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Two-phase sound in ^{3}He-^{4}He mixtures

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The properties of sound waves propagating in a superfluid two-phase system (liquid-helium mixture and its vapor) are investigated as the temperature is lowered for two ^{3}He mass concentrations $c=1\%$ and $c=0.1\%$ in the liquid. We see a crossover behavior from two independent sounds when $T=1.2$ K to one coupled mode when $T=0.7$ K. The crossover temperature depends on c and is higher for weak solutions. This is found to be related to the existence of exchanges which are initiated at the interface by the sound propagation and to the increase of an effective diffusion length L_{Deff} in the liquid: L_{Deff} is the range of the diffusion in the superfluid liquid of ^{3}He atoms and energy which have been exchanged at the interface. The main interest of ^{3}He-^{4}He superfluid mixtures is that L_{Deff} may become much larger than the usual diffusion lengths below 1.2 K, due to the rapid increase of the effective ^{3}He mean free path in weak solutions.

I. INTRODUCTION

Sound propagation in a two-phase liquid-vapor system becomes strongly dependent on the boundary conditions at the liquid free surface when the wavelength is much larger than the system depth. In classical fluids these conditions lead only to a sound velocity which is always very close to the vapor velocity: The large difference of densities means that the liquid is much stiffer than the vapor so that its state fits at any time to the state of the vapor by means of pressures equality at the interface, and the sound velocity will be driven by the vapor.

Below the lambda transition (2.17 K at the vapor pressure), ^{4}He becomes a superfluid known as He II. It behaves as though it is composed of two fluids, a superfluid component and a normal component, the superfluid component having no viscosity nor entropy: The superfluid-component ratio is 0 at the lambda transition, and increases towards 1 as the temperature is lowered. Different patterns of flow for the two components then result in different sound modes: In bulk helium the two components may flow together resulting in variations of total density and pressure, forming an ordinary compressional wave called first sound. The two components may also flow in opposite directions resulting in a temperature or entropy wave which is second sound. Because the expansion coefficient vanishes below 1.2 K, first sound has associated with it very small temperature changes at lower temperatures, and second sound has small density and pressure changes. Therefore, when liquid He II is in equilibrium with its vapor, temperature variations which can propagate in the superfluid may be bounded at the interface with pressure variations in the vapor along the coexistence curve. So the vapor sound and the second sound couple together resulting in a single propagating mode. Coupling takes place due to periodic evaporation and condensation of helium at the surface, and this new mode which is characterized by both temperature and pressure oscillations is thus called two-phase sound. Its velocity depends on the liquid depth in the waveguide and varies from the vapor velocity U_v to the second-sound velocity U_2 when the waveguide is filled.[1,2] The situation for superfluid helium differs markedly from that of a classical liquid in equilibrium with its vapor because the temperature fluctuations accompanying the evaporation-condensation process occur only at the interface for ordinary liquids, being localized by the condition of adiabacity. In He II an efficient means for heat transfer is provided by second sound and temperature fluctuations occuring at the surface may be detected at (or generated from) well submerged positions.

The case of superfluid ^{3}He-^{4}He mixtures is more complex since the ^{3}He concentration acts as a new degree of freedom: In previous experiments, which were run at $T=1.2$ K, the vapor sound and the second sound were found to propagate as independent modes for molar concentrations in the liquid larger than 1%,[3] but there was a crossover from these two independent modes to only one two-phase mode as the concentration was lowered to 0.01%.[4] In this survey, we report new experiments on liquid-vapor mixtures as the temperature is lowered from 1.2 to 0.7 K, the ^{3}He concentration in the liquid being kept constant: We see again a crossover phenomenon from two independent sounds at $T=1.2$ K to one delocalized mode at $T=0.7$ K.

Our goal is to investigate how temperature and ^{3}He concentration in the liquid drive this transition from one delocalized mode (analog to the pure He II two-phase sound) to two independent sounds in mixtures at high temperatures or ^{3}He concentrations. Indeed, this phenomenon is not well understood: It has been indicated[4,5] that ^{3}He and energy exchanges at the interface should be included in the hydrodynamics, but this is only a necessary condition. Evaporation-condensation processes always take place at the interface in helium mixtures due to any thermal excitation in either phase, but these phenomena do not disturb the propagation of sounds as long as their effects are localized at the interface (which is the case of a classical fluid). On the other hand, a simple new idea is that the propagation of sound will be strongly disturbed as soon as the range of diffusion phenomena from the interface in one phase can be equal to its own depth.

We find that the crossover occurs when the effective diffusion length L_{Deff} which can be defined in a superfluid ^{3}He-^{4}He mixture, becomes nearly equal to the superfluid liquid depth L: L_{Deff} measures the ability of the superfluid liquid to diffuse heat and matter which have been exchanged at the interface between the liquid and the vapor. Such "macroscopic" effects related to diffusion in one phase from the interface are allowed in our experiments by the ^{3}He-^{4}He mixtures superfluidity: As the ^{4}He atoms are condensed in the superfluid ground state at low temperatures, the normal component resumes to ^{3}He atoms and its relative flow is strongly enhanced; the effective mean free path of ^{3}He atoms which accounts for the existence of this counterflow between the components of the superfluid liquid therefore rises rapidly. L_{Deff} can thus become much larger than the usual diffusion lengths, and even larger than the superfluid liquid depth at low temperatures.

This paper is organized as follows. Section II will be concerned with experimental techniques on bulk two-phase sounds. Then experimental data will be displayed in Sec. III, and the discussion takes place in Sec. IV.

II. EXPERIMENTAL METHOD

A. General

The principle of our technique is to generate sound in an annular waveguide by applying a heat modulation to one resistor and detecting temperature oscillations with a bolometer placed symetrically in the channel. The signal is brought out of the noise with two PAR 124 matched to make a modulus lockin amplifier. The frequency is swept from 100 to 400 Hz with a Philips PM 5190 synthetiser driven by a microcomputer at a rate of 0.2 Hz/s. The amplitude detected is maximum when the sound wavelength almost equals the channel mean circumference, and the frequency at resonance is directly proportional to the sound velocity.

This method is suited to the emission as well as detection of second sound, but it also allows the generation and detection of acoustic sounds: heat pulses bring about evaporation of the film adsorbed on the emittor and generate pressure variations in the vapor which will be detected in a reversible way by condensing the film on the receiver. This is a way to calibrate the cell resonant frequencies, using the sound velocity of an ideal gas.

