analyzed the series both with the Roskies transform, as presented in Ref. 4 for isotropic percolation (see also Ref. 6), and with a graphical version of the method of Adler *et al.*[15] Both methods give plots of different Padé approximants to $1/\delta$ as a function of Ω for different choices of p_c in the series $P(p_c,\lambda)$ as $\lambda \to 1$. We have considered $0.7045 \le p_c \le 0.7060(S)$ and $0.5939 \le p_c \le 0.5969(T)$, extending the ranges quoted by De'Bell and Essam,[16] hereafter denoted by DBE-I. We expect that their D-log-Padé analysis will give reliable central estimates for p_c since the "thermal" confluent correction is[10] ~ 1.0.

The behavior of the Padé approximants is very similar on both lattices. Plots for some p_c choices are given in Fig. 1 (S) and Fig. 2 (T). For $p_c = 0.7055$ [Fig. 1(a)] and

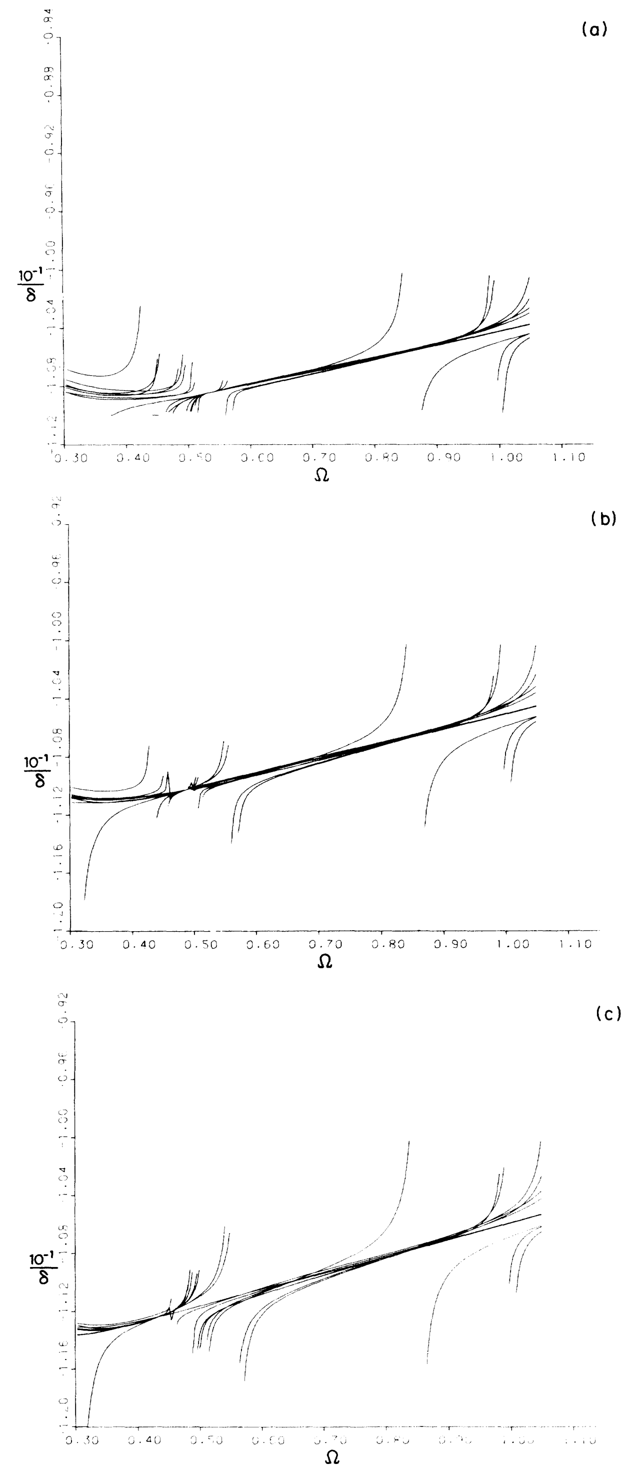

FIG. 1. Graph of different Padé approximants to $1/\delta$ as a function of Ω for different p_c choices on the square lattice: (a) $p_c = 0.7055$; (b) $p_c = 0.705\,25$; (c) $p_c = 0.705\,00$.

