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Theory of the Kapitza resistance

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Calculations are presented for the thermal-boundary resistance in an irregular interface between solid and liquid helium. The mechanism for energy transmission is due to the strain dynamics of surface defects which coherently excite in the adsorbed solid He layer phonons that, in turn, decay into the helium bath via diffusive processes. The topography of irregular surfaces is modeled by random distributions of islandlike defects on a flat substrate, and the energy transmission coefficient is obtained in general as a configurational average. The transmission coefficient is calculated for a blackbody source of thermal phonons and for defects of atomic dimensions. The Kapitza resistance is calculated in general for atomically irregular surfaces as a function of the source temperature, the surface density of defects, a statistical average of the dimensions of surface defects, and the Debye temperature of the solid He layer. The magnitude and temperature dependence of the calculated Kapitza resistance compare favorably with measurements in the range ($\sim 10^{-2}$–~ 2) K. The theory also accounts for other features of the anomaly such as the pressure dependence, the random yet bounded magnitude of the measurements, and the independence of these measurements from the superfluidity of liquid helium.

I. INTRODUCTION

Thermal-boundary resistance was first observed by Kapitza[1] in 1941. The corresponding heat transmission across the solid-liquid helium interface was subsequently attributed by Khalatnikov[2] to phonon scattering in a flat boundary between the two media. Extensive experimental and theoretical studies of the effect and of phonon transmission, reflection, and scattering processes at surfaces and interfaces have been made since. For recent reviews the reader is referred to Refs. 3–5.

Under special conditions of UHV or laser annealing, nearly ideal surfaces can be obtained[6–8] and for these surfaces the acoustic mismatch model[2] seems adequate for known ranges of frequency. In contrast many prepared surfaces do not behave in this ideal manner, and solid-liquid or solid-solid interfaces deviate significantly from the acoustic model, and give rise to the well-known Kapitza anomaly[3,4] across the solid-liquid helium interface and to phonon losses[9] and anomalous high-phonon backscattering at solid-solid interfaces.[10]

Furthermore, experiments where phonon transmission and phonon reflection are specifically investigated[3,5,11,12] show that there exist for nonideal surfaces two channels for phonon reflection and transmission, one which is specular in character that conserves the wave vector parallel to the interface, and another diffusive that accounts for most of the heat energy transmitted from the solid into the liquid-helium bath.

The effect of surface roughness on acoustic wave transmission between a liquid and an isotropic solid was considered by Lapin[13] and by Adamenko and Fuks.[14] More recently, Shiren[15] extended these calculations; this analysis opens up the critical cone and increases power transmission from one medium to the other, although it constrains contributions to $q\zeta < 1$ phonons in the perturbation calculation. q is the phonon wave vector and ζ depicts the dimensions of the surface irregularities. The model, furthermore, does not account for scattering measurements that show the thermal-boundary conductance to be dependent on the coupling of localized surface defects to the strain field of incident phonons.[16] The possibility of increased heat transmission due to localized strain dynamics of surface defects was pointed out some time ago.[17] To account for this observation, Kinder[18] considers surface defects as "glassy" centers that form two-level systems which incident phonons can excite. The absorption cross section is obtained in first-order perturbation, and within the model assumptions it is a direct measure of the energy transmission coefficient. This model is strictly valid, provided the strain dynamics of the defects can be neglected, and it is not clear if this can be justified in general for arbitrary frequencies and for high spatial densities of incident phonons. Its results compare favorably with measurements[19] in a band of low frequencies (~ 30 to ~ 60 GHz), but for larger frequencies, the absorption cross section is too large for the model approximations. In contrast, some measurements[5,20,21] show that the enhanced transmission in the Kapitza anomaly occurs via processes which involve these larger frequencies.

In a recent work by Khater[22] a theory for phonon scattering from surface defects in thermodynamically stable irregular surfaces is presented by analyzing the effect of phonon strain fields on the strain dynamics of defects, for all frequencies and arbitrary spatial densities of incident phonons. The topography of the irregular solid surface is modeled by a distribution of defects in the form of islands of random dimensions and separations on a flat substrate. The scattering matrix elements that mix the scalar amplitudes of the longitudinal L and transverse T phonon polarizations are calculated. The strain field on the islands is also obtained in exact analytic form, for all phonon frequencies.

The purpose of this paper is to present a model for the Kapitza resistance, which is extensively analyzed, with the aim of accounting for the measured order of magnitude of R_K and its temperature dependence, as well as other features of the anomaly. The proposed mechanism for energy transfer is assumed to be due to flexure[22] of the defects, that coherently generate two-dimensional phonons in the adsorbed solid helium layer at the solid-solid He layer interface, and the incoherent decay of these phonons into the liquid helium bath at the solid He layer-liquid He interface.