B. Experimental cell

The measurements were performed in a copper-brass channel with a rectangular cross section (see Fig. 1) used as the waveguide and cooled by a dilution refrigerator. Sounds propagate in the z direction, i.e., parallel to the interface. For a first study a cell was machined out with height $H = 1.0$ cm, width 1.0 cm, mean circumference 8.5 cm, and only one emittor and receiver hanging at 0.5 mm from the bottom. Then another resonator was machined out with emittors and receivers lying at both the top and the bottom of the cell in order to detect sound in each phase (height 1.0 cm, width 0.5 cm, mean circumference 10.1 cm).

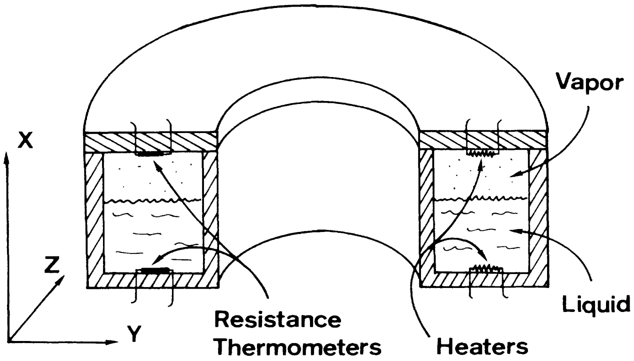

FIG. 1. Resonator transverse section.

The resistors were obtained by laying down a 100-μm thick strip of colloidal carbon[6] on an indissovable resin,[7] with the advantage of very weak heat mass and good thermal contact. Power levels of 200 μW were necessary because of the strong damping of the modes and a $(1/R)(\partial R/\partial T) = -0.5\ \mathrm{K}^{-1}$ relative sensitivity of the carbon bolometers at 1 K. These power levels were, however, weaker than those used by G. A. Williams *et al.*[3] within a fast-Fourier-transform technique. The temperature was measured with an external germanium diode[8] and regulated with a resistance bridge.[9]

The liquid depth L and mass concentration c can be found from the known geometry of the annulus and the measurement of the amounts of helium gas which are condensed, taking account of the atoms in the vapor. Pressure in calibrated volumes is measured with Baratron capacitive manometers and the final error on L and c is less than 1%.

C. Level alignment

For thick phases, the annular geometry is relatively insensitive to misalignments of the level because the sound modes average out changes in depth around the annulus. But this does not hold any longer when the liquid depth becomes too thin, so we had not only to level the annulus at room temperature, but to find a way to proceed during the experiments at low temperature.

This way was found by tilting the cryostat when the annulus was almost filled up with liquid. Nothing occurs when the annulus is completely filled up. But when there are about 200 μm lacking, one can see the second sound peak smoothly vanish and appear again at a frequency about 10 Hz smaller (see Fig. 2). This spectacular effect is the same for two misalignments symmetric with respect to one direction, which must be the level position (the peak frequency is then the same as if the cell were overfilled with liquid). It is not understood yet, but it is reproducible with pure He II and mixtures in both the annuli we used, and sensitive enough to get a level alignment with an error expected to be less than 10′ arc.

III. RESULTS

The experiments made with both waveguides provided information about localization, damping and the speed of sound.

A. Localization

Data were taken with emittors and receivers in each phase at the bottom or at the top of the waveguide. When the ^{3}He mass concentration in the liquid is 0.1% and $T = 1.2$ K, vapor sound and second sound propagate as independent modes in each phase. A sound in one phase cannot be excited with a heat modulation in the other phase so one can never detect two peaks from the same receiver unless both resistors are located at the interface.

Spectra obtained when $T = 0.85$ K with the same mixture are shown in Fig. 3: For the first time we can see in (d) two damped peaks from one receiver in the liquid; the lower-frequency peak is coupled second sound and the other one is delocalized coupled vapor sound. One can see that the coupled vapor sound can be excited as well as detected either in the vapor or in the liquid. So it propagates with pressure variations in one phase coupled with temperature variations in the other phase, while coupled second sound is localized in the liquid. As the temperature is lowered, acoustic sound, which is located in the vapor when $T = 1.2$ K, therefore spreads in the whole two-phase system when coupling occurs.

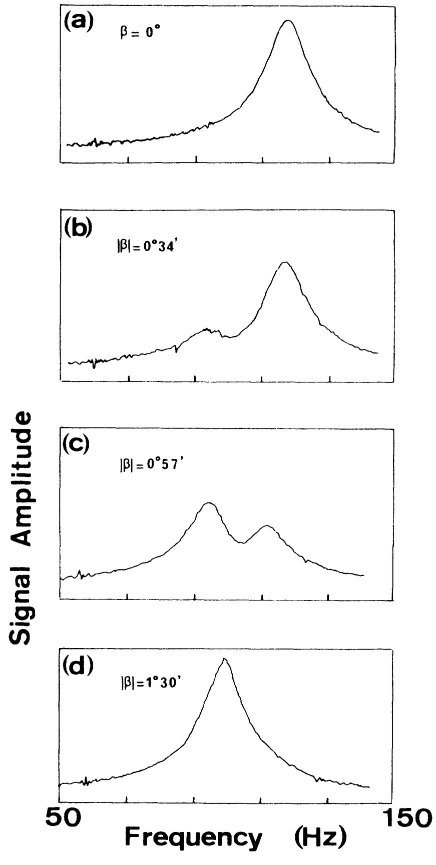

FIG. 2. Appearance of a second peak as the refrigerator is leaned from the right alignment with the angle β. The spectra appearance depends only on $|\beta|$. Signal amplitudes are displayed in arbitrary units.

B. Damping

The sound quality factors strongly depend, on the existence of an interface in the waveguide. Q varies in the range 20–100 with only gas or liquid in the waveguide, whereas it is in the range 2–5 when two phases coexist. Damping can be so strong that one can hardly measure quality factors and resonant frequencies when there are two peaks on the same spectrum.

