PHYSICAL REVIEW B VOLUME 35, NUMBER 13 1 MAY 1987

Localization and correlation effects in itinerant ferromagnets

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(Received 15 September 1986)

The effects of disorder, in particular of localization, have been studied in the vicinity of the ferromagnetic transition by making use of a generalized N-orbital model within a $1/N$ expansion technique. Fermi-liquid behavior is obtained in the $N\rightarrow\infty$ limit with two separate diffusion constants for spin- and charge-density fluctuations. The spin-diffusion constant associated with spin fluctuations and Stoner excitations is found to acquire a localization correction. While the dc conductivity in the ferromagnetic phase also acquires the same localization correction, the spin-wave stiffness constant does not. This indicates the system can exist in an insulating state and exhibit long-range magnetic order—a *ferromagnetic Anderson insulating* state.

I. INTRODUCTION

In this paper we introduce a model of interacting spin-half-fermions moving in a random potential and study some combined effects of disorder and interaction on the response properties of the system at zero temperature. A number of significant consequences of this interplay of disorder and interaction are already known. A very general consequence of impurity scattering is to change the plane-wave motion of a quantum-mechanical particle into a random-walk-like motion so that the particle-density fluctuation is diffusive on length scales larger than the mean-free path. Diffusing particles spend more time together and hence the mutual interaction effects are enhanced so much so that one finds, in one and two dimensions, significant departures from the normal Fermi-liquid-theory results for the quasiparticle lifetime,[1] the electrical conductivity[2] and the specific heat,[3] for example.

In a spin-half-fermion system with short-ranged interactions an important role is played by the spin-density fluctuations and so we should like to study the effect of disorder, in particular of localization, on its nature and also on the magnetic features that follow—such as the spin-wave instability. By being able to carry their spin bias to nearby regions, moving fermions, because of the interaction, bring about spin correlation in the system which can lead to the collective (spin-wave) mode. Clearly a system of ultralocalized fermions (i.e., single-particle excitations with a localization length ξ, of the order of the elastic mean-free path l) cannot support long-range order. It would be interesting, then, to understand the effects of localization on the spin-wave stiffness both in the weakly localized regime and close to the mobility edge (where $\xi \gg l$).

At the static level, disorder is known to enhance magnetic fluctuations as seen in experiments on Si:P in which a large growth of the magnetic susceptibility was observed at low temperatures.[4] Fukuyama[5] has shown, within a self-consistent renormalized theory of spin fluctuations with disorder, that the critical interaction strength is reduced. Similar results were found by Finkelstein[6] and by Castellani *et al.*[7] who showed that the effective triplet coupling constant scales to infinity (in a renormalized perturbation theory). These results have led to speculations that, in weak coupling, the system might develop local moments.

In this paper we want to study localization effects near the ferromagnetic transition. We make use of a generalized N-orbital model within a $1/N$ expansion technique to study these aspects in a consistent and systematic manner. Within this approach the maximally crossed diagrams of localization are of order $1/N$ whereas the ladder diagrams involving impurity lines (giving rise to the diffusion mode in particle-density fluctuations) are of order 1 just as are the ladder diagrams involving interaction lines (giving rise to spin fluctuations). Thus, the method is well suited to systematizing various pieces of physics. The use of $1/N$ as an appropriate expansion parameter was first demonstrated by Wegner[8] and Oppermann and Wegner[9] for the noninteracting problem and later by Oppermann[10] and Ma and Fradkin[11] for the spinless interacting case. Oppermann[12] has studied a random matrix model with a Heisenberg interaction within this scheme. As we show below, the ferromagnetic transition is accessible in the $N\rightarrow\infty$ limit. However, the mobility edge is not. The metal-insulator transition, within this scheme actually appears in a nonperturbative manner.[11]

We have evaluated the spin, charge, and current response functions of the system both in the paramagnetic and ferromagnetic phases. From the spin susceptibility one is able to obtain the Stoner criterion for the ferromagnetic transition. In the $N\rightarrow\infty$ limit only ladder diagrams with impurity and interaction lines are kept and this approximation thus neglects localization and treats interaction only at the mean-field level. The results are, therefore, appropriate for a three-dimensional system in the metallic limit. We obtain, as expected, typical Fermi-liquid behavior[13] (generalized to the ferromagnetic phase) with two diffusion constants—one for charge and one for spin.

We have also studied some of the physics at the $1/N$ level. We have evaluated the correction to spin-diffusion constant coming from the maximally crossed diagrams of localization. The effect of magnetization on the Cooper propagator (both with parallel and antiparallel spins) in the ferromagnetic phase has also been studied. We find that the localization correction to conductivity is unaffected by magnetization. On the other hand, there are no singular localization contributions to the static susceptibility and to the stiffness constant of the spin-wave mode. This indicates that the system can exist in a *ferromagnetic Anderson insulating* state with localized Stoner excitations and long-range ferromagnetic order.

II. THE MODEL

We consider a generalization of the model introduced by Wegner[8] by adding spin-dependent, short-ranged interactions. The N species of fermions interact with each other via two Hubbard-like interactions. One is a density-density interaction between fermions of opposite spin which are scattered into their respective species channels. The other term is an exchange interaction in which the species indices of the two fermions of opposite spin are exchanged. The Hamiltonian (for a jellium model) is:

$$H=\int d^3r \Bigg[\psi^\dagger_{\alpha\sigma}(\mathbf{r})(-\tfrac{1}{2}\nabla^2)\psi_{\alpha\sigma}(\mathbf{r}) + \frac{1}{\sqrt{N}}\epsilon_{\alpha\beta}(\mathbf{r})\psi^\dagger_{\alpha\sigma}(\mathbf{r})\psi_{\beta\sigma}(\mathbf{r}) + \frac{U_1}{N}\psi^\dagger_{\alpha\uparrow}(\mathbf{r})\psi_{\alpha\uparrow}(\mathbf{r})\psi^\dagger_{\beta\downarrow}(\mathbf{r})\psi_{\beta\downarrow}(\mathbf{r}) + \frac{U_2}{N}\psi^\dagger_{\alpha\uparrow}(\mathbf{r})\psi_{\beta\uparrow}(\mathbf{r})\psi^\dagger_{\beta\downarrow}(\mathbf{r})\psi_{\alpha\downarrow}(\mathbf{r}) \Bigg] , \qquad (1)$$

where repeated indices are summed, σ labels the two spins $\uparrow$ and $\downarrow$ and α,β label the orbital species. $\epsilon_{\alpha\beta}(\mathbf{r})$ is the local random potential with a white-noise probability distribution of width γ given by:

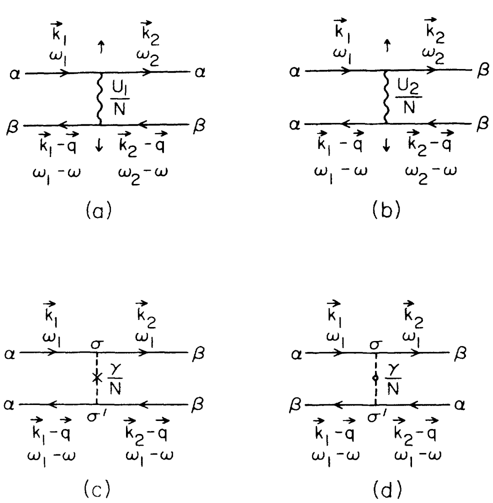

FIG. 1. Fermion scattering processes of the model. Density-density (a) and exchange (b) interaction vertices, and effective impurity scattering vertices (c) and (d).

$$P\{\epsilon_{\alpha\beta}(\mathbf{r})\}=\frac{1}{(2\pi\gamma)^{1/2}}\exp\left[-\int d^3r\,\epsilon^2_{\alpha\beta}(\mathbf{r})/\gamma\right] ,$$
$$\langle \epsilon_{\alpha\beta}(\mathbf{r})\rangle=0 \qquad (2)$$
$$\langle \epsilon_{\alpha\beta}(\mathbf{r})\epsilon_{\alpha'\beta'}(\mathbf{r}')\rangle=\gamma\delta^3(\mathbf{r}-\mathbf{r}')(\delta_{\alpha\alpha'}\delta_{\beta\beta'}+\delta_{\alpha\beta'}\delta_{\beta\alpha'}) .$$

For a spin-symmetric system the coupling constants U_1 and U_2 must be equal. In this case we have the full SU(2) spin symmetry of the Hubbard model. In the pure limit ($\gamma\rightarrow 0$) and with $U_1=U_2=U$ one obtains (in the $N\rightarrow\infty$ limit) the same results as for the Hubbard model[14] solved in the generalized Hartree [or random phase approximation (RPA)] approximation.[15]

Appropriate factors of $1/\sqrt{N}$ and $1/N$ are pulled out to keep the energy density finite in the $N\rightarrow\infty$ limit. After impurity averaging the scattering off impurities leads to an effective elastic scattering between fermions. The various scattering processes at work are shown in Fig. 1.

III. ONE-PARTICLE GREEN'S FUNCTION

In the $N\rightarrow\infty$ limit there are two terms which contribute to the self-energy (Fig. 2). The impurity-averaged one-particle Green's function is obtained by solving the Dyson equation self-consistently:

$$G^\sigma(\mathbf{k},\omega)=G^0(\mathbf{k},\omega)+G^0(\mathbf{k},\omega)[\Sigma^\sigma_{(a)}(\omega)+\Sigma^\sigma_{(b)}]G^\sigma(\mathbf{k},\omega) , \qquad (3)$$

where

$$G^0(\mathbf{k},\omega)=\frac{1}{\omega-\dfrac{k^2}{2}+i\,\mathrm{sgn}(\omega)} ,$$

$$\Sigma^\sigma_{(a)}(\omega)=\gamma\int\frac{d^3k}{(2\pi)^3}G^\sigma(\mathbf{k},\omega) ,$$

and

$$\Sigma^\sigma_{(b)}=n^{-\sigma}U .$$

The Green's function is independent of the orbital index and so it has been dropped. A straightforward extension of de Gennes's method[16] to include the spin-dependent shift in energy from the Hartree term leads to the result:

$$G^\sigma(\mathbf{k},\omega)=\frac{1}{[q^2_\sigma(\omega)-k^2]/2}$$
$$G^\sigma(\mathbf{r}-\mathbf{r}';\omega)=-\frac{1}{2\pi}\frac{\exp(iq_\sigma(\omega)\,|\,\mathbf{r}-\mathbf{r}'\,|\,)}{|\,\mathbf{r}-\mathbf{r}'\,|} , \qquad (4)$$

where

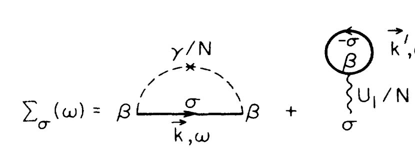

FIG. 2. Self-energy contribution in the $N\rightarrow\infty$ limit.

$$q_\sigma(\omega)=i/2l+\mathrm{sgn}(\omega)[2(\omega+\Omega_F)-\sigma\Delta]^{1/2}$$
$$(\mathrm{Im}[2(\omega+\Omega_F)-\sigma\Delta]^{1/2}\geq 0)\ .$$

Impurity scattering thus leads to a finite mean-free path $l=\pi/\gamma$. $\Delta=-(n^\uparrow-n\downarrow)U$ is the relative band shift and $\Omega_F-\sigma(\Delta/2)$ is the distance of the bottom of the σ band from the Fermi energy which is chosen to lie at zero of the energy scale. Δ and Ω_F are determined by solving self-consistently the equations for spin density $n^\uparrow-n^\downarrow\equiv m$ and the particle density $n^\uparrow+n^\downarrow\equiv n$. n is related to the Fermi momentum in the pure, noninteracting system by $n=k_F^3/3\pi^2$. From Eq. (4) we obtain the density of states:

$$N_\sigma(\omega)=\frac{1}{2\pi^2}[2(\omega+\Omega_F)-\sigma\Delta]^{1/2} \tag{5}$$

and the particle density:

$$\begin{aligned} n_\sigma &= \int d\omega\, N_\sigma(\omega) \\ &= \frac{1}{6\pi^2}(2\Omega_F-\sigma\Delta)^{3/2} \\ &\approx \frac{(2\Omega_F)^{3/2}}{6\pi^2}\left[1-\sigma\frac{3}{2}\frac{\Delta}{2\Omega_F}+\frac{3}{8}\left[\frac{\Delta}{2\Omega_F}\right]^2 \right. \\ &\qquad \left. +\sigma\frac{1}{16}\left[\frac{\Delta}{2\Omega_F}\right]^3+\cdots\right]\ . \end{aligned} \tag{6}$$