In Sec. II the strain energy of the harmonic flexure of an island is calculated using elasticity theory.[23] In Sec. III the elements of the proposed energy transfer mechanism are presented along with the basic assumptions made in the model. The energy transmission rate for the irregular interface involves configurational averages over random distributions of islands. This is given in general and calculated for atomically irregular solid surfaces. In Sec. IV the heat flux owing to a blackbody source of bulk phonons is calculated, and a formula for $R_K T^3$ is derived. It is a function of the absolute temperature T of the source, the Debye temperature of the solid He layer, the density of islands, and a statistical parameter which characterizes the atomically irregular solid surface. Numerical applications are presented in Sec. V, and the theory is compared to measurements.

II. SURFACE FLEXURE MODES

The topography of the irregular solid surface is modeled by a distribution of islands on a flat substrate. The islands are taken to be cylindrical, of random heights ζ, and random radii ξ. The linear separation between the islands is also a random quantity. In the laboratory frame the Cartesian x and z axes are oriented, respectively, parallel and perpendicular to the surface. The substrate and the vacuum occupy, respectively, the half-spaces $z<0$ and $z>0$. The islands are of the same material as the substrate, and are considered to be fixed at their circular base $z=0$ by bulk force constants.

Let incident and scattered L,T phonons be characterized by the source frequency ω and, respectively, by the wave vectors $\mathbf{q}_{0L}=(Q_{0L}^x,0,q_{0L}^z)$, $\mathbf{q}_{0T}=(Q_{0T}^x,0,q_{0T}^z)$ and $\mathbf{q}_L=(Q_L^x,0,q_L^z)$, $\mathbf{q}_T=(Q_T^x,0,q_T^z)$. The directions of propagation of the incident L,T phonons are arbitrary, and their polarization unit vectors are in the plane of scattering. Furthermore, let u_{0L},u_{0T} denote the scalar displacement amplitudes of the incident L,T phonons, respectively. The islands have an additional degree of freedom of flexure about the z axis, absent in an ideally flat surface. If u_F denotes its corresponding scalar displacement amplitude, then it can be shown[22] that this satisfies the condition

$$-i\frac{\partial u_F}{\partial z}\bigg|_{z=\zeta}\equiv[-A(q_{0L}^z)^2-B(Q_{0L}^x)^2-2q_{0L}^zQ_{0L}^x]\frac{v_L}{\omega}u_{0L}e^{-i\omega t}e^{iq_{0L}^z\zeta}$$
$$+\{Aq_{0T}^zQ_{0T}^x-Bq_{0T}^zQ_{0T}^x+[(Q_{0T}^x)^2-(q_{0T}^z)^2]\}\frac{v_T}{\omega}u_{0T}e^{-i\omega t}e^{iq_{0T}^z\zeta}\ , \tag{1}$$

where v_L and v_T are the longitudinal and transverse speeds of sound in the material, and the factors A and B are given by

$$A=[(Q_L^x)^2(Q_T^x)^2-(Q_L^x)^2(q_T^z)^2+2Q_L^xQ_T^xq_L^zq_T^z]/Q_T^xq_T^z\ , \tag{2}$$

$$B=[(-q_L^z)^2(Q_T^x)^2+(q_L^z)^2(q_T^z)^2+2Q_L^xQ_T^xq_L^zq_T^z]/Q_T^xq_T^z\ . \tag{3}$$

The Kapitza and the phonon reflection and transmission experiments are characterized by L and T phonons traveling parallel to each other from the source to the solid-liquid helium interface, and scattering predominantly in a specular manner,[3,11,12] since a well-prepared surface has a small density of islands. The boundary condition $u_F'\,|_{z=\zeta}$ simplifies hence from Eq. (1) to the form

$$u_F'\,|_{z=\zeta}=-i4q_{0L}^zQ_{0L}^x\frac{v_L}{\omega}u_{0L}e^{-i\omega t}e^{iq_{0L}^z\zeta}$$
$$-i2\left[\frac{\omega^2}{v_T^2}-2(Q_{0T}^x)^2\right]\frac{v_T}{\omega}u_{0T}e^{-i\omega t}e^{iq_{0T}^z\zeta}\ . \tag{4}$$

