Arbitrarily polarized model Fermi liquid

Tai Kai Ng and K. S. Singwi
Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60201
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We study the effects of spin polarization on the collective and single-particle properties of a model Fermi liquid—a model which is designed to simulate liquid ^{3}He. The study is based on an approximate theoretical scheme of Singwi, Tosi, Land, and Sjölander. Using this scheme, we have calculated the Landau parameters $F_{\sigma\sigma'}$ as a function of polarization for various densities, the so-called polarization potentials, the compressibility and magnetic susceptibility, and the zero-sound dispersion. This study of a model system may provide some guidance for future experimental and theoretical works on a more realistic system such as partially polarized liquid ^{3}He.

I. INTRODUCTION

In a previous paper,[1] hereafter referred to as I, we have carried out a detailed study of a model system of Fermi liquid whose particles interact *via* a hard-core potential and an attractive tail. The model is constructed to simulate liquid ^{3}He. In I we have calculated various static and dynamic properties of the model system within the approximate STLS (Ref. 2) (Singwi-Tosi-Land-Sjölander) scheme, which was originally proposed for the study of electron liquids. A qualitative agreement was found between the calculated and the experimental results.[1] In another paper,[3] using the same theoretical scheme, we have considered the case of a fully polarized model Fermi liquid and have compared some of its properties with those of an unpolarized model liquid. In this paper, we extend our model and the theory to investigate the corresponding system of a partially polarized model Fermi liquid, which is a subject of considerable interest because of the recently opened possibility of experimentation[4] on partially polarized liquid ^{3}He. The behavior of liquid ^{3}He as a function of polarization is not yet known experimentally. The aim here is to study the effect of polarization on various properties of our model system in the hope that it can provide some guidance for future experimental and theoretical works on more realistic systems.

II. LONGITUDINAL RESPONSE FUNCTIONS IN THE STLS SCHEME

We shall characterize a polarized system by two parameters: $c=a_0k_F$, which measures the density of the system[1] and $\delta=m/n$, where m and n are the magnetization and particle densities of the system, respectively. a_0 is the radius of the hard-core potential and k_F is the Fermi wave vector of the unpolarized system. The polarized system is assumed to be stabilized by a constant magnetic field B.

We apply a spin-dependent weak external potential $\Phi^{\sigma}_{\rm ext}(\mathbf{r},t)$ on the system. The induced density fluctuation $\bar{n}_\sigma$ in the system in linear-response theory is given by

$$\bar{n}_\sigma(q,\omega)=\sum_{\sigma'}\chi_{\sigma\sigma'}(q,\omega)\Phi^{\sigma'}_{\rm ext}(q,\omega)\ , \tag{1}$$

where $\chi_{\sigma\sigma'}(q,\omega)$ is the density-density response function between the two spin components σ and σ' of the system.

In the STLS scheme,[2,5] the density fluctuations are assumed to be given by a generalized RPA-type (random phase approximation) equation:

$$\bar{n}_\sigma(q,\omega)=\chi^{\sigma}_0(q,\omega)\left[\Phi^{\sigma}_{\rm ext}(q,\omega)+\sum_{\sigma'}V^{\sigma\sigma'}_{\rm eff}(q)\bar{n}_{\sigma'}(q,\omega)\right]\ , \tag{2}$$

where $\chi^{\sigma}_0(q,\omega)$ is the noninteracting polarizability of the spin-σ component and the $V^{\sigma\sigma'}_{\rm eff}(q)$ are the static effective interactions between particles with spin σ and σ'. The latter in the STLS scheme are given by

$$V^{\sigma\sigma'}_{\rm eff}(r)=-\int_r^\infty\frac{dV(r')}{dr'}g_{\sigma\sigma'}(r')dr'\ , \tag{3}$$

where $V(r)$ is the bare potential and $g_{\sigma\sigma'}(r)$ is the pair-correlation function between the σ,σ' spin components of the system. The pair-correlation functions $g_{\sigma\sigma'}(r)$ are related to the static structure factors $S_{\sigma\sigma'}(q)$ through

$$g_{\sigma\sigma'}(r)-1=\frac{1}{n_{\sigma\sigma'}}\int\frac{d^3q}{(2\pi)^3}e^{i\mathbf{q}\cdot\mathbf{r}}\gamma_{\sigma\sigma'}(q)\ , \tag{4a}$$

where $n_{\sigma\sigma'}=(n_\sigma n_{\sigma'})^{1/2}$ and

$$\gamma_{\sigma\sigma'}(q)=S_{\sigma\sigma'}(q)-\delta_{\sigma\sigma'}\ . \tag{4b}$$

The static structure factors are, in turn, related to the response functions defined in Eq. (1) by the fluctuation-dissipation theorem:

$$S_{\sigma\sigma'}(q)=\frac{1}{n_{\sigma\sigma'}}\int_0^\infty\frac{d\omega}{\pi}{\rm Im}\chi_{\sigma\sigma'}(q,\omega)\ , \tag{5}$$

where the response functions $\chi_{\sigma\sigma'}(q,\omega)$ are determined by solving Eq. (2) with the following results:

$$\chi_{\sigma\sigma}(q,\omega)=\frac{[1-V^{\bar{\sigma}\bar{\sigma}}_{\rm eff}(q)\chi^{\bar{\sigma}}_0(q,\omega)]\chi^{\sigma}_0(q,\omega)}{\Delta(q,\omega)} \tag{6a}$$

and

$$\chi_{\sigma\bar{\sigma}}(q,\omega)=\frac{\chi^{\sigma}_0(q,\omega)\chi^{\bar{\sigma}}_0(q,\omega)V^{\sigma\bar{\sigma}}_{\rm eff}(q)}{\Delta(q,\omega)}\ , \tag{6b}$$

where $\bar{\sigma}=-\sigma$ and

$$\Delta(q,\omega)=[1-V_{\rm eff}^{\uparrow\uparrow}(q)\chi_0^{\downarrow}(q,\omega)][1-V_{\rm eff}^{\downarrow\downarrow}(q)\chi_0^{\downarrow}(q,\omega)] -[V_{\rm eff}^{\uparrow\downarrow}(q)]^2\chi_0^{\uparrow}(q,\omega)\chi_0^{\downarrow}(q,\omega) . \quad (7)$$

Equations (3)–(7) constitute a set of self-consistent equations for $V_{\rm eff}^{\sigma\sigma'}(q)$, which, in general, can be solved only numerically. In the limit $\delta\to 0$, $\chi_0^{\sigma}(q,\omega)=\chi_0^{\bar{\sigma}}(q,\omega)$ and Eqs. (3)–(7) reduce to two decoupled sets of equations describing the density and spin response of the system. In this case the density and spin response functions are given by[1,2,5]

$$\chi_s(q,\omega)=\frac{\chi_0(q,\omega)}{1-V_{\rm eff}^{s}(q)\chi_0(q,\omega)} , \quad (8a)$$

$$\chi_a(q,\omega)=\frac{\chi_0(q,\omega)}{1-V_{\rm eff}^{a}(q)\chi_0(q,\omega)} , \quad (8b)$$

$\chi_0(q,\omega)$ being the usual Lindhard function and

$$V_{\rm eff}^{s}(r)=\tfrac{1}{2}[V_{\rm eff}^{\sigma\sigma}(r)+V_{\rm eff}^{\sigma\bar{\sigma}}(r)] , \quad (9a)$$

$$V_{\rm eff}^{a}(r)=\tfrac{1}{2}[V_{\rm eff}^{\sigma\sigma}(r)-V_{\rm eff}^{\sigma\bar{\sigma}}(r)] . \quad (9b)$$

In the limit $\delta=1$, $\chi_0^{\downarrow}(q,\omega)=0$ and we have

$$\chi^{\uparrow\uparrow}(q,\omega)=\frac{\chi_0^{\uparrow}(q,\omega)}{1-V_{\rm eff}^{\uparrow\uparrow}(q)\chi_0^{\uparrow}(q,\omega)} , \quad (10)$$

where only the density fluctuation of the spin-up component is present.[3] In the general case of arbitrary polarization, density and spin fluctuations are mutually coupled and one needs to solve the three coupled equations for $V_{\rm eff}^{\uparrow\uparrow}(q)$, $V_{\rm eff}^{\downarrow\downarrow}(q)$, and $V_{\rm eff}^{\uparrow\downarrow}(q)$ simultaneously.