The self-consistency equations are, then:

$$\begin{aligned} n^\uparrow-n^\downarrow &= m=\Delta/U \\ n^\uparrow+n^\downarrow &= n=k_F^3/3\pi^2\ . \end{aligned} \tag{7}$$

Using Eq. (6) we find the condition for a nonzero solution for Δ is:

$$\frac{1}{U}=\left.\frac{(2\Omega_F)^{1/2}}{2\pi^2}\right|_{\Delta\to 0}=\frac{k_F}{2\pi^2}=N(0)\ . \tag{8}$$

This is the Stoner criterion for the onset of ferromagnetism. In the $N\to\infty$ limit the critical interaction strength $U_c=1/N(0)$ is thus seen to be independent of disorder strength γ.

IV. TRANSVERSE MAGNETIC SUSCEPTIBILITY

In order to study the spin response of the system we evaluate the transverse magnetic susceptibility. A singularity in the static susceptibility indicates an instability of the ground state of the system. Furthermore, in as much as the inelastic scattering cross section [which effectively measures the willingness of say, a neutron, to lose energy in $(\mathbf{q},\omega)$ mode] is related to the imaginary part of the susceptibility, one can obtain information about the spin excitations in the system by looking at the peaks in $\mathrm{Im}\chi(\mathbf{q},\omega)$.

The transverse magnetic susceptibility for fermions in orbital α is given by the Kubo formula:[15]

$$\chi_\alpha^{-+}(\mathbf{r}t;\mathbf{r}'t')=\sum_\beta i\Theta(t-t')\langle\Psi_N\,|\,[a_{\alpha\downarrow}^\dagger(\mathbf{r},t)a_{\alpha\uparrow}(\mathbf{r},t),a_{\beta\uparrow}^\dagger(\mathbf{r}',t')a_{\beta\downarrow}(\mathbf{r}',t')]\,|\,\Psi_N\rangle \tag{9}$$

The retarded, two-particle Green's function of interest here can be obtained from the following time-ordered two-particle Green's function:

$$i\langle\Psi_N\,|\,Ta_{\alpha\uparrow}(\mathbf{r},t)a_{\beta\downarrow}(\mathbf{r}',t')a_{\beta\uparrow}^\dagger(\mathbf{r}',t')a_{\alpha\downarrow}^\dagger(\mathbf{r},t)\,|\,\Psi_N\rangle\ . \tag{10}$$

After impurity averaging we can Fourier transform spatially as well as temporally and obtain:

$$\langle\chi_\alpha^{-+}(\mathbf{q},\omega)\rangle=i\sum_\beta\langle G_{\alpha\beta}^{(2)}(\mathbf{q},\omega)\rangle\quad(\omega>0)\ . \tag{11}$$

To leading order $[O(1)]$ in the $N\to\infty$ limit only ladder diagrams involving U_2 interaction and impurity lines contribute to the impurity averaged two-particle Green's function. In order to sum the diagrams it is convenient to group them as shown in Fig. 3. The transverse susceptibility can be written in a form identical to the RPA susceptibility of the Hubbard model.[15]

$$\chi^{-+}(\mathbf{q},\omega)=\frac{I^{\uparrow\downarrow}(\mathbf{q},\omega)}{1-UI^{\uparrow\downarrow}(\mathbf{q},\omega)}\ , \tag{12}$$

where

$$I^{\uparrow\downarrow}(\mathbf{q},\omega)=i\int\frac{d\omega_1}{2\pi}\frac{J^{\uparrow\downarrow}(\mathbf{q},\omega;\omega_1)}{1-\gamma J^{\uparrow\downarrow}(\mathbf{q},\omega;\omega_1)}\ , \tag{13}$$

$$J^{\sigma\sigma'}(\mathbf{q},\omega;\omega_1)=\int\frac{d^3k_1}{(2\pi)^3}G^\sigma(\mathbf{k}_1,\omega_1)G^{\sigma'}(\mathbf{k}_1-\mathbf{q},\omega_1-\omega)\ . \tag{14}$$

The susceptibility is independent of the orbital index α and so it has been dropped. We now evaluate $I^{\uparrow\downarrow}(\mathbf{q},\omega)$. Using the expression given in Eq. (4) for the Green's function in the ferromagnetic phase we obtain

$$J^{\uparrow\downarrow}(\mathbf{q},\omega;\omega_1)=\frac{i}{2\pi}\frac{1}{q}\ln\left[\frac{q_\uparrow(\omega_1)+q_\downarrow(\omega_1-\omega)+q}{q_\uparrow(\omega_1)+q_\downarrow(\omega_1-\omega)-q}\right]\ . \tag{15}$$

In performing the ω_1 integral in Eq. (13) we divide it into

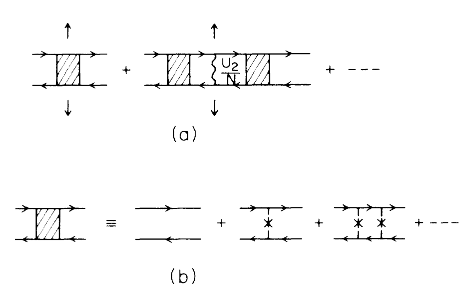

FIG. 3. Diagrams contributing to the transverse magnetic susceptibility in the $N\to\infty$ limit.