This result indicates that the L and T phonons contribute with different strengths to flexure. In particular, for phonons traveling near perpendicular incidence to the solid surface when $Q_{0T}^x(v_T/\omega)\approx Q_{0L}^x(v_L/\omega)\approx 0$ the L *phonons hardly strain the irregular surface, whereas the T phonons do.* In the presence of an energy transfer mechanism that couples the flexure mode to liquid-helium excitations, Eq. (4) implies that T phonons lose more energy than the L phonons when they scatter in the interface. This agrees with experimental measurements.[3,11,12,16]

Since T phonons are predominant over L phonons as the carriers of the elastic energy in the solid, Eq. (4) can be approximated formally to give

$$u_F'\,|_{z=\zeta}=2\frac{\omega}{v_T}\cos(2\alpha)u_{0T}e^{-i\pi/2}e^{iq_{0T}^z\zeta}e^{-i\omega t}\ , \tag{5}$$

where $\alpha=\sin^{-1}(Q_{0T}^xv_T/\omega)$ denotes the angle of incidence of the phonons. The boundary condition (5) is a harmonic force in the free circular surfaces of the islands.

The equation of motion for flexure of broad circular islands is given[23] by

$$\frac{\partial^2 u_F}{\partial t^2}+\frac{(1-\nu)EI_\xi}{\rho A_\xi}\frac{\partial^4 u_F}{\partial z^4}=0\ . \tag{6}$$

E and ν are, respectively, the Young's modulus of elasticity and Poisson's ratio, of the solid. I_ξ denotes the moment of area of the islands about their neutral axes. These quantities are well known in the theory of flexure of one-dimensional rods. A_ξ are the cross sectional areas of the islands.

The strain energy of harmonic flexure for the islands, $W(\xi,\zeta)$, is calculated using Eq. (6) and the appropriate boundary conditions at $z=0$ and at $z=\zeta$ from Eq. (5). For broad islands of atomic heights a good approximation is obtained in the form

$$W(\xi,\zeta)=4\frac{A_\xi}{V}p_\xi^4 v_T^2 \zeta^3\cos^2(2\alpha)\hbar\omega\ ,$$

with (7)

$$p_\xi=[\rho A_\xi/(1-\nu)EI_\xi]^{1/4}\ .$$

V is the volume of the solid bulk. ρ is its density.

III. ENERGY TRANSFER MECHANISM IN THE SOLID-LIQUID HELIUM SURFACE

The helium atoms adsorbed onto the solid surface form a solid layer which supports two-dimensional (2D) phonons[24,25] in the temperature range (0.01,2) K. This is the range in which the Kapitza resistance has been extensively measured. The layer is thermodynamically stable with a crystallographic structure that is incommensurate, in general, with that of the solid surface.

The hypothesis we make for the proposed energy transfer mechanism on the islands of an irregular solid surface comprises the following processes.

(i) The harmonic flexure of an island coherently generates 2D phonons in the solid He layer capping the island. This is reasonable from the measurements in Refs. 24 and 25 and the strong van der Waal's contact between He (layer) and solid atoms.

(ii) The 2D phonons on the solid He cap decay incoherently into the liquid He bath at the step edges $x=\pm\xi$, $z=\zeta$. The relatively weak contact between He (layer) and He (liquid) atoms, makes this a reasonable assumption.

The hypothesis is consistent with recent experimental results of Klitsner and Pohl.[21] See Sec. V for details.

The strain energy W_h of the solid He cap on an island under flexure is calculated using $W(\xi,\zeta)$. The repartition of the strain energy between the island and the cap is obtained, following process (i), in the form

$$W_h=\left[\frac{1}{\zeta}+\frac{8}{\xi}\right]d\frac{E'}{E}W(\xi,\zeta)\ . \tag{8}$$

E' is Young's modulus of elasticity of the solid He layer, and d is the layer thickness. Note that $E' \ll E$.

For the isolated system consisting of the solid and the adsorbed layer only, the 2D phonons are in thermal equilibrium.[24,25] In an R_K experiment, however, there is a net flux of heat across the interface. This implies that 2D phonons are excited in excess of their thermal equilibrium density, and following process (ii), that the return to equilibrium is governed by the decay of these phonons into the liquid He bath. The rate of energy transmission from a given island can be given consequently in a first-order approximation by

$$-\frac{d}{dt}W_h=\frac{W_h}{\tau}\ , \tag{9}$$

where τ is the lifetime of 2D phonons on the island. This can be calculated as

$$\tau=\frac{\pi}{4}\frac{\xi}{v}\ , \tag{10}$$

where v is the speed of sound of 2D phonons in the helium layer. It is significant to note that, for atomically irregular solid surfaces, τ of Eq. (10) and the frequencies ω of thermal phonons at low temperatures satisfy in general

$$\omega\tau \ll 1\ . \tag{11}$$

This result underlines the incoherent nature of the proposed energy transfer mechanism, in contrast with the coherent acoustic mismatch theory.[2] In particular, the proposed mechanism conserves neither ω nor the component of the bulk phonon wave vector that is parallel to the surface. These properties are experimentally observed for phonon transmission in the diffusive channel[3,11] (see Fig. 1).