Our model is characterized by a bare potential of the form

$$V(r)=\begin{cases} V_0 (V_0\to\infty), & r\le a_0 \\ -\varepsilon, & a_0<r\le a_1 \\ 0, & a_1<r . \end{cases} \quad (11)$$

We have studied two cases: (i) $\varepsilon=0$ (pure hard-core potential) and (ii) $\varepsilon\neq 0$ (hard core plus an attractive tail). In the latter case, values of ε and a_1 are fixed by requiring that the zeroth and first moments of the attractive part of the bare potential be equal to the corresponding moments of the Lennard-Jones potential for liquid ^{3}He.[1] Taking $a_0=0.9\sigma_0$, where $\sigma_0=2.556$ Å for ^{3}He, we get $a_1\simeq 2.05a_0$ and $\varepsilon\simeq 0.46\bar{\varepsilon}$; $\bar{\varepsilon}\sim 10.22$ K is the depth of the Lennard-Jones potential.

Using Eqs. (3) and (11), the effective interactions $V_{\rm eff}^{\sigma\sigma'}(q)$ can easily be shown to be

$$V_{\rm eff}^{\sigma\sigma'}(q)=\frac{4\pi(V_0+\varepsilon)}{q^3}g_{\sigma\sigma'}(a_0)[\sin(qa_0)-(qa_0)\cos(qa_0)] -\frac{4\pi\varepsilon}{q^3}g_{\sigma\sigma'}(a_1)[\sin(qa_1)-(qa_1)\cos(qa_1)] , \quad (12)$$

where the parameters $(V_0+\varepsilon)g_{\sigma\sigma'}(a_0)$ and $\varepsilon g_{\sigma\sigma'}(a_1)$ are to be determined self-consistently to determine $V_{\rm eff}^{\sigma\sigma'}(q)$. With Eq. (12), Eqs. (4)–(7) constitute a set of 6×6 nonlinear matrix equations for the above parameters. We have solved this set of equations in the limit $V_0\to\infty$ (see I for details) using Newton's method for several different densities and polarizations. The results are given in Tables I–III.

In Table I we give the results for the three parameters $V_0 g_{\sigma\sigma'}(a_0)$ for the case $\varepsilon=0$ for two densities $c=1.5$ and 1.7, and for several values of polarization. The parameters are measured in units of $E_F^0(c)$, which is the free-particle Fermi energy for the unpolarized liquid with density parameter c. Corresponding results for the $\varepsilon\neq 0$ case are shown in Table II, where results for $c=1.9$ are also given. Notice that we have only an accuracy of $\sim 5\%$ in our numerical calculation because of a limitation in computer time and also because our model is, anyway, of qualitative value only. In Table III we show a more complete set of results for the fully polarized ($\delta=1$) case with $\varepsilon\neq 0$. Note that in this case there is only one response function, $\chi^{\uparrow\uparrow}(q,\omega)$, and one effective interaction, $V_{\rm eff}^{\uparrow\uparrow}(q)$, to be determined.

In all cases our results show that the effective interactions are rather insensitive to the degree of polarization for a given density [except perhaps the attractive part in $V_{\rm eff}^{\uparrow\uparrow}(q)$]. It seems that this is a consequence of the very strong hard-core repulsion which dominates over the exchange effects at high density. Another important point which needs to be pointed out is that the results of the self-consistent solution are rather sensitive to the input parameters ε and a_1 (Ref. 1) of the bare potential. As a consequence, our results can be trusted only qualitatively. This is especially true for the case of spin response[1] in the unpolarized system because the difference of $V_{\rm eff}^{\sigma\sigma}(q)$ and $V_{\rm eff}^{\sigma\sigma'}(q)$ enters in the determination of $V_{\rm eff}^{a}(q)$.

TABLE I. Solution of the STLS equations for a pure hard-core potential expressed in units of the Fermi energy (see text).

δ	$V_0 g_{\uparrow\uparrow}(a_0)$	$V_0 g_{\uparrow\uparrow}(a_0)$	$V_0 g_{\uparrow\downarrow}(a_0)$
		$c=1.5$	
0.0	25.3	25.3	27.8
0.2	25.3	25.2	27.7
0.4	25.1	24.8	27.4
0.6	25.3	24.8	27.4
0.8	25.1	24.6	27.3
1.0	25.2		
		$c=1.7$	
0.0	38.4	38.4	40.3
0.2	38.0	38.1	39.9
0.4	38.0	37.7	39.7
0.6	38.2	38.0	40.0
0.8	38.0	37.9	39.7
1.0	37.9		

TABLE II. STLS solution for the hard-core with attractive tail potential, expressed in units of the Fermi energy (see text).

δ	$(V_0+\varepsilon)g_{\uparrow\uparrow}(a_0)$	$(V_0+\varepsilon)g_{\downarrow\downarrow}(a_0)$	$(V_0+\varepsilon)g_{\uparrow\downarrow}(a_1)$	$\varepsilon g_{\uparrow\uparrow}(a_1)$	$\varepsilon g_{\downarrow\downarrow}(a_1)$	$\varepsilon g_{\uparrow\downarrow}(a_1)$
			$c=1.5$			
0.0	23.0	23.0	26.4	1.11	1.11	1.54
0.2	23.0	22.7	26.3	1.19	1.01	1.54
0.4	23.0	22.4	26.3	1.25	0.85	1.53
0.6	22.9	21.7	26.2	1.30	0.61	1.51
0.8	23.2	20.0	26.2	1.32	0.13	1.46
1.0	23.3			1.34		
			$c=1.7$			
0.0	36.5	36.5	39.0	0.79	0.79	0.97
0.2	36.4	36.3	38.8	0.82	0.73	0.99
0.4	36.7	36.1	38.9	0.83	0.65	0.98
0.6	36.5	35.8	38.8	0.86	0.49	0.98
0.8	36.4	34.4	38.5	0.87	0.18	0.96
1.0	36.5			0.87		
			$c=1.9$			
0.0	30.3	30.3	32.0	0.59	0.59	0.67
0.2	30.1	30.1	31.8	0.61	0.58	0.67
0.4	30.1	30.1	31.9	0.60	0.51	0.68
0.6	29.9	30.0	31.9	0.61	0.41	0.69
0.8	29.6	29.0	32.0	0.62	0.18	0.69
1.0	29.3			0.62		

III. RESULTS AND DISCUSSIONS (FOR LONGITUDINAL RESPONSES)

A. Effective interactions

As mentioned above, the effective interactions are quite insensitive to the change in polarization. However, they have an interesting density dependence in the $\varepsilon\neq 0$ case.[1] The attractive part of the bare potential is found to be less important as density increases, which can be seen from the fact that the ratio $r=\varepsilon g(a_1)/(V_0+\varepsilon)g(a_0)$ decreases with increasing density. An interesting consequence of this is that a "dip" occurs in the small-q region of $V_{\rm eff}^{\sigma\sigma'}(q)$ at low density similar to what is found in the Pines-Aldrich polarization potentials.[6] However, this structure vanishes at high density. Examples of $V_{\rm eff}^{\sigma\sigma'}(q)$ are shown in Figs. 1 and 2 for $\delta=0.4$ and $c=1.5$ and 1.9, respectively, where the disappearance of the "dip" at higher density is obvious. The corresponding effective interactions $V_{\rm eff}^{\uparrow\uparrow}(q)$ for the fully polarized system are also shown in Fig. 3. It can be seen that qualitatively these effective interactions are similar for the same density. The change in magnitude is due mainly to the change in the density of states $N_\sigma(0)$, $N_\sigma(0)\propto k_F^\sigma\propto(1+\sigma\delta)^{1/3}$.