three regions—(i) $\omega_1 < 0$, (ii) $\omega_1 > \omega$, and (iii) $0 < \omega_1 < \omega$. In regions (i) and (ii) the sign functions in $q_\uparrow(\omega_1)$ and $q_\downarrow(\omega_1 - \omega)$ are the same whereas in region (iii) they are opposite. We extend the method used by de Gennes[16] to the ferromagnetic phase in performing the ω_1 integral in regions (i) and (ii). The contributions from regions (i) and (ii) are essentially found to give the static part, $I^{\uparrow\downarrow}(\mathbf{q},0)$ [an $O(\omega^2)$ term has been neglected]. Up to the quadratic term in q we find

$$I^{\uparrow\downarrow}(\mathbf{q},0)=\frac{k_F'}{2\pi^2}\left[1-\frac{q^2}{12k_F'^2}\frac{4k_F'^2l^2}{1+4k_F'^2l^2}\right], \tag{16}$$

where

$$k_F'/2\pi^2=[(2\Omega_F+\Delta)^{1/2}+(2\Omega_F-\Delta)^{1/2}]/4\pi^2 \approx 2\Omega_F/2\pi^2$$

is the spin-averaged density of states at the Fermi energy. In the paramagnetic phase $\Delta=0$ and $\Omega_F=\omega_F(=k_F^2/2)$. Substituting Eq. (16) into Eq. (12) we obtain the static susceptibility from which we can determine the spin-correlation length ξ_σ. We find

$$\xi_\sigma=\frac{1}{\sqrt{12}k_F}\left[\frac{UN(0)}{1-UN(0)}\right]^{1/2}\frac{2k_Fl}{(1+4k_F^2l^2)^{1/2}} . \tag{17}$$

Apart from the enhancement term $[UN(0)/1-UN(0)]^{1/2}$ due to correlation we find the length scale is set by the inverse Fermi momentum in the pure limit ($k_Fl \gg 1$) and by the mean-free path in the dirty limit $k_Fl \ll 1$). We can now write down an expression for the free energy of the form

$$F=\int d^3r[\tfrac{1}{2}AM^2(\mathbf{r})+\tfrac{1}{4}BM^4(\mathbf{r})+C\,|\nabla M(\mathbf{r})|^2] \tag{18}$$

provided we make the following identifications:

$$\frac{A}{B}=\frac{U_c-U}{U}\left[U_c=\frac{1}{N(0)}\right],$$

$$\frac{C}{B}=\frac{1}{12k_F^2}\frac{4k_F^2l^2}{1+4k_F^2l^2} .$$

The frequency-dependent term in $I^{\uparrow\downarrow}(\mathbf{q},\omega)$ comes from region (iii) in the ω_1 integral [Eq. (13)]. In the limit of small $\omega/2\Omega_F$ and $\Delta/2\Omega_F$ we obtain

$$I^{\uparrow\downarrow}(\mathbf{q},\omega)\,|_{\mathrm{iii}}=\frac{i\omega}{2\pi}\left[\frac{J^{\uparrow\downarrow}(\mathbf{q},\omega)}{1-\gamma J^{\uparrow\downarrow}(\mathbf{q},\omega)}\right],$$

$$J^{\uparrow\downarrow}(\mathbf{q},\omega)=\frac{i}{2\pi q}\ln\left[\frac{\frac{1}{l}+\frac{\omega-\Delta}{\kappa_F}+q}{\frac{1}{l}+\frac{\omega-\Delta}{\kappa_F}-q}\right];\ \kappa_F\equiv(2\Omega_F)^{1/2} . \tag{19}$$

We study the frequency-dependent part of $I^{\uparrow\downarrow}(\mathbf{q},\omega)$ given in Eq. (19) in various limits now.

(i) In the limit of small frequency ($\omega\tau \ll 1$), small magnetization ($\Delta\tau \ll 1$) (here $\tau\equiv l/\kappa_F$) and long wavelength ($1/q \gg l$) we obtain by keeping terms to first order in $\omega\tau$, $\Delta\tau$, and l^2q^2:

$$I^{\uparrow\downarrow}(\mathbf{q},\omega)\,|_{\mathrm{iii}}\approx\frac{i\omega}{2\pi\gamma\tau}\left[\frac{1}{Dq^2-i(\omega-\Delta)}\right], \tag{20}$$

where $D\equiv l^2/3\tau$ is the bare diffusion constant. Neglecting the q^2 term in the static part $I^{\uparrow\downarrow}(\mathbf{q},0)$ given in Eq. (16) and adding it to the frequency dependent part given above, we obtain:

$$I^{\uparrow\downarrow}(\mathbf{q},\omega)=\frac{\kappa_F}{2\pi^2}\left[\frac{Dq^2+i\Delta}{Dq^2+i\Delta-i\omega}\right]. \tag{21}$$

Substituting this in Eq. (12) we obtain the transverse magnetic susceptibility:

$$\chi^{-+}(\mathbf{q},\omega)=\frac{\bar{N}(0)}{1-U\bar{N}(0)}\left[\frac{D_\sigma q^2+i\Delta_\sigma}{D_\sigma q^2+i\Delta_\sigma-i\omega}\right], \tag{22}$$

where $\bar{N}(0)=\kappa_F/2\pi^2$ is the spin-averaged density of states at the Fermi energy in the ferromagnetic phase. $D_\sigma=[1-U\bar{N}(0)]D$ is the renormalized spin-diffusion constant and $\Delta_\sigma=[1-U\bar{N}(0)]\Delta$ is the renormalized band shift. If we identify the factor $1-U\bar{N}(0)$ with the Fermi-liquid constant $1+F^a$ (where F^a is the antisymmetric Landau parameter) then we obtain essentially the same Fermi-liquid description of the spin response of a dirty metal as in Ref. 13 but one which is generalized to the ferromagnetic regime as well.

In the paramagnetic phase $\Delta=0, \bar{N}(0)=N(0)$ and $\kappa_F=k_F$ and then Eq. (22) reduces to:

$$\chi^{-+}(\mathbf{q},\omega)\,|_{\mathrm{para}}=\frac{N(0)}{1-UN(0)}\left[\frac{D_\sigma q^2}{D_\sigma q^2-i\omega}\right] \tag{23}$$

which was first arrived at by Fulde and Luther[17] via a small $1/k_Fl$-type perturbation theoretic method which generates the same diagrams as our $1/N$ expansion in the $N\rightarrow\infty$ limit. The nature of spin fluctuations is thus diffusive on a scale larger than the mean-free path and this leads to a $T^{3/2}$-like contribution to the electronic specific heat.[18]