When coupling occurs, the second-sound damping in the liquid increases so much that it prevents the measure-

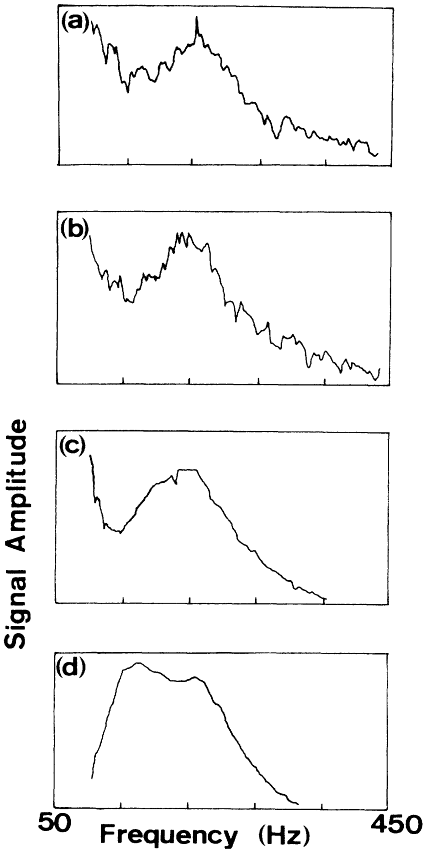

FIG. 3. Spectra obtained with $c = 1\%$, $T = 0.85$ K, and $L/H = 7\%$ when the emitter and the receiver are either in the vapor or in the liquid. These data were obtained with the cell being the second cell which was machined out (depth equals 1 cm). (a) Both resistors in the vapor. (b) Emitter in the liquid, receiver in the vapor. (c) Emitter in the vapor, receiver in the liquid. (d) Both resistors in the liquid. The damped peak in (a) is the two-phase sound. (b) and (c) show its delocalization because this mode can be excited in one phase and detected in the other phase. The first damped peak in (d) is second sound in the liquid, while the second one is again the two-phase sound. Signal amplitudes are displayed in arbitrary units.

ments of any deviation of the second-sound velocity from the one it should have if the cell were completely filled with liquid at the same temperature. Besides, vapor-sound quality factors are poor in the range of temperatures we investigated, and this prevented us from getting more experimental information on the variations of this mode damping.

C. Velocities

Data come from experiments run with only one emittor and one receiver located in the cell at 0.5 mm from the bottom. The sound velocities are plotted with respect to T for two ^{3}He concentrations and various depths of the liquid.

In a first experiment the liquid-vapor interface is leveled at the expected position of the resistors, i.e., $L/H=8\%$. Figure 4 shows data obtained when $c=1\%$. As T is lowered, the second sound vanishes, while the acoustic sound velocity departs from the value it should have if the cell were completely filled with vapor. We could not experiment at temperatures lower than 0.75 K because of technical obstacles. However, our main interest is to show the crossover between one to two modes and this crossover occurs when $T=0.95$ K (second-sound disappearance). Shown in the same figure are data obtained

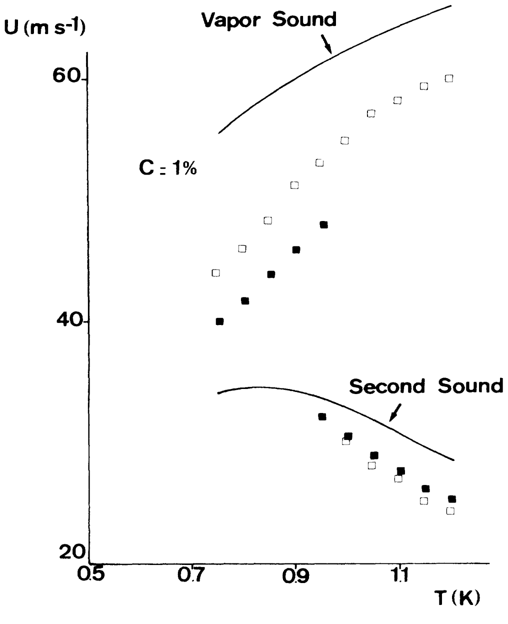

FIG. 4. Sound velocities are plotted with respect to temperature for two liquid depths, when $c=1\%$. All following data are obtained with the first cell which was machined out (depth equals 0.5 cm). Open squares represent $L/H=8\%$, both resistors being at the interface. Solid squares represent $L/H=11\%$, both resistors being in the liquid. Both localized modes can be detected when the resistors are at the interface. Due to strong damping, second sound vanishes as T is lowered. When both resistors are in the liquid, one can detect second sound at high temperatures, and delocalized two-phase sound at low temperatures.

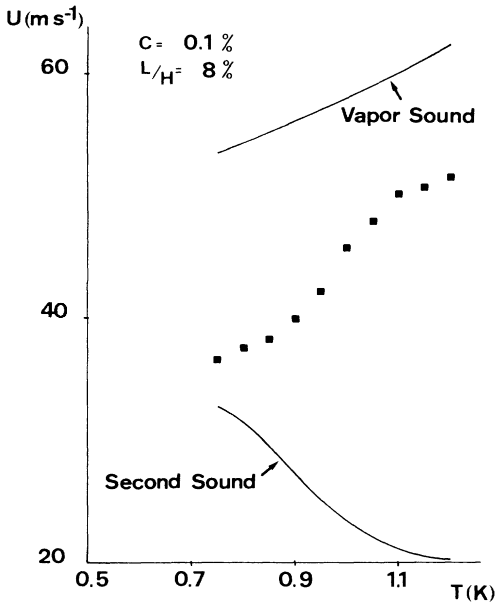

FIG. 5. Two-phase sound velocity is plotted with respect to temperature, when $c=0.1\%$ $L/H=8\%$: U departs from the vapor-sound velocity and decreases towards the second-sound velocity at same temperature. The second sound is damped too much and cannot be detected.

