Dynamics of droplet fluctuations in pure and random Ising systems

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Long-lived droplet fluctuations can dominate the long-time equilibrium dynamics of long-range-ordered Ising systems, yielding nonexponential decay of temporal spin autocorrelations. For the two-dimensional pure Ising model the long-time decay is a stretched exponential, $\exp(-\sqrt{t/\tau})$, where t is time and τ a correlation time. For systems with quenched random-exchange disorder the spatially averaged correlation decays as a power of time, t^{-x}, with the exponent x in general being nonuniversal. For systems with quenched random-field disorder the decay is slower still, as $\exp[-k(\ln t)^{(d-2)/(d-1)}]$, where k is a nonuniversal number and d is the dimensionality of the system. The low-frequency noise from this slow dynamics may be experimentally detectable, as is the analogous noise in spin-glass ordered phases.

I. INTRODUCTION AND SUMMARY

The symmetry that is spontaneously broken in the low-temperature ordered phase of an Ising system is a discrete symmetry. The low-frequency, long-length-scale fluctuations in the ordered phase arising from this broken symmetry are droplets of reversed spins surrounded by domain walls. Like the analogous Goldstone modes of a system with a spontaneously broken *continuous* symmetry, these droplet fluctuations can dominate long-distance and long-time correlation functions of ordered phases with discrete symmetries. Here, for simplicity, we discuss Ising systems, but the ideas we present apply quite generally to any system with a spontaneously broken discrete symmetry.

Droplet fluctuations have not received much attention in the literature, perhaps because their effect on thermodynamic observables in Ising systems without quenched disorder is only to produce some essential singularities at points of multiphase coexistence.[1] Recently, Abraham[2] showed that the anomalous long-distance decay of spatial spin-spin correlations in the ordered phase of the exactly solvable two-dimensional Ising model[3] is due to droplet fluctuations. Here we extend this work to discuss both spatial and, especially, temporal correlations in equilibrium of d-dimensional ferromagnetic Ising systems without disorder (Sec. II), with quenched random exchange disorder (Sec. III), and with random fields (Sec. IV). The Ising spin glass has already been considered in a separate publication,[4] and here we contrast the results with the ferromagnetic systems.

Droplet fluctuations occur only for temperatures, T, less than the ordering temperature, T_c, where multiphase coexistence is possible. Figure 1 depicts a droplet fluctuation D. The droplet is a "down" domain embedded in the "up"-magnetized phase. The domain is surrounded by a domain wall and contains sites i and j. The free energy of this droplet fluctuation, F_D, consists of the free energy of the domain wall and, for systems with random or uniform fields, the free-energy cost of flipping the interior of the domain. In the cases we will be considering the probability is very small that there are other nearby droplets present with which this one would interact. Therefore a droplet may be treated as a simple two-state system at temperature T, either present, with probability

$$p_D \approx \frac{e^{-F_D/T}}{1+e^{-F_D/T}} , \tag{1.1}$$

or absent, with probability $1-p_D$.

Because it flips the two spins simultaneously, a droplet fluctuation contributes an amount proportional to $p_D(1-p_D)$ to the equal-time correlation between spins at site i and site j *both* inside the droplet:

$$G_{ij} \equiv \langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle . \tag{1.2}$$

[Here and throughout this paper the angular brackets denote an average within the ordered phase being considered (e.g., the "up" phase in Fig. 1). This average may be implemented in the thermodynamic limit either by boundary conditions that select the phase or by an

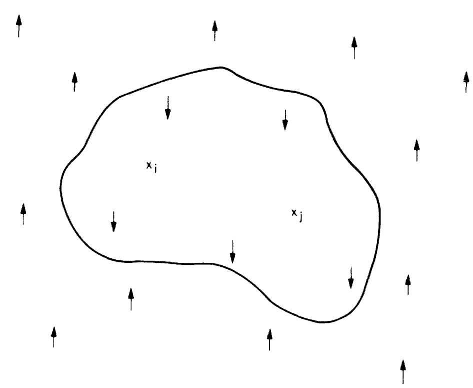

FIG. 1. A droplet fluctuation of a "down" domain within "up" phase. The droplet is surrounded by a domain wall (solid line) and contains sites i and j.

infinite-time average with an initial condition in that phase.] One naively expects $G_{ij} \approx \exp(-r_{ij}/\xi)$, where r_{ij} is the distance between sites i and j and ξ is a correlation length. Droplet fluctuations do not appear to alter this, except for introducing a power-law prefactor (see Sec. II) in any physical dimension for ferromagnetically ordered Ising systems, with or without disorder. In contrast, for the Ising spin glass in which droplet excitations are much more common, we obtained[4]

$$\overline{|G_{ij}|} \sim r_{ij}^{-\theta} , \tag{1.3}$$

where $\theta \leq (d-1)/2$, and it appears that $0<\theta<1$ for $d=3$. The overbar denotes an average over realizations of the disorder.