(ii) We consider now the limit when $\Delta \gg q\kappa_F$ and $\omega \ll \Delta$ to study the effect of disorder on the spin-wave instability[19] in the system. The frequency-dependent part of $I^{\uparrow\downarrow}(\mathbf{q},\omega)$ given in Eq. (19) is

$$I^{\uparrow\downarrow}(\mathbf{q},\omega)\,|_{\mathrm{iii}}=\frac{i\omega}{2\pi}\frac{\kappa_F}{i\pi\Delta} \tag{24}$$

which is quite independent of disorder strength. Together with the static part, $I^{\uparrow\downarrow}(\mathbf{q},0)$ we obtain

$$I^{\uparrow\downarrow}(\mathbf{q},\omega)=\frac{\kappa_F}{2\pi^2}\left[1+\frac{\omega}{\Delta}-\frac{\eta q^2}{12\kappa_F^2}\right], \tag{25}$$

where $\eta\equiv4\kappa_F^2l^2/(1+4\kappa_F^2l^2)$ contains the only effect of disorder (up to the $N\rightarrow\infty$ level). In the pure limit ($\kappa_Fl \gg 1$) the factor η approaches 1. The condition for spin-wave instability is obtained from Eq. (12)

$$1-UI^{\uparrow\downarrow}(\mathbf{q},\omega)=0 . \tag{26}$$

Using $\kappa_F/2\pi^2=1/U$ from Eq. (8), we obtain

$$\omega = \eta \Delta \frac{q^2}{12\kappa_F^2} \tag{27}$$

as the equation for the spin-wave mode. The stiffness constant of the spin-wave mode is thus proportional to the factor η.

This discussion generalizes Fermi-liquid theory for a dirty system to a ferromagnetic phase. The expression found here for the stiffness constant is valid in so far as the spin-correlation length, ξ_σ is smaller than the localization length. This is the case in the weakly localized regime. However, as the localization length grows shorter, magnetic fluctuations will be enhanced. This effect is discussed in Sec. VII.

V. DENSITY CORRELATION FUNCTION

The response of an electronic system to an external scalar potential $\phi^{\rm ext}(\mathbf{r},t)$ is contained in the retarded density correlation function:

$$\Pi_{\alpha\beta}^{\sigma\sigma'}(\mathbf{r},t;\mathbf{r}',t') = -i\Theta(t-t')\langle \Psi N \mid \times [n_{\alpha\sigma}(\mathbf{r},t), n_{\beta\sigma'}(\mathbf{r}',t')] \mid \Psi_N \rangle \tag{28}$$

via linear response theory:

$$\delta\langle n_{\alpha\sigma}(\mathbf{r},t)\rangle = \int_{-\infty}^{\infty} dt' \int d^3r' \sum_{\beta,\sigma} \Pi_{\alpha\beta}^{\sigma\sigma'}(\mathbf{r},t;\mathbf{r}',t') e\phi^{\rm ext}(\mathbf{r},t') \; . \tag{29}$$

The retarded density correlation function can be obtained from the following time-ordered two-particle Green's function

$$i\langle \Psi_N \mid T\psi_{\alpha\sigma}(\mathbf{r},t)\psi_{\beta\sigma'}(\mathbf{r},t')\psi_{\alpha\sigma}^\dagger(\mathbf{r},t)\psi_{\beta\sigma'}^\dagger(\mathbf{r}',t') \mid \Psi_N \rangle \; . \tag{30}$$

After impurity averaging and Fourier transformation we find that in the $N \to \infty$ limit diagrams which give leading order contribution to the direct term involve effective interactions (which are sums of bubble diagrams) with vertex corrections (Fig. 4). In the spin-diagonal case we get the exchange term as well which involves impurity ladders. From their diagrammatic representation in Fig. 4 we obtain the following expressions for the effective interaction, $U_{\rm eff}^{\sigma\sigma'}(\mathbf{q},\omega)$ between diffusing fermions and the vertex correction $\Gamma^{\sigma\sigma}(\mathbf{q},\omega,\omega_1)$

$$U_{\rm eff}^{\sigma,\sigma}(\mathbf{q},\omega) = -\frac{U^2 I^{-\sigma,-\sigma}(\mathbf{q},\omega)}{1-U^2 I^{-\sigma,-\sigma}(\mathbf{q},\omega) I^{\sigma,\sigma}(\mathbf{q},\omega)} \; , \tag{31}$$

$$U_{\rm eff}^{\sigma,-\sigma}(\mathbf{q},\omega) = \frac{U}{1-U^2 I^{-\sigma,-\sigma}(\mathbf{q},\omega) I^{\sigma,\sigma}(\mathbf{q},\omega)} \; , \tag{32}$$

$$I^{\sigma,\sigma}(\mathbf{q},\omega,\omega_1) = \frac{1}{1-\gamma J^{\sigma,\sigma}(\mathbf{q},\omega;\omega_1)} \; , \tag{33}$$

where $I^{\sigma,\sigma}(\mathbf{q},\omega)$ is given by an expression similar to Eq. (13) and $J^{\sigma,\sigma}(\mathbf{q},\omega)$ is given in Eq. (14). We obtain:

$$\Pi^\sigma(\mathbf{q},\omega) = \sum_{\beta,\sigma'} \Pi_{\alpha\beta}^{\sigma\sigma'}(\mathbf{q},\omega)$$

$$= \frac{-U^2 I^{-\sigma,-\sigma}(\mathbf{q},\omega)[I^{\sigma,\sigma}(\mathbf{q},\omega)]^2}{1-U^2 I^{-\sigma,\sigma}(\mathbf{q},\omega) I^{\sigma,\sigma}(\mathbf{q},\omega)} - I^{\sigma,\sigma}(\mathbf{q},\omega) + \frac{U I^{\sigma,\sigma}(\mathbf{q},\omega) I^{-\sigma,-\sigma}(\mathbf{q},\omega)}{1-U^2 I^{-\sigma,-\sigma}(\mathbf{q},\omega) I^{\sigma,\sigma}(\mathbf{q},\omega)} \tag{34}$$

$$= -\frac{I^{\sigma,\sigma}(\mathbf{q},\omega)[1-U I^{-\sigma,-\sigma}(\mathbf{q},\omega)]}{1-U^2 I^{-\sigma,-\sigma}(\mathbf{q},\omega) I^{\sigma,\sigma}(\mathbf{q},\omega)} \; . \tag{35}$$