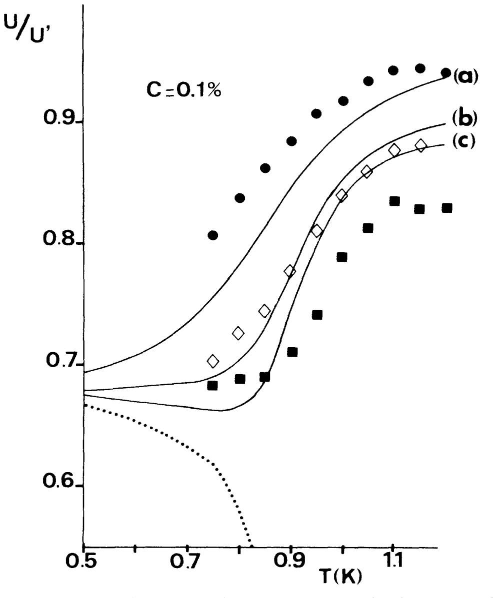

FIG. 6. U is the two-phase sound velocity and U' is the vapor-sound velocity which is measured when the cell is filled with gas only. The ratio U/U' is plotted with respect to temperature for $c=0.1\%$ and various depths of the liquid, solid lines being the hydrodynamic predictions: (a) and solid circles represent $L/H=2\%$; (b) and open rhombs represent $L/H=5\%$; (c) and solid squares represent $L/H=8\%$. The dotted line displays the variation of the ratio U_2/U', denoting with U_2 the second sound velocity if the cell were completely filled with liquid. The two-phase sound velocity departs from U' and decreases towards U_2 as the temperature is lowered or the cell is filled with liquid.

when both resistors are in the liquid: one can detect second sound above 0.95 K and coupled delocalized acoustic sound below.

Strong coupling occurs below 1.2 K when $c=0.1\%$. Data are plotted on Fig. 5: second sound is too strongly damped, but one can see the coupled acoustic sound velocity decrease towards the second-sound velocity. So at $T=0.75$ K, there is only one two-phase sound with a velocity close to the second sound velocity.

When the cell is completely filled with vapor, the sound velocity U' depends on temperature T, and has been measured with respect to T in our waveguide. In order to cancel out this variation in the data, we plotted on Figs. 6–8 U/U' and U/U_2. Corrections to U' related to the varying ^{3}He vapor concentration with T are taken into account. Data are plotted for $c=0.1\%$ and $c=1\%$ and various depths of the liquid. Solid lines show hydrodynamic predictions which will be discussed soon. The crossover temperature depends on the depth of superfluid liquid and strongly on the ^{3}He concentration. We can see that the second-sound velocity does not show the wide variation predicted by hydrodynamics because of a strong damping which prevents detecting this mode at low temperatures.

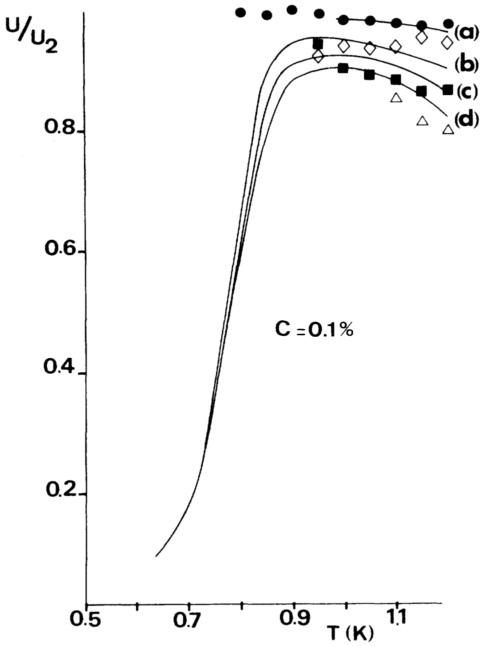

FIG. 7. U is the coupled second-sound velocity in the two-phase system and U_2 is the second-sound velocity which is measured when the cell is completely filled with liquid. The ratio U/U_2 is plotted with respect to temperature for $c=0.1\%$ and various depths of the liquid, solid lines being the hydrodynamic predictions: (a) and solid circles represent $L/H=50\%$; (b) and open rhombs represent $L/H=20\%$; (c) and solid squares represent $L/H=14\%$; (d) and open triangles represent $L/H=11\%$. Coupled second sound vanishes due to strong damping before its velocity can depart widely from U_2 and decrease to 0 as it is predicted by the hydrodynamics. As the liquid phase is made thinner, we can yet see that the coupled second-sound velocity is smaller and that this mode vanishes at higher temperature.

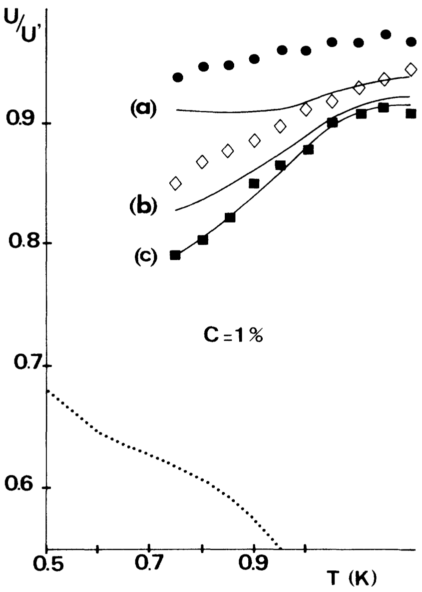

FIG. 8. U is the two-phase sound velocity and U' is the vapor-sound velocity which is measured when the cell is filled with gas only. The ratio U/U' is plotted with respect to temperature for $c=1\%$ and various depths of the liquid, solid lines being the hydrodynamic predictions: (a) and solid circles represent $L/H=2\%$; (b) and open rhombs represent $L/H=5\%$; (c) and solid squares represent $L/H=8\%$. The dotted line displays the variation of the ratio U_2/U', denoting with U_2 the second-sound velocity if the cell were completely filled with liquid. The two-phase sound velocity departs from U' and decreases, but this variation is less pronounced than if $c=0.1\%$, showing that coupling is stronger as the ^{3}He mass concentration is made smaller.

IV. DISCUSSION

We will first discuss the main features of the hydrodynamic description we developed for two-phase sound in a waveguide,[5] with particular emphasis on the supposition of equilibrium for the interface, and then we discuss new features of this crossover phenomenon.