The energy transmission coefficient χ_1 across the area A of the interface due to the proposed mechanism involves configurational averages over random distributions of islands in the irregular solid surface. For a phonon of energy $\hbar\omega$ incident at an angle α, this is given by

$$\chi_1(\omega,\alpha)=\frac{1}{A}\int\int N(\xi,\zeta)d\xi\, d\zeta\frac{W_h}{\tau}\ , \tag{12}$$

where $N(\xi,\zeta)$ denotes the surface distribution of islands having heights ζ and radii ξ. χ_1 identifies the diffusive channel in the present model. There is another transmission coefficient $\chi_2(\omega,\alpha)$ that identifies the acoustic mismatch channel from the flat parts of the solid surface. Since the energy transmission in these two channels

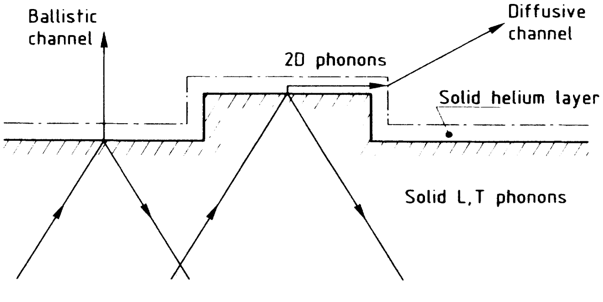

FIG. 1. Schematic representation of the ballistic and diffusive channels. The diffusive mechanism consists of flexure of the island, excitation of the 2D phonons in the solid helium layer, and their annihilation into liquid helium excitations.

occurs simultaneously with no interference effects,[3,11,12] then the total transmission coefficient can be written as the sum

$$\chi(\omega,\alpha)=\chi_1(\omega,\alpha)+\chi_2(\omega,\alpha) \ . \tag{13}$$

To calculate $\chi_1(\omega,\alpha)$, ξ and ζ are recast in terms of the solid lattice constant a,

$$\xi=ma, \quad \zeta=na \ . \tag{14}$$

m and n denote the number of atomic layers corresponding, respectively, to the radius and height of the islands. For any prepared solid surface it is a reasonable approximation to suppose that the surface treatment introduces a configurational scaling between m and n. With no loss of generality this is considered as

$$m=\beta n^s \ . \tag{15}$$

This form is the simplest that satisfies the necessary condition that $m=0$ when $n=0$, for the scaling factors $\beta>0$ and $s>0$. The scaling (15) folds the double distribution $N(\xi,\zeta)d\xi\, d\zeta$ into a single dimensionless one. Substituting (7)–(10) and (14)–(15) into (12) yields $\chi_1(\omega,\alpha)$ explicitly:

$$\chi_1(\omega,\alpha)=\varepsilon\beta^{-3}CE'\cos^2(2\alpha)\hbar\omega\int_1^{n_{\max}}P(n)y(n)dn \ , \tag{16}$$

where $P(n)dn$ is the normalized probability of finding an island with height n, and radius m following (15). The limits of the integral, 1 and $n_{\max}$, depict minimum and maximum island heights that are available in the irregular solid surface. ε is the surface density of islands. The factor $y(n)$ is given explicitly by the expression

$$y(n)=d\left[\frac{n^{2-3s}}{a}+8\beta^{-1}\frac{n^{3-4s}}{a}\right] . \tag{17}$$

C is a product of constant terms and $y(n)$ is dimensionless.

For well-prepared solid surfaces, as those obtained by cleavage or electropolishing, it is reasonable to suppose that the irregularities consist mostly of monatomic step islands on a local flat substrate.[26,27] The normalized probability is consequently approximated in these surfaces by the form

$$P(n)dn\equiv\delta(1)dn \ . \tag{18}$$

Substituting (18) into (16) yields

$$\chi_1(\omega,\alpha)=\varepsilon\beta^{-3}(1+8\beta^{-1})CE'\frac{d}{a}\cos^2(2\alpha)\hbar\omega \ . \tag{19}$$

It should be noted that the final expression for $\chi_1(\omega,\alpha)$ is not very sensitive to the choice of a particular form for the normalized probability distribution, nor to the scaling approximation (15).