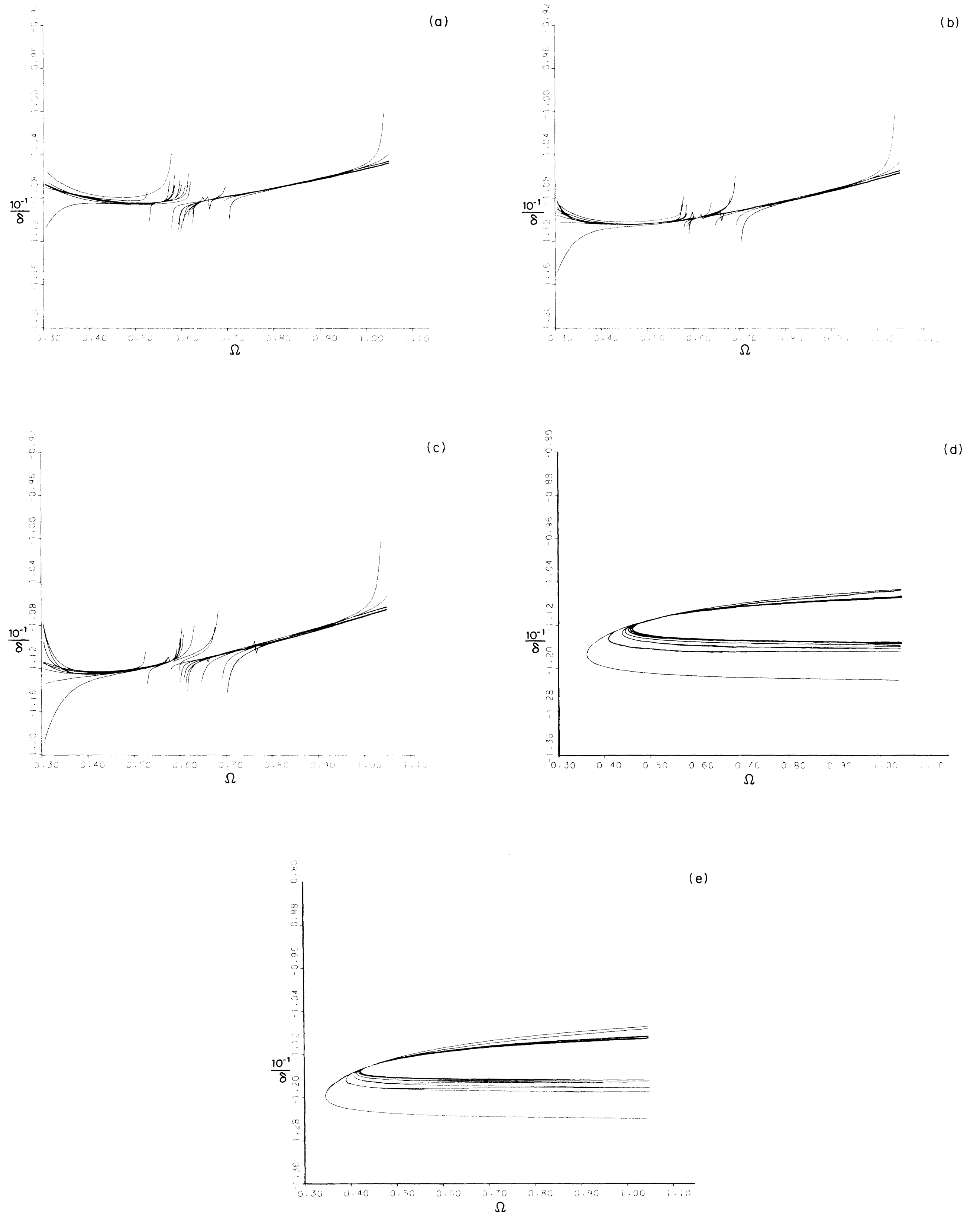

FIG. 2. Graph of different Padé approximants to $1/\delta$ as a function of Ω for different p_c choices on the triangular lattice: (a) $p_c=0.5957$; (b) $p_c=0.5954$; (c) $p_c=0.5951$; (d) $p_c=0.5954$, method of Ref. 15; (e) $p_c=0.5949$, method of Ref. 15.