A. Hydrodynamics

The description starts with linearized hydrodynamics in the superfluid, taking into account heat and mass currents.[10,11] With usual notations

$$\frac{\partial}{\partial t}(\rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n) + grad(\rho) = 0 \ , \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n) + grad(p) = 0 \ , \tag{2}$$

$$\frac{\partial}{\partial t}(\rho c) + div\,(\rho c \mathbf{v}_n + \mathbf{g}) = 0 \ , \tag{3}$$

$$\frac{\partial}{\partial t}(\rho s) + div(\rho s \mathbf{v}_n + \mathbf{q}/T - \mathbf{g}Z/\rho T) = 0 \ , \tag{4}$$

$$\frac{\partial}{\partial t}(\mathbf{v}_s) + grad\,(\mu - Zc/\rho) = 0 \ . \tag{5}$$

T and p are the temperature and pressure; ρ, ρ_s, and ρ_n

are, respectively, the total, superfluid, and normal densities (^{3}He atoms belong to the normal component); $\mathbf{v}_s$ and $\mathbf{v}_n$ are the superfluid and normal velocities; s and μ are the entropy and chemical potential per unit mass. We also use the usual relations for Z/ρ and μ as functions of the ^{3}He and ^{4}He chemical potentials μ_3 and μ_4,

$$Z/\rho=\mu_3-\mu_4 \ , \tag{6}$$

$$\mu=c\mu_3+(1-c)\mu_4 \ . \tag{7}$$

Besides, the exchanges at the interface which are initiated by the sound propagation will bring about the existence of temperature and concentration gradients close to the interface. Diffusion phenomena therefore occur in each phase and the bulk ^{3}He current $\mathbf{g}$ and heat flux $\mathbf{q}$ which account for it are given by

$$\mathbf{g}=-\rho D\,[grad\,(c)+(k_T/T)grad\,(T)] \ , \tag{8}$$

$$\mathbf{q}/T-\mathbf{g}Z/\rho T=\mathbf{g}\left[\frac{\partial s}{\partial t}+(k_T/T)\frac{\partial}{\partial t}(Z/\rho)\right] -(\kappa/T)grad(T) \ , \tag{9}$$

where D and k_T are, respectively, the diffusion and thermal diffusion coefficients, while κ is the thermal conductivity.

Taking advantage of the large difference between the propagation and diffusive wave vectors (respectively, k_z and k_D, $2\pi/k_Z$ being almost equal to the resonator mean circumference as it is specified in Sec. II A), the resolution of this set of equations in the superfluid leads to the existence of three hydrodynamic modes: first and second sound, and a diffusive mode of heat and matter related to the effective diffusive coefficient D_{eff} and the diffusive wavelength L_{Deff}. From Ref. 5 we have

$$D_{\text{eff}}=\left\{D\left[1-(2ck_T/T\bar{s})\frac{\partial}{\partial c}(Z/\rho)+(c^2/\bar{s}^2)\left[\frac{\partial}{\partial c}(Z/\rho)\right]^2(k_T/T)^2\right] +(\kappa/\rho T)(c^2/\bar{s}^2)\frac{\partial}{\partial c}(Z/\rho)\right\}\Bigg/\left[1+(c^2/\bar{s}^2)\frac{\partial s}{\partial T}\frac{\partial}{\partial c}(Z/\rho)\right] \ , \tag{10}$$

with

$$\bar{s}=s-c\frac{\partial s}{\partial c} \ . \tag{11}$$

Then

$$L_{\text{Deff}}=(2D_{\text{eff}}/\omega)^{1/2}, \quad k_d=2\pi/L_{\text{Deff}} \ . \tag{12}$$

$k_D \gg k_z$ means that L_{Deff} is small compared to the sound wavelength: Because this later equals the cell mean perimeter, this condition is always fulfilled and allows the pressure, temperature and concentration variations in the superfluid to be expressed as

$$\delta P=\delta P_1 \ , \tag{13}$$

$$\delta T=\delta T_2+\delta T_d \ , \tag{14}$$

$$\delta c=\delta c_2+\delta c_d \ , \tag{15}$$

where for second sound

$$\delta c_2=(c/\bar{s})\frac{\partial s}{\partial c}\delta T_2 \ , \tag{16}$$

and for the diffusive mode

$$\delta T_d=-(c/\bar{s})\frac{\partial}{\partial c}(Z/\rho)\delta c_d \ . \tag{17}$$

The liquid thermal expansion is expected to be vanishingly small so there is no coupling between first and second sound in the superfluid.

So it is necessary to match at the interface the three hydrodynamic modes in the superfluid with the three modes in the vapor, which are acoustic sound, heat diffusion, and ^{3}He diffusion. The interface position being eliminated with the continuity equation (1), we need six boundary conditions to get the dispersion relation: three equations come from conservation at the interface of momentum ($\delta P=\delta P'$), ^{3}He, and entropy. The next three come from the connection between flows and forces at the interface.

The general case is that this connection involves nine Onsager coefficients related to the irreversibility of exchanges at the interface. Several authors[12] have shown that evaporation and energy transport processes occuring at the interface of helium due to incidence and conversion of sound waves may be connected with appreciable energy dissipation. The reason is, on one hand, that evaporation processes are very important for He II because the heat of vaporization is relatively small compared with other liquids; on the other hand, the heat transport is extremely high due to counterflow of the normal fluid and the superfluid. Thus even small deviations from equilibrium such as those taking place in sound waves give rise to appreciable evaporation rates and dissipation.

Nevertheless, in the limit of small frequencies (100–400 Hz), one can get a good agreement between theory and experiments by neglecting this dissipation and expecting thermodynamic equilibrium at the interface. We think this is due to much lower frequencies than in previous sound conversion experiments: First, the interface is closer to equilibrium; secondly, diffusion lengths are much longer. Dissipation at the interface which was dominant in the high-frequency case will therefore become small compared to the diffusive effects in the superfluid when coupling occurs, and taking account of it should lead only to small corrections on the velocities (see Ref. 2 for a complete survey of the case of pure He II). Then for equilibrium, the interface mass and energy currents are only determined by the bulk properties of the two media and we can get the following (prime will refer to all parameters related to the vapor phase):

$$\delta\mu_3=\delta\mu_3' \;, \tag{18}$$

$$\delta\mu_4=\delta\mu_4' \;, \tag{19}$$

$$\delta T=\delta T' \;. \tag{20}$$

One may then use the six following potentials which describe a standing wave in the transverse direction x and a traveling wave in the z direction. These potentials satisfy the boundary conditions at the top and the bottom of the cell ($x=0$ and $x=H$) which are $\mathbf{g}=0$ and $\mathbf{q}=0$:

$$\phi_1=A_1\cos(k_{1x}x)\exp[i(k_zz-\omega t)], \quad \text{with } \delta P=-\rho\frac{\partial\phi_1}{\partial t} \quad \text{and } k_{1x}^2+k_z^2=\omega^2/U_1^2 \;, \tag{21}$$

$$\phi_2=A_2\cos(k_{2x}x)\exp[i(k_zz-\omega t)], \quad \text{with } \mathbf{v}_n=grad(\phi_2) \quad \text{and } k_{2x}^2+k_z^2=\omega^2/U_2^2 \;, \tag{22}$$