In order to get some quantitative feeling of the results of our model calculation, we have shown in Fig. 3 the effective interaction, marked by crosses, as calculated by Krotscheck *et al.*[7] for a fully polarized liquid ^{3}He at normal density. It is seen that our effective interaction for $c=1.5$ (for an explanation for the identification of $c=1.5$ with normal-liquid-^{3}He density, see the section on Landau parameters) is indeed qualitatively very similar to the one calculated by these authors. Note that in making this comparison we have taken $a_0=2.56$ Å and $E_F(\delta=0)\simeq 5$

TABLE III. Solution of the STLS equation in the fully polarized liquid expressed in units of the Fermi energy (see text).

C	$(V_0+\varepsilon)g_{\uparrow\uparrow}(a_0)$	$\varepsilon g_{\uparrow\uparrow}(a_1)$
1.3	12.1	1.93
1.4	14.4	1.61
1.5	23.3	1.34
1.6	33.5	1.08
1.7	36.5	0.87
1.8	33.8	0.73
1.9	29.3	0.62

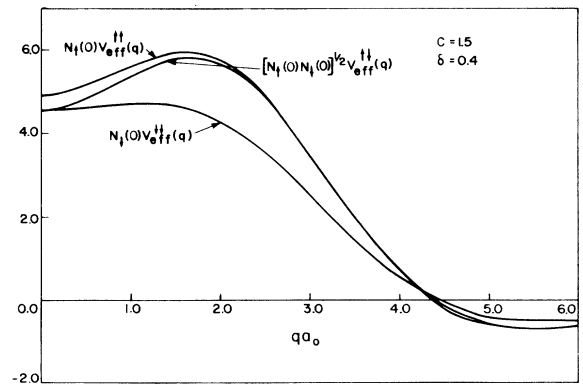

FIG. 1. Spin-dependent dimensionless effective interactions $[N_\sigma(0)N_{\sigma'}(0)]^{1/2}V_{\rm eff}^{\sigma\sigma'}(q)$ vs qa_0 for $c=1.5$ and $\delta=0.4$.

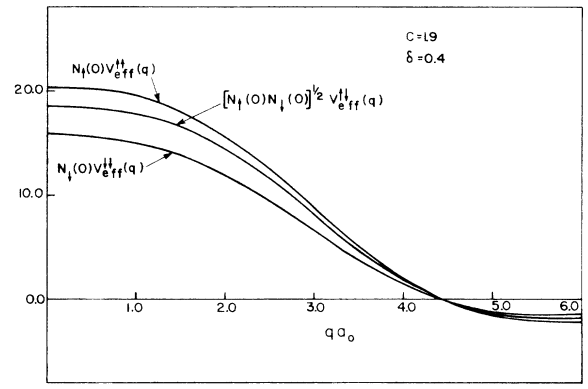

FIG. 2. Spin-dependent dimensionless effective interactions $[N_\sigma(0)N_{\sigma'}(0)]^{1/2}V^{\sigma\sigma'}_{\rm eff}(q)$ vs qa_0 for $c=1.9$ and $\delta=0.4$.

K for our model system for $c=1.5$. With this choice and using the fact that $N_\uparrow(0)=(k_F^\uparrow)^3/4\pi^2E_F^\uparrow$ and $n_\uparrow=(k_F^\uparrow)^3/6\pi^2$, the conversion of units can be done easily. This has been used to convert the numbers of Fig. 4 of Ref. 7 to our dimensionless units.

B. Landau parameters

The calculated Landau parameters $F_{\uparrow\uparrow}=N_\uparrow(0)V^{\uparrow\uparrow}_{\rm eff}(q=0)$, $F_{\downarrow\downarrow}=N_\downarrow(0)V^{\downarrow\downarrow}_{\rm eff}(q=0)$, and

$$F_{\uparrow\downarrow}=[N_\uparrow(0)N_\downarrow(0)]^{1/2}V^{\uparrow\downarrow}_{\rm eff}(q=0)$$

are shown in Figs. 4 and 5 as functions of polarization for the case $\varepsilon\neq0$ for two different densities. It is seen that in both cases $F_{\uparrow\uparrow}$ increases smoothly as polarization increases, whereas $F_{\downarrow\downarrow}$ and $F_{\uparrow\downarrow}$ stay rather flat at low polarization but drop rapidly to zero in a narrow range of δ close to unity. Such a behavior is a consequence of the characteristic change in the density of states $N_\sigma(0)$ as a

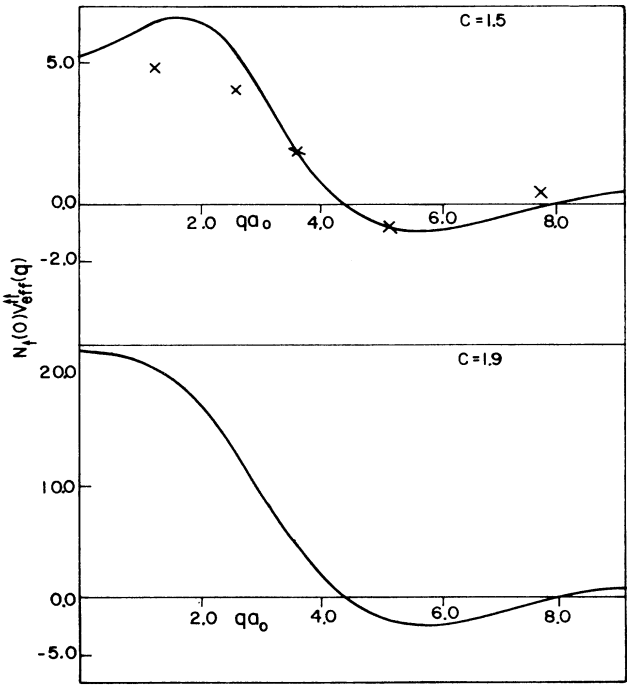

FIG. 3. Dimensionless effective interaction $N_\uparrow(0)V^{\uparrow\uparrow}_{\rm eff}(q)$ vs qa_0 for two densities $c=1.5$ and 1.9 in the fully polarized case. Crosses are representative points of the effective interaction as obtained by Krotscheck *et al.* (Ref. 7) for fully polarized liquid ^{3}He at normal density.

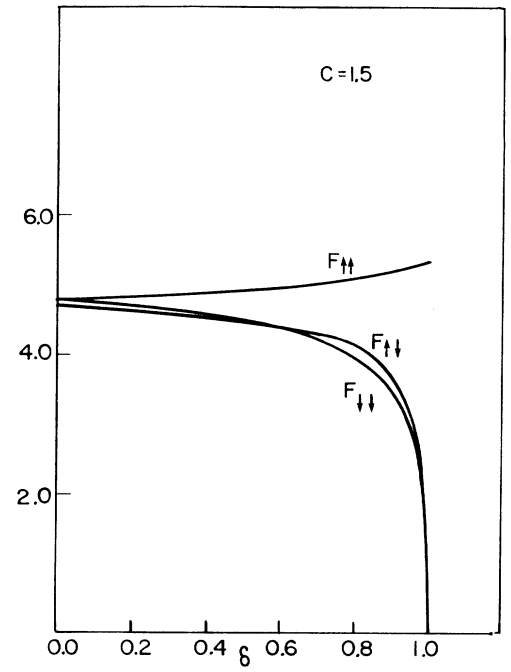

FIG. 4. Landau parameters $F_{\sigma\sigma'}$ vs polarization for density $c=1.5$.