The effects of droplet fluctuations on *dynamics* are much more dramatic. The droplet fluctuation D in Fig. 1 contributes[5] an amount proportional to $p_D(1-p_D)$ to the temporal autocorrelation function

$$C_i(t) \equiv \langle S_i(0)S_i(t)\rangle - \langle S_i\rangle^2 \tag{1.4}$$

for spins i within D. The time dependence of this contribution depends on the dynamics of the droplet fluctuation. For a pure two-dimensional Ising model with no conservation laws constraining its dynamics, we find (Sec. II below) a "stretched exponential" decay of $C_i(t)$,

$$|\ln C_i(t)| \sim t^{1/2} ,$$

due to the long-lived, large droplet fluctuations. This result is special to spatial dimensionality $d=2$; for $d \geq 3$ the naively expected exponential decay of $C_i(t)$ does not appear to be significantly altered by the droplet fluctuations.

For Ising systems with quenched disorder, the domain wall surrounding the droplet D is pinned to the disorder. The droplet must evolve continuously by motion of the surrounding domain wall. The free energy of the droplet as a function of its volume is shown schematically in Fig. 2. When the droplet is present, with volume V_D, its free energy is F_D. It is then metastable, with an activation barrier of additional free energy B_D that must be crossed in order to remove the droplet. The lifetime of the droplet fluctuation, τ_D, is therefore activated:

$$\tau_D \sim \exp(B_D/T) , \tag{1.5}$$

and the contribution of the droplet D to $C_i(t)$ is proportional to $p_D(1-p_D)\exp(-t/\tau_D)$. For long times t, only the long-lived droplets with $\tau_D \gtrsim t$ contribute significantly to $C_i(t)$ and thus the long-time behavior is dominated by *rare droplets.*

We obtain nontrivial behavior for the average spin-autocorrelation function $\overline{C_i(t)}$ for all the cases we consider: For Ising systems with random dilution or random-exchange disorder (not so disordered that the system becomes a spin glass), we find that for long times

$$\overline{C_i(t)} \sim t^{-x(t)} , \tag{1.6}$$

where the exponent $x(T)$ is generally nonuniversal, depending on the type, strength, and distribution of the disorder as well as the temperature. For many systems $x(T) \to 0$ for $T \to 0$, yielding very slow low-temperature dynamics. If the system has a continuous phase transition at $T=T_c$, we expect $x(T)$ to approach, for $T \to T_c$, a universal value (possibly infinite) determined by the universality class of that phase transition. For random-field Ising systems we find an even slower decay,

$$\overline{C_i(t)} \sim \exp[-k(\ln t)^{(d-2)/(d-1)}] , \tag{1.7}$$

where the number k depends on a variety of properties of the system. In deriving (1.6) and (1.7) we have tried to identify the type of droplet fluctuation which is dominant at long times. If we have failed, the true temporal decay of the correlations must be slower still, with our results serving as lower bounds.

For a spin-glass phase, we have previously found

$$\overline{C_i(t)} \sim (\ln t)^{-\theta/\psi} , \tag{1.8}$$

with $0<\theta/\psi \leq 1$, arising from a consistent picture of the nature of the spin-glass phase and its droplet excitations.[4] This result is consistent with the experimentally observed[5] $1/f$ low-frequency power spectrum for the magnetization noise in certain spin glasses. We hope that similar experimental observations of the equilibrium fluctuations in random-exchange and random-field systems will be consistent with our predictions (1.6) and (1.7).