IV. HEAT CONDUCTION AND THE KAPITZA RESISTANCE

When a thermal source provides the heat flux $\dot{Q}$ in an experiment, the phonons that travel to the solid-liquid helium interface have a blackbody spectrum.[2,3] The heat flux across the interface due to the flexure of the islands can be hence written as

$$\frac{\dot{Q}_1}{A}=\int_0^{\infty}\chi_1(\omega,\alpha)n(\omega)D(\omega)d\omega\int_0^{\pi/2}\tfrac{1}{2}v_T\sin\alpha\cos\alpha\, d\alpha \ , \tag{20}$$

where $n(\omega)$ is the Bose-Einstein factor and $D(\omega)$ is the Debye density of states of the solid bulk phonons. T phonons carry most of the elastic energy in the $\dot{Q}$ field, and the calculations are restricted to their contribution.

Substituting for $\chi_1(\omega,\alpha)$ from Eq. (19) and integrating gives

$$\frac{\dot{Q}_1}{A}=\varepsilon\beta^{-3}(1+8\beta^{-1})CT^4E'd \ . \tag{21}$$

C is again a product of constant terms. T in Eq. (21) refers to the temperature of the thermal source.

The mechanical and thermal properties of the solid He layer can be related. In particular, the Debye temperature of the solid layer is nearly that of the solid helium bulk at the same interatomic separation in the respective solid phases.[28] E' can thus be recast in terms of the Debye temperature Θ_h of solid He. With some algebra Eq. (21) becomes hence

$$\frac{\dot{Q}_1}{A}=\varepsilon\beta^{-3}(1+8\beta^{-1})CT^4\Theta_h^2 d \ , \tag{22}$$

where C is a product of constant terms given in final form by

$$C=\frac{\pi^3}{180}\left[\frac{4\pi}{9}\right]^{2/3}mvG(v')\frac{\rho}{(1-v')E^2a}\frac{k^6}{\hbar^5} \ , \tag{23}$$

$$G(v')=3\frac{(1-2v')(1+v')}{(1-v')}\left[1+2\left[\frac{2(1-v')}{(1-2v')}\right]^{3/2}\right]^{2/3} .$$

m is the mass line density of the helium solid at a given molar volume, v' is the Poisson's ratio for solid He, and k is Boltzmann's constant.

The form for the heat flux owing to flexure, given by Eq. (22), depends on the density of the islands and on the parameter β characteristic of the statistical properties of the irregular solid surface. The condition $\varepsilon\rightarrow0$ corresponds to an ideally flat surface, and it is straightforward to show that the heat flux due to the proposed mechanism goes to zero in this limit,

$$\operatorname{Lim}\frac{\dot{Q}_1}{A}=0, \quad \text{as } \varepsilon\rightarrow0 \ . \tag{24}$$

In this case the energy transmitted reduces to the acoustic mismatch heat flux $\dot{Q}_2/A$ since the corresponding $\chi_2(\omega,\alpha)$ persists for the flat interface, which result is consistent with R_K measurements in nearly ideal surfaces.[6]

The heat flux $\dot{Q}_1/A$ also depends on the Debye temperature of the adsorbed solid He layer. The molar volume of this solid layer is not well known since the state and topography of irregular solid surface are generally unknown. Nevertheless, the molar volume should fall between maximum and minimum allowable values for solid

He. This point is significant because it allows one to treat Θ_h in Eq. (22) as a random variable for different surfaces. At sufficiently low temperatures Θ_h varies with temperature;[29-31] $\Theta_h = \Theta_h(T)$ is then one of a family of curves bounded by loci that correspond to the minimum and maximum molar volumes.

The Kapitza resistance due to the proposed mechanism can be obtained using Eq. (22) in the form

$$R_K T^3 = \frac{\beta^3(1+8\beta^{-1})^{-1}}{2\varepsilon C d(2\Theta_h^2 + T\Theta_h \partial\Theta_h/\partial T)} , \tag{25}$$

provided $\dot{Q}_1 \gg \dot{Q}_2$, which is satisfied for reasonable values of ε and β. The acoustic mismatch contribution should be retained, otherwise, to yield the corresponding thermal boundary resistance.

V. NUMERICAL ANALYSIS AND CONCLUSIONS

The constant terms of Eq. (25) are calculated, using Eq. (23) for C, by taking values typical of a material solid, and of solid helium:[32] $\rho = 3\ \mathrm{g\,cm^{-3}}$, $\nu = 0.34$, $E = 10^{12}\ \mathrm{dyn\,cm^{-2}}$, $\nu' = 0.31$, $v = 4.2\times10^5\ \mathrm{cm\,sec^{-1}}$ (for the predominating transverse 2D phonons), $m = 7.4\times10^{-17}\ \mathrm{g\,cm^{-1}}$, and its variation with molar volume can be neglected, $d = 2\times10^{-8}$ cm for the solid He layer, and $a = 3\times10^{-8}$ cm for the lattice constant of the solid.