$p_c = 0.5957$ [Fig. 2(a)] we have $1/\delta \sim 0.108$ and $\Omega \sim 0.85$. The p_c choices 0.705 25 [Fig. 1(b)] and 0.5954 [Fig. 2(b)] give $1/\delta \sim 0.110$ and the Ω values substantially unchanged. For $p_c = 0.7050$ [Fig. 1(c)] and $p_c = 0.5951$ [Fig. 2(c)] we have $1/\delta \sim 0.112$. The interpretation of these figures is somewhat marred by the presence of apparently defective Padé approximants between the intersection regions, leading both to uncertainty in the Ω value at the first intersection region and to a possible question as to whether there are in fact two distinct regions. If there is only one region, then the $1/\delta$ values would be < 0.108 and the Ω values near 0.7. To clarify this situation we ran studies with the method of Ref. 15, which we think is more reliable near a confluent correction of 1.0 but should be able to clearly distinguish whether one or two regions are present elsewhere. In Fig. 2(d) we give this analysis for $p_c = 0.5954$, where the intersection region falls at $1/\delta \sim 1.105$ and $\Omega \sim 0.5$, and in Fig. 2(d) the plot for $p_c = 0.5949$, where $1/\delta \sim 1.14$ and $\Omega \sim 0.45$. In both cases the plots from the second method give results consistent with those of the first only if the two-intersection region interpretation is invoked.

Our overall conclusions for two dimensions are

$$0.7045 \le p_c \le 0.7060 \; (S) ,$$

$$0.5945 \le p_c \le 0.5960 \; (T) ,$$

$$0.108 \le 1/\delta \le 0.114 ,$$

$$0.30 \le \Omega \le 0.60 .$$

The p_c ranges overlap, but one wider than, those of DBE-I. The implications of the δ and Ω results will be discussed below.

IV. THREE-DIMENSIONAL ANALYSIS FOR δ AND Ω

In three dimensions we have considered simple cubic site percolation. The first question here concerns the value of p_c for this lattice; De'Bell and Essam[17] hereafter (denoted by DBE-II) quote $p_c = 0.434 \pm 0.004$, but express