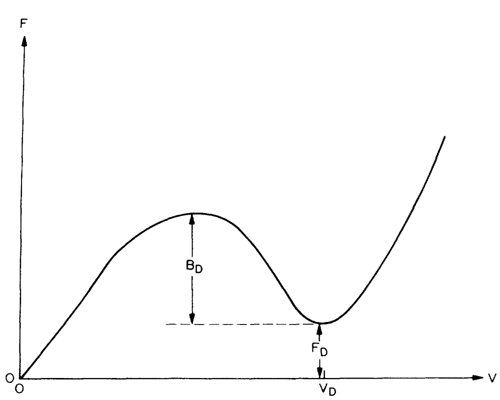

FIG. 2. The free energy, F, of an activated droplet fluctuation as a function of its volume, V. The metastable state with volume V_D and free energy F_D is separated from the ground state of no droplet ($V=0$) by an activation barrier of height B_D.

II. PURE ISING SYSTEMS

Let us first consider droplet fluctuations in Ising systems without any disorder. The free energy, F_D, of a large droplet, D, of one phase in the other is just the surface tension, σ, times the total area of the domain wall surrounding the droplet. Here we are discussing droplets larger than the correlation length, ξ; these have $F_D \gg T$ and therefore small Boltzmann probability. However, these droplets are also large and rather long-lived and thereby dominate long-distance and long-time correlations in sufficiently low dimension d.

Away from the coexistence curve (i.e., for magnetic field $H \neq 0$ or $T > T_c$), the equal-time spin-spin correlation function has the Ornstein-Zernike form at long distances:

$$G_{ij} \approx A e^{-r_{ij}/\xi} / r_{ij}^{(d-1)/2} , \tag{2.1}$$

where the amplitude A and the correlation length ξ are field and temperature dependent. At coexistence, droplet fluctuations alter this form for $d \leq 2$. Only droplets that contain both sites i and j contribute to G_{ij}. For $d \geq 2$ and r_{ij} large, the lowest free energy and thus most probable such droplets are long, thin, and tubelike, in order to minimize the total domain-wall area.[2,6] For $d > 2$, only tubes with small cross sections contribute, even for large r_{ij}. Their total contribution to G_{ij} has precisely the Ornstein-Zernike form (2.1),[6] with T/ξ being the free energy per unit length of the "tube" and the factor of $r_{ij}^{-(d-1)/2}$ arising from the reduction of the transverse fluctuations of the tube due to the restriction that it must run from site i to site j.

For $d = 2$, the elongated droplets that dominate G_{ij} at coexistence consist of two domain walls running roughly parallel, which can now be widely separated at little energy cost and much entropy gain. The breakdown of the Ornstein-Zernike form for $T < T_c$ and $d = 2$ is due to this widening of the droplets.[2] The resulting form is[2,3]

$$G_{ij} \approx A e^{-2r_{ij}\sigma/T} / r_{ij}^2 .$$

Thus $d = 2$ serves as an upper critical dimension where droplet fluctuations begin to alter the long-distance form of equal-time correlation functions. If we allow ourselves to consider $1 < d < 2$, we find that a roughly "spherical" droplet containing both i and j has domain wall area and thus free energy proportional to $\sigma r_{ij}^{(d-1)}$. Such droplets contribute to G_{ij} in proportion to their Boltzmann weight, leading to

$$G_{ij} \sim \exp[-(r_{ij}/\xi)^{(d-1)}] \tag{2.2}$$

for large r_{ij}, where ξ is a correlation length. Thus droplet fluctuations give rise to nonexponential decay of spatial correlations for the unphysical dimensionality range $1 < d < 2$. We will now show that the decay of correlations with *time* becomes nonexponential for $1 < d < 3$.

There are a variety of different types of dynamics that an Ising system can have.[7] If the system has a conservation law, then the local autocorrelation function for the conserved quantity and any other quantity directly coupled to it decays diffusively, as $t^{-d/2}$. For pure Ising systems, droplet fluctuations can play an important role in the low-frequency dynamics only in the absence of an order-parameter conservation law; that is, for model A in the classification of Hohenberg and Halperin.[7] We will henceforth restrict consideration to such systems.