$$\phi_3=A_3\cos(k_{dx}x)\exp[i(k_zz-\omega t)], \quad \text{with } k_{dx}=(1+i)(\omega/2D_{\text{eff}})^{1/2} \;, \tag{23}$$

$$\phi'=A_1'\cos[k_x'(H-x)]\exp[i(k_zz-\omega t)], \quad \text{with } \delta P'=-\rho'\frac{\partial\phi'}{\partial t} \quad \text{and } (k_x')^2+k_z^2=\omega^2/(U')^2 \;, \tag{24}$$

$$\delta T_d'=A_2'\cos[k_{Tx}'(H-x)]\exp[i(k_zz-\omega t)], \quad \text{with } k_{Tx}'=(1+i)(\omega\rho'C_P'/2\kappa')^{1/2} \;, \tag{25}$$

$$\delta c_d'=A_3'\cos[k_{dx}'(H-x)]\exp[i(k_zz-\omega t)], \quad \text{with } k_{dx}'=(1+i)(\omega/2D')^{1/2} \;. \tag{26}$$

From the interface boundary conditions we then get the dispersion relation

$$k_{2x}^2H+(U_6^2/U_2^2)\bar{k}^2H+\beta(1-i)k_{2x}^2L\bar{k}^2H/\omega^{1.5}=0 \;, \tag{27}$$

with

$$\bar{k}^2H=\frac{\rho'}{\rho}k_1^2L+(k_x')^2(H-L) \;. \tag{28}$$

The physical bearings of U_6 and β are discussed below. Both parameters are complex functions of thermodynamic and transport coefficients and their complete definition will therefore be found in Ref. 5.

B. Discussion of the dispersion relation

Equation (28) arises from the boundary condition $\delta P=\delta P'$ at the interface. It means that acoustic sound in the vapor and first sound in the liquid couple together resulting in an acoustic sound averaged over the densities and depths of each phase. Because of $\rho \gg \rho'$ we find again the case of a classical fluid in equilibrium with its vapor and the average acoustic sound velocity is very close to the vapor velocity.

Equation (27) couples this average acoustic sound with second sound in the liquid. As in Ref. 2, U_6 is a velocity characteristic of the liquid-vapor coupling and is related to the vapor density ρ' and the heat of vaporization at the interface. In pure He II,[2]

$$U_6^2=s'\left[\frac{\partial P}{\partial T}\right]_{co}\bigg/\rho\left[\frac{\partial s}{\partial T}\right]_T \;. \tag{29}$$

As $(\partial P/\partial T)_{co}$ in pure He II, U_6 measures in mixtures the amplitude of evaporation-condensation process at the interface, i.e., the amplitude of pressure oscillations in the vapor set up by temperature oscillations in the liquid. From thermodynamics of ^{3}He-^{4}He mixtures,[13] we have plotted U_6 in Fig. 9 with respect to T when $c=1\%$, $c=0.1\%$, and for pure ^{4}He: ρ', latent heats, and, therefore, U_6 are expected to decrease when T goes to 0. Moreover, β is related in particular to transport coefficients and is plotted in Fig. 10. β is the coupling parameter of the sounds because (1) if $\beta\rightarrow\infty$, i.e., for small transport coefficients, then $k_{2x}^2L\bar{k}^2H\rightarrow 0$, i.e., second sound and the average acoustic sound behave as independent modes, and (2) if $\beta\rightarrow 0$, i.e., for high transport coefficients then

$$k_{2x}^2L+(U_6/U_2)^2\bar{k}^2H=0 \;,$$

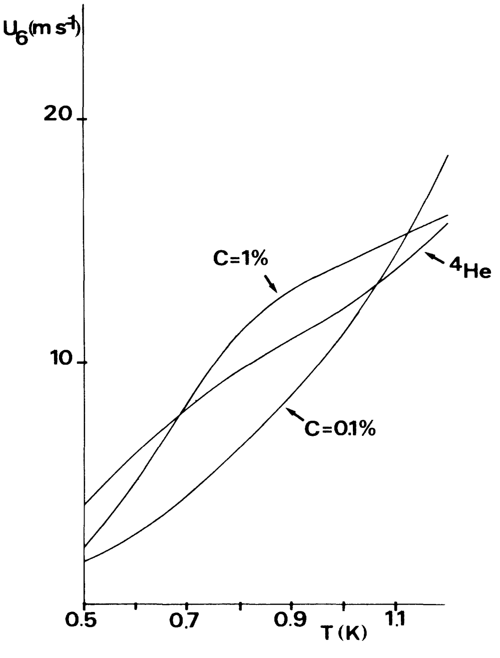

FIG. 9. U_6 vs temperature.

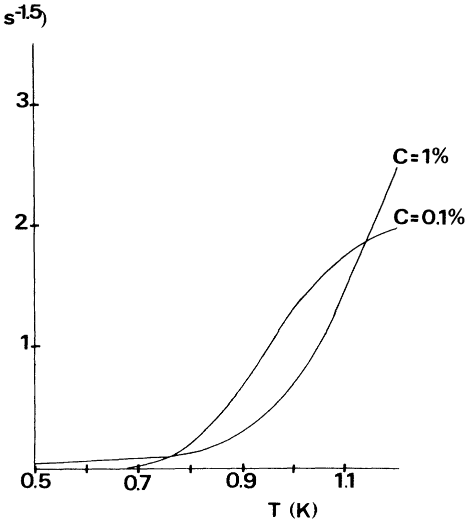

FIG. 10. β vs temperature.

which is the dispersion relation in pure ^{4}He.[3]

The solid lines in Fig. 6–8 are plotted from Eq. (27) and tabulated values found in the literature. The hydrodynamic predictions fit fairly well with the measured velocities, taking account that discrepancies must come from poorly known parameters in the mixtures. Moreover, it predicts a strong damping of second sound as the temperature is decreased.[5]

C. The diffusive length L_{Deff}

A striking feature is that coupling occurs while neither U_6 nor β or transport coefficients show any wide variation in the range 0.7–1.2 K excepted D_{eff} and L_{Deff} which we have plotted in Fig. 11. L_{Deff} is related to D_{eff} which is driven by $\bar{s}=s-c(\partial s/\partial c)$. In fact, with $c=0.1\%$ when $T=1.2$ K,

$$D_{\text{eff}} \simeq \kappa/\rho C_P \ , \tag{30}$$

and when $T=0.5$ K

$$D_{\text{eff}} \simeq (\kappa/\rho T \bar{s}^2) c^2 (\partial/\partial c)(Z/\rho) \ . \tag{31}$$

For weak solutions,[10]

$$\bar{s}=s_{40}+kc/m_3 \ \text{ and } \ c^2\frac{\partial}{\partial c}(Z/\rho)=kTc/m_3 \ , \tag{32}$$

s_{40} being the entropy of pure ^{4}He, so as T is lowered and $c \rightarrow 0$, $\bar{s} \rightarrow s_{40}$ and therefore decreases to 0 very rapidly (see Table I). This accounts for the large increase of D_{eff} and L_{Deff} when T is lowered, and for a wider variation when $c=0.1\%$ than when $c=1\%$.