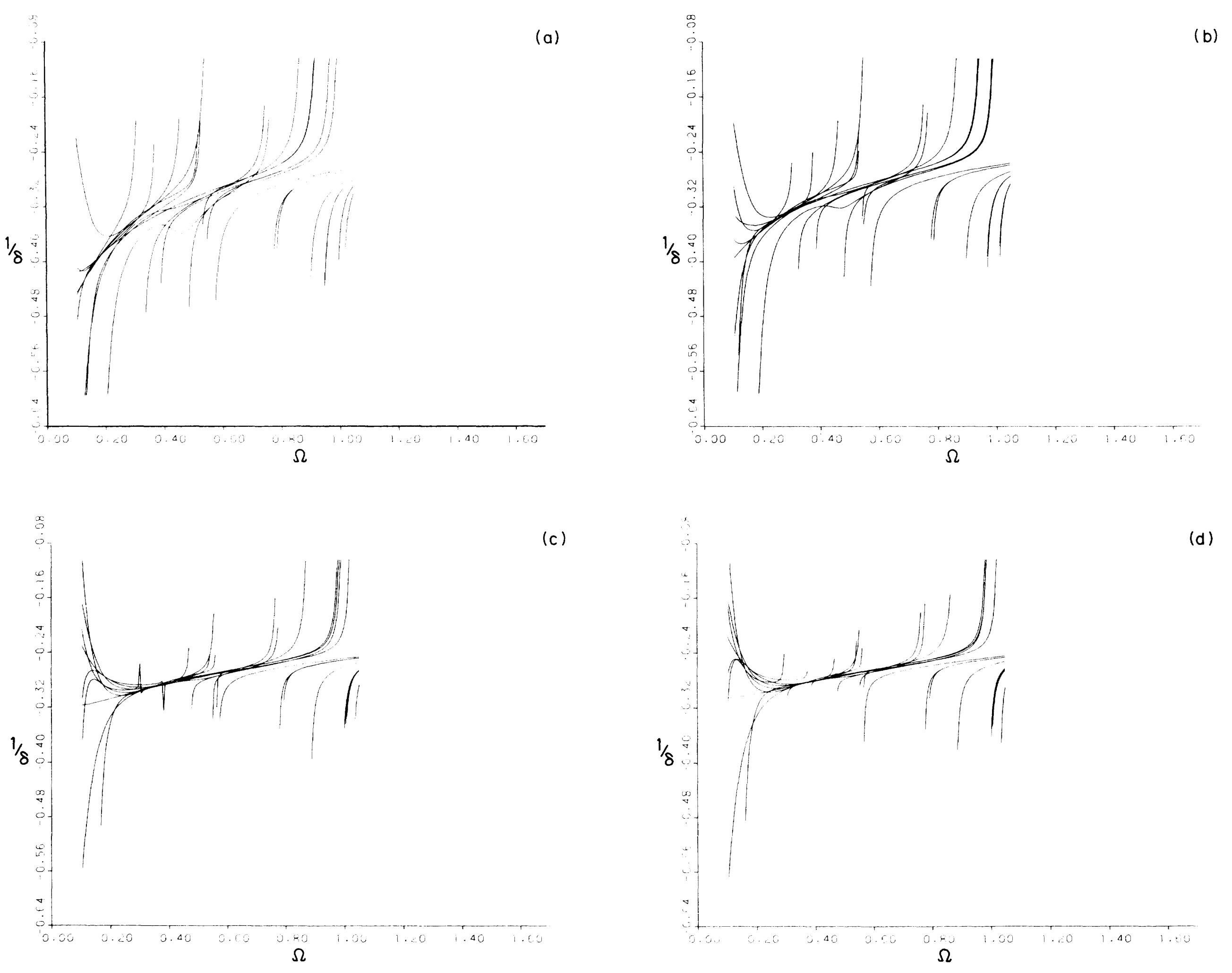

FIG. 3. Graph of different Padé approximants to $1/\delta$ as a function of Ω for different p_c choices on the simple cubic lattice: (a) $p_c = 0.4315$; (b) $p_c = 0.4325$; (c) $p_c = 0.4338$; (d) $p_c = 0.4345$.

doubts based on problems with the convergence of the Padés. Their alternative choice is $p_c = 0.4313 \pm 0.0015$, and Ref. 14 quotes $p_c = 0.435 \pm 0.0015$. We find [Fig. 3(a)] very poor convergence for $p_c = 0.4315$ and much improved convergence for $p_c > \sim 0.4325$. We plot graphs of $1/\delta$ as a function of Ω for several p_c choices that give reasonable convergence. In all these graphs there is also the question of whether there is one or two convergence regions. However, unlike two dimensions we have no clear alternative information. Therefore, we shall complete two analyses: one based on the assumption that there are two regions, one centered at $0.2 < \Omega < 0.4$ and another at $\sim \Omega = 0.6$; and a second based on the assumption that there is a single region with $0.3 < \Omega < 0.6$.