Substituting these values in Eqs. (23) and (25) yields

$$R_K T^3 = \frac{1}{0.3}\,\frac{\beta^3}{\varepsilon(1+8\beta^{-1})}\,\frac{1}{(2\Theta_h^2 + T\Theta_h \partial\Theta_h/\partial T)} . \tag{26}$$

A reasonable theoretical estimate for well-prepared surfaces is $\varepsilon \sim 1\%$, and the statistical parameter is taken as $\beta \sim 10$ from low-energy electron diffraction measurements. The temperature variation of $\Theta_h(T)$ for the hcp solid-helium phases are also known,[30,31] at different molar volumes.

In Fig. 2, plots are presented for the thermal-boundary resistance for a solid-liquid ^{3}He interface. The molar volume of the ^{3}He layer is taken in the dense phase of $11.42\ \mathrm{cm^3\,mol^{-1}}$. $\beta = 10$ and $\varepsilon = 1\%$. At sufficiently low temperatures the Debye temperature tends to a constant value, and we have taken $T \sim 0.1$ K to be where this occurs. The upper plot that corresponds to the pressure of 1 psi is calculated for a helium layer thickness $d = 2\times10^{-8}$ cm in Eqs. (25) and (26). In our model the higher pressure of liquid helium results in an increase of the layer thickness with no change in its molar volume. The lower plot in Fig. 2 is consequently obtained from the upper plot by increasing the layer thickness to $d = 3.2\times10^{-8}$ cm, to correspond to the higher pressure of 395 psi. There is good agreement in both cases with the experimental measurements of Anderson *et al.*[33]

In Fig. 3 is presented the plot for the thermal boundary resistance for a solid-liquid ^{4}He interface. The molar volume of the ^{4}He layer is taken in the dense phase of $12.23\ \mathrm{cm^3\,mol^{-1}}$. $\beta = 10$ and $\varepsilon = 1.8\%$. At sufficiently low temperatures the Debye temperature tends to a constant value, and we have taken $T \sim 0.06$ K to be where this occurs. The points are the experimental data of Anderson *et al.*[33]

The effect of surface preparation on the Kapitza resistance has been measured in a series of reproducible experiments by Johnson and Anderson[34] between copper and liquid ^{3}He. In Fig. 1 of the above reference, curve *B* is for a machined surface mechanically lapped to optical flatness and then electropolished. Curve *C* is for a similar surface which was also annealed under high vacuum as a final step. It is reasonable to suppose in this instance that the surface density of defect islands is greater for *B* than for *C*. The plots marked *B* and *C* in our Fig. 4 correspond to putting $\varepsilon = 1\%$ and 0.33%, respectively, in Eq. (26) where other terms are unchanged from our Fig. 1 for a solid-liquid ^{3}He interface. The theory compares well with the experimental measurements of Johnson and Anderson for

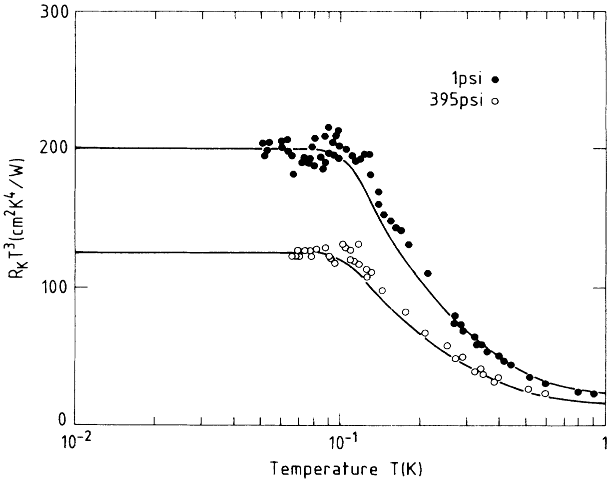

FIG. 2. Plots of $R_K T^3$ with temperature for a solid-liquid ^{3}He interface at two different pressures. The molar volume of the solid ^{3}He layer is $11.42\ \mathrm{cm^3\,mol^{-1}}$. $\beta = 10$ and $\varepsilon = 1\%$. The points are the experimental measurements of Anderson *et al.* (Ref. 33). See text for details.