The physical meaning of this increase of D_{eff} (and then of L_{Deff}) for weak solutions is that the ^{3}He diffusion is strongly enhanced by the counterflow of the superfluid components. ρ_{n4} and ρ_3 being, respectively, the normal density of ^{4}He atoms and the density of pure ^{3}He, the normal component velocity in the superfluid liquid is

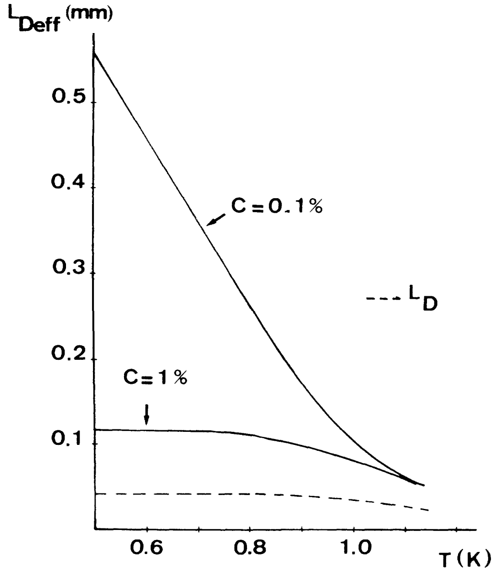

FIG. 11. The effective diffusion length L_{Deff} in the superfluid mixture vs temperature for two mass ^{3}He concentrations in the liquid $c=1\%$ and $c=0.1\%$, when the frequency is 200 Hz. The dashed line displays the variation of the nominal diffusion length $L_D=\sqrt{2D/\omega}$.

$$v_n \simeq v_s \rho_s/(\rho_{n4}+c\rho_3) \ . \tag{33}$$

This equation shows that the normal component flow is enhanced for weak solutions as ρ_{n4} decreases, i.e., as the ^{4}He atoms are condensed in the superfluid ground state, and this holds until the ^{4}He thermal excitations become ballistic for very low temperatures or ^{3}He concentrations. An effective mean free path l_{eff} must therefore be associated with the ^{3}He atoms: l_{eff} is the ^{3}He mean free path in the superfluid liquid between a couple of collisions which takes into account the existence of the normal component flow to which the ^{3}He atoms take part. As soon as this effect plays a leading part on the ^{3}He diffusion, we get roughly

$$D_{\text{eff}} \sim l_{\text{eff}} \sim \rho_s/(\rho_{n4}+c\rho_3) \ , \tag{34}$$

with at $T=0.5$ K:

$$D_{\text{eff}} \sim l_{\text{eff}} \sim 1/c \ . \tag{35}$$

It is here noticeable that the previous phenomenologic description displays the strong dependence of D_{eff} with the ^{3}He concentration which has been yielded from a

TABLE I. $\bar{s}$ (10^3 erg/K^{-1}g^{-1}), see Eq. (11).

T \ C	1%	0.1%
1.2 K	916	545
0.5 K	774	28

cumbersome hydrodynamic calculus [Eqs. (31) and (32)] for weak solutions and such low temperatures that the entropy s_{40} of ^{4}He atoms has become small compared to the contribution of ^{3}He in the mixture.

Coupling between the acoustic sound and the second sound can then be related in ^{3}He-^{4}He mixtures to the increase of L_{Deff}, the crossover being driven by the dimensionless parameter L_{Deff}/L (L represents the superfluid depth). L_{Deff} is the range in the superfluid liquid of exchange phenomena occuring at the interface: Exchanges of heat and matter always take place at the interface,[12] but these phenomena play no part in the propagation of two-phase sound unless this range becomes comparable to the superfluid depth.

To understand why mixtures do not behave like pure He II, suppose that the interface were an adiabatic glass which could prevent exchange between each phase: One can guess (and the hydrodynamics where **g** and **q** are neglected show[3]) that both vapor sound and second sound would propagate as independent modes. Vapor, as well as superfluid, would be in equilibrium at the interface, temperature oscillations δT_2 being counterbalanced by second sound concentration variations δc_2 in the liquid.

As soon as ^{3}He exchanges are set possible, this balance is broken in the superfluid on a range equal to L_{Deff}.

(1) Pressure oscillations in the vapor bring out temperature variations in the liquid near the interface: If $L \gg L_{\text{Deff}}$, local equilibrium can still exist on a range equal to L_{Deff} near the interface so that temperature oscillations in the liquid are not excited. As soon as $L_{\text{Deff}} \sim L$, local equilibrium cannot take place any longer at the interface so that conversion of pressure oscillations in the vapor to temperature oscillations in the liquid occurs as in pure He II. Because of $U_2 < U'$, this coupled sound then propagates in a new medium with a weaker stiffness, and thus its velocity is decreased.

(2) Because of heat transfer and evaporation-condensation at the interface, the balance between δT_2 and δc_2 for second sound is broken as deeper in the liquid as L_{Deff} becomes larger. Dissipation due to the diffusion damps the second sound while the acoustic sound is excited by means of sound conversion at the interface, and becomes delocalized over the whole waveguide. The two-phase sound velocity is then an average of each phase velocity and depth as it is for a classical fluid in equilibrium with its vapor.

One can check from experimental data that the crossover from two propagative modes to one propagative and one diffusive mode occurs with damping of the liquid mode, showing that the dissipation occurs in the liquid. Moreover, investigations made when the liquid progressively fills up the waveguide show that second sound is more strongly coupled when L is lowered, i.e., L_{Deff}/L is increased. Both modes are more strongly coupled when c and T are lowered, i.e., when L_{Deff} is increased.