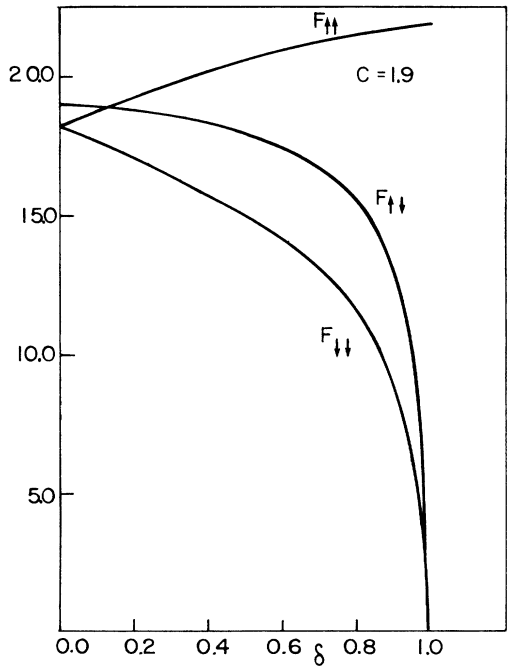

FIG. 5. Landau parameters $F_{\sigma\sigma'}$ vs polarization for density $c=1.9$.

function of polarization. A similar behavior is also found for the case $\varepsilon=0$, except that in this case the Landau parameters are larger in magnitude and the difference between $F_{\uparrow\uparrow}$ and $F_{\uparrow\downarrow}$ is larger in the unpolarized case[1] (i.e., more magnetic).

In Fig. 6 are shown the Landau parameters $F_{\uparrow\uparrow}$ for the fully polarized case as a function of density. The latter is measured with respect to a reference density $n_0^{-1}=(4\pi/3)a_0^3$ for the sake of convenience. The Landau parameter $F_0^s=F_{\uparrow\uparrow}+F_{\uparrow\downarrow}$ in the unpolarized case is also shown for comparison. Qualitatively, the density dependence of both these parameters is very similar, except that F_0^s is larger than F_0, in agreement with the predictions of other authors.[8,9] In order to get a feeling to what extent one can trust our predictions for F_0^s and hence for F_0, we have made a comparison of the calculated F_0^s with what is actually observed in liquid ^{3}He. In making this comparison one has to decide on the choice of the value of the parameter c for the model system. In making this choice, we have been guided by the consideration that the static structure factor $S(q)$, which is a microscopic quantity, for the model system should resemble as closely as possible (in particular, the peak height) the observed $S(q)$ in ^{3}He at normal density. This led us to the value $c\simeq 1.5$. The crosses in Fig. 6 are the experimental values of Greywall[10] for three different densities. For higher densities as the system approaches the solidification point the theory cannot obviously be trusted.

It is interesting to note that the absolute value of the compressibility κ remains rather constant throughout the whole range of polarization in our calculation. This is true for all values of density we have considered. The compressibility as a function of polarization is shown for different densities in Fig. 7 for the case $\varepsilon\neq 0$. Variation of compressibility with polarization is more pronounced in other theories.[8,11]

The magnetic susceptibility is shown in Fig. 8 as a function of polarization for different densities for the case $\varepsilon\neq 0$. The drop in susceptibility as polarization increases is a characteristic feature of a paramagnon-type theory. This is in sharp contrast to the results obtained from an "almost localized" approach,[12] where the susceptibility is found to increase with polarization when the system is close to localization. The rapid drop in susceptibility as $\delta\rightarrow 1$ is again the result of a rapid change in the density of states $N_\downarrow(0)$ as $\delta\rightarrow 1$.

C. Zero-sound dispersion

Zero-sound dispersion for the system is obtained by solving the equation

$$\Delta(q,\omega)=0\ . \tag{13}$$

Interestingly enough, we find that for all cases we have considered, the zero-sound velocity is quite insensitive to the change in polarization, which can also be understood from the fact that the compressibility κ is insensitive to changes in polarization. We have shown in Fig. 9 the zero-sound dispersion for various polarizations for the density $c=1.7$ in the case $\varepsilon\neq 0$. Similar behavior is also obtained for the case $\varepsilon=0$, except that the sound velocities are higher.[1] It is interesting to point out that at low

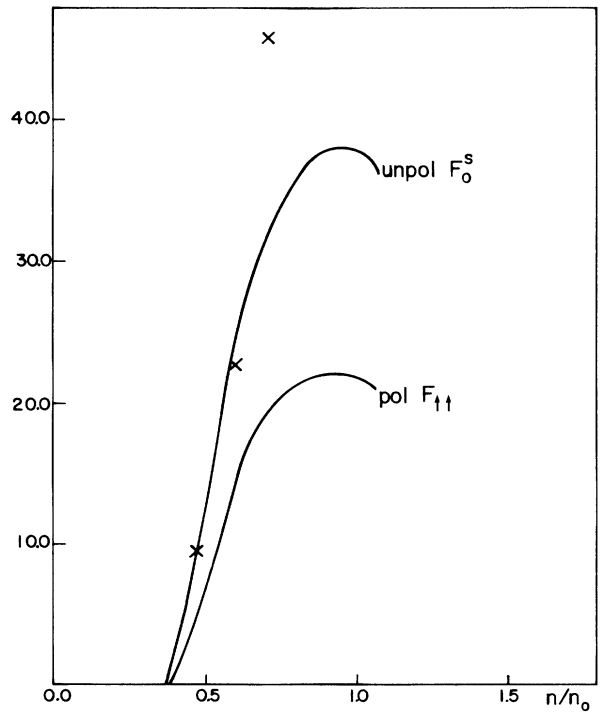

FIG. 6. Landau parameters $F_{\uparrow\uparrow}$ (fully polarized) and F_0^s (unpolarized) vs density n/n_0. Crosses are the experimental values of Greywall (Ref. 10).

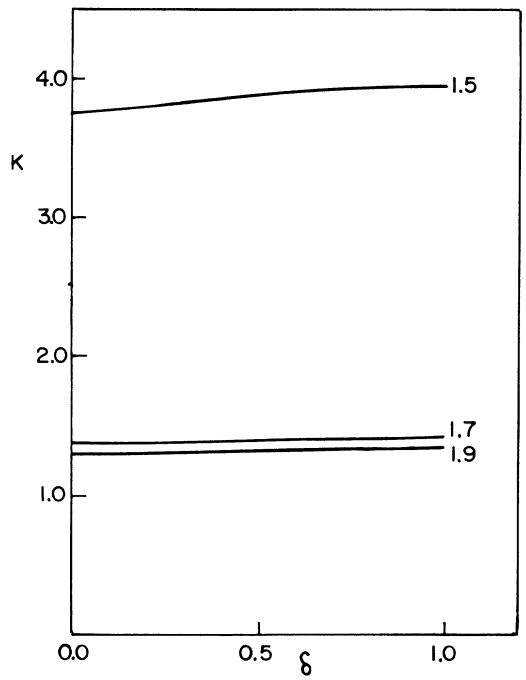

FIG. 7. Compressibility κ vs polarization δ for three different values of c. The figure is drawn in arbitrary units.