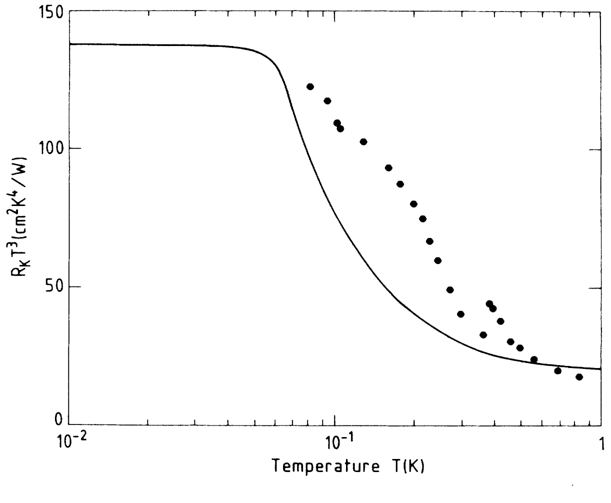

FIG. 3. Plot of $R_K T^3$ with temperature for a solid-liquid ^{4}He interface. The molar volume of the solid ^{4}He is $12.33\ \mathrm{cm^3\,mol^{-1}}$. $\beta = 10$ and $\varepsilon = 1\%$. The points are the experimental measurements of Anderson *et al.* (Ref. 33).

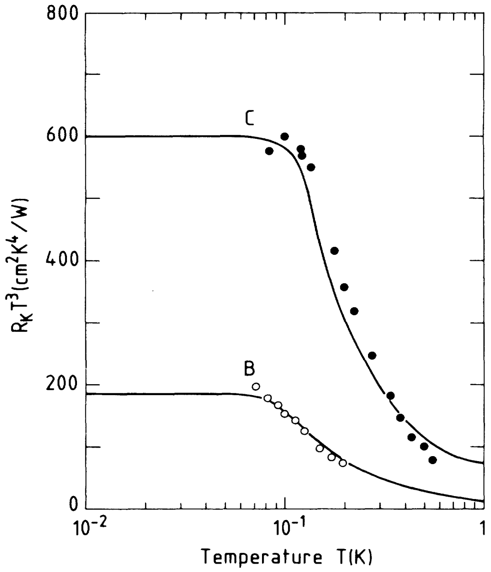

FIG. 4. Plots of $R_K T^3$ with temperature for a solid-liquid ^{3}He interface at two different surface densities of the defects, $\varepsilon = 1\%$ for curve B and $\varepsilon = 0.33\%$ for curve C; all other terms are the same as in Fig. 2. The points are the experimental measurements of Johnson and Anderson (Ref. 34). See text for details.

their B and C surfaces.

In Fig. 5, plots of $R_K T^3$ with temperature are presented for a solid-liquid ^{3}He interface in the range 0.1 to 1.9 K. The 1, 2, and 3 plots correspond, respectively, to the molar volumes 11.42, 15.29, and 19.05 cm^3 mol^{-1} of the hcp solid ^{3}He phase. $\varepsilon = 1\%$ and $\beta = 10$, in the irregular solid surface. Similarly, in Fig. 6, plots of $R_K T^3$ are presented for a solid-liquid ^{4}He in the same temperature range. The 1, 2, and 3 plots correspond now, respectively, to the molar volumes 12.23, 16.90, and 20.93 cm^3 mol^{-1} of the hcp solid ^{4}He phase. ε and β are the same as in Fig. 5.

The present theory accounts for the random and bounded variation of the thermal-boundary resistance with surface preparation, since in Eq. (26) the quantities ε, β, and Θ_h are all random yet bounded variables. Also, the phonons emitted into helium by process (ii), assumed for the transfer mechanism in our model, have random directions consistent with the observed cosine law.[35] The theory accounts, furthermore, for the known observation that R_K is independent of the superfluidity of He II.[3] The heat flux $\dot{Q}_1/A$ calculated for the diffusive channel in Eq. (22) depends on the mechanical properties of the solid helium layer. These properties, for both solid ^{3}He and ^{4}He, are independent of the superfluid character of liquid helium.[32] The model accounts for why the heat transmission is similar for thin He films and solid He.[36] In solid He, as in the liquid under high pressures, the affected layer in which phonons are excited is larger than that in thin films.