If we assume two regions we find $p_c = 0.4325$ [Fig. 3(b)] gives $1/\delta = 0.34 \pm 0.04$, $\Omega = 0.30 \pm 0.20$, and $p_c = 0.4338$ [Fig. 3(c)] gives much better convergence to $1/\delta = 0.29 \pm 0.03$ and $\Omega = 0.33 \pm 0.16$. For $p_c = 0.4345$ [Fig. 3(d)] just above the DBE-II range, we find $1/\delta = 0.28 \pm 0.03$, but Ω a little higher at 0.36 ± 0.16. This choice of p_c is well into the central estimate region recently proposed in Ref. 14 from extended susceptibility series of 17 and 16 terms. Their chosen γ is 1.564 ± 0.025, and if the standard Padé analyses of these authors (who also add several cautionary remarks on the difficulty of this specific problem) are to be reconciled with the bond cubic estimate $\gamma = 1.60 \pm 0.04$ (see Sec. V) even a value of 0.436–0.437 is not unreasonable. We conclude that $1/\delta = 0.285 \pm 0.035$ and $\Omega = 0.35 \pm 0.18$. If we assume a single region we find $\Omega = 0.5 \pm 0.2$ and $1/\delta = 0.27 \pm 0.03$. We suggest the overall estimate $\Omega = 0.4 \pm 0.2$.

V. TOWARD A Δ_1 ESTIMATE IN THREE DIMENSIONS

Previous Δ_1 estimates in three dimensions are[15] $0.6 \le \Delta_1 \le 1.1$ and estimates of $\Omega = \Delta_1 \nu$ from RFT. These are[18] $\Omega = 0.49 \pm 0.01$ from an ϵ expansion and $\Omega = 0.58 \pm 0.01$ from a loop expansion. Using the quantum-Reggeon-spin-model ν value of Brower *et al.*,[19] which is 1.271 ± 0.007, this gives $\Delta_1 = 0.62 \pm 0.02$ and $\Delta_1 = 0.74 \pm 0.02$, the former being barely consistent with the Ref. 15 estimate. We note that Cardy[18] prefers the ϵ-expansion choice.

We have attempted to find an improved Δ_1 estimate from reanalyses of the lattice RQS series of Ref. 19, which were considered too short for a confluent analysis in the past but turn out to be quite useful, and new confluent singularity analyses of some three-dimensional directed percolation series.

The RFT series are 10 terms long, and we chose to take the series for $\chi_{00}(k)$, which has a dominant exponent $\nu(\eta_1)$. For the central K_c choice of Ref. 19, we find $\nu(1+\eta) \sim 1.57$, but no evidence of any confluent correction. The graph of $\nu(\eta+1)$ as a function of Δ_1 [Fig. 4(a), $K_c = 1/2.428$] has a lack of structure that is reminiscent of the three-dimensional spin-$\frac{1}{2}$ Ising graphs at the T_c one obtains from an analysis which neglects confluent corrections to scaling and gives exponents that violate hyperscaling [see Figs. 2(a) and 5(b) of Adler[20]]. In both cases there is good Padé convergence, but no intersection of the