A droplet fluctuation containing site i contributes to the spin-autocorrelation function $C_i(t)$ at time t only if its lifetime is of order t or greater. For long times, t, the dominant droplet fluctuations contributing to $C_i(t)$ for the pure system are nearly spherical[8] (circular for $d = 2$) and of large radius, r. The domain wall surrounding such a large droplet with $r \gg \xi$ moves more or less deterministically in response to its own surface tension, as has been discussed by Lifshitz.[9] This motion of the domain wall is driven by its curvature and leads to a shrinking of the droplet with time according to

$$\frac{dr}{dt} \sim -\frac{1}{r} . \tag{2.3}$$

The resulting lifetime of a droplet initially of radius r is proportional to r^2, and the time typically taken to form such a droplet is of the same order. Thus the minimum size of droplet fluctuations that contribute to $C_i(t)$ is proportional to $\sqrt{t}$. The free energy of a droplet of radius r is proportional to its surface area and thus to $r^{(d-1)}$, so we see that the maximum Boltzmann probability of a droplet fluctuation that contributes significantly to $C_i(t)$ is $\exp[-(t/\tau)^{(d-1)/2}]$, where τ is a correlation time (that diverges as $T \to T_c$). For $1 < d < 3$ these droplet fluctuations therefore dominate $C_i(t)$ at long times, yielding

$$C_i(t) \sim \exp[-(t/\tau)^{(d-1)/2}] . \tag{2.4}$$

From (2.4) and (2.2) we see that dynamic correlation functions are more sensitive to droplet fluctuations than are static correlation functions. The upper critical dimension where droplet fluctuations begin to alter the long-time exponential decay of time correlations is $d = 3$. For the two-dimensional Ising model with nonconserved (model A) dynamics, we therefore predict a "stretched exponential" decay of the spin-autocorrelation function for $T < T_c$:

$$C_i(t) \sim \exp(-\sqrt{t/\tau}), \quad d = 2 . \tag{2.5}$$

This prediction should not be too difficult to confirm with Monte Carlo simulations. It is noteworthy that this stretched exponential or Kohlrausch form for the temporal correlation function appears in this simple system, without glassy behavior, hierarchies, or even activation barriers playing any role.[10]

The above discussion of the shrinking of a droplet only considers the deterministic part of the evolution (2.3). A more careful treatment should include the stochastic part of the evolution. Locally, the domain wall surrounding the droplet has a random normal velocity due to thermal noise, in addition to the deterministic, curvature-driven term in (2.3). The resulting Langevin equation for the evolution of the droplet radius is

$$\frac{dr}{dt} \approx -\frac{\Gamma}{r} + \frac{\eta(t)}{r^{(d-1)/2}} , \tag{2.6}$$

where Γ is a measure of the interface mobility and η is the thermal noise, whose amplitude is independent of r for large r. The reduction of the effect of the noise on r by a factor of $r^{(d-1)/2}$ is due to averaging over the entire surface area of the droplet. The deterministic lifetime of a droplet of radius r is proportional to r^2, as discussed above, but there is always the possibility that the droplet lives much longer than this: The probability of the droplet still being present should decay exponentially as $\exp[t/\tau_s(r)]$ for long times, $t \gg r^2$, where r is the maximum size attained by the droplet over its history. We will call $\tau_s(r)$ the "stochastic lifetime" of the droplet.

In order for a droplet to live much longer than its deterministic lifetime, the noise term in (2.6) must, on average, cancel the deterministic drift term. This requires $\eta \approx \Gamma r^{(d-3)/2}$, which for $d < 3$ is more probable for larger

r. Thus if we consider a droplet whose maximum radius is r_m and which has existed for a very long time $t \gg r_m^2$, its most probable history is one in which over the bulk of its lifetime its radius is near r_m. The probability of this occurring decays with time as $\exp[-t/\tau_s(r_m)]$, where $\tau_s(r_m) \sim r_m^{(3-d)}$. Thus the stochastic lifetime of a droplet, $\tau_s(r)$, is, for $d>1$, much shorter than the deterministic lifetime of the droplet. If we consider the contribution of these extra-long-lived droplets to the autocorrelation function $\overline{C_i(t)}$, the droplets with $r_m \sim \sqrt{t}$ again dominate at long times for $1<d<3$, and their contribution has the same form as found above (2.4). Presumably a careful analysis for $d=3$ would yield interesting, nontrivial corrections to the exponential temporal decay analogous to those found for the spatial correlations[2,3] in $d=2$.

For $d>3$, long-lived small droplets, of radius of the order of the correlation length, are found to dominate $C_i(t)$ at long times, yielding a simple exponential decay. Thus the large-scale droplet fluctuations become too improbable to play a leading role in the low-frequency dynamics for $d>3$.