The processes (i) and (ii) proposed for the transfer mechanism in our model can be generalized. In (i) it is sufficient to define a region of adsorbate atoms surrounding the defect where adsorbate phonons are excited by the

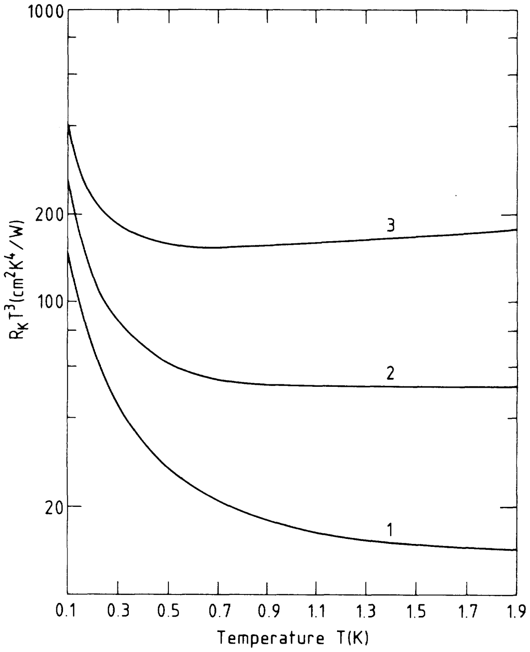

FIG. 5. Plots for $R_K T^3$ with temperature for a solid-liquid ^{3}He interface in the range 0.1 to 1.9 K. The 1, 2, and 3 plots correspond, respectively, to the molar volumes 11.42, 15.29, 19.05 cm^3 mol^{-1} of the hcp solid ^{3}He phase. $\beta = 10$ and $\varepsilon = 1\%$.

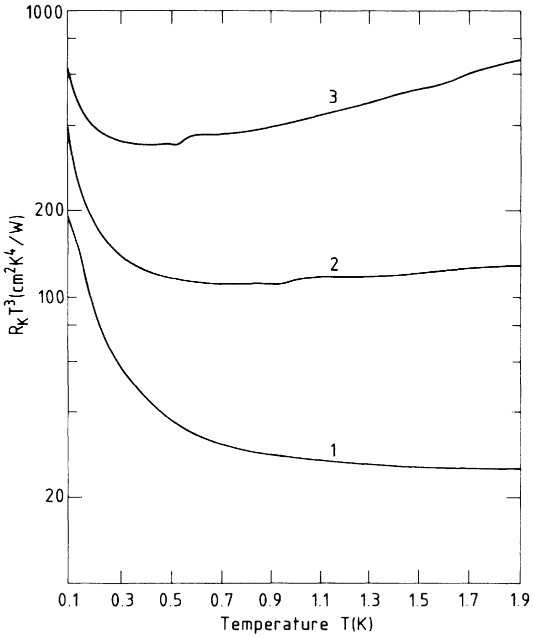

FIG. 6. Plots for $R_K T^3$ with temperature for a solid-liquid ^{4}He interface in the range 0.1 to 1.9 K. The 1, 2, and 3 plots correspond, respectively, to the molar volumes 12.23, 16.90, and 20.93 cm^3 mol^{-1} of the hcp solid ^{4}He phase. $\beta = 10$ and $\varepsilon = 1\%$.

flexure dynamics of defects. The notion of a solid adsorbate layer is not a prerequisite but the manner of variation with temperature of the elastic properties of the adsorbate atoms in this region is needed. Also, in (ii) the scattering of excited adsorbate phonons need not be limited to the scattering of 2D phonons at the step edges of the defects but may occur generally via the structural irregularities within the adsorbate films. There is evidence for such diffuse scattering,[21] with limits of a few hundred angstroms. The order of magnitude of τ, the relaxation time for process (ii) given above by Eq. (10), is unaffected by this generalization.

The transmission coefficient χ_1 for the diffusive channel obtained in approximate form in Eq. (19) varies linearly with the energy of incident phonons $\hbar\omega$. This approximation is adequate for thermal phonons from a blackbody source at low temperatures, and for defects of atomic dimensions. The detailed theory for phonon scattering in irregular surfaces[22] gives, however, physically appropriate limits for scattering when $\omega \rightarrow 0$ and $\omega \rightarrow \infty$. In these limits $q\xi \ll 1$ and $q\xi \gg 1$, respectively, and the corresponding phonons are shown to scatter effectively from flat surfaces without exciting flexure on the defects. χ_1 of Eq. (19) here contrasts with t_1 of Eq. (1) in Ref. 18. In Kinder's work the transmission coefficient and the model are strictly valid, provided the strain dynamics of the defects are neglected. In our work this problem is specifically investigated. The energy transmission is shown to be due to the flexure of defects in the solid surface that coherently generate phonons in the adsorbed helium layer, and to the diffusive decay of these phonons into the liquid helium bath. The energy transmission coefficient is given in general and calculated for atomically irregular surfaces. The theoretical results for the Kapitza resistance compare favorably with measurements.

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