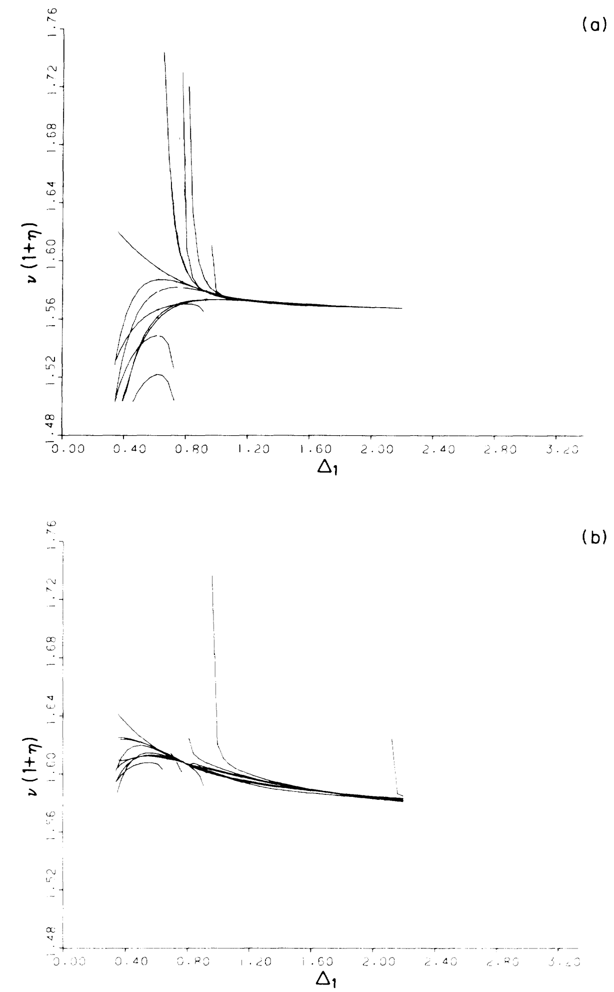

FIG. 4. Graph of different Padé approximants to $\nu(1+\eta)$ as a function of Δ_1 for different K_c choices in the RQS model: (a) $K_c = \frac{1}{2.428}$; (b) $K_c = \frac{1}{2.4222}$.

Padé approximants in the Roskies transform graph. When the K_c value is increased to well above the error bars of Ref. 19, the lack of structure is replaced by two intersection regions, one at $\Delta_1 \sim 0.80 \pm 0.10$, where $\gamma(1+\eta) = 1.61 \pm 0.01$ for $K_c = \frac{1}{2.4222}$ [Fig. 4(b)] with a lower $\Delta_1 \sim 0.65 \pm 0.10$ and higher $\nu(1+\eta) = 1.615 \pm 0.010$ for $K_c = \frac{1}{2.4210}$. Both graphs have a second convergence region near $\Delta_1 = 1.8$.

These graphs are reminiscent of Figs. 2(c), 5(c), and 6(c) of Ref. 20, where the exponent values at the intersection of the various Padé approximants were those that agreed with hyperscaling and the field theoretic values. We thus conclude that $\Delta_1 \sim 0.73 \pm 0.10$ and $\nu(1+\eta) \sim 1.61 \pm 0.02$ for the Reggeon spin model. Using the η value of RFT given by Cardy,[18] $\eta = 0.26$, we find $\nu = \frac{1.61}{1.26} = 1.28$, rather than the 1.271 of Ref. 19, and this gives $\Delta_1 \sim 0.63 \pm 0.02$ or $\sim 0.74 \pm 0.02$, depending on the Ω choice. Both of these Δ_1 estimates are consistent with the direct estimate from the spin series, but we see that consistency with the series would favor the loop choice. The ν estimate is closer to the series value of 1.28 ± 0.03 of DBE-II than the 1.271 choice was.

There are several well-behaved three-dimensional directed percolation series. We choose to study two series for directed bond percolation on the sc lattice, the resistivity series of Bhatti[21] and the 13-term mean size series $S(p)$ of DBE-II, since these have the same critical point. It has been conjectured[22] that Δ_1 will be the same for the susceptibility (mean size) and resistive susceptibility for isotropic percolation, with strong support coming from 2D directed percolation; thus, we will certainly make the same assumptions here.

The p_c estimate of DBE-II is 0.382 ± 0.001. From the series of Bhatti[21] we have plotted γ_r versus Δ_1 for several p_e choices in and out of this range and find

$$p_c=0.382 \Longrightarrow \gamma_r=2.70\pm0.04,\quad \Delta_1=0.95\pm0.2\ ;$$

$$p_c=0.383 \Longrightarrow \gamma_r=2.78\pm0.04,\quad \Delta_1=0.8\pm0.2\ ;$$

$$p_c=0.3835 \Longrightarrow \gamma_r=2.83\pm0.03,\quad \Delta_1=0.7\pm0.2\ .$$

The plot for $p_c=0.3830$ is given in Fig. 5.