III. RANDOM-EXCHANGE SYSTEMS

In this section we consider droplet fluctuations in Ising ferromagnets with quenched random-exchange disorder. We assume the random-exchange disorder is not so strong as to destroy the ferromagnetic long-range order, so the average free energy of a roughly circular or spherical[8] droplet of radius r is $A_d \sigma r^{d-1}$, where σ is the average surface tension and A_d is the surface area of a unit circle or sphere in d dimensions. However, because of the quenched fluctuations in the strength of the local ferromagnetic coupling, there are rare places where a droplet of radius r can be formed whose free energy is considerably less than average. This occurs when the domain wall surrounding the droplet passes through regions of particularly weak ferromagnetic coupling and thus reduced local domain-wall free energy. Such a droplet, once formed, is long lived because the domain wall is pinned to the weakly ordered regions and in order to dissolve the droplet the domain wall must move away from these favorable locations and cross a large free-energy barrier.

The droplets that appear to dominate the long-time behavior of the autocorrelation function $\overline{C_i(t)}$ have free energies that are below average by an amount proportional to r^{d-1} and also have activation barriers proportional to r^{d-1}. For each such droplet, let us denote the ratio of its actual free energy to the average free energy of a droplet of the same size and shape as f. In other words, the average surface tension over the entire domain wall surrounding the droplet must be $f\sigma$. This occurs, for a near circular or spherical droplet, with a probability proportional to $\exp[-p(f)r^{d-1}]$, where the function $p(f)=0$ for $f=1$ and increases as f decreases. For systems with a nonzero probability for the local domain-wall free energy to be negative or zero, $p(f)$ goes to a finite, positive value for $f \to 0$. This occurs in systems with either random antiferromagnetic couplings present or diluted systems where isolated, disconnected clusters can occur. If, on the other hand, there is a positive lower bound on the local ferromagnetic coupling, then $p(f)$ diverges for $f \le f_c$, where the local surface tension in the weakest possible coupled regions is $f_c \sigma$.

The droplets that appear to dominate $\overline{C_i(t)}$ at long times are nearly spherical[8] and have $f<1$, i.e., the region around the surface of the droplet has on average a reduced ferromagnetic coupling. Typically, the interior of such a droplet has average coupling, so the activation barrier for removing the droplet will be $b(f)r^{d-1}$, where $b(f)$ is of order σ for $f=0$ and decreases monotonically with f, vanishing for $f \to 1$. Of course, droplets with even larger barriers occur when the entire interior of the droplet has above average coupling, but the probability of this occurring is of order $\exp(-r^d)$ and therefore much smaller than the events we are considering.

The relaxation time for a droplet of radius r and $f<1$ is $\tau_D \sim \exp[b(f)r^{d-1}/T]$. Only those droplets with $\tau_D \gtrsim t$ contribute to $\overline{C_i(t)}$, and of these, those with $\tau_D \simeq t$ are dominant. For a given f, these are the droplets with

$$r^{d-1} \approx \frac{T\ln(t/t_0)}{b(f)} , \tag{3.1}$$

where t_0 is a microscopic time. Now the Boltzmann probability of such a droplet being excited is simply $\exp(-f\sigma A_d r^{d-1}/T)$, so we find that for large times, t, the average autocorrelation function is proportional to a steepest-descents integral:

$$\overline{C_i(t)} \sim \int df\, t^{-[f\sigma A_d + Tp(f)]/b(f)} . \tag{3.2}$$

Thus we find a power-law decay of correlations at long times,

$$\overline{C_i(t)} \sim t^{-x(T)} , \tag{3.3}$$

with the exponent

$$x(T) = \min_f \{[f\sigma A_d + Tp(f)]/b(f)\} \tag{3.4}$$

being the minimum value taken on by the exponent in the integral in (3.2). Since $b(f) \to 0$ for $f \to 1$, the minimum never occurs at $f=1$, it is always exponentially rare droplets with $f<1$ that dominate at long times.

The actual value of the exponent $x(T)$ will depend on the temperature and, in general, on nonuniversal details of the system. However, if there is a critical point, then $x(T)$ should approach, for $T \to T_c$, a value determined by the universality class of the critical point. If, as occurs in two-dimensional Ising systems,[11] the random-exchange disorder is irrelevant at the critical point and the critical point is in the same universality class as the pure system, then $x(T)$ should diverge for $T \to T_c$. On the other hand, for systems in which the random exchange disorder is relevant at criticality, $x(T)$ presumably approaches a finite, universal value for $T \to T_c$. This exponent is a new property of the random-exchange fixed point. It would be interesting to have an estimate of its value for three-dimensional (3D) Ising systems, in which random-exchange disorder is relevant.[12]