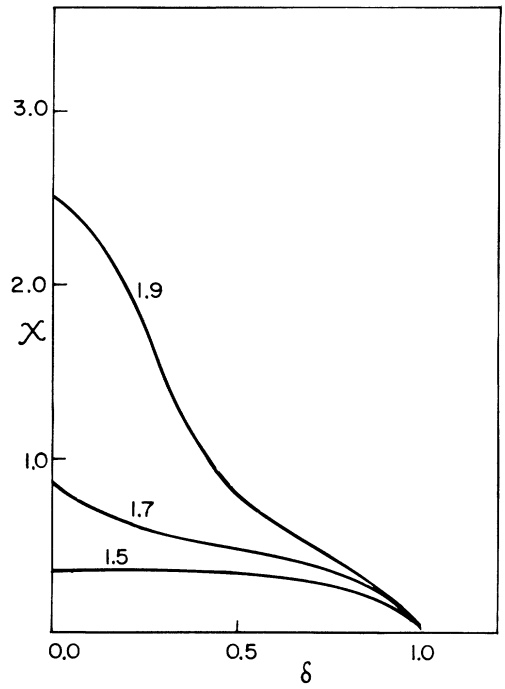

FIG. 8. Magnetic susceptibility χ vs polarization δ for three different values of c. The figure is drawn in arbitrary units.

density ($c \lesssim 1.6$) there also exists in the zero-sound mode a region of positive dispersion at small q similar to the one predicted by Aldrich *et al.*[13] in the case of unpolarized liquid ^{3}He at normal pressure. However, the dispersion becomes predominately negative in the high-density regime in our calculation.[1] This is again a consequence of the appearance of a "dip" in $V_{\rm eff}^{\sigma\sigma'}(q)$ at small q at low density which vanishes at high density. Another point worth mentioning is that in a partially polarized system, since the density and spin fluctuations are always mutally coupled, it should be possible to induce a zero-sound mode by applying a space-time–varying magnetic field with proper wavelength and frequency.

D. Pair-correlation functions (structure factors)

Pair-correlation functions $g_{\uparrow\uparrow}(r)$ and $g_{\uparrow\downarrow}(r)$ for an unpolarized liquid with density $c=1.5$ are shown for both $\varepsilon=0$ and $\varepsilon\neq 0$ in Figs. 10 and 11, respectively, in the region $r>a_0$. A weakness of the present theory is that $g(r)\neq 0$ for $r<a_0^{\downarrow}$, which reflects the fact that the present theory is essentially a pseudopotential-type theory valid mainly for long distances or small q. However, as can be seen from Figs. 10 and 11, the pair-correlation functions have, qualitatively, a very reasonable behavior even for $r\to a_0$ when compared with the calculations of Manousakis *et al.*[11] A comparison between Figs. 10 and 11 shows that the attractive part of the bare potential is very effective in producting substantial changes in $g_{\sigma\sigma'}(r)$, as was pointed out in Sec. II. In Fig. 12 we show the pair-correlation functions (at $r>a_0$) for density $c=1.5$, $\varepsilon\neq 0$, and $\delta=0.6$. Notice the rather large changes in the three pair-correlation functions as the system gets polarized. However, it is interesting that, despite the fact that the individual pair-correlation functions $g_{\sigma\sigma'}(r)$ for different σ and σ' are rather strongly dependent on polarization, the total density-density pair-correlation function $g(r)$ is very insensitive to changes in polarization. The change in $g(r)$ is less than 5% over most region of space throughout the whole range of polarization. This is true for both $\varepsilon=0$ and $\varepsilon\neq 0$ cases, and remains true also in the corresponding static structure factors $S_{\sigma\sigma'}(q)$. In Fig. 13 we show the static structure factors $S(q)$ for several different densities in the fully polarized ($\delta=1$) case with $\varepsilon\neq 0$. The similarity between these and the corresponding results for the unpolarized case can be seen clearly by comparing Fig. 13 with Fig. 17 of I (see also Ref. 11). Notice that there is a small plateau in $S(q)$ in the region of small q for $c=1.5$. This is another consequence of the "dip" we find in the small-q region of $V_{\rm eff}^{\sigma\sigma}(q)$ at low density. The plateau disappears at higher density as the structure in $V_{\rm eff}^{\sigma\sigma}(q)$ vanishes.[1]

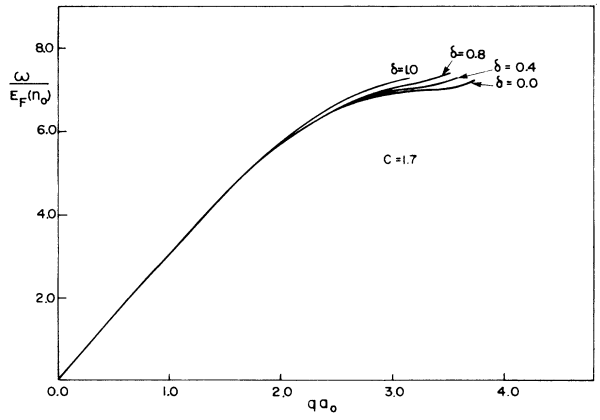

FIG. 9. Zero-sound dispersion $\omega/E_F(n_0)$ vs qa_0 for four different values of polarization, $E_F(n_0)$ being the free-particle Fermi energy for the unpolarized light at density $n=n_0$.

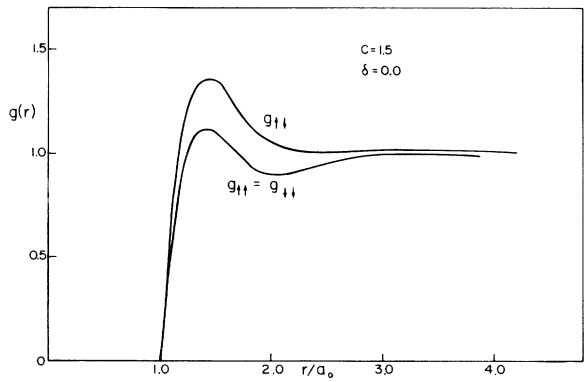

FIG. 10. Pair-correlation functions $g_{\uparrow\uparrow}(r)$ and $g_{\uparrow\downarrow}(r)$ vs r/a_0 for $c=1.5$ and $\delta=0.0$ in the case $\varepsilon=0$ (pure hard-core potential).

IV. QUASIPARTICLE PROPERTIES

The self-energy for quasiparticles in an arbitrarily polarized Fermi liquid can be calculated approximately using the following expression:[14]

$$\Sigma_\sigma(p)=-\sum_{\sigma'}\int \psi_{\rm eff}^{\sigma\sigma'}(q)g^0_{\sigma'}(p-q)\frac{d^4q}{(2\pi)^4}\ , \tag{14}$$

where p,q are energy-momentum vectors, $g^0(k)$ is the free-particle Green's function, and

$$\psi_{\rm eff}^{\sigma\sigma}(q)=\bar{T}^{\sigma\sigma}(q)+\{[V_{\rm eff}^{\sigma-\sigma}(q)]^2\chi_0^{-\sigma}(q)[1+V_{\rm eff}^{\sigma\sigma}(q)\chi_0^{\sigma}(q)]+[V_{\rm eff}^{\sigma\sigma}(q)]^2\chi_0^{\sigma}(q)[1-V_{\rm eff}^{-\sigma-\sigma}(q)\chi_0^{-\sigma}(q)]\}\frac{1}{\Delta(q)}\ , \tag{15a}$$

where $\bar{T}^{\sigma\sigma}(q)$ is an appropriately averaged T matrix,[14] and

$$\Delta(q)=[1-V_{\rm eff}^{\uparrow\uparrow}(q)\chi_0^{\uparrow}(q)][1-V_{\rm eff}^{\downarrow\downarrow}(q)\chi_0^{\downarrow}(q)]-[V_{\rm eff}^{\uparrow\downarrow}(q)]^2\chi_0^{\uparrow}(q)\chi_0^{\downarrow}(q)\ . \tag{15b}$$

$$\psi_{\rm eff}^{\sigma-\sigma}(q)=[V_{\rm eff}^{t}(q)]^2\chi^t(q)\ . \tag{16a}$$