From the $S(p)$ series we find

$$p_c=0.3817 \Longrightarrow \gamma=1.56\pm0.02,\quad \Delta_1=1.05\pm0.10\ ;$$

$$p_c=0.382 \Longrightarrow \gamma=1.57\pm0.02,\quad \Delta_1=0.92\pm0.10\ ;$$

$$p_c=0.3822 \Longrightarrow \gamma=1.58\pm0.02,\quad \Delta_1=0.85\pm0.15\ ;$$

$$p_c=0.3824 \Longrightarrow \gamma=1.61\pm0.02,\quad \Delta_1=0.82\pm0.15\ ;$$

$$p_c=0.3826 \Longrightarrow \gamma=1.62\pm0.02,\quad \Delta_1=0.75\pm0.15\ .$$

The convergence degrades above $p_c=0.3826$ for the $S(p)$ series, and we conclude from these two studies that p_c for the bond cubic problem is slightly above the DBE-II estimate (we suggest $p_c=0.3825\pm0.001$), and that $\gamma=1.60\pm0.04$, $\gamma_r=2.74\pm0.07$, $\Delta_1=0.78\pm0.22$ give exponent estimates that are consistent with this choice.

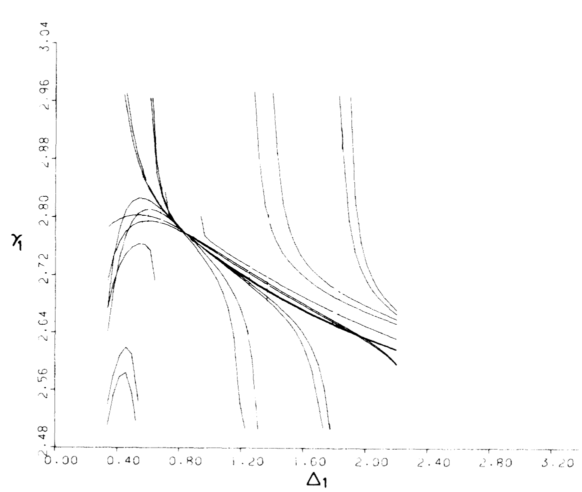

FIG. 5. Graph of different Padé approximants to γ_r as a function of Δ_1 for $p_c=0.3830$ on the simple cubic lattice.

From these diverse analyses we may conclude that $0.65<\Delta_1<0.85$, with the possibility of a slightly lower estimate if the ϵ expansion choice were to be preferred over the loop and series ones.

VI. TESTING THE SCALING RELATION

We now compare the Ω estimates from Secs. III and IV with the values obtained from the relation $\Omega=\Delta_1/\beta\delta$. We begin by comparing our $1/\delta$ estimates with those obtained via the scaling relation $\delta-1=\gamma/\beta$. In two dimensions we take $\gamma=2.277\,21\pm0.000\,01$ from Ref. 10 and $\beta=0.28\pm0.01$ from DBE-II to find $\delta-1=8.13\pm0.28$ $\Longrightarrow 1/\delta=0.1095\pm0.0040$, quite in agreement with our estimate of $1/\delta=0.111\pm0.03$. The convergence for the β and γ series is much tighter since $\Delta_1=1.0$.[10] In three dimensions we take our $\gamma=1.60\pm0.04$, together with $\beta=0.59\pm0.02$ to predict $1/\delta=0.27\pm0.015$. This overlaps with our range of $1/\delta=0.285\pm0.035$, and is directly consistent with the $1/\delta$ estimates from the higher p_c and Ω choices. It is in perfect agreement with our estimate made by assuming a single intersection region, although the 2D results would seem to imply that the two-region interpretation is the correct choice. We note that the β value used for the prediction was not taken from a confluent singularity analysis. As we will shortly require a β estimate, we make an alternate calculation using the relation[9] $\beta=\frac{1}{2}\nu(\frac{1}{2}Dz-\eta)$ with the RFT $\eta=0.26$, our and DBE-II's $\nu=1.28$ and the RFT $z=1.16$ to find $\beta=\frac{1}{2}(1.28[\frac{1}{2}(2\times1.16)-0.26])\sim0.58$, which is closer to the site estimates of DBE-II, and thus see that neglect of confluent corrections does not affect the β estimate very much. This β gives a $1/\delta$ estimate Of 0.26, also within our range.