In the paramagnetic phase $I^{\sigma,\sigma}(\mathbf{q},\omega)$ is spin independent and so we obtain from Eq. (35)

$$\Pi(\mathbf{q},\omega) = -\left[\frac{I(\mathbf{q},\omega)}{1+UI(\mathbf{q},\omega)} \right] . \tag{36}$$

In the limit of long wavelength $(1/q \gg l)$ and small frequency $(\omega\tau \ll 1)$, $I(\mathbf{q},\omega)$ has the diffusive form:

$$I(\mathbf{q},\omega) = N(0) \left[\frac{Dq^2}{Dq^2 - i\omega} \right] , \tag{37}$$

with $D = l^2/3\tau$. Substituting Eq. (37) in Eq. (36) we obtain the density correlation function in the paramagnetic phase:

$$\Pi(\mathbf{q},\omega) = \frac{N(0)}{1+UN(0)} \left[\frac{D_\rho q^2}{D_\rho q^2 - i\omega} \right] , \tag{38}$$

where $D_\rho = [1+UN(0)]D$ is the renormalized diffusion constant for charge. From the definition of density correlation function we know that the rate of change of fermion density with change in chemical potential (a static, uniform electric field) is given by:

$$\frac{\delta N}{\delta \mu} = -\Pi(\omega=0, \mathbf{q}\to 0) = \frac{N(0)}{1+UN(0)} \; . \tag{39}$$

If we identify $1+UN(0)$ as a Fermi-liquid constant via $1+UN(0) = 1+F^s$ where F^s is the symmetric Landau parameter, then the effect of interaction can be absorbed in

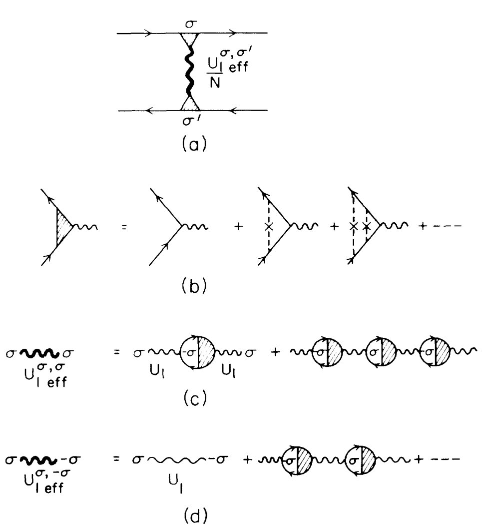

FIG. 4. (a) The density correlation function in the $N \to \infty$ limit involves the effective (U_1) interaction and vertex correction (b); the effective dynamic interactions (c) and (d).

this definition and we can write the density correlation function in the familiar Fermi-liquid form:

$$\Pi(\mathbf{q},\omega)=-\frac{N(0)}{1+F^s}\left[\frac{D_\rho q^2}{D_\rho q^2-i\omega}\right]. \tag{40}$$

In the ferromagnetic phase one obtains essentially the same expression for the spin-averaged density correlation function as in Eq. (40) with $N(0)$ replaced by $\kappa_F/2\pi^2$, the spin-averaged density of states (to order $\Delta/2\Omega_F$) at the Fermi-energy in the ferromagnetic phase.

VI. CONDUCTIVITY

The conductivity can be obtained from the current-current correlation function

$$\sigma^\sigma_{\mu\nu}(\omega)=\sum_{\sigma'}\frac{[R^{\sigma\sigma'}_{\mu\nu}(\omega)]^T-[R^{\sigma\sigma'}_{\mu\nu}(0)]^T}{-i\omega}\quad \omega>0\ , \tag{41}$$

where

$$[R^{\sigma\sigma'}_{\mu\nu}(\omega)]^T=\sum_\beta i\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}\int\frac{d\omega_1}{2\pi}\int\frac{d\omega_2}{2\pi}k_1^\mu k_2^\nu\langle G^{(2)}(\mathbf{k}_1\omega_1\sigma\alpha,\mathbf{k}_2\omega_2\sigma'\beta;\mathbf{k}_1\omega_1-\omega\sigma\alpha,\mathbf{k}_2\omega_2+\omega\sigma'\beta)\rangle\ . \tag{42}$$

Because of the integration over the two momentum vertices k_1^μ and k_2^ν, the only nonvanishing contribution to $[R^{\sigma\sigma'}_{\mu\nu}(\omega)]^T$ (in the $N\rightarrow\infty$ limit) comes from the disconnected piece in the exchange series (with $\sigma'=\sigma$). Hence we obtain essentially the same result as for the noninteracting case:

$$\begin{aligned}\sigma^\sigma_{\mu\nu}(\omega)&=\frac{1}{\omega}\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d\omega_1}{2\pi}k_1^\mu k_1^\nu G^\sigma(\mathbf{k}_1,\omega_1)G^\sigma(\mathbf{k}_1,\omega_1-\omega)\,\Big|_{\omega\text{-dependent part}}\\&=\delta_{\mu\nu}\frac{N^\sigma(0)D}{1-i\omega\tau}\ ,\end{aligned} \tag{43}$$

where $N^\sigma(0)$ is the density of states of the spin σ band at the Fermi-energy [Eq. (5)]. The spin-averaged conductivity (to order $\Delta/2\Omega_F$) is thus

$$\bar{\sigma}_{\mu\nu}(\omega)=\delta_{\mu\nu}\bar{N}(0)\frac{D}{1-i\omega\tau} \tag{44}$$

which can be written in the Fermi-liquid form as:

$$\begin{aligned}\bar{\sigma}_{\mu\nu}(\omega)&=\frac{\bar{N}(0)}{1+U\bar{N}(0)}\frac{D_\rho}{1-i\omega\tau}\\&=\frac{\delta\bar{N}}{\delta\mu}\frac{D_\rho}{1-i\omega\tau}\ .\end{aligned} \tag{45}$$