$\chi^t(q)$ is the transverse-spin susceptibility. $V_{\rm eff}^t(q)$ is an approximate effective interaction for transverse-spin fluctuation, where[14]

$$\chi^t(q)=\frac{\chi_0^t(q)}{1-V_{\rm eff}^t(q)\chi_0^t(q)} \tag{16b}$$

and

$$\chi_0^t(q)=\int\frac{d^4p}{(2\pi)^4}g_0^{\sigma}(p)g_0^{-\sigma}(p+q)\ . \tag{16c}$$

A previous study of Lowy and Brown[15] on an electron gas shows that $\bar{T}^{\sigma\sigma}(q)$ can be identified roughly with $V_{\rm eff}^{\sigma\sigma}(q)$. We shall adopt this approximation in our calculation. However, the transverse-spin response function and the effective interaction $V_{\rm eff}^t(q)$ are not derivable within the STLS scheme. All we know is that in the limit $\delta=0$ (unpolarized), the transverse-spin response function is related to the longitudinal-spin response function in a simple way because of isotropy in the spin space;[14] and in the limit $\delta=1$, $\chi^t(q)=0$.

In the following we shall study in detail the quasiparticle properties for the case of a fully polarized liquid and compare them with the corresponding results in the unpolarized case. In the second part of this section we shall consider a simple interpolation scheme for the effective interaction $V_{\rm eff}^t(q)$ for the case of a partially polarized liquid. Quasiparticle effective masses for an arbitrarily polarized system will be studied thereafter.

A. Fully polarized systems

In the fully polarized case only up-spin particles are present, and Eqs. (14)–(16) reduce to

$$\Sigma_\uparrow(p)=-\int\psi_{\rm eff}^{\uparrow\uparrow}(q)g_\uparrow^0(p-q)\frac{d^4q}{(2\pi)^4}\ , \tag{17a}$$

where

$$\psi_{\rm eff}^{\uparrow\uparrow}(q)=\frac{V_{\rm eff}^{\uparrow\uparrow}(q)}{1-V_{\rm eff}^{\uparrow\uparrow}(q)\chi_0^{\uparrow}(q)}\ . \tag{17b}$$

Equation (17) is very similar to the one used by Krotscheck *et al.*[7] in their calculation of the effective mass in the fully-spin-polarized liquid ^{3}He. Spin fluctuations are absent in the latter.

Effective masses and renormalization constants on the Fermi surface were evaluated numerically in the case $\varepsilon\neq0$ for several different densities with results shown in Table IV, where the corresponding results for the unpolarized system are also shown for comparison. Mathematical details of the calculation can be found in Appendix B of I. Here we shall concentrate on the final results only.

Table IV shows that the effective masses on the Fermi

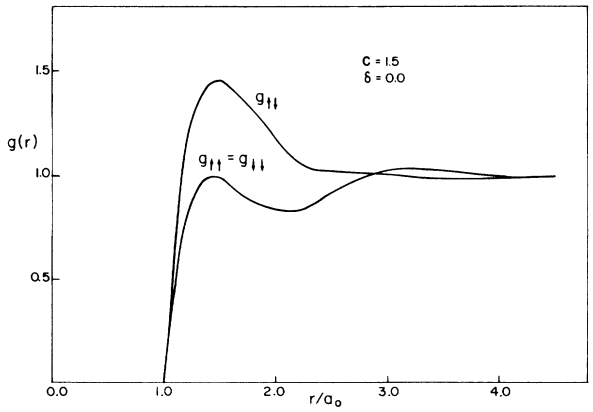

FIG. 11. Pair-correlation functions $g_{\uparrow\uparrow}(r)$ and $g_{\uparrow\downarrow}(r)$ vs r/a_0 for $c=1.5$ and $\delta=0.0$ in the case $\varepsilon\neq0$.

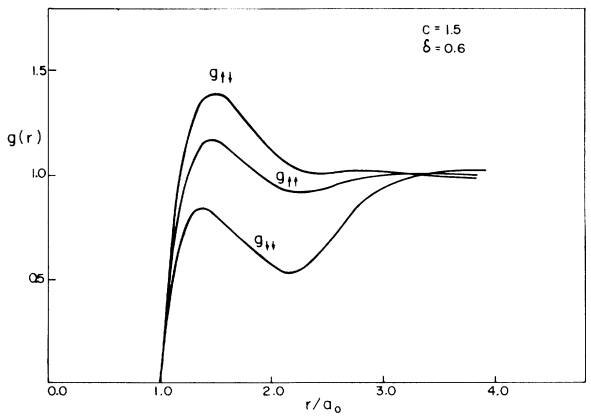

FIG. 12. Pair-correlation functions $g_{\sigma\sigma'}(r)$ vs r/a_0 for $c=1.5$ and $\delta=0.6$ in the case $\varepsilon\neq0$.

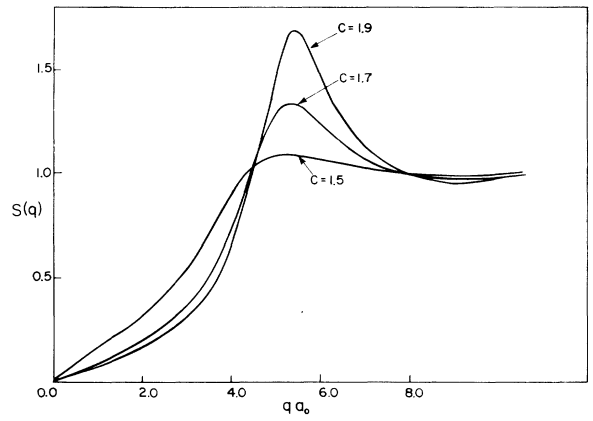

FIG. 13. Static structure factor $S(q)$ vs qa_0 for three different densities in the fully polarized case.

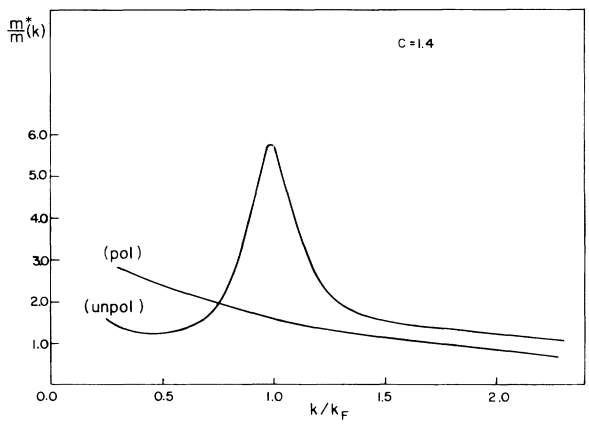

FIG. 14. "On-shell" effective mass $m^*(k)/m$ vs k/k_F for both unpolarized and fully polarized liquids for $c = 1.4$.

surface for the spin-polarized case are significantly smaller than the corresponding ones for the unpolarized case due to the absence of spin-fluctuation effects. Also, a spin-fluctuation contribution is found to be more important at high density. This is a consequence of the rather rapid increase of magnetic susceptibility as a function of density in our theory.