In two dimensions, we use $\Delta_1=1.0$, and β and $1/\delta$ as above, to predict $\Omega=\Delta_1/\beta\delta=1.00\times\frac{0.111}{0.28}=0.39\pm0.02$. This is in good agreement with the measured $\Omega=0.45\pm0.15$. In three dimensions, we use $\Delta_1=0.75\pm0.10$, $\beta=0.58$, and $1/\delta=0.285\pm0.04$ to predict $\Omega=\Delta_1/\beta\delta=0.75\times\frac{0.28}{0.58}=0.37\pm0.10$. If we use the ϵ expansion[18] $\Delta_1=0.63\pm0.02$, we predict $\Omega=0.63\times\frac{0.285}{0.59}$ $=0.31\pm0.05$. The loop expansion $\Delta_1=0.74\pm0.02$, gives $\Omega=0.36\pm0.05$. These Ω values are both well within the range of our two-intersection region estimate, $\Omega=0.35\pm0.18$, and of our average Ω and toward the lower end of the Ω estimate from the single intersection region estimate, $\Omega=0.5\pm0.2$. We conclude that the scaling relation $\Omega=\Delta_1/\beta\delta$ is satisfied in both two and three dimensions for directed percolation. We exclude the possibility of a more relevant Ω than that given by $\Delta_1/\beta\delta$ with a large amplitude but cannot exclude the possibility that there is an accidental degeneracy between an Ω derived from the thermal correction and one from another source.

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[1]M. George and J. J. Rehr (unpublished).

[2]J. Adler, J. Phys. A (to be published).

[3]J. Adler, A. Aharony, Y. Meir, and A. B. Harris, Phys. Rev. B **34**, 3469 (1986).

[4]J. Adler, M. Moshe, and V. Privman, in *Annals of the Israel Physical Society,* edited by G. Deutscher, R. Zallen, and J. Adler (Hilger, London, 1983), Vol. 3, see especially, p. 409.

[5]A. Margolina, Z. V. Djordjevic, D. Stauffer, and H. E. Stanley, Phys. Rev. B **28**, 1652 (1983).

[6]J. Adler, Phys. Rev. B **31**, 4693 (1985).

[7]Saleur (unpublished).

[8]M. den Nijs (private communication).

[9]J. L. Cardy and R. L. Sugar, J. Phys. A **13**, L423 (1980).

[10]J. W. Essam, K. De'Bell, J. Adler, and F. N. Bhatti, Phys. Rev. B **33**, 1982 (1986).

[11]J. A. M. S. Duarte, Port. Phys. **15**, 119 (1984).

[12]J. A. M. S. Duarte, J. Phys. (Paris) **47**, 383 (1986).

[13]M.C.T.P. Carvalho and J.A.M.S. Duarte, Z. Phys. B **62**, 239 (1986).

[14]J.A.M.S. Duarte and H. J. Ruskin, J. Phys. (Paris) **47**, 843 (1986).

[15]J. Adler, M. Moshe, and V. Privman, J. Phys. A **14**, L363 (1981); see also J. Adler and I. G. Enting, J. Phys. A **17**, 2233 (1984).

[16]K. De'Bell and J. W. Essam, J. Phys. A **16**, 385 (1983).

[17]K. De'Bell and J. W. Essam, J. Phys. A **16**, 3553 (1983).

[18]J. L. Cardy, Phys. Lett. **67B**, 97 (1977).

[19]R. C. Brower, M. A. Furman, and M. Moshe, Phys. Lett. **76B**, 218 (1978).

[20]J. Adler, J. Phys. A **16**, 3585 (1983).

[21]F. M. Bhatti, J. Phys. A **17**, 1771 (1984).

[22]J. Adler, J. Phys. A **18**, 307 (1985).