A last striking feature of the two-phase sound experiments, which is also predicted by the hydrodynamics, is that as the temperature is lowered, the vapor mode velocity seems to decrease towards the second-sound velocity one could detect if the cell were completely filled with the liquid mixture. The hydrodynamics point out that this is related to the decrease of U_6, i.e., of the vapor density ρ' and the vaporization latent heat as the temperature is lowered. Indeed, as this latent heat is made smaller with a vanishing vapor pressure in the gas phase, pressure variation in the vapor will be squeezed by even a weak evaporation-condensation process at the interface. In other words, as U_6 is made smaller, poor pressure variations are set up at the interface by temperature variations in the liquid [corresponding to a vanishing $(\partial P/\partial T)_{co}$ in pure He II], which means that poor exchanges occur and there is no more diffusion in the liquid though L_{Deff} remains larger than L. It behaves as though the two-phase system were a single superfluid liquid phase, and the result is therefore a mode of temperature variations in the liquid coupled to very weak pressure variations in the vapor, where velocity is necessarily close to the second-sound velocity.

This crossover between vapor sound and second sound to the two-phase sound can be related to previous experiments run on second sound and fourth sound. When pure He II is contained in a porous solid matrix such as a packed fine powder, whose pores are small enough that the viscous wavelength λ_d for the normal component is larger than the pore diameter δ, the normal fluid is locked by viscous forces. However, sound waves can still propagate through motion of the superfluid component, and the mode which propagates in such a system (called a superleak) is fourth sound. So in bulk He II we have two propagating modes, first and second sound, and in the powder we have only one which is fourth sound as the helium normal component is clamped. Several authors[14] asked which of these two modes, first or second sound, disappears when the helium is clamped. With several packed fine powders of various diameters δ, some motion of $\mathbf{v}_n$ was allowed: Then, as the clamping is tightened the attenuation for both modes increases until a certain point when the attenuation of first sound starts decreasing. In the limit of complete clamping, first sound becomes fourth sound and second sound becomes a diffusion wave with vanishingly small diffusive velocity, due to dissipation occuring in the liquid and related to the increase of λ_d/δ. The transition between first and second sound to fourth sound thus appears to be driven by a dimensionless parameter λ_d/δ related to dissipation in the liquid in the same way that the crossover from vapor sound and second sound to the two-phase sound is driven by L_{Deff}/L.

V. CONCLUSION

Coupling of vapor sound with superfluid second sound in a waveguide has been related in the limit of large wavelengths and for ^{3}He-^{4}He mixtures to the increase of a diffusive length L_{Deff} in the superfluid when either T or the ^{3}He mass concentration is decreased. L_{Deff} is the range in the superfluid of diffusion phenomena which is initiated by heat and matter exchanges at the interface. The main property of the ^{3}He-^{4}He mixtures is that L_{Deff} may become much larger than the usual diffusion lengths because of the counterflow of the superfluid components which strongly enhances the ^{3}He diffusion at low temperatures.

L_{Deff}/L is then a dimensionless parameter which accounts for the dissipation which occurs in the liquid due to this diffusion and drives the crossover from two independent modes to one two-phase sound (L being the superfluid depth). This crossover behaves like the one which exists in superleaks from first and second sound to fourth sound. In the limit of large wavelengths, the account of a range in the liquid of exchange phenomena which occur at the interface allows an understanding of the whole set of situations for a wave traveling in our waveguide.

(1) In a classical liquid in equilibrium with its vapor the thermal diffusion length is generally small compared to the liquid depth L, which means that temperature fluctuations are localized at the interface. Therefore no strong damping occurs and sound is only an average on densities and depths of each phase mode.

(2) In pure He II, an efficient means for heat transfer is provided by second sound so that the range in the liquid of an evaporation-condensation process occuring at the interface equals the liquid depth. Strong coupling therefore occurs, and there is only one traveling mode, the other being diffusive.

(3) Two-phase mixtures then allow us to manage the transition between a "classical behavior" (heat transfer by means of diffusion in the liquid) to a "superfluid behavior" (heat transfer by means of thermal traveling wave in the liquid), varying either the temperature or the ^{3}He concentration which drives the effective diffusion length L_{Deff}, L_{Deff} being successively smaller or larger than the liquid depth.

The previous survey also shows that diffusion phenomena which are often neglected in superfluids can play a major part in several circumstances. The dissipation they set up in a superfluid liquid in equilibrium with a normal fluid could define a new limit for working of ^{3}He-^{4}He dilution refrigerators at ultralow temperatures.

Such an account of diffusion phenomena which are brought about by exchanges at the interface should also be applied to third-sound propagation and coupling with the vapor. In a recent experiment on third sound,[15,16] a type of waveguide was developed which featured a plate spacing that was variable. This apparatus, used with time-of-flight techniques, allowed, in particular, a detailed study of the propagation of the acoustic sound in the vapor made as a function of plate spacing, the film thickness being the same for all plate separations. For spacings up to 10 μm, which may be labeled the gas-wave onset, the gas wave is found to be completely absent. Then from the onset to 150 μm, i.e., 80 times the mean free path in the vapor as the authors pointed out, the gas speed is found to be smaller than the ideal gas law sound speed by a factor of 7–8 at onset (being yet larger than the third sound speed) to 1 for spacings around 150 μm. This wave (which the authors called lethargic) suggested to them that even these large spacings were not macroscopic as far as the wave was concerned and that the gas wave attenuation which occured should be produced by a relaxation mechanism involving the vapor above the film.

These unusual low-speed measurements for the gas wave, the fact that dissipation is found to occur in the vapor and that the encountered plate spacings are in the range of thermal diffusion lengths at the same temperature and frequency (40 μm), suggest to us that the same coupling we investigated for bulk systems could exist between third sound in a superfluid film and the acoustic sound in the vapor in the limit of large wavelengths. The thermal diffusion length in the vapor being compared to the vapor depth could then account in this experiment for a new crossover from two waves to one in the same way that it was in the superfluid liquid of a two-phase bulk mixture system. We suggest therefore that new effects as those investigated by G. J. Jelatis *et al.* should occur due to diffusion phenomena in the vapor. A formalism like the one we developed for bulk two-phase systems may be able to account for the effects which have been seen in the gas, and may suggest new effects on the velocity and attenuation of this sound.

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