It is interesting to point out that we find $m^*/m > 1$ for all densities in our calculation in the fully polarized case. This is in contradiction to the prediction by several other authors[8,9,16] and suggests that higher-order Landau parameters may be important in the polarized system in liquid ^{3}He in order that the forward-scattering amplitude sum rule be satisfied.[8,9] The renormalization constants on the Fermi surface are found to be small in both polarized and unpolarized liquids, indicating that the single-particle properties are strongly renormalized in both cases. We have also performed an "on-shell" calculation of the momentum-dependent effective mass for $c = 1.4$ in the fully polarized case. Our results are shown in Fig. 14, where the corresponding results for the unpolarized case are also shown for comparison. Notice that the large peak on the Fermi surface in the unpolarized case disappears for the polarized system due to "freeze-out" spin fluctuation. The corresponding E_k-vs-k results are also shown in Fig. 15, where E_k is the "on-shell" quasiparticle energy. Note that we have renormalized the spectrum so that $E_k = 0$ on the Fermi surface. It is also interesting to point out that a large "bump" appears in the $m^*/m\,(k)$-vs-k curve for $k > k_F$ at higher density when the coupling of single-particle excitations to the zero-sound mode becomes important.[3,17]

We have also calculated the transport coefficients (thermal conductivity and viscosity) as a function of density in the s-p approximation for the fully polarized case. We do not give here any details of the calculation, since it is quite standard.[18] We give the final results which are shown in Figs. (16)a and (16)b. The general behavior of the transport coefficients is found to be similar to the one found by Hess *et al.*,[9] except our values are a few times larger because of the rather different values of the effective mass we have used. We have also examined the superfluid transition temperature in the fully polarized liquid in the s-p approximation and find that our model liquid stays normal at all densities we have considered, a result in agreement with the prediction by other authors on polarized liquid ^{3}He.[8,9]

B. Arbitrarily polarized system

To study quasiparticle properties for an arbitrarily polarized system, we need to know the transverse-spin response function $\chi^{-+}(q,\omega)$ defined by

$$\chi^{-+}(q,\omega) = \int d\mathbf{r}\, e^{i\mathbf{q}\cdot\mathbf{r}} \int dt\, e^{i\omega t} \langle T\sigma^-(\mathbf{r},t)\sigma^+(0,0)\rangle \ . \tag{18}$$

TABLE IV. Effective masses and renormalization factor on the Fermi surface. $(m^*/m)^1$unpol is calculated with full effect of spin and density fluctuations. $(m^*/m)^2$unpol is calculated with spin-fluctuation terms excluded.

C	$(m^*/m)^1$unpol	$(m^*/m)^2$unpol	(m^*/m)pol	Z unpol	Z pol
1.4	1.53	1.36	1.30	0.41	0.57
1.5	1.55	1.36	1.30	0.30	0.39
1.6	1.56	1.36	1.30	0.23	0.29
1.7	1.63	1.37	1.31	0.20	0.25
1.8	1.76	1.40	1.33	0.18	0.24
1.9	1.99	1.45	1.35	0.17	0.24

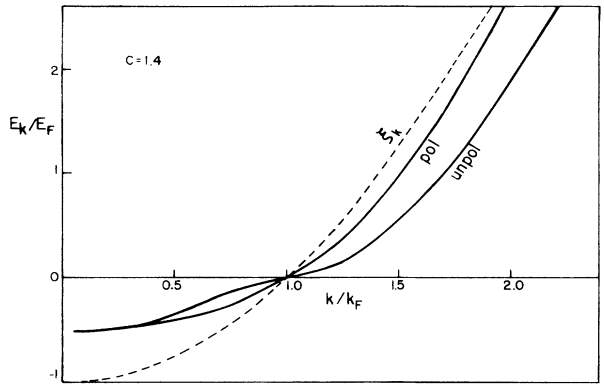

FIG. 15. Quasiparticle spectrum E_k vs k/k_F for both unpolarized and fully polarized liquids for $c=1.4$. Dashed curve is the free-particle spectrum ξ_k. E_k and ξ_k are measured in units of the free-particle Fermi energy $E_F=\hbar^2k_F^2/2m$.

Since it is very difficult to obtain χ^{-+} in a microscopic theory, we shall construct it phenomenologically in such a way that it satisfies some general exact requirements. Our aim in this study is to have a rough, qualitative idea of the transverse-spin response and quasiparticle properties in a partially polarized system.

In accordance with the generalized RPA structure of our present theory, we assume $\chi^{-+}(q,\omega)$ also to have a GRPA (generalized RPA) form, as in Eq. (16), where, for an arbitrarily polarized system,

$$\chi_0^{-+}(q)=\frac{-1}{V}\sum_k \frac{n_{k\uparrow}-n_{k+q\downarrow}}{\omega+\varepsilon_k-\varepsilon_{k+q}-\Delta} \ . \tag{19}$$

$n_{k\sigma}$ is the usual Fermi-Dirac distribution function for the σ component, $\varepsilon_k=k^2/2m$, and

$$\Delta=E_{F\downarrow}-E_{F\uparrow} \ , \tag{20}$$

where $E_{F\sigma}$ is the Fermi energy for the spin component σ. Δ consists of a sum of two terms, the external field contribution $2\mu B$ and the contribution from internal molecular fields arising from the interaction between particles.

In the limit $\delta\to 0$, $\Delta\to 0$, and

$$V_{\rm eff}^t(q)=V_{\rm eff}^{\uparrow\uparrow}(q)-V_{\rm eff}^{\uparrow\downarrow}(q) \tag{21}$$

because of spin isotropy.[14]

In the case of arbitrary polarization, $V_{\rm eff}^t(q)$ should remain the same when the spin labels $\uparrow$ and $\downarrow$ are interchanged. We make the following *ansatz*,

$$V_{\rm eff}^t(q)=\tfrac{1}{2}[V_{\rm eff}^{\uparrow\uparrow}(q)+V_{\rm eff}^{\downarrow\downarrow}(q)]-V_{\rm eff}^{\uparrow\downarrow}(q) \ , \tag{22}$$

for an arbitrarily polarized system, where the $V_{\rm eff}^{\sigma\sigma'}(q)$ are determined in Sec. II.

Notice that Eq. (22) is only one of the many possibilities which satisfy the above two criteria. The uncertainty thus arising in our results will be discussed at the end of the section.

The spin-wave dispersion is obtained by solving the equation

$$1-V_{\rm eff}^t(q)\chi_0^{-+}(q,\omega)=0 \ . \tag{23}$$

Results for the case $\varepsilon=0$, and $c=1.5$ for various values of the polarization, are shown in Fig. 17. Notice that at $q=0$, $\omega=2\mu B$ because of conservation of magnetization. Effective masses on the Fermi surface are calculated using Eqs. (14)–(16). Results for $\varepsilon=0$ and for two different densities are shown in Fig. 18(a). For small δ, the change in m^*/m is proportional to δ, but has the opposite sign for $\uparrow$ and $\downarrow$ spin fermions,[19] while for large δ both $(m^*/m)_\uparrow$ and $(m^*/m)_\downarrow$ decrease because of "freeze-out" of spin-fluctuation contributions. Notice, however, that the "peak" in $(m^*/m)_\downarrow$ is not observable from a specific-heat experiment which involves both $(m^*/m)_\uparrow$ and $(m^*/m)_\downarrow$. In Fig. 18(b) we show the change in the linear specific-heat term as a function of polarization, defined by

$$\Delta C_v=\frac{C_v(\delta=0)-C_v(\delta)}{C_v(\delta=0)} \ . \tag{24}$$

It is clear that ΔC_v is structureless over the whole range of polarization we have considered.