VII. 1/N CORRECTIONS: LOCALIZATION EFFECTS ON FERROMAGNETIC FLUCTUATIONS

In order to see the effect of localization on the spin-diffusion constant we need to include the maximally crossed diagrams of order $1/N$ (Fig. 5) when evaluating $I^{\uparrow\downarrow}(\mathbf{q},\omega)$ [see Eq. (12) and Fig. 3]. If we perform a resummation in which maximally crossed diagrams separated by impurity ladders are summed up, we get

$$\begin{aligned}I^{\uparrow\downarrow}(\mathbf{q},\omega)=i\int_{-\infty}^{\infty}\frac{d\omega_1}{2\pi}\\ \times\Bigg[&\frac{J^{\uparrow\downarrow}}{1-\gamma J^{\uparrow\downarrow}}+\frac{1}{1-\gamma J^{\uparrow\downarrow}}L^{\uparrow\downarrow}\frac{1}{1-\gamma J^{\uparrow\downarrow}}\\&+\frac{1}{1-\gamma J^{\uparrow\downarrow}}L^{\uparrow\downarrow}\frac{\gamma}{1-\gamma J^{\uparrow\downarrow}}L^{\uparrow\downarrow}\frac{1}{1-\gamma J^{\uparrow\downarrow}}\\&+\cdots\Bigg]\ .\end{aligned} \tag{46}$$

A. Paramagnetic phase

Since we are concerned here with the correction to the diffusion constant, we consider only the frequency-dependent part of $I^{\uparrow\downarrow}(\mathbf{q},\omega)$ which comes from the $0<\omega_1<\omega$ region of the ω_1 integral. In this interval

$$1-\gamma J^{\uparrow\downarrow}(\mathbf{q},\omega;\omega_1)\approx\tau(Dq^2-i\omega)\ll 1$$

so that $J^{\uparrow\downarrow}\approx 1/\gamma$. If we replace $J^{\uparrow\downarrow}$ by $1/\gamma$ in the first term in Eq. (46), the series can easily be summed to yield: $1/\gamma(1-\gamma J^{\uparrow\downarrow}-\gamma L^{\uparrow\downarrow})$. Therefore

$$I^{\uparrow\downarrow}(\mathbf{q},\omega)-I^{\uparrow\downarrow}(\mathbf{q},0)=i\frac{\omega}{2\pi}\frac{1}{\gamma}\left[\frac{1}{\tau(Dq^2-i\omega)-\gamma L^{\uparrow\downarrow}}\right]. \tag{47}$$

The static part $L^{\uparrow\downarrow}(0,0)$ vanishes due to an exact cancella-

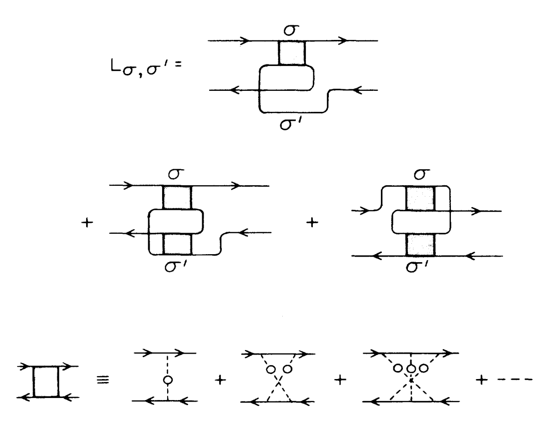

FIG. 5. Maximally crossed diagrams of order $1/N$.

tion which is a consequence of particle number conservation.[11] So does the term linear in ω and one is left with:

$$L^{\uparrow\downarrow}(\mathbf{q},\omega)=4\pi N(0)Dq^2(\gamma\tau^3)\int\frac{d^dQ}{(2\pi)^d}\left[\frac{1}{Dq^2-i\omega}\right]. \tag{48}$$

Substituting this in Eq. (47) we see that the diffusion constant acquires a correction due to localization effect.

$$I^{\uparrow\downarrow}(\mathbf{q},\omega)-I^{\uparrow\downarrow}(\mathbf{q},0)=N(0)\left[\frac{i\omega}{D^l(\omega)q^2-i\omega}\right], \tag{49}$$

where $D^l(\omega)$ is given by

$$D^l(\omega)=D\left[1+\frac{1}{4\pi^2N(0)DL_z}\ln\left[\frac{\omega}{D\Lambda^2}\right]\right] \quad \text{quasi-2D}$$
$$=D\left[1+\frac{1}{2\pi^3N(0)D}\left[\frac{\omega}{D}\right]^{1/2}-\text{const}\right] \quad 3\text{D}\,. \tag{50}$$

Here it is understood that ω is to be replaced by the infrared cutoff, Ω_c as $\omega\to 0$. L_z is the size of the quasi-2D system in the z direction. If we express $I^{\uparrow\downarrow}(\mathbf{q},0)$ as $I^{\uparrow\downarrow}(\mathbf{q},0)=N(0)[1-O(q^2)]$, then from Eq. (49) it follows that (for $\omega<\Omega_c$)

$$I^{\uparrow\downarrow}(\mathbf{q},\omega)=N(0)\left[\frac{D^lq^2}{D^lq^2-i\omega}\right]+O(q^2)\,. \tag{51}$$

Substituting this in Eq. (12) we obtain the transverse magnetic susceptibility:

$$\chi^{-+}(\mathbf{q},\omega)=\frac{N(0)}{1-UN(0)}\left[\frac{D^l_\sigma q^2}{D^l_\sigma q^2-i\omega}\right], \tag{52}$$

where $D^l_\sigma=[1-UN(0)]D^l$. In the small-frequency ($\omega\tau\ll 1$, $\omega<\Omega_c$) and long-wavelength ($1/q\gg l$) limit the spin fluctuations are thus diffusive with a spin-diffusion constant which involves (up to this level) the Fermi-liquid renormalization factor, $1-UN(0)$ and the diffusion constant, D^l for the noninteracting problem.