At low temperatures $T \ll T_c$, the functions $p(f)$ and $b(f)$ are only weakly temperature dependent. If $p(f=0)$ is finite, then at sufficiently low temperatures the minimum in (3.4) occurs at $f=0$ and we have $x(T)=Tp(f)/b(f)|_{f=0}$. Therefore, such systems, which have regions with antiferromagnetic or zero coupling giving $f=0$, have *very* slow dynamics at low temperatures, with $x(T)\to 0$ for $T\to 0$. When $x(T)<1$, the low-frequency magnetization noise

$$C(\omega)\sim 1/\omega^{1-x(T)} \tag{3.5}$$

is divergent, becoming $1/f$ noise as $x(T)\to 0$. These results should apply to diluted or weakly frustrated Ising ferromagnets or antiferromagnets. In systems with only ferromagnetic couplings and thus a minimum possible value, f_c, of f, the low-temperature decay exponent will be dominated by the extreme case $f=f_c$ and we expect $x(T)\to f_c\sigma A_d/b(f_c)$ for $T\to 0$. In general, $x(T)$ increases with T, but this is not always the case: If, at low temperature for $d=3$, f_c is sufficiently near unity, then $x(T)$ will be so large that it must decrease to attain the finite universal value expected in the limit $T\to T_c$.

The form of the equal-time spatial correlation functions will not be strongly affected by random-exchange disorder. However, the exponential decay length of $\overline{G_{ij}}$ will be affected by the disorder. For $d>2$ an optimization over the free energy for a long tube of cross section of order unity, will yield a decay length for $\overline{G_{ij}}$ which is larger than that for $\overline{\ln G_{ij}}$. In addition, the prefactor will no longer be of the Ornstein-Zernike form. For $d<2$, a result of the form (2.2) will obtain with the decay length given by a surface free-energy minimization as for the droplet dynamics. In $d=2$, as for the pure case,[2] a more delicate balance of terms is needed to establish the detailed form of the decay.

IV. RANDOM-FIELD SYSTEMS

In this section we consider the dynamics of droplet fluctuations in Ising systems in the presence of quenched random-field disorder. We only consider $d>2$ and sufficiently weak random fields, so that the long-range order which occurs in the absence of random fields is not destroyed.[13] For these systems the free energy of a large droplet excitation consists of a sum of a surface term due the domain wall and a bulk term due to the random fields acting over the entire interior of the droplet. The droplets that appear to dominate the long-time behavior of the autocorrelation function $\overline{C_i(t)}$ are the thermally active droplets. Such droplets have typical surface free energies that are almost exactly compensated for by an unusual random-field configuration, so the total excitation free energy of the droplet is of the order of the temperature, T, and the Boltzmann probability of the droplet being excited is of order unity.

The typical surface free energy of a roughly circular or spherical droplet[8] of radius r is again $A_d\sigma r^{d-1}$. In order to compensate for this cost in free energy to create the droplet, the net effect of all the random fields in the interior of the droplet must favor the droplet being present. The typical contribution of these interior random fields to the total droplet free energy is of order $hmr^{d/2}$, where h is the root-mean-square random-field strength and m is the order parameter. Note that in order to have two-phase coexistence the average value of the random field must vanish, and we are only considering systems with short-range correlations between the quenched random fields. For large r, the distribution of the random-field free energy is Gaussian for free energies much smaller than hmr^d, so the probability of the random-field free energy precisely (to within order T) canceling the surface free energy is proportional to

$$p\sim\exp(-c\sigma^2 r^{d-2}/h^2m^2)\ , \tag{4.1}$$

where c is a number that depends on d and the probability distribution of the random fields. This is therefore the density of thermally active droplets of radius r. These droplets, which will be excited with relatively high probability, will dominate the long-time dynamics.

The relaxation time for these thermally active droplets is determined by thermal activation over the free-energy barriers that must be crossed in order to produce or remove them. These barriers are of order σr^{d-1} for droplets of radius r, so the relaxation time of the droplet is $\tau_D\sim\exp(a\sigma r^{d-1}/T)$, where a, like c, depends on the dimensionality and the distribution of the random fields. To contribute significantly to $\overline{C_i(t)}$ a droplet must be of radius

$$r\gtrsim\left[\frac{T\ln t}{a\sigma}\right]^{1/(d-1)} . \tag{4.2}$$

Those droplets satisfying condition (4.2) contribute in proportion to their probability (4.1) of being thermally active. Those with radii approximately giving an *equality* in (4.2) dominate, yielding

$$\overline{C_i(t)}\sim\exp\left[\frac{-c\sigma^2}{h^2m^2}\left[\frac{T\ln t}{a\sigma}\right]^{(d-2)/(d-1)}\right] . \tag{4.3}$$

Note that this temporal decay of the autocorrelations is slower than any power of time and therefore yields $1/f$ noise in the order-parameter fluctuations.