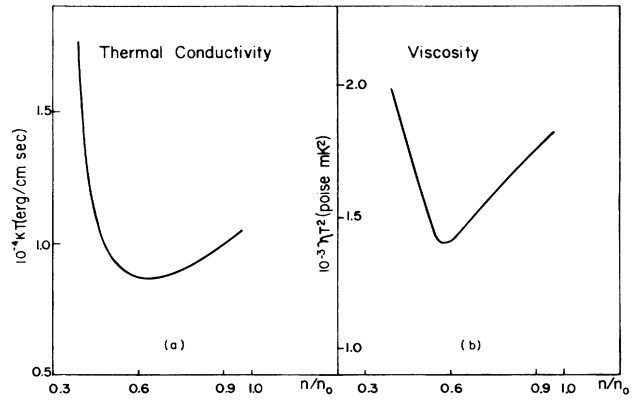

FIG. 16. (a) Thermal conductivity κT vs density n/n_0 in the fully polarized case. (b) Viscosity ηT^2 vs density in the fully polarized case.

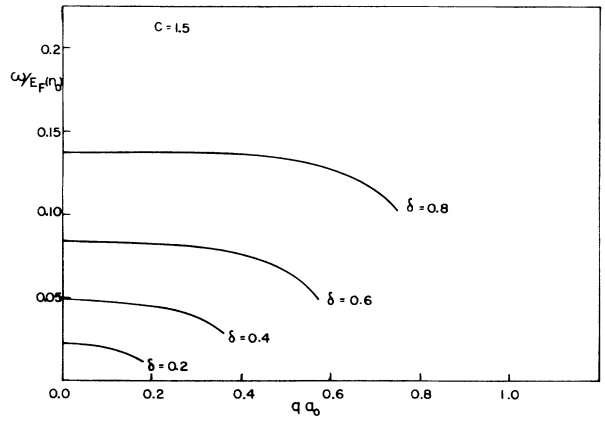

FIG. 17. Spin-wave dispersion: $\omega/k_F(n_0)$ vs qa_0 in the case $\varepsilon=0$ and $c=1.5$ for four different polarizations.

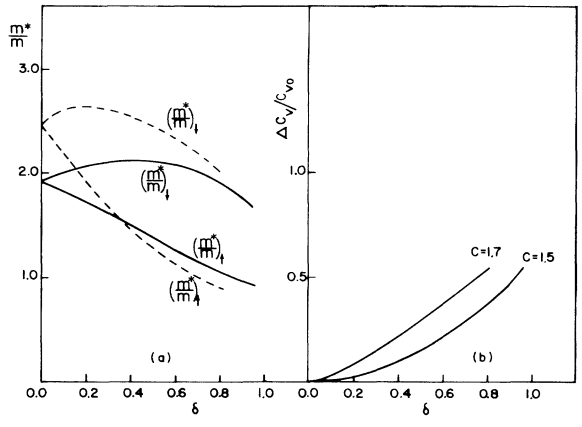

FIG. 18. (a) Effective masses $(m^*/m)_\sigma$ vs polarization in the case $\varepsilon=0$ for 2 different densities: (i) ——, $c=1.5$; (ii) — — —, $c=1.7$. (b) Deviation in linear specific-heat term ΔC_v vs polarization in the case $\varepsilon=0$ for two different densities $c=1.5$ and 1.7.

We would now like to comment on the validity of our choice of the effective interaction (22) and the results which follow. It is quite obvious from Table I that in the $\varepsilon=0$ case the effective interactions $V_{\rm eff}^{\sigma\sigma'}(q)$ are all very insensitive to changes in polarization. Therefore, any reasonable way of constructing $V_{\rm eff}^{t}(q)$ from $V_{\rm eff}^{\sigma\sigma'}(q)$'s which satisfy the two criteria stated above would leave $V_{\rm eff}^{t}(q)$ very similar. However, this is not the case for $\varepsilon\neq 0$, especially in the small-q region, where the effect of the attractive part of the interaction is strongest. $V_{\rm eff}^{t}(q)$ depends strongly the way it is constructed and also on the parameters a_1 and ε which specify the shape of the bare potential. It is for this reason that we have only considered the case $\varepsilon=0$ in this part of the section.

V. CONCLUDING REMARKS

In this paper we have examined within the STLS scheme effects of polarization on a number of properties of a model Fermi-liquid system which is intended to simulate liquid ^{3}He. Because of the intrinsic complications brought about by partial polarization, such an analysis is extremely difficult to carry out for a realistic system or with any other known microscopic theories. With the model system, the price we have paid is that we can only offer qualitative answers since we have found that quantitative properties of the system depend rather sensitively on the shape of the input bare potential. In the following we shall examine some of the conclusions we have arrived at for the model system in relation to the realistic system of liquid ^{3}He.

One rather unexpected result we have obtained is the insensitivity of the effective interactions $V_{\rm eff}^{\sigma\sigma'}(q)$ of our model system to changes in polarization. We believe that this is a consequence of the hard-core potential, which induces correlations so strong that exchange effects become comparatively insignificant. For the same reason, we have found that the total density response of the model system is very insensitive to polarization, although "partial" density-response functions $\chi_{\sigma\sigma'}(q,\omega)$ depend strongly on polarization because of the difference in the density of the two spin components. In the realistic system of liquid ^{3}He, the interparticle potential is weaker, so we expect that there will be a stronger spin dependence in effective interactions and larger changes in total density responses with change in polarization, although we expect these changes to be still rather small.

One possible cause for our effective interactions to be insensitive to the degree of polarization could be that in the STLS scheme the exchange effects, which enter the theory *via* the pair-correlation function $g_{\sigma\sigma'}(r)$, are not taken into account properly, although from our previous experience in the electron-liquid case this does not seem to be the case. There is, however, a major drawback in the STLS scheme in that the particle-hole interaction is local, whereas one knows that the exchange part of this interaction is nonlocal in character.

In our theory the predicted behavior of magnetic-susceptibility and single-particle properties is qualitatively in agreement with the predictions by a number of authors who have used rather different formalisms.[8,19] However, due to the sensitivity of the magnetic response to the input bare potential in our theory, our results can be trusted only in a very broad sense when compared with the realistic system of liquid ^{3}He.

A basic weakness of our theory consists of the problem of incorporating single-particle and collective properties of the system in a self-consistent manner. The approximate expression [Eq. (14)] we have used to calculate the self-energy takes into account effects of multiparticle scatterings and collective motions, but these single-particle renormalization effects are absent in the STLS theory which determines collective motion. As a result, in the static and long-wavelength limit the expressions for the compressibility and the magnetic susceptibility in the STLS scheme do not reduce to the usual Landau form. The renormalization effects, in contrast to the electron gas, are found to be strong in our model system, and as a result one expects strong modification of collective motion. So far, we have not been able to discover a simple and physically transparent mathematical treatment which handles single-particle and collective properties of a Fermi liquid in a self-consistent manner. Simple, approximate self-consistent schemes such as replacing the bare propagators by approximate dressed propagators (see Ref. 20, for example) in the Lindhard function seem to indicate that the single-particle renormalization effects would be weakened by self-consistency. However, the validity of such an approach in the present context is not clear.

An as yet unsolved problem in our approach is the determination of the transverse-spin response in a partially polarized system. So far, we have taken only a phenomenological approach to the problem. A microscopic theory for the transverse-spin response is needed to account fully for the properties of a partially polarized Fermi system.

Finally, one should keep in mind that the present-day

experimentally obtained partially polarized liquid ^{3}He is not really a system in equilibrium. With all these complications, it is certain that there is still a long way to go before a quantitative theory of partially polarized liquid ^{3}He can be formulated.

ACKNOWLEDGMENT

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