B. Ferromagnetic phase

We have also studied the effect of magnetization on the Cooper propagator by evaluating the contribution from maximally crossed diagrams in parallel and antiparallel spin channels using Green's functions in the ferromagnetic phase. The relevant integral is

$$C^d_{\sigma\sigma'}(\omega,\Delta)=\int\frac{d^dQ}{(2\pi)^d}\,\frac{\gamma}{1-\gamma J^{\sigma\sigma'}(\mathbf{Q},\omega,\Delta;\omega')}\,;$$
$$\omega'>0,\omega'-\omega<0\,. \tag{53}$$

It is of interest to evaluate this integral because the localization correction to conductivity and transverse susceptibility would involve $C^d_{\sigma\sigma}(\omega)$ and $C^d_{\uparrow\downarrow}(\omega)$, respectively. From Eqs. (15) and (4) we obtain, in the limit of small $\omega/2\Omega_F$ and $\Delta/2\Omega_F$:

$$J^{\sigma\sigma'}(\mathbf{Q},\omega,\Delta;\omega')=\frac{i}{2\pi}\,\frac{1}{Q}\ln\left[\frac{\dfrac{i}{l}+\dfrac{\omega-(\sigma-\sigma')\Delta/2}{\kappa_F}+Q}{\dfrac{i}{l}+\dfrac{\omega-(\sigma-\sigma')\Delta/2}{\kappa_F}-Q}\right] \tag{54}$$

As is obvious, in the parallel spin channel ($\sigma=\sigma'$), the Cooper propagator is going to be independent of (small) magnetization $m(=\Delta/U)$. Therefore, the localization correction to conductivity[20] remains essentially unchanged. Up to this level of approximation it then appears that the ferromagnetic system can undergo a metal-insulator transition and the system can exist in a *ferromagnetic Anderson insulator* state. The Cooper propagator in the antiparallel spin channel is, however, not independent of magnetization and we obtain

$$C^d_{\uparrow\downarrow}(\omega,\Delta)=\frac{\gamma}{\tau}\int\frac{d^dQ}{(2\pi)^d}\left[\frac{1}{DQ^2-i(\omega-\Delta)}\right]. \tag{55}$$

We see that as $\omega\to 0$, the band shift Δ acts as the infrared cutoff. The scale dependence which comes in through the lower limit of Q integration is, therefore, removed provided $\Delta>\Omega_c$.

The maximally crossed diagrams (Fig. 5) give only a contribution of order q^2 and so in view of Eq. (25) we expect a correction to the spin-wave stiffness constant. However, because of the presence of Δ in Eq. (55) this correction is not singular and scale dependent unlike the correction to conductivity discussed earlier. This indicates that while the conductivity vanishes at the mobility edge due to localization, the stiffness constant does not. The system can thus exhibit long-range magnetic order even in the insulating state.

We now discuss the Stoner excitations in the system which involve spin-flip transitions between the two spin bands. The excitation spectrum for Stoner excitations has a gap equal to Δ and therefore we are interested in frequencies close to Δ. If $\omega\approx\Delta$ then in Eq. (55) we see that $C^d_{\uparrow\downarrow}(\omega,\Delta)$ is going to become singular. In fact a straight forward extension to the ferromagnetic phase of the discussion in Sec. VII A leading to Eq. (51) results in

$$I^{\uparrow\downarrow}(\mathbf{q},\omega,\Delta)=\bar{N}(0)\left[\frac{D^l(\omega-\Delta)q^2}{D^l(\omega-\Delta)q^2-i(\omega-\Delta)}\right]. \tag{56}$$

From Eq. (50) we see that for $\omega\approx\Delta$, the diffusion constant associated with Stoner excitations has exactly the same localization correction as does the *dc* conductivity. In view of their lifetime $(=1/D^lq^2)$ we then conclude that the Stoner excitations in the system become localized as the mobility edge is crossed.

VIII. CONCLUSION

The main purpose of this paper is to study the effects of disorder, particularly of localization, on the properties of a system of fermions with spin-dependent, short-ranged interactions in the vicinity of the ferromagnetic transition. In the $N\to\infty$ limit we find typical Fermi-liquid-like behavior (generalized to the ferromagnetic phase) with two diffusion constants. Fluctuations in spin and charge densities diffuse with their respective diffusion constants. The stiffness constant of the spin-wave mode is found to acquire a correction due to disorder. The critical interaction strength for the ferromagnetic transition is independent of disorder in this limit. Enhancement of interaction effects due to disorder are believed[5] to cause a lowering of the critical interaction strength. When localization physics is included we find that there is a localization correction to the spin-diffusion constant associated with spin fluctuations and Stoner excitations. As the mobility edge is approached spin fluctuations and Stoner excitations become more and more localized. This indicates the development of local moments in the system. While the dc conductivity in the ferromagnetic phase also acquires the same localization correction, the spin-wave stiffness constant is found not to. Therefore, it appears that the system can exist in an insulating state and exhibit long-range magnetic order—a ferromagnetic Anderson insulating state.

An analogous situation is found in a dirty superconductor.[21] Superconductivity can persist even in the Anderson localized regime and the ground state can have phase coherence even though the single-particle excitations are localized. In fact, quite generally, thermodynamic properties of a system (e.g., magnetism, superconductivity, superfluidity) are decoupled to some extent from the dynamical response properties (e.g., conductivity).

In a recent paper Ma, Halperin, and Lee[22] have studied the properties of a localized superconductor and showed that they can be understood in terms of a simpler model: a random anisotropic spin one-half Heisenberg model with a random field. For our problem a similar argument can be devised. However, since the *interactions are repulsive* and the *impurity scattering is spin independent* the effective Heisenberg model does not couple to a random field. Thus all one is left with is a quantum spin one-half model with (time independent) random exchange constants. Thus, at finite temperature, one expects itinerant ferromagnets with nonmagnetic impurities to behave like random ferromagnets. Since the specific-heat exponent, α is negative the Harris criterion implies that the critical properties of the finite temperature transition will not be affected by this normal disorder even in the limit when all states are localized. Fluctuations, however, are enhanced resulting in a lowering of the critical temperature[23] and an enhancement of the critical region.

In this paper we have studied exactly the $N \rightarrow \infty$ limit and localization corrections of order $1/N$ to the response properties of the system have been included. There are other important corrections to appear already in order $1/N$ to the spin diffusion constant and the critical interaction strength U_c for example. We expect that U_c will become smaller due to disorder induced enhancement of interaction. These effects are currently being studied.

ACKNOWLEDGMENTS

This work has been supported in part by the National Science Foundation–Materials Research Laboratory (NSF-MRL) program, Grant No. NSF-DMR-83-16981/24 at the MRL and by NSF Grant No. NSF-DMR-84-15063 at the Physics Department of the University of Illinois at Urbana-Champaign.

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[23]Note that Fukuyama (Ref. 5) finds a rise of T_c in the weak localization regime.