For the experimentally studied random-field system of dilute Ising antiferromagnets in a uniform magnetic field,[14] the thermally active droplets are those that couple strongly to the field due to having much more dilution on one sublattice than the other. They therefore have large magnetic moments. Thus the long-lived droplet fluctuations enter directly into the fluctuations of the total magnetization, even though the magnetization is not the order parameter for these systems. Thus the uniform, as well as the staggered, magnetization should exhibit $1/f$ noise.

The averaged equal-time spatial correlations in random-field systems will be affected by droplet fluctuations, but for $d\geq 3$ only the decay length of $\overline{G_{ij}}$ will be altered. In the unphysical dimensionality range $2<d<3$, however, roughly spherical thermally active droplets like those dominating the dynamics will yield

$$\overline{G_{ij}}\sim\exp[-(r_{ij}/\xi)^{d-2}]\ . \tag{4.4}$$

V. CONCLUSIONS

We have shown that the long-time dynamics in the ordered phase of random systems with a discrete broken symmetry is dominated by droplet excitations. In all the systems considered, the behavior of the autocorrelations is most naturally described as a function of the logarithm of the time. The average over the system (or over disorder) is dominated by *rare regions* with anomalously long relaxation times. In random ferromagnets the dominant regions are exponentially rare in $\ln t$, yielding

$$\overline{C_i(t)} \sim e^{-k(\ln t)^y} , \tag{5.1}$$

with y depending only on the nature of the randomness [$y=1$ for random exchanges, $y=(d-2)/(d-1)$ for random fields] and k on details of the distribution, the temperature, etc. For spin glasses, on the other hand, the dynamics is dominated by regions which are only power-law rare in $\ln t$, yielding the decay as a power of $\ln t$ as in Eq. (1.8).[4] For random-field magnets where $y<1$, we expect $1/f$ noise for all $T<T_c$, as for spin glasses.

Randeria, Sethna, and Palmer[15] have argued that in the paramagnetic phase of spin glasses anomalously strongly coupled regions yield a decay of the form (5.1) with $y=d/(d-1)$ for a range of temperatures *above* the phase transition that they call a "Griffiths phase."[15,16] Extension of their results to other random magnets yields the same form with $y=d/(d-1)$ in any part of the paramagnetic phase where there are Griffiths singularities.[16] Depending on the distribution of the randomness, the Griffiths phase either extends from T_c up to a temperature $T_G>T_c$, or up to infinite temperature. Note, however, that since the decay in the paramagnetic phase is faster than any power of time, this behavior may be difficult to observe.

We comment, lastly, on the possibilities for experimental tests of our results in random ferromagnets. If the randomness is weak, then the asymptotic behavior will only be reached at long times, so it may be advantageous to work with a relatively strongly disordered system. A measure of the strength of the disorder in a diluted ferromagnet is the suppression of T_c below its value in the undiluted system. The disorder is strong for systems whose T_c is suppressed by, say, 40% or more. For a random-field system it is likewise preferable to work with strong random fields. For dilute antiferromagnets,[14] the suppression of T_c by a uniform field is a good measure of the strength of the random field generated. Thus we would like a system whose zero-field T_c is considerably suppressed by dilution and then use a field that suppresses T_c well (say, 30% or more) below its zero-field value. Recall that for these systems the magnetization and staggered magnetization noise should exhibit similar behavior.

The prediction in Sec. III, that a universal power law of time should obtain at long times near T_c for 3D diluted Ising magnets, may be rather difficult to verify since there will be a crossover from a critical power law to the noncritical but universal power law at a time $\tau \sim (T_c-T)^{-z\nu}$ with z and ν the critical exponents of the diluted magnet.

In all of these systems, there are potential problems associated with nonequilibrium effects. However, one generally expects that, provided $\omega t_w \gg 1$, the behavior at frequency ω will not depend much on the waiting time, t_w, for which the system has been equilibrated. This behavior was found in spin glasses in Ref. 5.

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